

# Econ/Demog C175 Lab 5: Population and Climate Modeling

This week's lab will use a simplified version of Nordhaus's climate model. We will not get too much in to the details of the model. Instead we will use it to focus on the potential effect of population growth on climate change.

Our questions will be:

1. How does the model specify the relationship between population and climate?
2. What would happen if we slowed population growth? How big an effect would it have on global temperature if we slowed growth by 50%? How about if we just had zero population growth from now on?
3. Is part of the benefit of slower population growth offset by capital deepening?
4. How does the effect of slower population growth compare to an acceleration in decarbonization of the economy?

## The model

As discussed in the Nordhaus reading, Integrated Assessment Models are used to try to understand relationships between economic activity and climate change. The goal is not to get exact predictions but rather to get a useful approximation of the magnitudes involved. The questions can be what-if. For example, if we were able to reduce the emissions by 10%, how would that change the climate, and how would that change our standard of living? Or the questions can be more prescriptive. For example, how high would carbon taxes need to be in order to maximize the welfare of humanity over the next 2 centuries?

Today our goal is to focus mostly on population and climate, and so we will leave many of the other interesting economic questions aside. (These include: How does climate change feedback and hurt the economy? How should one take into account the welfare of different generations living at different points in time? How do we account for risk-aversion, the perhaps small chance that things could get very, very bad?).

We're going to use a simplified Integrated Assessment Model developed by Panagiotis Tsigaris and Joel Wood, at Thomson Rivers University in British Columbia. Special thanks to them for sharing their R code, a very slightly adapted version of which we will use below.

- i. A Solow-like production function, with time varying inputs.

$$Y(t) = A(t)K(t)^a L(t)^{1-a}$$

Population will be exogenous, but capital (per person) will depend on the usual balance of savings, depreciation, and population growth.  $A(t)$  is a term representing productivity (or "technology").

(There is also an additional term  $D(t)$ , which will appear in iv. below, but which is left out for now to emphasize the similarity to what we did with Solow.)

- ii. Carbon emissions ( $E(t)$ ) as a function of economic output

$$E(t) = \sigma(t)Y(t)$$

Emissions are tons of carbon released. A crucial quantity here is  $\sigma(t)$ , which is the carbon intensity of the economy, telling us the average carbon emission per \$ unit of output. Declining  $\sigma(t)$  means that the economy is “getting greener” in one sense. But if the economy grows quickly, then the increase in output  $Y$  could outweigh the decrease in emissions intensity  $\sigma$ .

and

- iii. Temperature change as a function of cumulative carbon emissions

$$C(t) = C_0 + \sum_0^t E(\tau)$$

and

$$T(t) = \beta * C(t)$$

The climate part of this model is very simple. It just says that temperature deviation from normal rises linearly at rate  $\beta$  with emissions.

(Warning: I’m not a climate scientist and don’t know how realistic this is. But, as we will see, the projected temperature increase is similar in this simplified model as with Nordhaus’s more sophisticated approach.)

- iv. A “damage” function that makes the economy less productive as temperature rises.

$$Y(t|damage) = D(t)Y(t)$$

We’re mostly going to ignore this for now, although it is in the code and has some effect, especially over the very long term. Getting this right is important for assessing the Social Cost of Carbon, and is a very interesting scientific topic on its own in terms of studying the impacts of warming.

## The R Code version of the model

Here is the full model, in R, as calibrated by Tsigaris and Wood to be close to the estimates of more complicated models. You don’t need to understand the code in detail, just understand that it is a dynamic model that begins with initial conditions and loops forward in time. For our purposes, I’ve parameterized the function to allow change in population and in decarbonization. The default values are given in the function definition, but can be changed as we do later in the Lab.

(Note: the model begins in 2010, and so it gives us 10 extra years to adapt, which we no longer really have. But in theory we could update all of the initial conditions ...)

```
library(data.table) # if you're working on this using the RStudio desktop version, make sure to install
solow_climate_fun <- function(gL0 = 0.022547, dL = -0.052,
                              gsigma0 = -.01, dsigma = -.0002)
{
  ## CONSTANTS ##
  ## capital's share of income
  a<-0.3
  ## savings rate
  s<-0.25
  ## Depreciation rate
  d<-0.1
  ## initial GDP, & Capital
  Y0<-63.69
```

```

K0<-135
##Initial population, pop growth rate, and parameter reducing pop growth
L0<-6.838 ## billions
## next lines commented out because they are arguments in fun
## gL0<-0.022547 ## Initial growth rate, about R
## dL<-0.052 ## exponential rate of decline in R, i.e,  $R(t+1) = R(t) \exp(-dL * 1)$ 
## Initial Total Factor Productivity, TFP growth rate, and parameter affecting TFP growth
A0<-3.955
gA0<-0.015323
dA<-0.011
## Initial emissions intensity, intensity growth, and parameter affecting intensity growth
sigma0 <- 0.549 ## this is carbon output in some unit of tons per $ of output
## next lines commented out because they are arguments in fun
## gsigma0<- -0.01 ## initial rate of decline in sigma
## dsigma <- -0.0002 ## rate of change in rate of decline
## Damage Parameters ## we'll ignore these, but they do play a small role.
theta1<-0.0023888849
theta2<-2
theta3<-0.00000507029
theta4<-6.754
## Damage to TFP
gamma<-0.001
## Damage through depreciation
d1<-0.01 ## this is the same constant "d" we had in solow model
## CCR Cumulative Carbon
beta<-1.8 ## multiplier to convert cumulative carbon to temperature increase
C0<-530 ## total carbon output up to 2010

## ### BASE MODEL ###
T<-191
year<-c(2010:2200)

## initialization
k<-rep(NA,T) # rep means 'NaN' replication 'T' number of times
A<-rep(NA,T)
gAt<-rep(NA,T)
sigma<-rep(NA,T)
gsigmat<-rep(NA,T)
y<-rep(NA,T)
D<-rep(NA,T)
L<-rep(NA,T)
gLt<-rep(NA,T)
E<-rep(NA,T)
Ccum<-rep(NA,T)
Temp<-rep(NA,T)
y.nd<-rep(NA,T)
k.nd<-rep(NA,T)
## starting values (for notations see loop step, line 218 onwards)
Ccum[1]<-C0
Temp[1]<-beta*Ccum[1]/1000
D[1]<-1/(1+theta1*(Temp[1]^theta2))
A[1]<-A0
gLt[1]<-gL0

```

```

k[1]<-((s*A[1]*D[1])/(d+gL0))^(1/(1-a))
gAt[1]<-gA0
sigma[1]<-sigma0
gsigmat[1]<-gsigma0
y[1]<-D[1]*A[1]*k[1]^a
L[1]<-L0
gLt[1]<-gL0
E[1]<-sigma[1]*y[1]*L[1]/3.67
k.nd[1]<-((s*A[1])/(d+gL0))^(1/(1-a))
y.nd[1]<-A[1]*k.nd[1]^a

## now we loop thru
for (t in 2:T){
  ## with climate damages
  Ccum[t]<-Ccum[t-1]+E[t-1] ## cumulative emissions
  Temp[t]<-beta*Ccum[t]/1000 ## temperature increase in celsius
  D[t]<-1/(1+theta1*(Temp[t]^theta2)) ## "damages"; line 108
  gAt[t]<-gAt[t-1]/(1+dA) ## change in growth rate of TFP
  A[t]<-A[t-1]*(1+gAt[t]) ## TFP
  gLt[t]<-gLt[t-1]*(1+dL) ## change in growth rate of pop, dL < 0, gLt shrinks
  L[t]<-L[t-1]*(1+gLt[t]) ## Pop
  k[t]<-((s*A[t]*D[t])/(d+gLt[t]))^(1/(1-a)) ## capital per person in solow steady state
  gsigmat[t]<-gsigmat[t-1]/(1+dsigma) ## change in emissions intensity
  sigma[t]<-sigma[t-1]*(1+gsigmat[t]) ## emissions intensity
  y[t]<-D[t]*A[t]*k[t]^a ## income per capita
  E[t]<-sigma[t]*y[t]*L[t]/3.67 ## total emmissions (3.67 converts carbon to CO2)

  ## without climate damages i.e., D=1
  k.nd[t]<-((s*A[t])/(d+gLt[t]))^(1/(1-a))
  y.nd[t]<-A[t]*k.nd[t]^a
}
out = data.table(year, Ccum, Temp, D,
                  A, L, k,
                  sigma, y, E, k.nd, y.nd)
return(out)
}

```

## The baseline case, “Business As Usual”

```

out = solow_climate_fun() ## without arguments, uses default
year = out$year

```

Let’s walk through some of our exogenous inputs in the baseline scenario

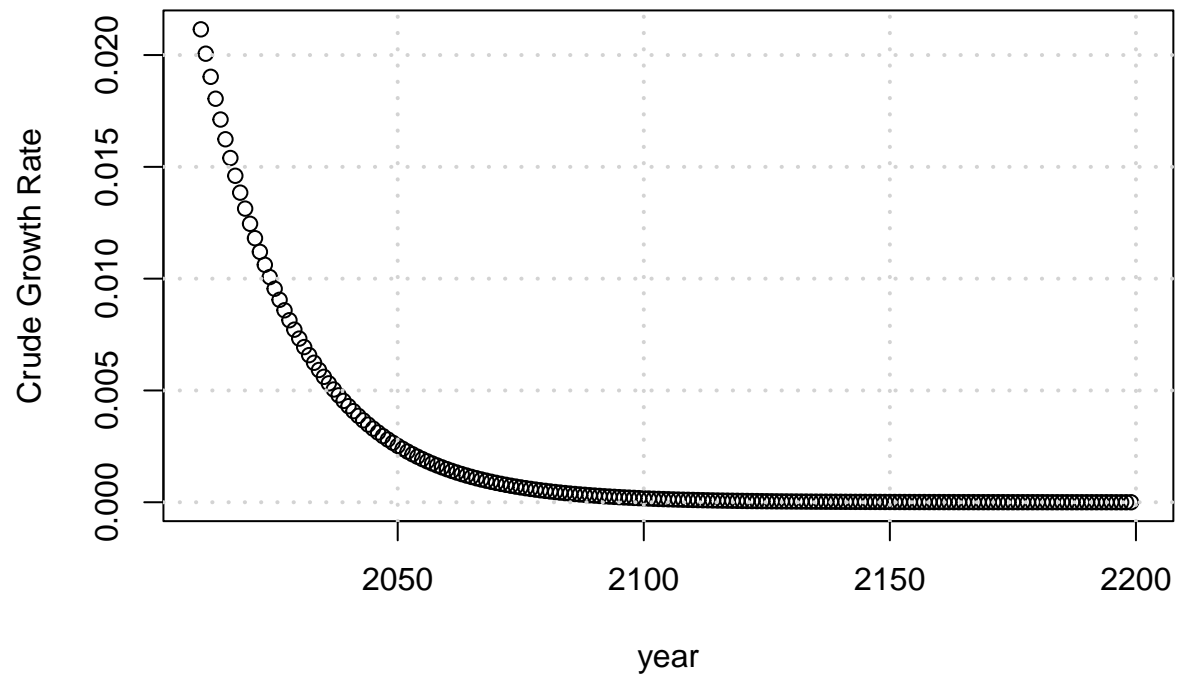
- Population growth rates

```

L = out$L
R = c(diff(log(L))/1, NA) ## calculate exponential growth rate as in Lab 1.
plot(year, R, ylab = "Crude Growth Rate",
      main = "Exponential Pop Growth Rate 'R'")
grid(lwd = 2) # lwd stands for line width

```

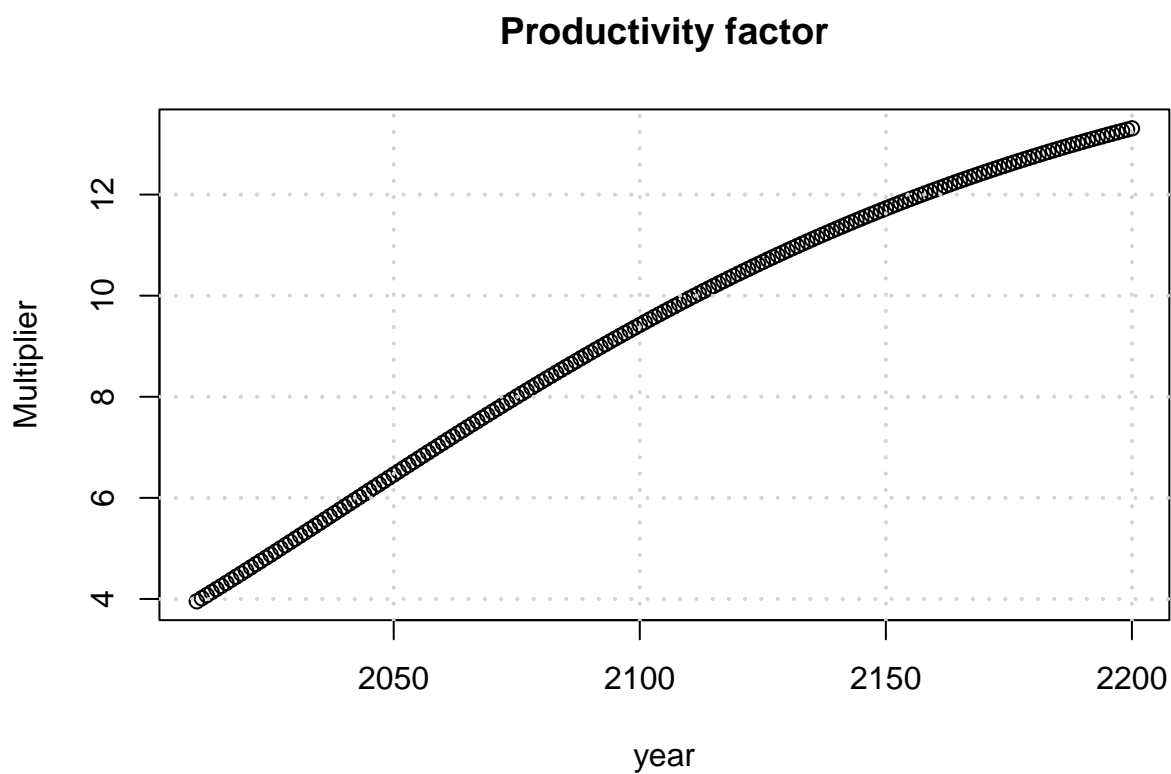
## Exponential Pop Growth Rate 'R'



We see pop growth rate declines quickly to very low levels in 2050 and nearly zero by 2100.

- Productivity

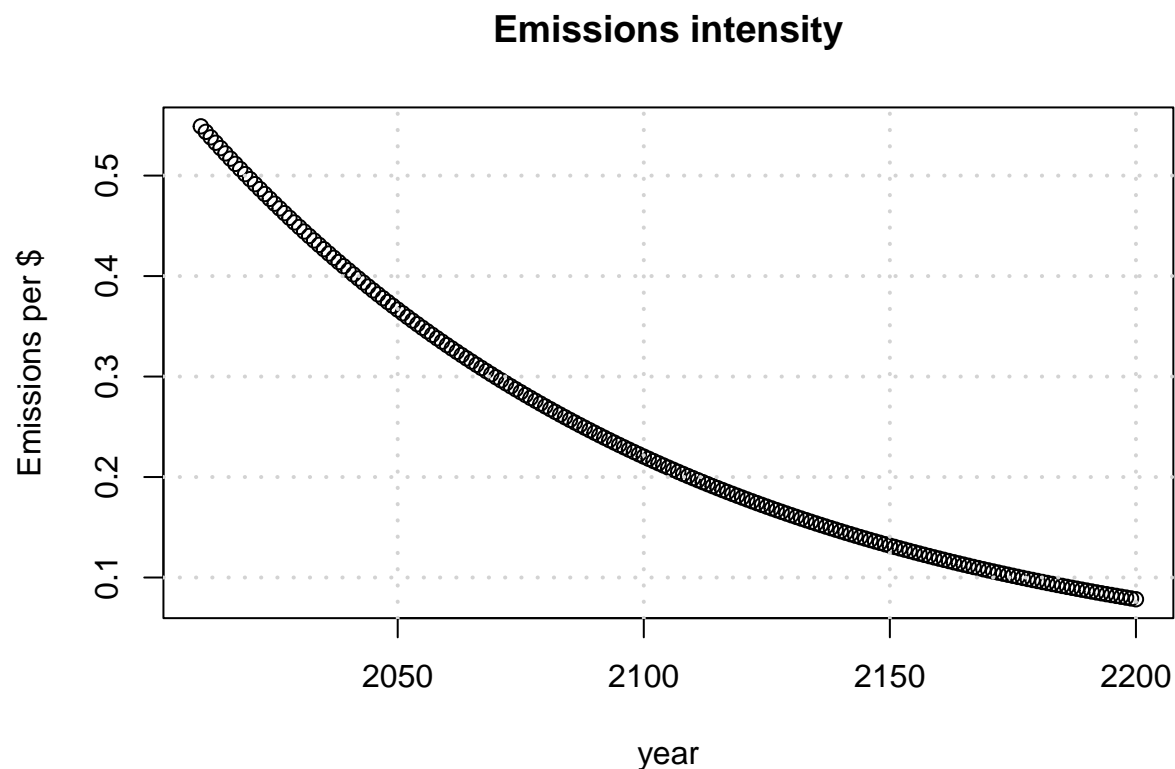
```
A = out$A
plot(year, A, ylab = "Multiplier",
      main = "Productivity factor")
grid(lwd = 2)
```



We see that productivity increases roughly linearly until about 2100, after which it slows a bit.

- Emissions intensity

```
sigma = out$sigma
plot(year, sigma, ylab = "Emissions per $",
      main = "Emissions intensity")
grid(lwd = 2)
```



Emissions decline fairly linearly, but rate of decline slows late in projection.

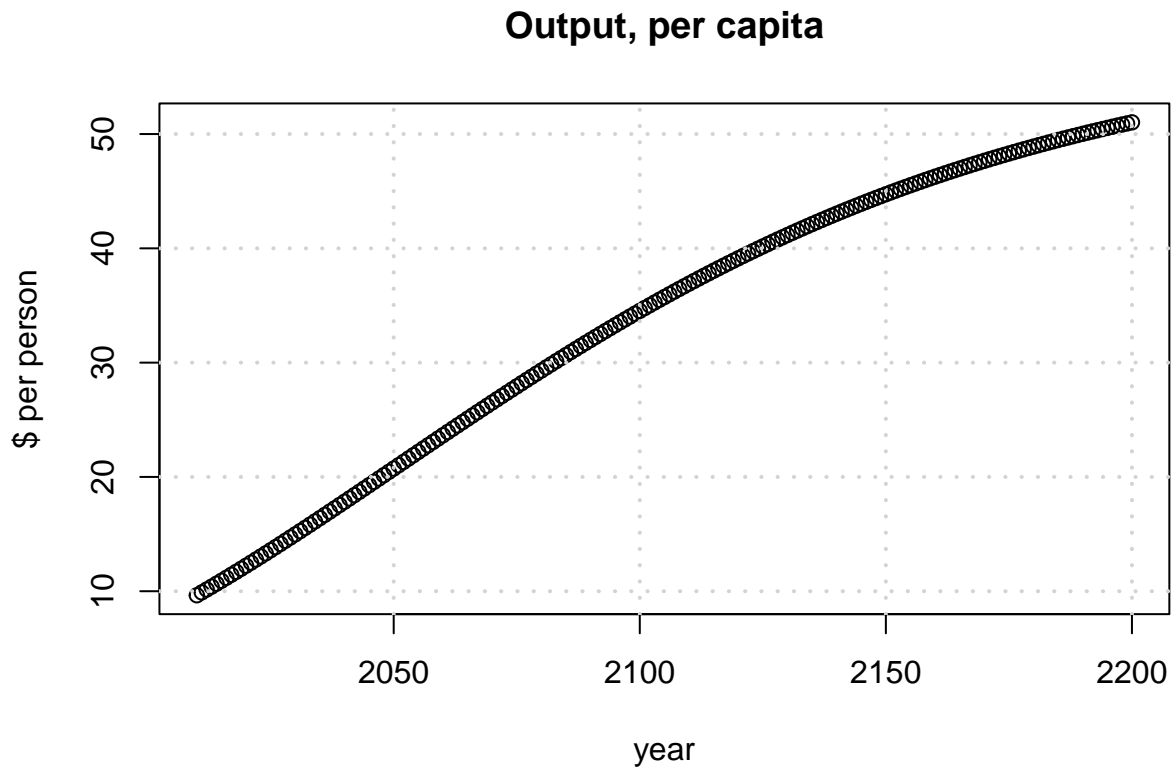
Now we're ready to look at what will happen to the climate and the economy. Before we begin, let's make some guesses:

Q1 Will per capita output  $y$  in 2100 compared to 2010 output be:

A. A lot lower because of climate change damages. B. The same because technology is assumed constant C. Much higher ( $> 3x$ ) because we assumed a lot of productivity improvement, so much so that climate change damages won't make much dent. D. Impossible to say

Let's see:

```
y = out$y
plot(year, y, ylab = "$ per person",
      main = "Output, per capita")
grid(lwd = 2)
```



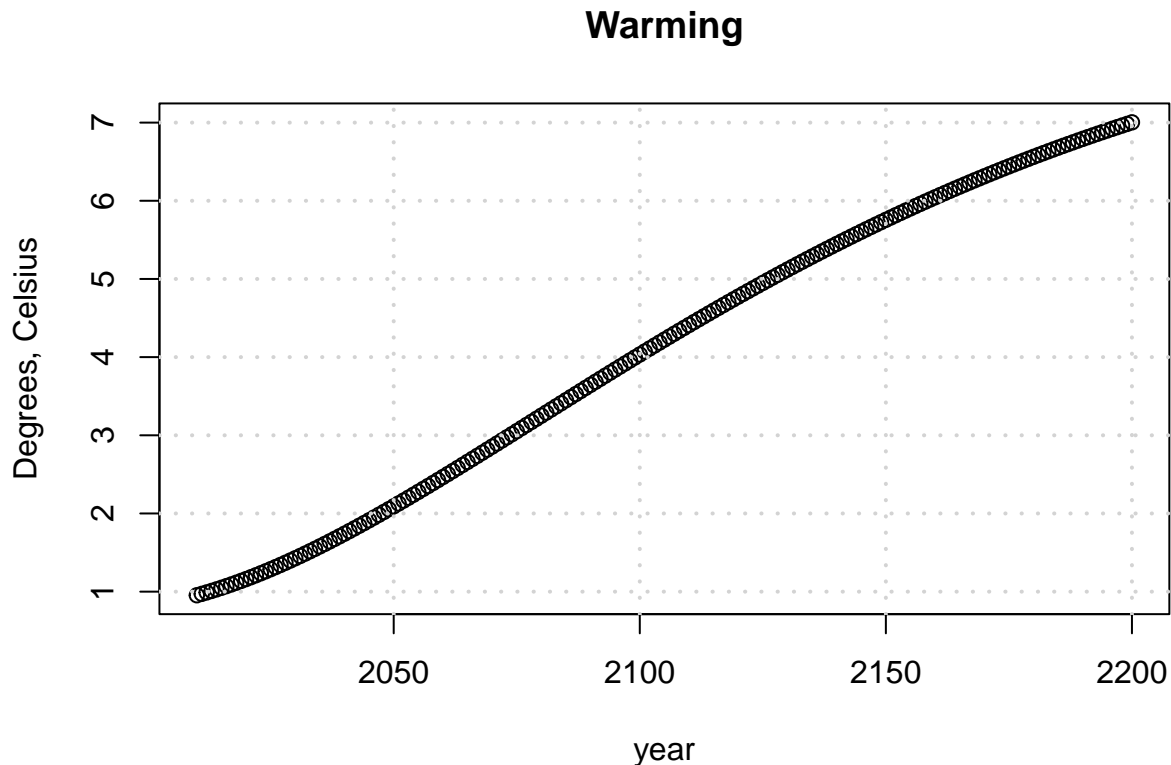
It looks like the answer is “C”

Q2 How high will temperature rise? A. Zero, because this is the so-called baseline case. B. By 2 degrees Celsius because that is what is considered still not-catastrophic C. By much more than 2 degrees, because baseline is business-as-usual, and if we continue on the path we are on, there will be a lot of warming.

Let's see:

```
Temp = out$Temp
plot(year, Temp, ylab = "Degrees, Celsius",
      main = "Warming")
grid(lwd = 2)
```





It looks like the answer is, once again, “C”.

Let’s compare to the Nordhaus reading. In his Figure 5, on page 2003, you can see the “Temperature trajectories for different objectives.” The blue line shows “Base”, which has 2 degrees of warming by 2050 and 4 degrees by 2100. This is very close to what the simple model we are using is predicting. (This is probably not evidence that either model is right, but rather an indication that the model defaults of our simple model were chosen so that their output would be close to the standard model, Nordhaus’s.)

## Slower population growth

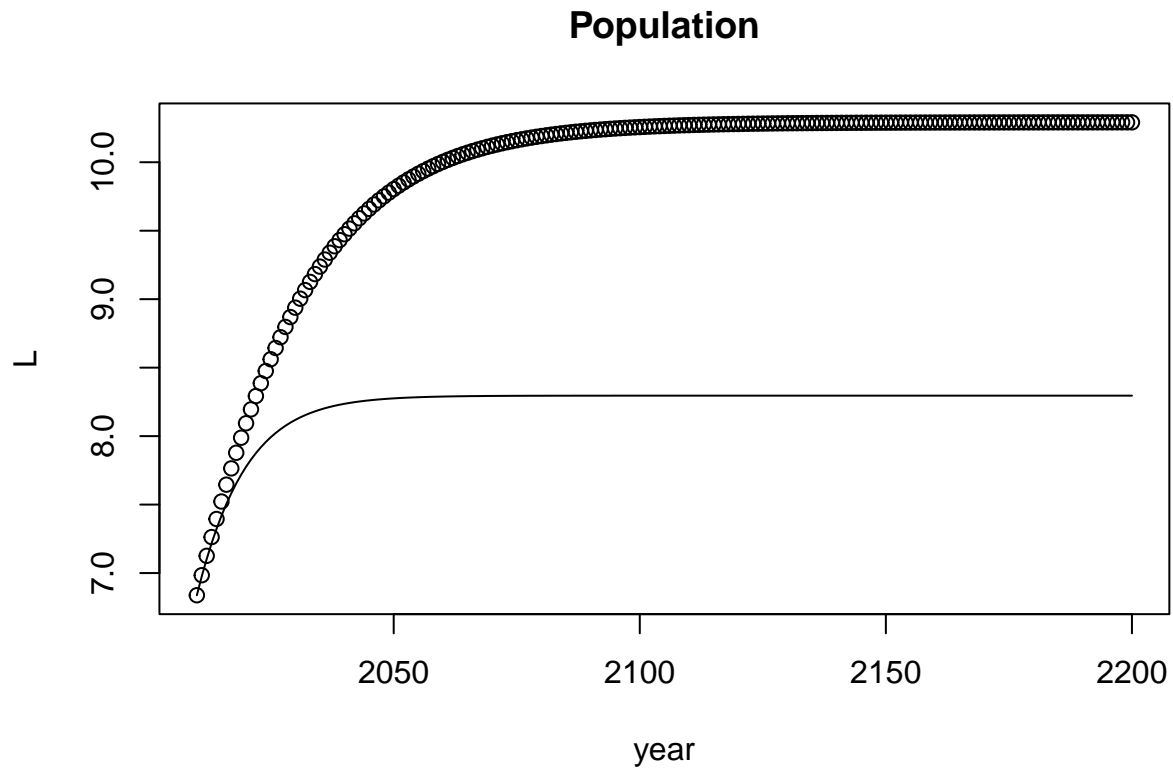
As we saw above when we plotted population growth rates, the baseline model assumes a leveling off of population growth, beginning at a bit more than 2% a year and declining continuously to about zero population growth by 2100.

Let’s consider the case of speeding up the world demographic transition so that the growth rate itself declines exponentially at twice the rate. I call this the “slow pop” scenario to indicate that population is growing more slowly.

```
out.slow.pop <- solow_climate_fun(gL0 = 0.022547, ## we leave starting pop growth as is
                                dL = -0.052*2, ## and multiply rate of change in growth rate by 2
                                gsigma0 = -.01, dsigma = -.0002) # solow_climate_fun function defined earlier is now
```

Let’s see how much difference this made in population.

```
L.slow <- out.slow.pop$L
plot(year, L, main = "Population")
lines(year, L.slow)
```



It looks like there would only about 8.3 billion people rather than 10.3 billion. This is a difference of about 20%.

Since the climate model depends on cumulative emissions and it doesn't matter when the emissions occurred, it is useful to think about the number of "person-years" in the two scenarios. To find the person-years lived from 2010 to 2100, we add up the people alive in each year and sum them up.

```
pyl.base = sum(L[year %in% 2010:2100])
pyl.slow = sum(L.slow[year %in% 2010:2100])
print("base")
```

```
## [1] "base"
```

```
print(pyl.base)
```

```
## [1] 863.8091
```

```
print("slow")
```

```
## [1] "slow"
```

```
print(pyl.slow)
```

```
## [1] 740.1995
```

```
print("pop years lived ratio")
```

```
## [1] "pop years lived ratio"
```

```
print(pyl.slow / pyl.base)
```

```
## [1] 0.8569017
```

It looks like person-years will decline by about 15%. How about emissions?

```
E = out$E
E.slow = out.slow.pop$E
sum.E.base = sum(E[year %in% 2010:2100])
sum.E.slow = sum(E.slow[year %in% 2010:2100])
print("E ratio")
```

```
## [1] "E ratio"
```

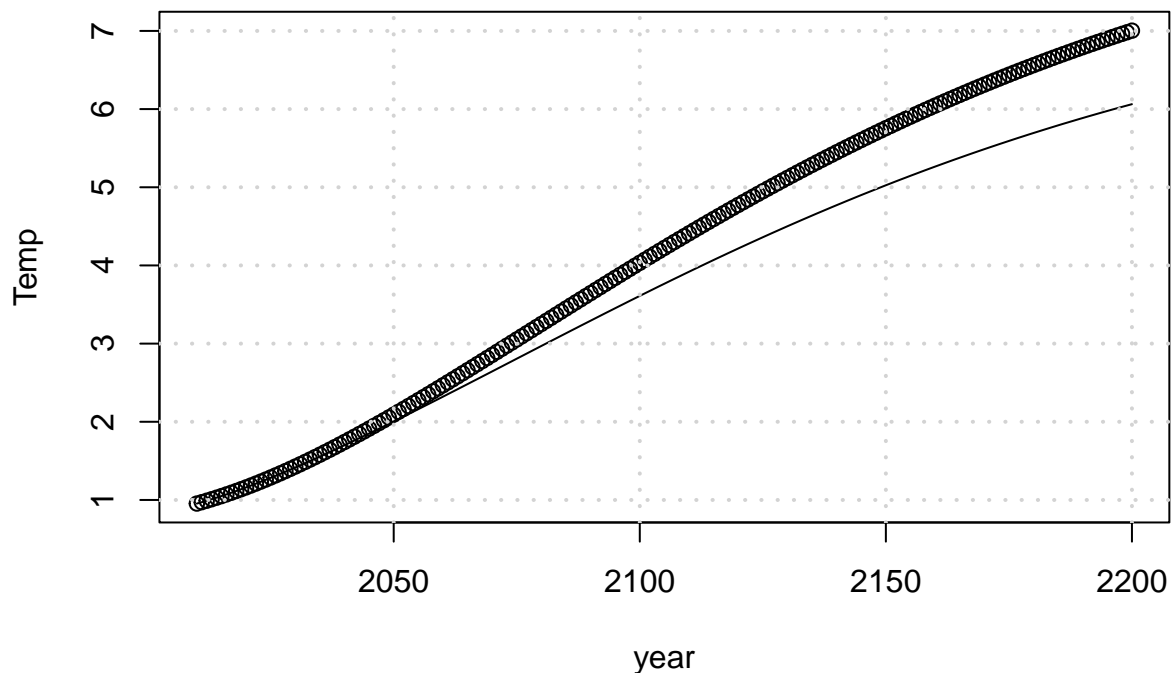
```
print(sum.E.slow / sum.E.base)
```

```
## [1] 0.8620572
```

Also, about 15%! (OK, 14% ...)

And now we can check on warming

```
Temp.slow = out.slow.pop$Temp
plot(year, Temp)
lines(year, Temp.slow)
grid(lwd = 2)
```



It looks like we have about +4 degrees in 2100 in the baseline, and about +3.6 degrees in the slower population growth scenario. This seems at first to be a decline of temperature increase by about 10%, since  $3.6/4.0 = 0.9$ . But we started in 2010 already with +1 degree of warming. So the ratio we want is  $(3.6 - 1)/(4 - 1) = 0.87$ . Very close.

Note that by 2200 the smaller population scenario produces about 1 degree less warming.  $(6 \text{ degrees} - 1 \text{ degree by 2010}) / (7 - 1) = 0.83$ .

So overall, even with this complicated model, incremental warming is roughly proportional to population size.

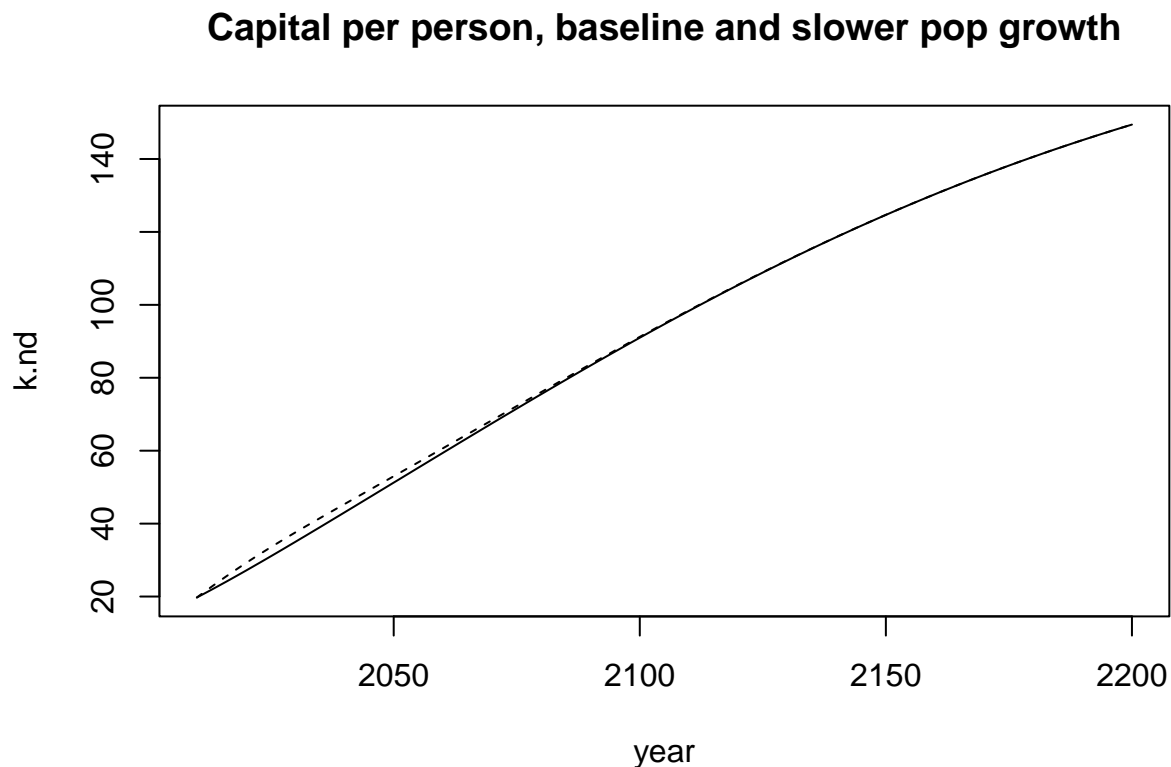
## Capital deepening?

If population growth slows, then there will be, according to the Solow model, capital deepening – assuming savings rates and depreciation stay constant. In principle, this could lessen the environmental benefits of slower human population growth, but it’s not clear that it would be a very big effect.

How much does capital deepen in the slower population growth scenario?

To see the effect more clearly, we’ll look at “k.nd” (the steady state ignoring climate damages “D”, which we have pretty much ignored already.)

```
k.nd = out$k.nd
k.nd.slow = out.slow.pop$k.nd
plot(year, k.nd, type = "l", main = "Capital per person, baseline and slower pop growth")
lines(year, k.nd.slow, lty = 2)
```



It looks like there’s a bit of capital deepening at the beginning but not much. How can this be?

We’ll explore this in “graded questions” at the end. The answer, we’ll see, boils down to the differences in growth rates being very small (fractions of a percent) relative to the depreciation rate (10%).

(Note: the simplified implementation of the model assumes that capital is always at its steady state level. One could also reprogram so that capital is accumulated directly from the previous period, depreciated, and recalculated per person. I don’t think the answer probably would be very different. But an adventurous mini-project could check on this.)

## Accelerating decarbonization

Carbon emissions are modeled as a changing fraction of economic output. Every million dollars of output produces some number of tons of carbon emissions. The baseline assumption that we graphed above is that emission rates per unit of output start out declining at 1% per year, a pace that slows very gradually later on. What if we started out with emission decline that is twice as fast, 2%? What would temperature be in 2100?

Let's start with a guess.

Q. What do we think will happen to emissions if we have much faster decarbonization beginning at twice the rate of baseline??

A. Even with slower emissions per unit of GDP, productivity is increasing so fast that increasing GDP will mean ever increasing emissions.

B. Over the long term, the effect of lower emissions per unit of output will dominate increasing output, resulting – eventually – in ever declining emissions.

C. We're doomed. There's nothing we can do. And nothing matters anymore.

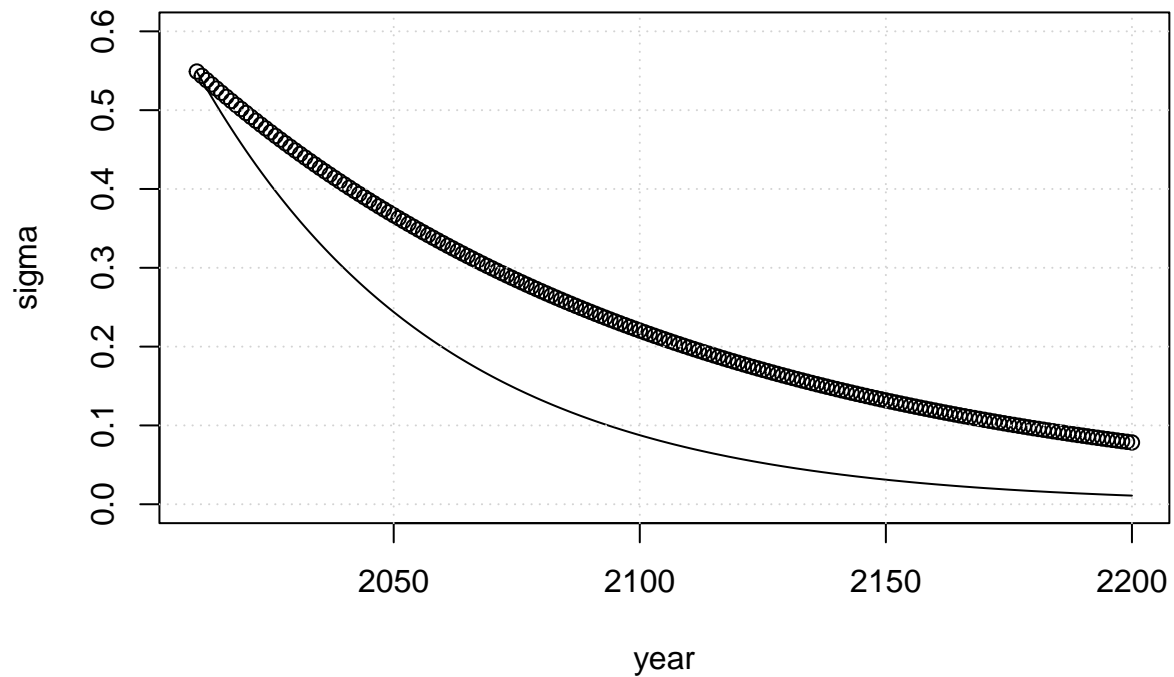
D. A & C.

```
out.fast.decarb <- solow_climate_fun(gL0 = 0.022547, ## we leave starting pop growth as is
                                   dL = -0.052, ## and multiply rate of change by 2
                                   gsigma0 = -.01*2, dsigma = -.0002)
```

Let's plot the carbon intensity in the two scenarios just to compare

```
sigma = out$sigma
sigma.fast = out.fast.decarb$sigma
## note: here "fast" is referring to decarbonization,
## whereas before "slow" was referring to population growth. Sorry!
plot(year, sigma, ylim = c(0,.6), main = "Carbon intensity, by scenario")
lines(year, sigma.fast)
grid()
```

## Carbon intensity, by scenario

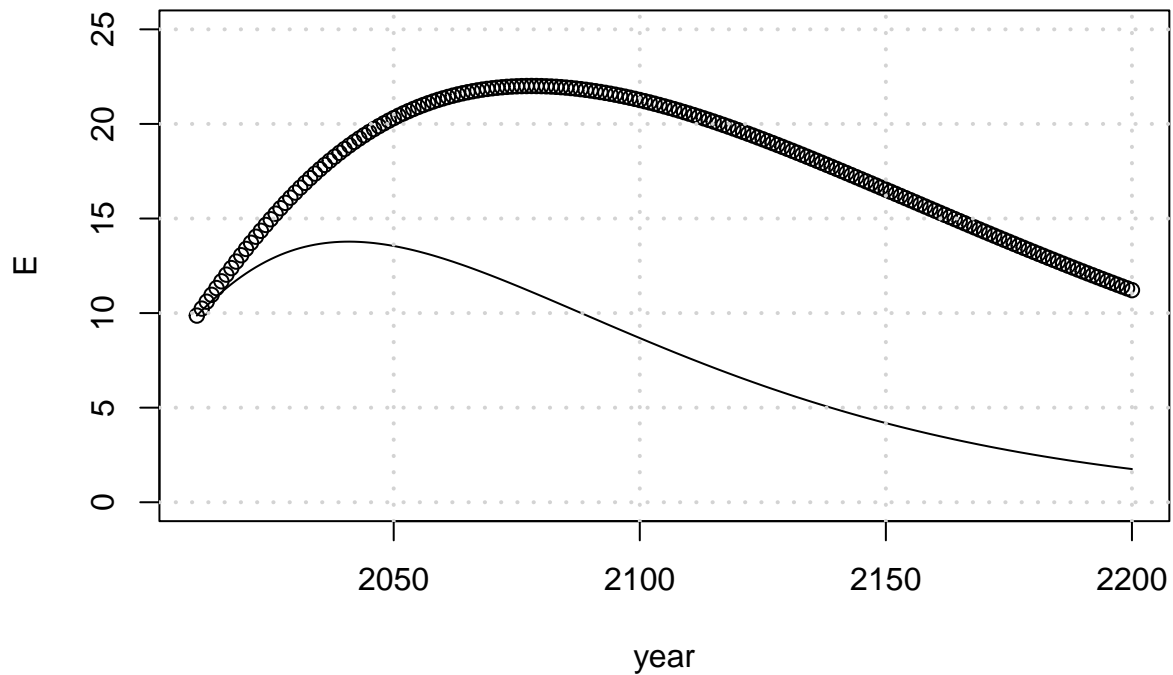


It looks like emissions intensity (per \$ of output) get very low by 2100 in the rapid decrease. But they're still quite far from zero well past 2050.

And let's see how total annual emissions are affected. (Remember this is emissions intensity times the total size of the economy.)

```
E = out$E
E.fast = out.fast.decarb$E
plot(year, E, ylim = c(0, 25), main = "Emissions, by scenario")
lines(year, E.fast)
grid(lwd = 2)
```

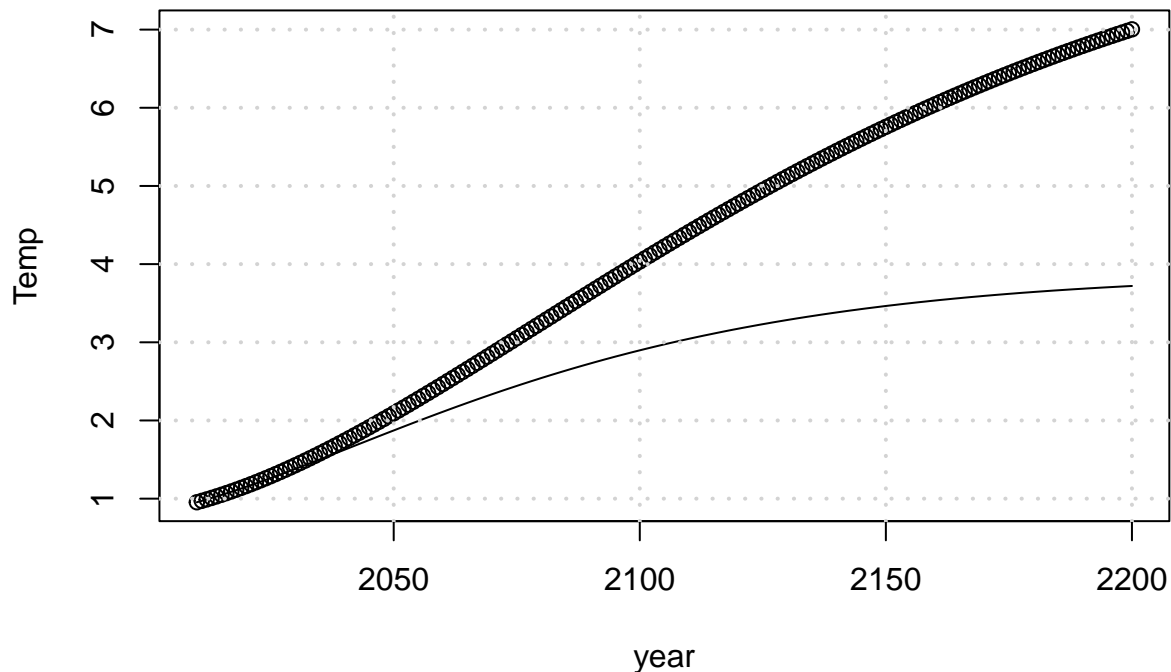
## Emissions, by scenario



We see that annual emissions rise and fall in both cases. Eventually the decline in emissions intensity dominates the increase in output. With faster decline in emission intensity – our “fast” scenario – we get what looks to be much lower emissions. But we can also see that even in the “fast” scenario emissions don’t fall below their current level until almost 2100.

And finally, temperature. Could decarbonization save us?

```
Temp.fast = out.fast.decarb$Temp
plot(year, Temp)
lines(year, Temp.fast)
grid(lwd = 2)
```



This story seems good-news, bad-news, bad-news. The good news is that by 2100, there would be about 1 degree less of warming – which is a pretty big effect. The bad news is that we would still get about the same amount of warming by 2050.

The biggest benefit in terms of climate of faster decline emissions intensity appears to be that it helps us avoiding the enormous temperature increases far in the future, beyond the year 2100 into the 22nd century. If we care a lot about our great-grand-children, then this is a very big benefit. Although the model also predicts they'll be very rich. So, it's not entirely clear we should sacrifice enormously now for them. (This raises the whole issue of how to compare generations, which is very interesting but beyond the scope of what we're looking at here.)

One more bit of optimism: a much faster rate of de-carbonization is not implausible. Figure 6 (and Table 3) in the Nordhaus reading shows world carbon intensity is already decreasing at something like 1.8% per year. A combination of technological advances in energy production and world-wide action on carbon pricing holds some hope for making rapid progress.

Here are the take-away lessons I got from writing this lab:

- Population matters, but not as much as I expected. I think the reason for this is that although emissions are more-or-less proportional to population size in the model, population size just doesn't change that much if we accelerate the approach to zero population growth.
- Declines in emission intensity are potentially much more important. As modeled, the lower bound on sigma is zero, and the faster we get there, the less warming there will be.
- Substantial global warming is very hard to avoid. This is because we have a substantial amount of cumulative emissions even today, and they are going to continue at their present rates for some time, no matter what we do.
- Nordhaus's call to optimize and choose the best path should not be seen as just giving up and accepting



whatever comes our way. Just the opposite. It is a call to act now to avoid very bad outcomes. But at the same time a caution that draconian reductions in standard of living now may not be the best thing. Instead, we should aim for a path that balances the costs and the benefits. His optimal path is has about 3 degrees of warming by 2100, remarkably close to the one we obtained by doubling the speed of decarbonization.

## Graded questions

### 1. Solow review

Recall that in equilibrium, the “birth and death of capital lines” cross so that

$$(n + d)k = sy = sk^a$$

- a) Use some algebra to solve for equilibrium value of  $k$ .

The equilibrium value for  $k$  can be solved by:

$$(n + d)k = sk^a n + d = sk^{a-1} \frac{n + d}{s} = k^{a-1} \frac{n + d}{s} = k^{a-1} \left( \frac{s}{n + d} \right)^{\frac{1}{1-a}} = k$$

- b) Compare this to r-code for  $k[t]$  in the loop of the function. What if any differences to do you see?

The R code for  $k[t]$  is set to  $((sA/t/D[t])/(d+gLt[t]))^{1/(1-a)}$  which is capital per person in solow steady state. The format of the equations looks similar.

- c) Let  $a = .3$ ,  $d = .1$ , and  $s = .25$ . What will be the ratio of steady state capital per person with a population growth rate of 0.25% to that of a population with growth rate 0.5%? (These is roughly in line with the difference between the two scenarios for 2010 to 2060.) Is this much of a difference? (1 sentence is fine.)

The ratio of the steady state will be:

$$\left( \frac{s}{n + d} \right)^{\frac{1}{1-a}} = k \left( \frac{0.25}{0.0025 + 0.1} \right)^{\frac{1}{1-0.3}} = k \left( \frac{0.25}{0.005 + 0.1} \right)^{\frac{1}{1-0.3}} = k$$

```
k1<- (0.25 / (0.0025 + 0.1)) ** (1 / (1 - 0.3))
print(k1)
```

```
## [1] 3.574094
```

```
k2<- (0.25 / (0.005 + 0.1)) ** (1 / (1 - 0.3))
print(k2)
```

```
## [1] 3.453149
```

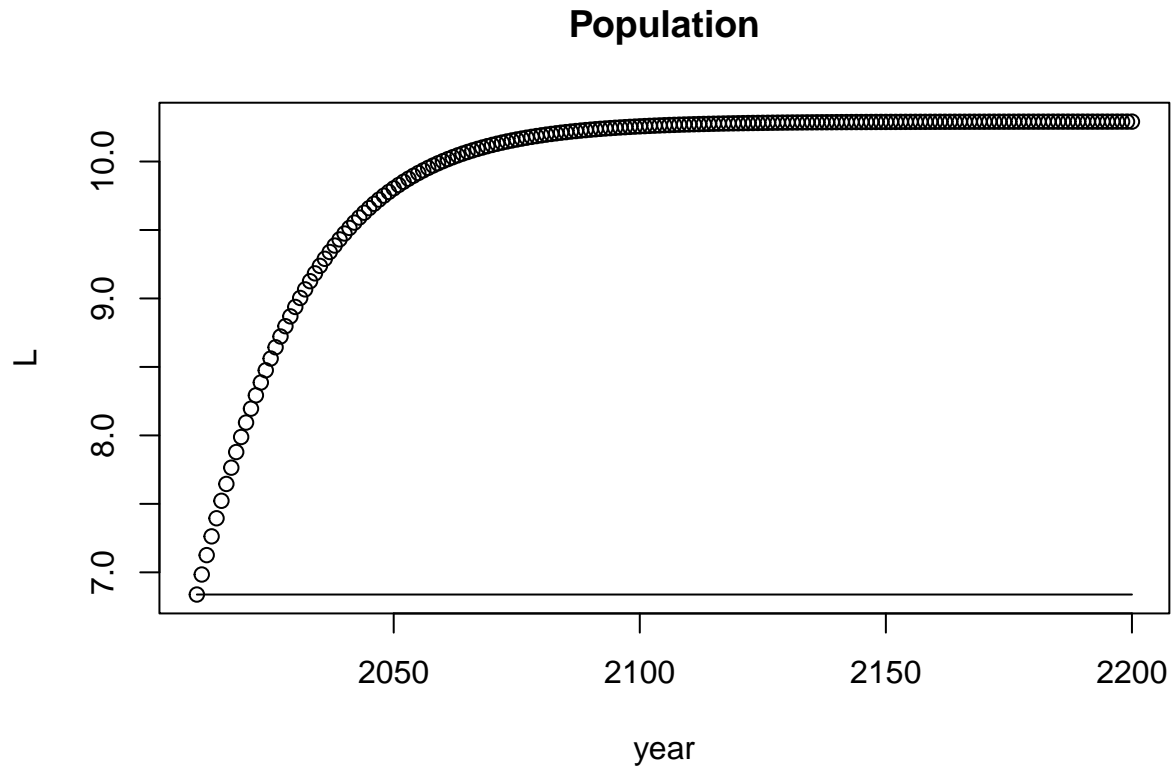
The respective  $k$  values are 3.574094 and 3.453149 for a population growth of 0.25 and 0.5 percent. This is not much of a difference.

### 2. Zero population growth

- a. Simulate zero population growth by modifying the “slow” scenario above. Hint 1: you need to make population growth even slower. (Big) Hint 2: try changing the parameters “gL” and “dL” to zero. Hint 3: you can check your simulation by plotting the out\$L to make sure it is a constant.

```
out.slow.pop <- solow_climate_fun(gL0 = 0.0, ## we leave starting pop growth as is
                                dL = 0.0, ## and multiply rate of change in growth rate by 2
                                gsigma0 = -.01, dsigma = -.0002) # solow_climate_fun function defined earlier is now
```

```
L.slow <- out.slow.pop$L
plot(year, L, main = "Population")
lines(year, L.slow)
```

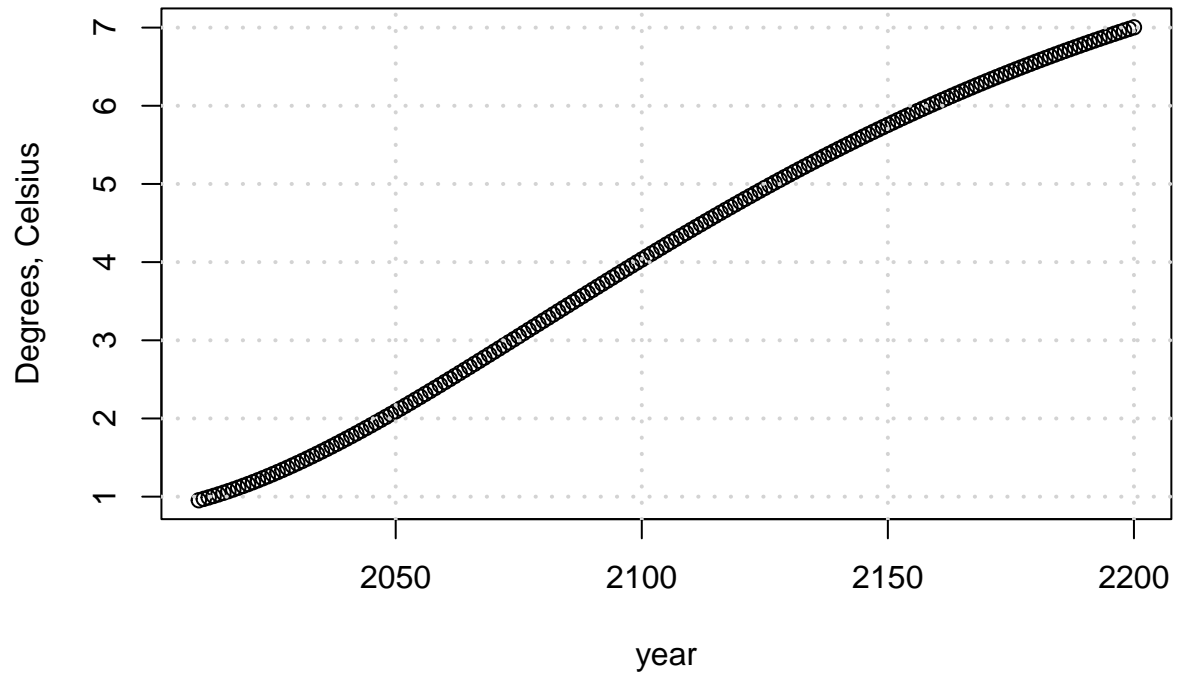


b. How much warming will there be? Is this more or less than the 2 degrees Celsius that is considered dangerous?

[Note: we'll see in a few weeks when we study age structure that even a sudden shift to of parents to having exactly 2 children would not stop population growth immediately. ]

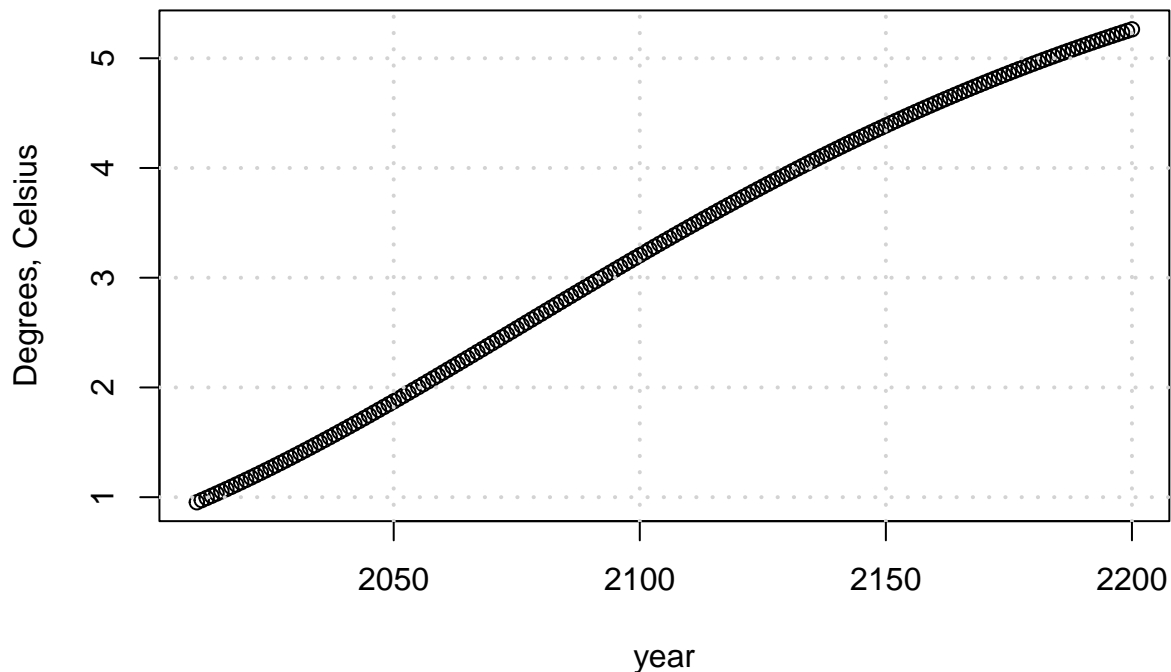
```
Temp = out$Temp
plot(year, Temp, ylab = "Degrees, Celsius",
      main = "Warming")
grid(lwd = 2)
```

## Warming



```
Temp = out.slow.pop$Temp
plot(year, Temp, ylab = "Degrees, Celsius",
      main = "Warming with slow/zero")
grid(lwd = 2)
```

## Warming with slow/zero



The temperature has in general decreased it's maximum to a peak of 5 degrees Celsius. The difference in total is just over 5, which is more than the 2 degrees Celsius that is considered dangerous.

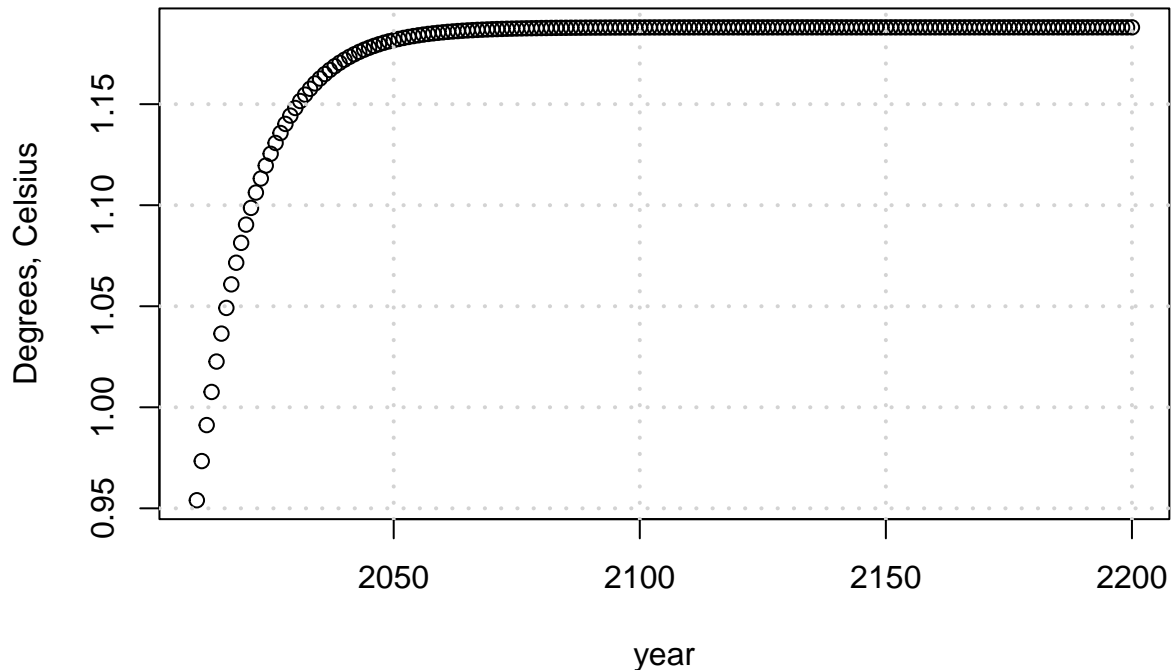
3. How much decarbonization is necessary?

- a. Try different rates of change of sigma (by varying the parameter "gsigma0") until you find one that will keep world temperature rise below 2 degrees. What value is enough? (Optional: a sentence on whether this numbers seems realistic).

```
out.dec.pop <- solow_climate_fun(gL0 = 0.0, ## we leave starting pop growth as is
                                dL = 0.0, ## and multiply rate of change in growth rate by 2
                                gsigma0 = -0.1, dsigma = -.0002) # solow_climate_fun function defined earlier is now

Temp = out.dec.pop$Temp
plot(year, Temp, ylab = "Degrees, Celsius",
      main = "Warming with slow/zero varying gsigma0")
grid(lwd = 2)
```

### Warming with slow/zero varying gsigma0



A gsigma of -0.1 gave a temperature difference of less than 2. gsigma is denoted as the initial decline in sigma, so considering it's a higher negative number, it seems realistic, but very optimistic.

b. Why is Nordhaus's optimal trajectory higher than 2 degrees? A few sentences is fine.

Nordhaus's optimal trajectory is higher than 2 degrees as he was trying to be pragmatic and realistic about the current status of the environment, such as ice sheets melting or any other additional damage caused by natural causations.

4. There will clearly be a Tragedy of the Commons if each firm is allowed to make their own independent judgement about how much carbon to emit. According to Nordhaus, why is it not sufficient to let each country enforce its own carbon emissions policy on its own firms? [1 or 2 sentences is fine.]

It is not sufficient to let each country handle it's own carbon emission policy, because it should really be a global effort. It will become an issue when property rights are poorly defined they are not compatible with other nation's prospective goals.

Congratulations! You've finished Lab 5.