Advanced Econometrics 2 Assignment 1 Problem 3

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Problem 3a

$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t + \theta \epsilon_{t-1}$

$(Y_t - \mu) = \phi_1(Y_{t-1} - \mu) + \phi_2(Y_{t-2} - \mu) + \epsilon_t + \theta \epsilon_{t-1}$

$E(Y_t - \mu)(Y_t - \mu) = E(Y_t - \mu)^2 = \phi_1 E(Y_{t-1} - \mu)(Y_t - \mu) + \phi_2 E(Y_{t-2} - \mu)(Y_t - \mu) + E[\epsilon(Y_t - \mu)] + \theta E[\epsilon_{t-1}(Y_t - \mu)]$

$\gamma_0 = \phi_1 \gamma_1 + \phi_2 \gamma_2 + E[\epsilon_t(Y_t - \mu)] + \theta E[\epsilon_{t-1}(Y_t - \mu)]$

$E[\epsilon_t(Y_t - \mu)] = \sigma^2$

$\theta E[\epsilon_{t-1}(Y_t - \mu)] = \theta E(\epsilon_{t-1}\phi_1 y_{t-1}) + \theta E(\epsilon_{t-1}\phi_2 y_{t-2}) + \theta E(\epsilon_{t-1}\theta \epsilon_{t-1}) + \theta E(\epsilon_{t-1}\epsilon_t)$

$= \theta^2 E(\epsilon_{t-1}^2) = \theta \phi_1 E(\epsilon_{t-1} y_{t-1}) + 0 + 0$

$= \theta^2 \sigma^2 + \theta \phi_1 E(\epsilon_{t-1} y_{t-1})$

$= \theta^2 \sigma^2 + \theta \phi_1 E[\epsilon_{t-1}(\phi_1 y_{t-2} + \phi_2 y_{t-2} + \epsilon_{t-1}\theta \epsilon_{t-2})]$

$= \theta^2 \sigma^2 + \theta \phi_1 \sigma^2$

$\gamma_0 = \phi_1 \gamma_1 + \theta_2 \gamma_2 + \sigma^2 + \theta \phi_1 \sigma^2$

$\gamma_1 = E(Y_t - \mu)(Y_{t-1} = \mu) = \phi_1 E(Y_{t-1} - \mu)(Y_{t-1} - \mu) + \phi_2 E(Y_{t-2} - \mu)(Y_{t-1} - \mu) + E[\epsilon(Y_{t-1} - \mu)] + \theta E[\epsilon_{t-1}(Y_{t-1} - \mu)]$

$\gamma_1 = \phi_1 \gamma_0 + \phi_2 \gamma_1 + \theta \sigma^2$

$\gamma_2 = E(Y_t - \mu)(Y_{t-2} - \mu) = \phi_1 E(Y_{t-1} - \mu)(Y_{t-2} - \mu) + \phi_2 E(Y_{t-2} - \mu)(Y_{t-2} - \mu) + E[\epsilon_t(Y_{t-2} - \mu)] + \theta E[\epsilon_{t-1}(Y_{t-2} - \mu)]$

$\gamma_2 = \phi_1 \gamma_1 + \phi_2 \gamma_0$

for n > 2

$$\gamma_n = \phi_1 \gamma_{n-1} + \phi_2 \gamma_{n-2}$$