Econ/Demog C175 Lab for Week 9: Micro-economic Models of Fertility and an Investigation of Historical Switzerland

In this lab, we first go further into the cost-of-time model of fertility, answering questions based on theory and experimentation with the cost_of_time “app”. Then, we look at fertility and socio-economic data from historical cross-section provinces in French-speaking Switzerland, trying to see how our models may or may not be consistent with historical experience.

> Note: The first part of the lab includes some mathematical derivations that we did not fully cover in class. You should understand what analysis is done (and why), but you don’t need to be able to derive this yourself. To display the equations you may need to click on them or below them in the R Notebook.

> Further note: The graded questions are interspersed throughout the lab. They should be numbered consecutively.

> Final note: The computing with data part of the lab is short this time and should take less time than the first parts on the cost_of_time.

Part 0. Preliminaries

The data to be used in the lab is a built-in R dataset so we don’t need to download it.

```
# Do not edit this chunk, but *do* press the green button to the answer key for the quiz
info (the unreadable string below)
tot = 0
answer.key = "eJytVU1v2zAMvedXELukAzoDjZMeNgxFtgFDLztsxYAdFZuJhcqSq4+k/vcjGc2t513WGgiS0K
ae9B7Jp+LBpnaH/qq4go9A34uict5jFQtlw+nPizef3iyKRtuYw224h+BadBZh7zw0aDrQe+hdgkrZZYRah86oHm
KD57edOkjuouBH4aRDgFpFFTAWcBshNC6ZGlTXofKQgrYHWbzk1ReS/3YJlasRqibZ+0Vx0Ee0oHbuiAUdT9sQfa
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5YM4I6eF0lE5NHUAFcVaVOUPh8g9IrUXo1VXol0g5KS/jFq31E2kcxpjGy64Ttaja2gnTnE17CLkVo9KGhpbwpYJ
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1pYyNl2pqeka6NNjU9GhICPmIY1aCUGpTTGpTjGkj4g3L6SzorVAaPxI3fTVuunK0IgvQLwyWRr1zbEldWzGALWe
IQHbWaMgSrTB/0mN1a2K2n7NbjWV6fi93zkKl6GLLowDh3DyoKDZ7SD0Dqe0oaD/RUhPVsIqzHncidQoWOPGDKU+
votnM+qnMlBuobob6ZUt+MqUt4R+d7SBj+NPUO2XWowj0YHdEr8x6Wp4Z0yGO/vJkQ3sxGWJC2JL24y9DEtT6bC2
FrGivtQ4SePZIGijueOtIuitaRP+xwzz+UScabvKWF7LqbkULXotD1VKHrcetL+LOAQBrXQcbvJxkQ9tOiX8+mgS
B9RYtHxQVRhiyj8xhSK3bB2K0L7Bl1ohNoOlAPNUlgXIf1M56l+HY5vdDKfINlnjn8TDzIuM8H7rgELC1vt9eRvI
36wU66vZztlslId2x59KkInEac7xlBYzDKJhKSHZ4uvYHtSthOLpVyfKnk8IVs57plMtJtOHv832z7f9O9eU63FL
oT/z6/GMY8hy+kO5efZ6TX0F0L3Ymhl2NDz+EL6c7l3BnpNXQ3Qndi4mW2yIHu5hV05/LtjPTfdH8DxdbEMg=="
library(quizify)
source.coded.txt(answer.key)
```

Part 1. Implications of the Cost-of-Time Model of Fertility Choice

The Cost_of_time app computes the optimal bundle (N, X) of N children and X-dollars worth of consumer goods. The app plots two choices, one based on the initial values of $(w, p, c, andI)$ and one for separate values $(w', p', c', andI')$. The app is available at

```
browseURL("http://shiny.demog.berkeley.edu/josh/")
```

under cost_of_time. We recommend resizing the window of the app and playing with the font size (cmd-shift+, cmd-shift- on a mac) until the graph is fairly square.

You can see the effect of changing any single value by changing any one of the “Parameters to perturb.”

You can see how the effect of these changes (e.g., increasing wages) depends on the values of the other parameters by first changing the initial parameter values, and then changing one of the “Parameters to perturb”.

Background to the Model

We imagine potential parents choosing between the number of children they want to have and the amount of “other goods” they want to consume.

Prices

For simplicity, we let there just be one kind of “other goods”, which we denote X. And we let the price be 1 dollar per unit of X.

The price of children has two components:

1. A fixed dollar expenditure per child p for food, clothing, shelter, schooling, toys, etc.

2. A time cost of c hours per child. This dollar value of the time cost depends on the parents wage.

Income and the Budget Constraint

Parents face a budget constraint. Their household income consists of 'outside income' I and wage income $wL = w(T - cN)$.

1. 'Outside income' I includes sources like interest on assets, inheritance, as well as the wage-earnings of someone who cannot spend their time with children.

2. Wage income is the hourly wage rate w times the number of hours worked L. The number of hours worked is equal to the total time available for work T, minus the hours spent taking care of children. (In our simple model, there is no leisure.)

If the division of labor is such that men never take care of children, and women divide their time between child-care and market work, then one can think of I as "male earnings" and $w(T - cN)$ as "female earnings."

The family's expenditures must satisfy the budget constraint

$$\text{Income} = \text{Consumption}.$$

In our model, this means

$$I + w(T - cN) = X + pN.$$

We can re-arrange as

$$I + wT = X + (p + cw)N,$$

where the left-hand side is now "Full Income" and the right hand side gives the consumption including the time-cost of children. The full cost per child is thus $p + cw$.

Utility

The choice of how many children and how much "other goods" is made so as to make the parents as happy as possible. We can use the same Cobb-Douglas formulation that we used for production functions earlier in the semester as a convenient model for utility. We let the utility of a particular choice of X and N be

$$U(X, N) = X^a N^{1-a},$$

where $0 < a < 1$.

The Optimal 'Bundle' of Goods

Parents will choose X and N to maximize U, subject to their budget constraint. There are many methods for solving this constrained optimization problem, including:

- Numerical search. (We try all combinations of X and N that satisfy the budget constraint). This is what we did in class.
- Substitution. We rewrite our budget constraint so that we express one variable in terms of another. Then we substitute this expression into the utility function and maximize utility of this single variable function. (We will do this below.)

- Lagrange multipliers. This is a general method for constrained optimization. Good to know, but beyond the mathematics we're using in this course.

Solving for the Optimum by Substitution

The purpose of showing the derivation is to give you a sense of how this is done. You don't need to be able to do this yourselves.

1. Express X in terms of N.

From the budget constraint,

$$I + wT = X + (p + cw)N,$$

we can write

$$X = (I + wT) - (p + cw)N$$

2. Substitute into U

$$U(X, N) = X^a N^{1-a},$$

$$U(X, N) = U(N) = [(I + wT) - (p + cw)N]^a N^{1-a}$$

3. Maximization (the rate of change at the optimum should be zero)

At the maximum, $U'(N) = 0$. But it is a bit easier to work with $\log U$, which will reach a maximum at the same value of N.

$$\log U(N) = a \log [(I + wT) - (p + cw)N] + (1 - a) \log N$$

$$\frac{d \log U(N)}{dN} = \frac{-a(p + cw)}{[(I + wT) - (p + cw)N]} + \frac{(1 - a)}{N}$$

When $\frac{d \log U(N)}{dN} = 0$,

$$\frac{(1 - a)}{N} = \frac{a(p + cw)}{[(I + wT) - (p + cw)N]}$$

4. Solving for the optimal values

Solving for N gives us

$$N^* = (1 - a)\frac{I + wT}{p + cw}.$$

We obtain X^*, by substituting N^* into our earlier expression.

$$X^* = (I + wT) - (p + cw)N^* = (I + wT) - (1 - a)(I + wT) = a(I + wT).$$

(Because of the properties of the Cobb-Douglas utility function, the amount of X chosen depends only on full income and is independent of the price of children.)

Analysis of the solution

With our solution

$$N^* = (1-a)\frac{I+wT}{p+cw}$$

in hand, we can now ask what effect changes in any of the parameters will have on the choice of N.

For example, increasing a will clearly reduce N^*. We can write

$$\frac{\partial N}{\partial a} < 0$$

(The symbol ∂ indicates a partial derivative, differentiating only with respect to one variable.)

Likewise, we can see that

$$\frac{\partial N}{\partial I} > 0$$

And since p and c are both in the denominator,

$$\frac{\partial N}{\partial c} < 0$$

and

$$\frac{\partial N}{\partial p} < 0$$

.

The only effect that is ambiguous in sign is

$$\frac{\partial N}{\partial w} \lessgtr 0$$

.

Some additional mathematical manipulation shows that

$$\frac{\partial N}{\partial w} < 0, \text{ when } \frac{T}{c} < \frac{I}{p}.$$

.

So, while in class, we questioned the usefulness of the cost-of-time model in making predictions about the effect of a wage increase on fertility, we see here that the model makes clear predictions about the other variables. Increases in outside income I unambiguously increase fertility. Increases in the price of children through c or p unambiguously reduce fertility. And changes in taste, by changing the relative preferences a for children vs. goods, are also unambiguous in their effect.

For the purposes of this class, you do not need to be able to follow every step in the mathematical derivation. What is important to understand is that it is possible to analyze this model and discover in a quite general way what predictions the model makes and what the limits of the model are. The tractability of the model is what makes it so useful for enhancing our understanding.

Graded Questions on the Cost-of-Time Model of Fertility Choice

For the following questions, we will leave the initial values of w, p, c, and I unchanged. (Tip: you can always refresh your browser to return the app to its starting state.)

1. Effect of a wage increase, changing w' from \$15 to \$30:

If we increase the hourly wage w' from \$15 to \$30, what happens to fertility? [1 sentence with actual values.]

It decreases from 2.6 to 2.2.

What happens to utility (the values of the indifference contours)? [1 sentence with actual values.]

It goes from 90 to 100.

What happens to consumption of goods? [1 sentence with actual values.]

The consumption rises from 960 to 1320.

2. If we increase outside income I' from 1000 dollars per week to 2,000 dollars per week, what happens to fertility? [1 sentence with actual values.]

It increases fertility from 2.6 to 4.2.

3. If we reduce fixed costs p' from \$100 per week to \$50 per week, what happens to fertility? [1 sentence with actual values.]

Increases from 2.6 to 3.2.

4. If we increase the time costs per child c' from 10 to 15 hours per week, what happens to fertility [1 sentence with actual values.]

Decreases from 2.6 to 2.

5. For which of the above does the direction of the effect depends on initial parameter values, and for which do you think the direction of the effect is unambiguous? (Hint: you can verify the analytical answers given in the previous section by experimentation.) [A sentence for each parameter.]

- Wage: Initial values seem to be important
- Outside Income: Unambiguous. There is a relationship between outside income and fertility, where increasing income increases fertility.
- Fixed costs: Unambiguous. There is an inverse relationship between fixed costs and fertility. Where an increase in fixed costs causes a decrease in fertility.
- time costs: Unambiguous. Again, there is an inverse relationship between time cost and fertility. A decrease in time cost increases fertility.

6. Now change the initial parameter value of c from 10 hours per child to 2 hours per child before increasing wages from \$15 per hour to \$30 per hour.

What is the effect of a wage increase on fertility now? [1 sentence with actual values.]

The wage increases form 4.9 to 5.5.

Describe the relative size of the income and substitution effects that must be true for this effect to hold. [1 sentence saying which effect must dominate, no numbers needed.]

The income effect mostly influences where the changes in wage and move us to a higher indifference contour with higher fertility and consumption rates.

7. Review of concepts. For each of the following scenarios, specify how you could operationalize the scenario by changing a parameter, and what effect the change in the parameter would be expected to have on fertility. (For example, if the scenario was a cut in the inheritance tax, you could say that on average this would increase non-wage income I, and that the effect of increasing I would be unambiguously to increase fertility.)

A. The government reduces the costs of elementary schooling (e.g. by getting rid of school fees). What parameter would you change, and what effect would this have on fertility?

Decrease the per child fixed costs and time costs. This would increase fertility.

> B. Male income falls, but we live in a society in which only women take care of children. What parameter would you change, and what effect would this have on fertility?

Decrease outside income. This would would cause a decrease in fertility.

> C. Men becoming more involved in child care. One way of operationalizing is to increase T, the time that could be spent working or taking care of children. What other parameter would one want to change? How could these changes be made in a way that had no effect on fertility?

Men's outside income falls, thus an increase in total earnings would balance things out to make it so that there is no effect on fertility.

> D. Infant mortality falls. If we conceptualize the costs per child as costs per surviving child, then this effectively lowers the costs of children. What parameters would you change, and what effect would these changes have on fertility?

Lower fixed costs and time costs. This would increase fertility rate.

Part 2. Fertility Decline in Switzerland

In this homework, we take advantage of the wonderful coincidence that R has a built in data set from the literature on the historical demographic transition. The dataset has fertility, mortality, and a number of economic and social indicators for 47 French-speaking provinces of Switzerland in the years around 1888.

The goal of this homework is to see empirically what the determinants of lower fertility appear to be – and to see which of our economic theories seem to be applicable in this case.

2.1 Understanding the variables

Read results of help(swiss) and answer the following T/F questions

```
help(swiss)
```

The fertility measure goes from 0 to 100, with 100 being an estimate of the biological maximum fertility (under normal conditions). If you're interested in the details, see (http://opr.princeton.edu/archive/pefp/indices.aspx (http://opr.princeton.edu/archive/pefp/indices.aspx)).

Q1.1 Agriculture is the percentage of land area dedicated to farming. A. True B. False

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer1.1 = "B"
quiz.check(answer1.1)
```

```
## Your  answer1.1 : B
## Correct.
## Explanation:  It's the percent of males involved in agriculture as occupation.
```

Q1.2 Education and examination are measured only for males A. True B. False

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer1.2 = "A"
quiz.check(answer1.2)
```

```
## Your  answer1.2 : A
## Correct.
## Explanation:  True, but higher male education is probably associated
## .with higher female education -- so it is probably not incorrect to
## .think of this as a proxy for the education of both parents and of
## .children of both sexes.
```

Q1.3 No measure of income is available A. True B. False

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer1.3 = "A"
quiz.check(answer1.3)
```

```
## Your  answer1.3 : A
## Correct.
## Explanation:  Yes, a common problem with historical analysis.
```

Q1.4 “Catholic” is the fraction, not percentage, of people who are Catholic. A. True B. False

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer1.4 = "B"
quiz.check(answer1.4)
```

```
## Your  answer1.4 : B
## Correct.
## Explanation:  True, but not particularly important.
```

Q1.5 What percent of children die before the age of 1 in Moutier?

```
swiss["Moutier","Infant.Mortality"]
```

```
## [1] 20.3
```

A. 0.203 B. 20.3 C. 2.03

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer1.5 = "B"
quiz.check(answer1.5)
```

```
## Your  answer1.5 : B
## Correct.
## Explanation:  A lot of children died in their first year -- and even
## .more before they turned age 5.
```

Q1.6 What is the district with the highest examination scores?

```
swiss[which.max(swiss$Examination),]
```

	Fertility <dbl>	Agriculture <dbl>	Examination <int>	Education <int>	Catholic <dbl>	Infant.Mortality <dbl>
V. De Geneve	35	1.2	37	53	42.34	18

1 row

A. The Valley of Geneva B. Bern C. Some very rural place

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer1.6 = "A"
quiz.check(answer1.6)
```

```
## Your  answer1.6 : A
## Correct.
## Explanation:  Geneva is also presumably the most industrially developed.
```

2. Theoretical Expectations

Before analyzing the Swiss data, the first questions ask you to write down what you expect the relationship between fertility and the other variables. Use the cost-of-time model, Becker's quantity-quality model, or your own independent reasoning to provide the reasons for your prediction.

Here is a model answer to show the format to use for the other parts.

Infant mortality

(Prediction: Higher infant mortality would be associated with higher fertility. My reasoning is that it would discourage investments in child quality by making the price of quality per surviving child high. So parents would have many, "low" investment children, rather than few, "high" investment children.)

Graded Questions on Expected Effects in Switzerland

> 8. Catholic

This could limit the fertility rate and potentially decrease it, due to the strong belief of having no children/sex before marriage. This would increase the price per child, and we would expect lower fertility.

> 9. Education (Hint: distinguish between the effects of parents being more educated, and parents providing more education for their children).

Higher education typically means higher wages. This would reduce the cost of child. Thus, it means a lower fertility rate.

> 10. Examination

Higher exam score may indicate that the region is well educated and well funded. By similar reasoning of 9, we should predict a lower fertility rate.

> 11. Agriculture

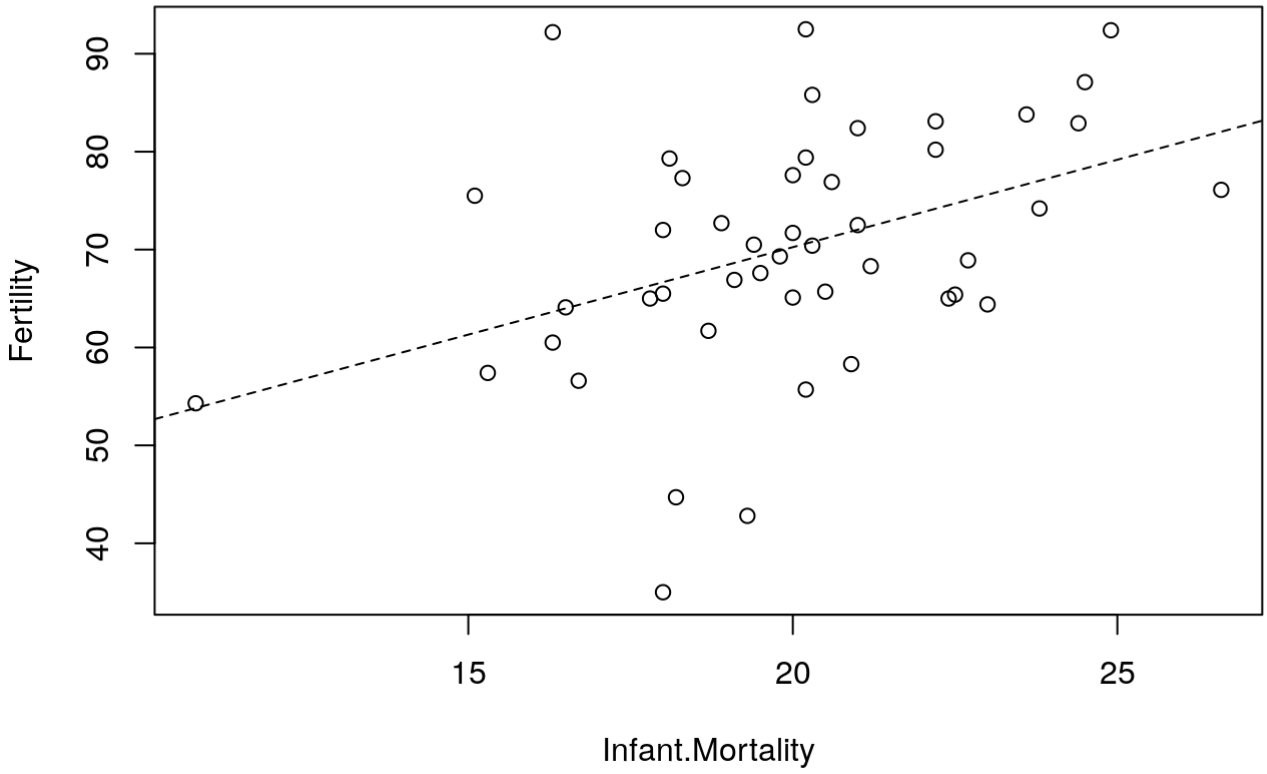
This is labor intensive, so it doesn't require much education. This means an increase in cost of child and higher level of food production, thus we would expect higher fertility rate.

3. Analysis

What is the observed relationship between mortality and fertility?

Plot it, using fertility as the outcome on the Y-axis

```
plot(Fertility ~ Infant.Mortality, data = swiss)
abline(lm(Fertility ~ Infant.Mortality, data = swiss), lty = 2)
```



Q3.1 Higher infant mortality is associated with

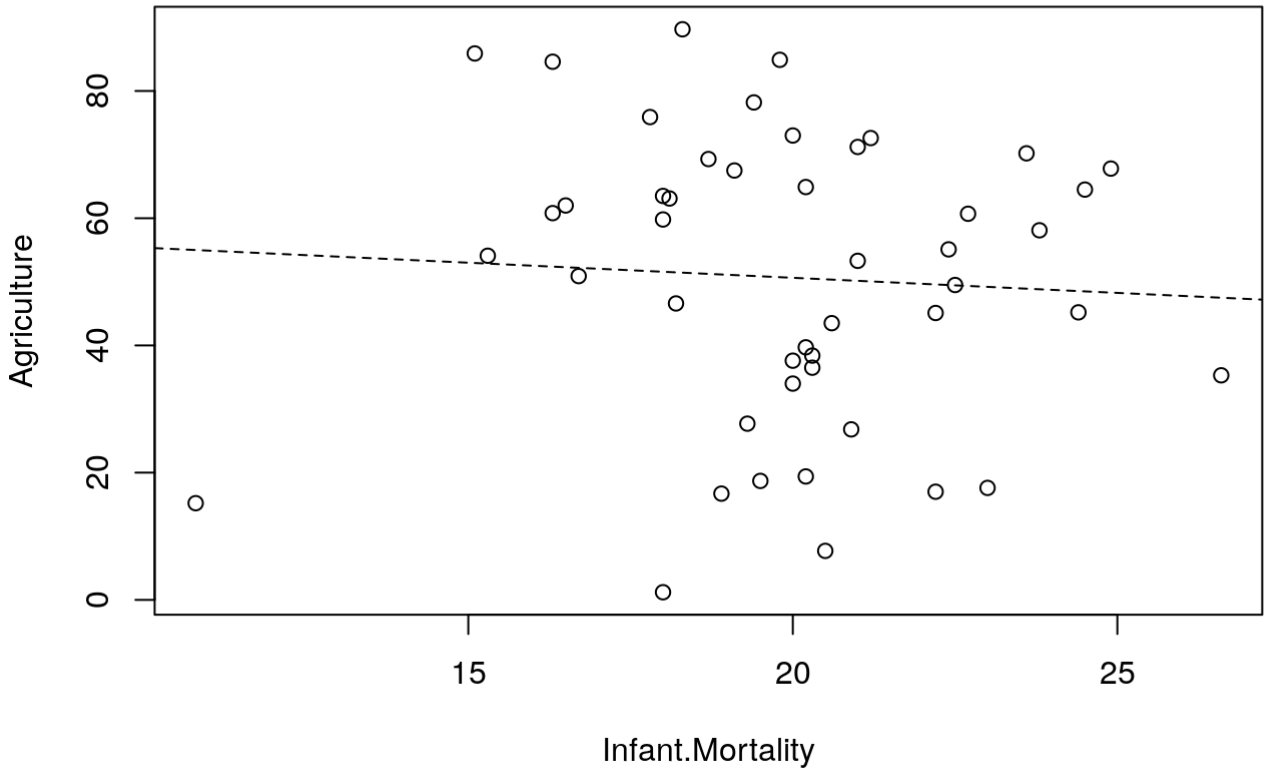
A. Higher fertility B. Lower fertility C. No clear pattern in fertility

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer3.1 = "A"
quiz.check(answer3.1)
```

```
## Your  answer3.1 : A
## Correct.
## Explanation:  This is consistent with our expectations above.
```

2. What is the relationship between “Agriculture” and fertility?

```
## Cut and paste and adapt the code for Infant.Mortality above
plot(Agriculture ~ Infant.Mortality, data = swiss)
abline(lm(Agriculture ~ Infant.Mortality, data = swiss), lty = 2)
```



Q3.2 Higher proportions working in agriculture are associated with

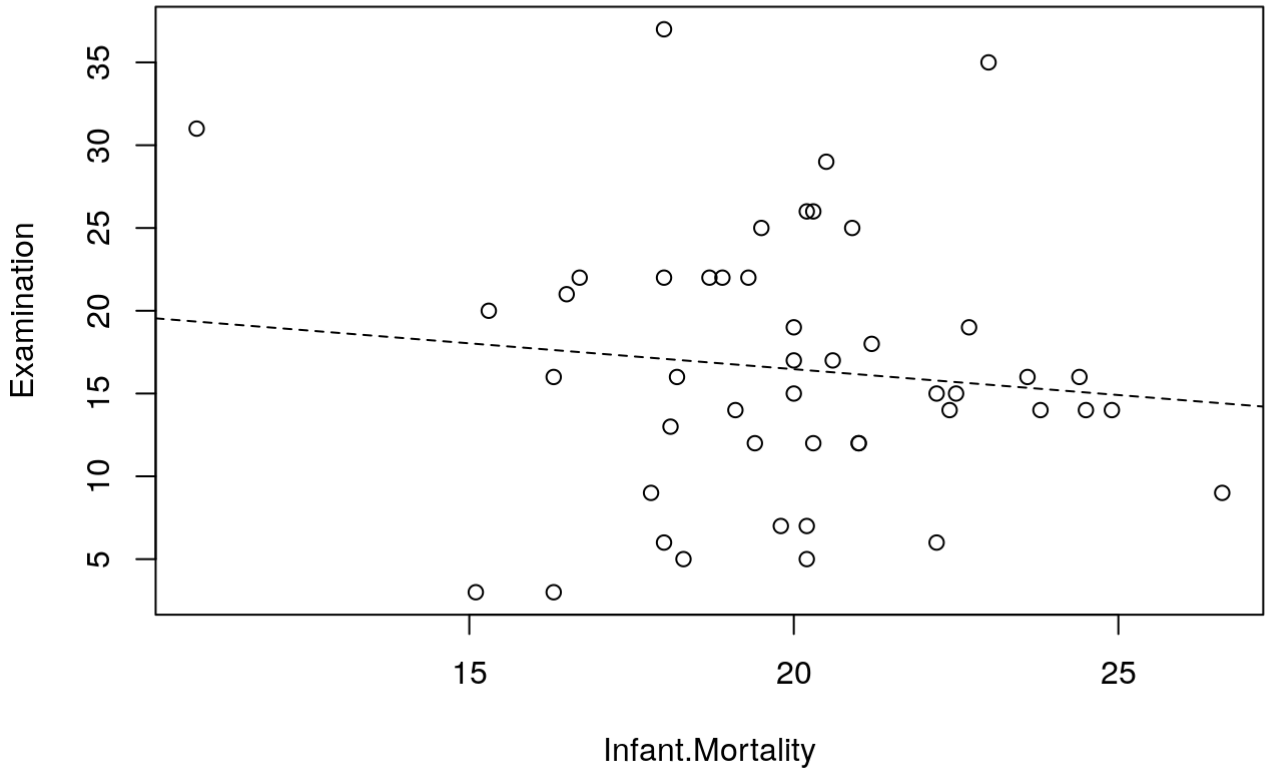
A. Higher fertility B. Lower fertility C. No clear pattern in fertility

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer3.2 = "A"
quiz.check(answer3.2)
```

```
## Your  answer3.2 : A
## Correct.
## Explanation:  Is this consistent with your expectations above?
```

3. What is the relationship between “Examination” and fertility?

```
## Cut and paste and adapt the code for Infant.Mortality above
plot(Examination ~ Infant.Mortality, data = swiss)
abline(lm(Examination ~ Infant.Mortality, data = swiss), lty = 2)
```



Q3.3 Higher examination scores are associated with

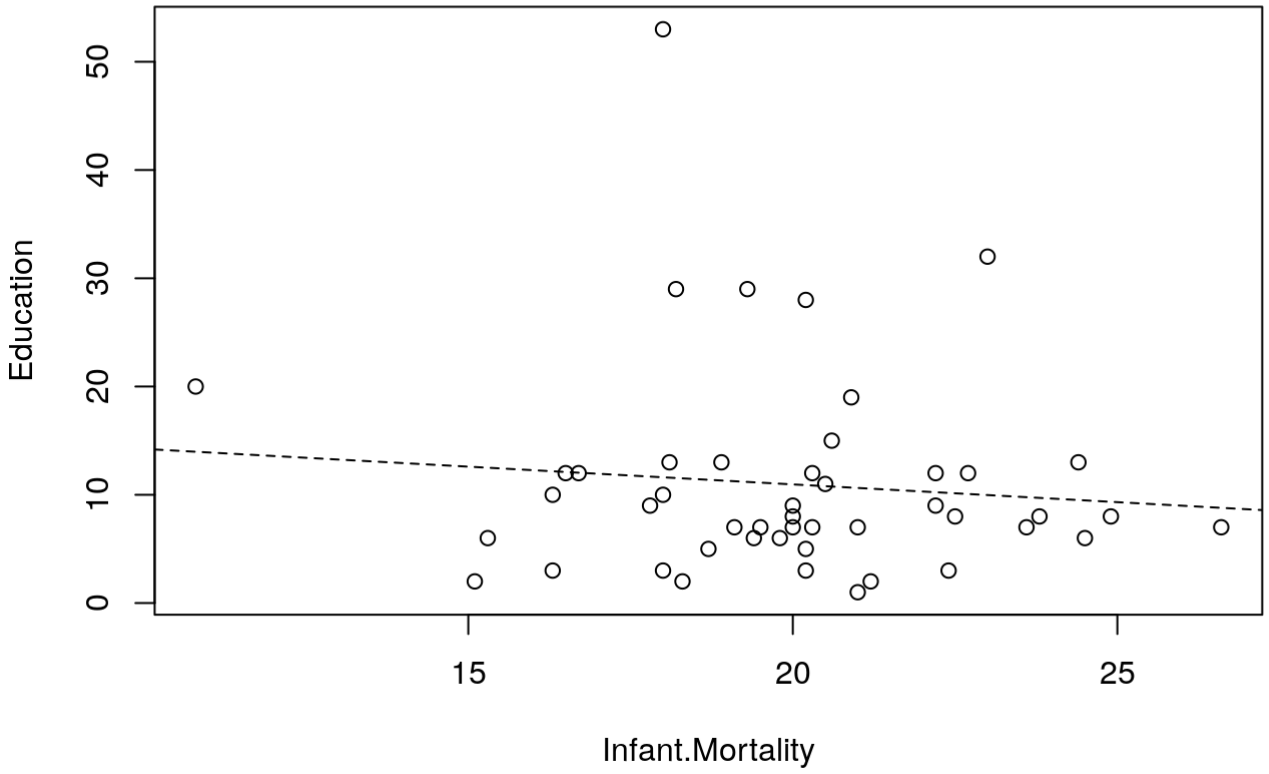
A. Higher fertility B. Lower fertility C. No clear pattern in fertility

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer3.3 = "B"
quiz.check(answer3.3)
```

```
## Your  answer3.3 : B
## Correct.
## Explanation:  Is this consistent with your expectations above?
```

4. What is the relationship between “Education” and fertility?

```
## Cut and paste and adapt the code above for Infant.Mortality above
plot(Education ~ Infant.Mortality, data = swiss)
abline(lm(Education ~ Infant.Mortality, data = swiss), lty = 2)
```



Q3.4 More education is associated with

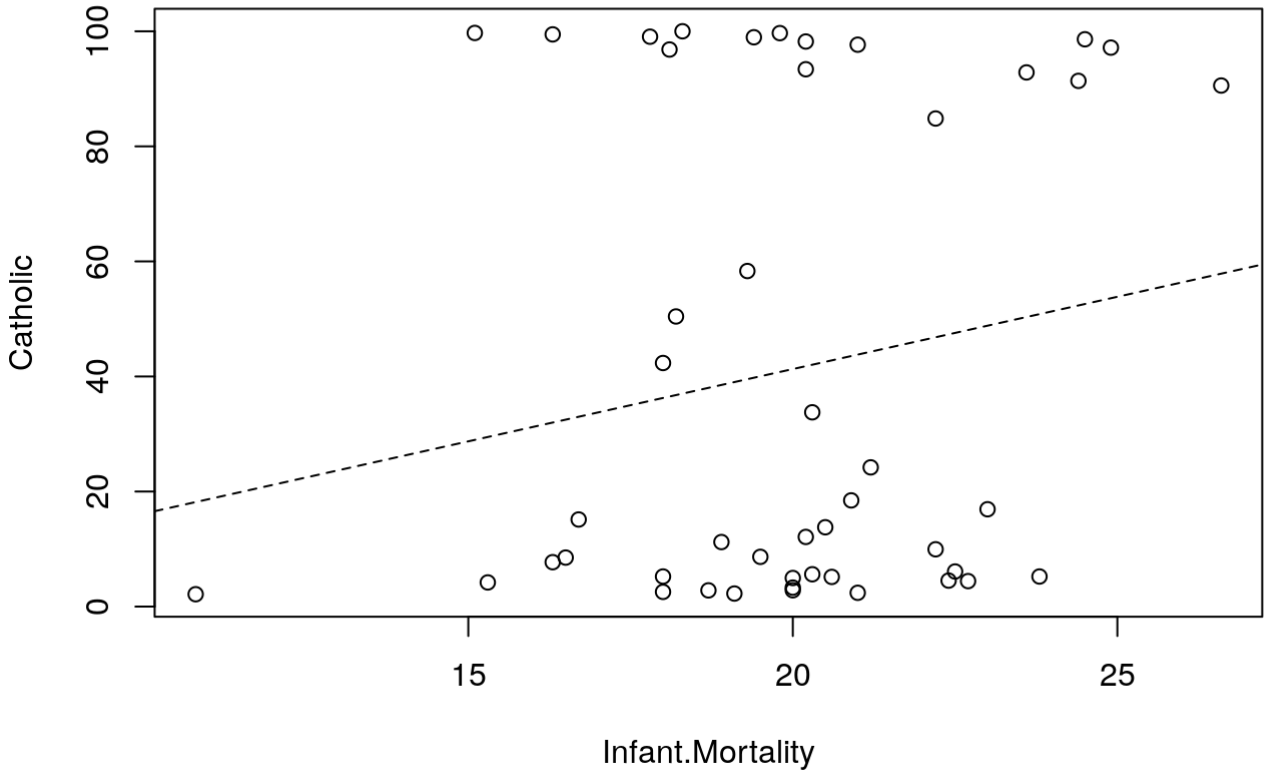
A. Higher fertility B. Lower fertility C. No clear pattern in fertility

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer3.4 = "B"
quiz.check(answer3.4)
```

```
## Your  answer3.4 : B
## Correct.
## Explanation:  Is this consistent with your expectations above?
```

5. What is the relationship between “Catholic” and fertility?

```
## Cut and paste and adapt the code above for Infant.Mortality above
plot(Catholic ~ Infant.Mortality, data = swiss)
abline(lm(Catholic ~ Infant.Mortality, data = swiss), lty = 2)
```



Q3.5 More Catholic provinces tend to have

A. Higher fertility B. Lower fertility C. No clear pattern in fertility

```
##  "Replace the NA with your answer (e.g., 'A' in quotes)"
answer3.5 = "A"
quiz.check(answer3.5)
```

```
## Your  answer3.5 : A
## Correct.
## Explanation:  Is this consistent with your expectations above?
```

Graded Questions on Observed and Expected Effects in Switzerland

12. Write a paragraph [< 100 words] summarizing which features (if any) of the Swiss data were consistent with your expectations (from our micro-economic models and your own ideas) and which features (if any) were inconsistent. Feel free to provide any further interpretation.

My analysis and predictions were pretty close with the empirical data. Only the Catholic affiliation was inconsistent. But that may be due to the inconsisties of the data as it looks like there might be missing values or not enoughd data points.

Congratulations! You have completed the lab for week 9.