Advanced Econometrics 2 Assignment 1 Problem 1

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Problem 1a

Show that $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t$ can be rewritten as $Y_t = \Sigma_{j=0}^{\infty}(\Sigma_j^{i=0}\lambda_1^{j-i}\lambda_2^i)e_{t-j}$

Since the roots of $(1 - \phi_1 z - \phi_2 z^2) = 0$ lie outside the unit circle we know that the process is stable and also that $\lambda_{1,2} < |1|$, which will allow us to invert the lag polynomial and also allow us to take advantage of the geometrics series.

$Y_t = (1 - \lambda_1 L)^{-1}(1 - \lambda_2 L)^{-1}\epsilon_t = (1 + \lambda_1 L + \lambda_1^2 L^2 + ...)(1 + \lambda_2 L + \lambda_2^2 L^2 + ...)\epsilon_t$

after expanding and collecting like terms we have. . .

$= [1 + (\lambda_1 + \lambda_2)L + (\lambda_1^2 + \lambda_2^2)L^2 + (\lambda_1^3 + \lambda_1^2\lambda_1 + \lambda_1\lambda_2^2 + \lambda_2^3)L^3 + ...]\epsilon_t$

$= \Sigma_{j=0}^{\infty}(\Sigma_{i=0}^{j}\lambda_1^{j-i}\lambda_2^{j})L^j\epsilon_t = \Sigma_{j=0}^{\infty}(\Sigma_{i=0}^{j}\lambda_1^{j-i}\lambda_2^{j})\epsilon_{t-j}$

Problem 1b

$j = 0$

then we have $\lambda_1^0\lambda_2^0 = 1$ and by definition $c_1 + c_2 = 1$

$j = 1$

then we have $c_1\lambda_1 + c_1\lambda_2 = (\frac{\lambda 1}{\lambda_1 - \lambda_2})\lambda_1 + (\frac{\lambda_2}{\lambda_2 - \lambda_2})\lambda_2 = \frac{\lambda_1^2(\lambda_2 - \lambda_1) + \lambda_2^2(\lambda_1 - \lambda_2)}{(\lambda_1 - \lambda_2)(\lambda_2 - \lambda_1)}$

$= \frac{(\lambda_1 + \lambda_2)(2\lambda_1\lambda_2 - \lambda_1^2 - \lambda_2^2)}{2\lambda_1\lambda_2 - \lambda_1^2 - \lambda_2^2} = \lambda_1 + \lambda_2$

$= \Sigma_{i=0}^{1}\lambda_1^{1-i}\lambda_2^i$

$j = 2$

$c_1\lambda_1^2 + c_2\lambda_2^2 = (\frac{\lambda_1}{\lambda_1 - \lambda_2})\lambda_1^2 + (\frac{\lambda_2}{\lambda_2 - \lambda_1})\lambda_2^2 = \frac{\lambda_1^3(\lambda_2 - \lambda_1) + \lambda_2^3(\lambda_1 - \lambda_2)}{(\lambda_1 - \lambda_2)(\lambda_2 - \lambda_1)}$

$= \frac{\lambda_1^3\lambda_2 + \lambda_1\lambda_2^3 - \lambda_1^4 - \lambda_2^4}{2\lambda_1\lambda_2 - \lambda_1^2 - \lambda_2^2} = \frac{(\lambda_1^2 + \lambda_2^2 + \lambda_1\lambda_2)(2\lambda_1\lambda_2 - \lambda_1^2 - \lambda_2^2)}{2\lambda_1\lambda_2 - \lambda_1^2 - \lambda_2^2} = \lambda_1^2 + \lambda_2^2 + \lambda_1\lambda_2$

$= \Sigma_{i=0}^{2}\lambda_1^{2-i}\lambda_2^i$

Problem 1c

$\lambda_1 + \lambda_2 = \phi_1$

$\lambda_1 \lambda_2 = -\phi_2$

$\phi_1^2 + \phi_2 = (\lambda_1 + \lambda_2)^2 - \lambda_1 \lambda_2 = \lambda_1^2 + \lambda_2^2 + 2\lambda_1 \lambda_2 - \lambda_1 \lambda_2$

$= \lambda_1^2 + \lambda_2^2 + \lambda_1 \lambda_2 = c_1 \lambda_1^2 + c_2 \lambda_2^2$

which is the same as the expression for $j = 2$ in part b