$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$	$(\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$	$(\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv$	$(\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta)$	$\equiv$	$(\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

9/10 propositional logic

Knowledge representation : constrains, domains, variables (like these)

↓ a new kind

propositional logic

propositional logic ~~KB~~ System :

Knowledge Base : KB ("△") all true

Operations { tell : a new proposition
ask : if the KB entails a query

factorabe → amenable to inference

proposition (命题) : A sentence which is either True or False

{ Atomic : 大写表示，只含一个 proposition
complex : atomics + logic connectives

$\wedge$ and

$\vee$ or

$\neg$ not

$\Rightarrow$ if ... then

$\Leftrightarrow$ bidrectional if ... then

Syntax (句法，语法) Semantic (语义的)

↓

Here : true / false

propositional model : an assignment of every atomics

truth table

logically valid : true for all models
invalid / unsatisfiable : false for all models
satisfiable / contingent : true in some models .

entailment : $\alpha \models \beta$ iff $M(\alpha) \subseteq M(\beta)$

<u>inference</u> { model checking (1)
theorem proving (2)

(1) Complexity : 2^k (k = #atomics) 最大32 propositions.

Check : every model in α , in β too ? < yes $\models$; no $\not\models$

(2) arguments (论证) , proof-like

need to Search for a proof

→ { inint state : Δ
actions : $\boxed{\text{inference rules}}$
trasactions : results of
goal test : prove or disprove
cost : uniform

inference rules : { soundness (稳健性) : for every $\alpha \vdash \beta$, $\alpha \models \beta$
Completeness (完整性) : for every $\alpha \models \beta$, $\alpha \vdash \beta$

resolution rule :

$$\frac{l_1 \vee l_2 \vee \dots \vee l_k , \quad m}{l_1 \vee l_{i-1} \vee l_{i+1} \vee \dots \vee l_k}$$

remember to remove redundant : $A \vee A$ 简化为只有 A

refutable complete : Δ is inconsistant $\Leftrightarrow$ derive [ ]

proof by contraddiction :

$\alpha \models \beta$ iff $\alpha \wedge \neg\beta$ is unsatisfiable

$\boxed{\text{To show } \Delta \models \alpha, \text{ we show that } \Delta \wedge \neg\alpha \text{ is unsatisfiable}}$

CNF: logically equivallent normalized form

注意：每次只能用两个proposition，否则不sound.

$\neg(\neg\alpha) \equiv \alpha$

$\neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$ 同样的，$\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$

$(\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$

流程：

① To CNF

② 列出 KB，$\wedge$分开括号，编号

③ 列出 $\neg$goal，编号

④ 两两 resolve，对[ ]编号

⑤ Therefore $\Delta \wedge \neg S$ is unsatisfiable

So $\Delta \models S$