Econ/Demog C175 Lab 3: Solow model analysis

```
# Do not edit this chunk, but *do* press the green button to the answer key for the quiz info (the unre
tot = 0
answer.key = "eJytlE1v2zAMhu/+FVwu2QBXQH3fIesuuxTDVqDYUbEZW4gsufqIm38/UpIzd2mxHXoxbMl6ST58KfFk4rhHdytu4
library(quizify)
source.coded.txt(answer.key)
```

Overview

In this lab we will work with the neo-classical growth model developed by Solow and Swann. Our goals are:

1. To derive some analytical properties of the model, assuming a Cobb-Douglas production function.
2. To understand the dynamics of income and savings using a simulation app.
3. To understand how the capital intensification may or may not lead to increasing inequality, depending on the nature of the production function.

The order of this lab will be slightly different in that we will start with some mathematical derivations and then go on to do some computing afterwards.

(In order to display the equations, you may need to click on them with the mouse.)

1. Introduction

In the Solow model, the capital per worker and output per worker are in their steady state when investment exactly balances population growth and depreciation. This is when

$$sy(k) = (n + d)k.$$

2. Analytical questions

Note: In this lab, we are beginning with some analytic, pencil and paper problems. Please answer them on Gradescope. You can even open up the Gradescope page using an R-command.

```
browseURL("https://gradescope.com/courses/352627")
```

(Note: if this doesn't work, please navigate manually as usual.)

1. Let the production function be Cobb-Douglas, with $y(k) = k^{0.4}$. Assume that the savings rate is 20 percent, population growth is 1 percent, and depreciation is 4 percent.

(Technical note: The production function often has a constant "A", such that $y(k) = A * k^{\alpha}$. For simplicity we omit this term, or, equivalently, assume A = 1.)

For example, for the steady-state level of capital per worker, we substitute the production function $y(k) = k^a$ into the steady-state condition above to give

$$sy(k) = sk^{\alpha} = (n + d)k$$

Re-arranging, then gives us a formula for k in the steady state:

$$\left(\frac{s}{n+d}\right)^{\frac{1}{1-\alpha}} = k$$

You can then substitute the values given in the problem and provide your numerical answer.)

a) What is the steady-state level of capital per worker?

```
k <- (20.0 / (1 + 4)) ^ (1 / (1 - 0.4))
k
```

```
## [1] 10.07937
```

$$\left(\frac{20}{4+1}\right)^{\frac{1}{1-0.4}} = k = 10.07937$$

b) What is the steady-state of output per worker?

```
y_k <- k ^ {0.4}
y_k
```

```
## [1] 2.519842
```

$$y(k) = k^{0.4} = 2.519842$$

c) What is the steady-state level of consumption per worker?

```
(1-0.2) * y_k
```

```
## [1] 2.015874
```

$$sy(k) = (1 - 0.2)2.519842 = 2.015874$$

2. Now assume population growth is instead -0.5 % (approximately the growth rate when every couple has 1.7 children), but that all other parameters stay the same.

a) What is the new steady-state output per worker? Is it higher or lower than with faster population growth? [A numerical answer and 1 sentence response is fine.]

```
y_k <- ((20.0 / (-0.5 + 4)) ^ (1 / (1 - 0.4))) ^ 0.4
y_k
```

```
## [1] 3.196254
```

3.196254. The new steady state output per worker is higher with a faster population growth.

b) What is the new steady-state of consumption per worker? Is it higher or lower than with faster population growth? [A numerical answer and 1 sentence response is fine.]

```
(1-0.2) * y_k
```

```
## [1] 2.557003
```

2.557003, which is also higher with a faster population growth.

3. Questions with the App

For the following question, you may find it useful to experiment with the "Solow_2017" app available at:

```
browseURL("http://shiny.demog.berkeley.edu/josh/solow_2017/")
```

(You can also use the app to get a rough check your answers to the analytical questions above.)

Assume your initial parameters are the default s, n, d, alpha, d, and k values on the app.

3. Assume a technological innovation, like the availability of electricity, increases output per person at all levels of the capital/labor ratio by 30% (You can implement this in the app by moving the slider on "Output level y"' from 1 to 1.3).

a) Describe in words what happens in the short run to output. (E.g., how large is the immediate increase in output?) [1 sentence]

   In the short run, ouput increases by around 30% as well, indicating that the increse in technology has made workers more productive.

b) Describe in words what happens in the long-run. (If the long-run steady state is higher than the short-term level of output right after the technology change, what is causing this additional increase?) [2 sentences]

   In the long run, output increase even more by a slightly higher new steady state. This indicates that the saving function also shifts upward and ahs lead to this slight increase.

4. Growth and Inequality (skip in 2021 and later)

(Note: We're going to skip this section this year, but you are welcome to do as self-study, especially if you're an econ major.)

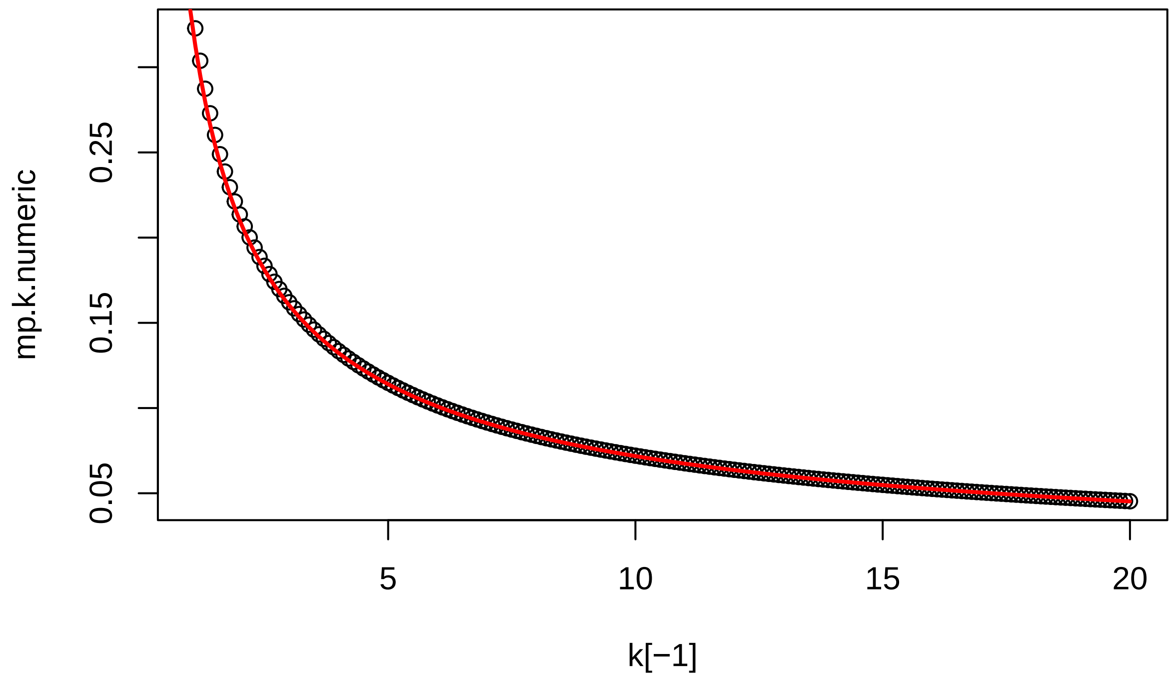
In this part of the lab we will use R to calculate the shares of income from capital and from labor using two different production functions. In a competitive market, the returns to capital and labor are equal to their marginal product. In our case, we are working with per capita income, so the only variable is the amount of capital per worker, k. The derivative (change) of the production with respect to k is the marginal product and the rate of return of capital.

i. Cobb-Douglas

Here we use a production function of the form

$$y(k) = k^{\alpha}$$

```
k <- seq(1, 20, .1)
alpha <- 1/3
y.of.k <- k^alpha
## numerical slope = rises/runs
mp.k.numeric = diff(y.of.k)/diff(k)
plot(k[-1], mp.k.numeric)
## analytic derivative, taking derivatitive of k^alpha with respect to k.
## (don't worry about this if you haven't had calculus)
mp.k.analytic = alpha * k^(alpha - 1)
lines(k, mp.k.analytic, col = "red", lwd = 2)
```




```
## we see they match quite well
```

Q1.1 Is the marginal product on capital A. Constant with increases in capital B. Rising with increases in capital C. Declining with increases in capital

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer1.1 = 'C'
quiz.check(answer1.1)
```

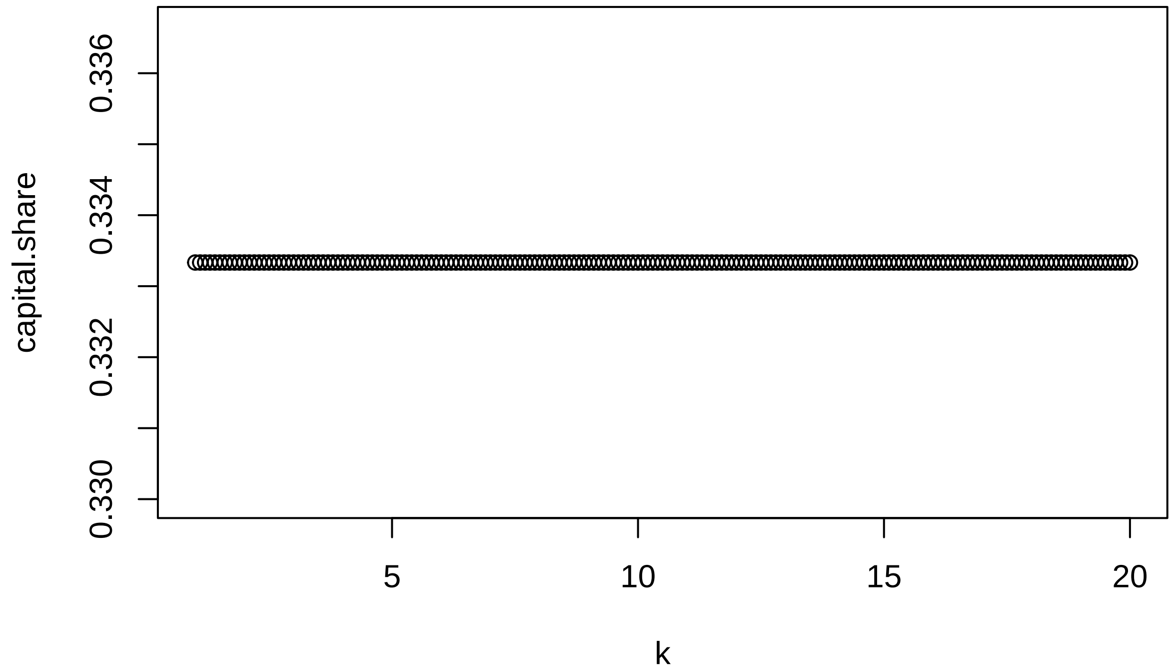
```
## Your  answer1.1 : C
## Correct.
## Explanation:  The marginal product on capital is equal to the slope of
## .the production function, which our plot shows is declining with
## .increases in capital.
```

Now let's see what happens to capital's share of total output. We assume here that the rate of return is equal to its marginal product. So as we increase the capital per person, we will have two countervailing forces: the amount of capital will increase, but the rate of return will decrease. Let's see what effect dominates or if the two effects cancel each other out.

```
mp.k <- mp.k.analytic
output.from.capital.per.worker <- mp.k * k
total.output.per.worker <- y.of.k
output.from.labor.per.worker <- y.of.k - mp.k * k
capital.share <- mp.k * k / y.of.k
```

Now let's see what happens to capital share as we increase k

```r
plot(k, capital.share)
```



Q1.2 Does the capital share of income A. Rise with increases in capital B. Stay constant with increases in capital D. Fall with increases in capital

```r
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer1.2 = 'B'
quiz.check(answer1.2)
```

```
## Your  answer1.2 : B
## Correct.
## Explanation:  With Cobb-Douglas production the rate of increase in
## .capital is exactly balanced with the decline in the rate of return,
## .keeping the capital share of income constant.
```

ii. An alternative production function

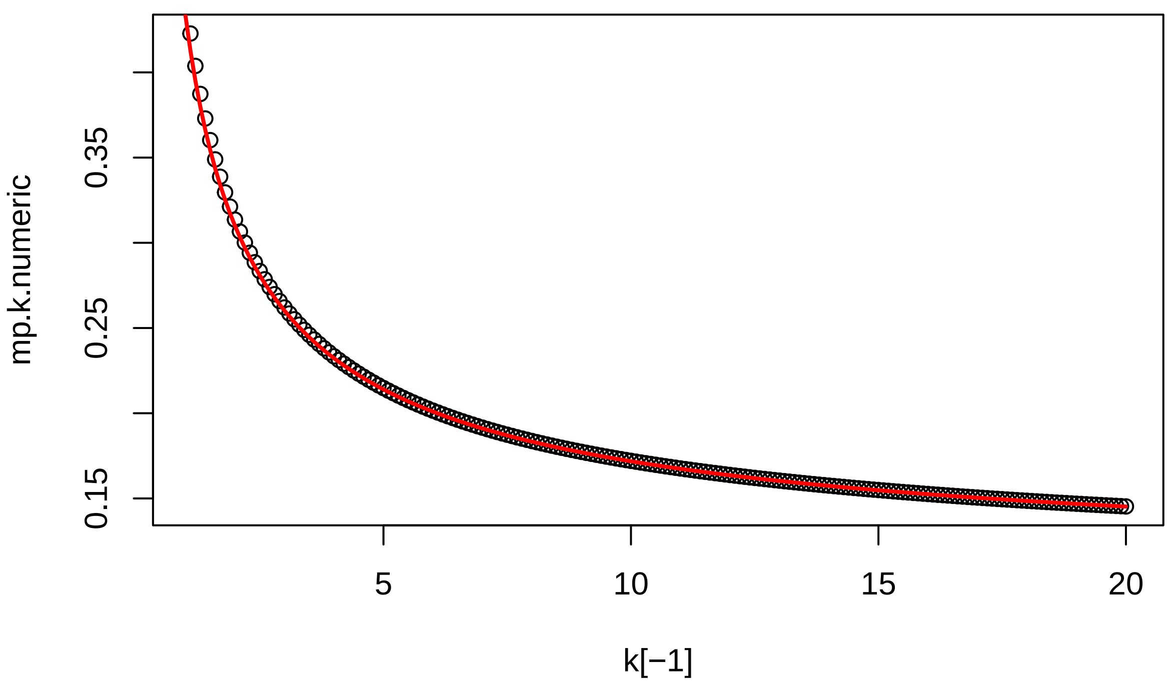
Let's modify the Cobb-Douglas production function slightly so that it is

$$y(k) = k^{\alpha} + k/10$$

Modify the code below to work with this new production function. (Hint: the derivative with respect to k of $k^{\alpha} + k/10$ is $\alpha * k^{\alpha-1} + 1/10$.)

```r
k <- seq(1, 20, .1)
alpha <- 1/3
```

```r
y.of.k <- k^alpha + (k / 10) ### <--- MODIFY THIS LINE.
## numerical slope = rises/runs
mp.k.numeric = diff(y.of.k)/diff(k)
plot(k[-1], mp.k.numeric)
## analytic derivative, taking derivatitive of k^alpha with respect to k.
## (don't worry about this if you haven't had calculus)
mp.k.analytic = alpha * k^(alpha - 1) + 0.1 ### <--- MODIFY THIS LINE, TOO!
lines(k, mp.k.analytic, col = "red", lwd = 2)
```




```r
## you should see that they match quite well
```

Q2.1 Is the marginal product on capital still declining A. Yes, but perhaps less quickly B. No, it is no longer declining

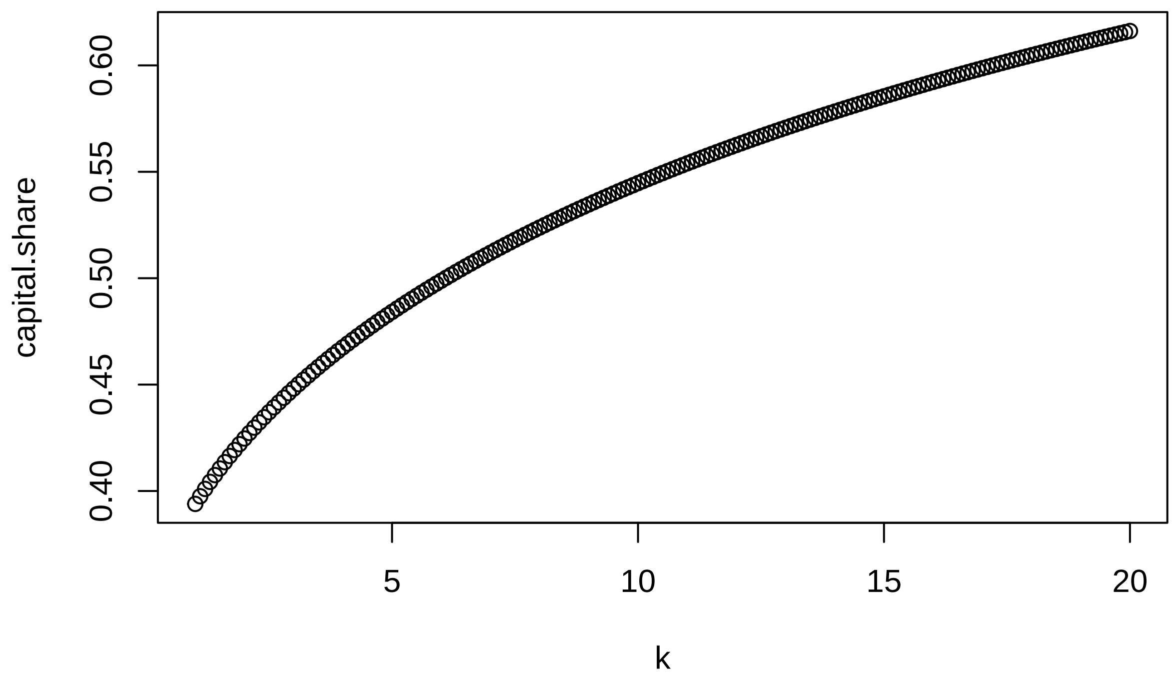
```r
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer2.1 = 'A'
quiz.check(answer2.1)
```

```
## Your  answer2.1 : A
## Correct.
## Explanation:  Yes, it is still declining. If you compare with
## .Cobb-Douglas, the rate of return looks like it declines more slowly,
## .especially for large values of k.
```

Now let's see what happens to capital's share of total output.

```r
## Note: here the variables carry over from the previous chunk. So, as
## long as you have executed all of the chunks up to here, they will
```

```r
## be from the 2nd production function.
mp.k <- mp.k.analytic
output.from.capital.per.worker <- mp.k * k
total.output.per.worker <- y.of.k
output.from.labor.per.worker <- y.of.k - mp.k * k
capital.share <- mp.k * k / y.of.k
## graph our result to see what happens to capital share as we increase k
plot(k, capital.share)
```



Q2.2 Does the capital share of income A. Rise with increases in capital B. Stay constant with increases in capital D. Fall with increases in capital

```r
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer2.2 = 'A'
quiz.check(answer2.2)
```

```
## Your  answer2.2 : A
## Correct.
## Explanation:  We now see that the more capital we have per worker,
## .the greater the capital share of the economy is! This is what Piketty
## .argues is likely to happen. As growth rates fall, capital per worker
## .will increase, and with it an increase in the capital share of the
## .economic output. Note that Piketty's view on this is empirically
## .driven and runs counter to what many theorists, who believe more in
## .the Cobb-Douglas-ness of the world.
```

Part 5: Lab write up.

Make sure to use Gradescope to submit your answers.

```
browseURL("https://gradescope.com/courses/352627")
```

1. (Question 1 in the analytical section at the beginning of lab)
2. (Question 2 in the analytical section at the beginning of lab)
3. (Question 3 in the questions with the app section above)
4. What level of population growth would maximize income per capita? Is this a plausible goal for a society? (Hint:you don't need calculus for this problem. Try thinking about it with a diagram.) [1 or 2 sentences is enough here.]

   A low population growth (negative growth) would maximize income per capita. Specifically, a negative growth rate that is larger than the deprecation rate would make the intersection between saving curve and $(n+d)k$ curve as far as possible. Obviously, a negative growth rate is not a plausible goal for society.

Robots and depreciation? (skip this year)

Many are worried about the effects of robots on inequality. But perhaps robots will also depreciate more quickly than earlier forms of productive capital. (The next two questions are on this topic.)

5. (skip) What would happen to capital per worker if we invested our savings in fast depreciating robots instead of slower-depreciating traditional productive capital? (Please make the (unrealistic?) assumption that per dollar robots have the same effect on production as traditional capital.) [Two or three sentences is fine.]
6. (skip) Assume, as per Piketty (and per our 2nd example of a production function above), that the capital share of output increases with capital intensification and decreases when capital per worker declines. Would our hypothetical case of faster depreciation increase inequality or reduce it? [Two or three sentences is fine.]

Immigration

A country is considering two immigration policies. What would Solow's model predict for each of these policies? (Note: both of these examples leave out consideration of the human capital of migrants, which in a more realistic model could be important.)

7. Allow a one-time wave of immigrants, but only those who bring exactly the amount of capital needed to leave the steady state capital/labor ratio unchanged. What would happen to per capita income in the short and long-run? (Hint: think about the time path of k) [Answer in a sentence or two.]

   There would be no change in both the short and the long run, as immigrants bring in capital with them making capital and labor ratio unchanged.

8. Allow a one-time wave of penniless immigrants who bring no capital at all. What would happen to per capita income in the short and long run? (Hint: think about the time path of k). [Answer in a sentence or two.]

   In the short run, there will be a decrease in per capita, as new immigrants are a crutch to the economy with no captial. But in the long run, it will eventually stabilize back to steady state of the original per capita income.

Congratulations! You are finished with Lab 3.