Advanced Econometrics 2 Assignment 1 Problem 2

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Problem 2a

$Y_t - Y_{t-1} = (1 - \theta L)\epsilon_t$

since $|\theta| < 1$ we can invert $(1 - \theta L)$ and expand using a geometric series

$\rightarrow (Y_t - Y_{t-1})(1 - \theta L)^{-1} = \epsilon_t$

$\rightarrow (Y_t - Y_{t-1})(1 + \theta L + \theta^2 L^2 + \theta^3 L^3 + ...) = \epsilon_t$

$\rightarrow (Y_t + \theta L Y_t + \theta^2 L^2 Y_t + ...) - (Y_{t-1} + \theta L Y_{t-1} + \theta^2 L^2 Y_{t-1} + ...) = \epsilon_t$

$\rightarrow Y_t + [Y_{t-1}(\theta - 1) + Y_{t-2}(\theta^2 - \theta) + ...] = \epsilon_t$

subtract the series from both sides and on the right hand side distribute the -1 to flip the theta terms

$\rightarrow Y_t = [Y_{t-1}(1 - \theta) + Y_{t-2}(\theta - \theta^2) + ...] + \epsilon_t$

$\rightarrow Y_t = \Sigma_{i=1}^{\infty}(\theta^{i-1} - \theta^i)Y_{t-1} + \epsilon_t$

let $\pi_i = \theta^{i-1} - \theta^i$, then we have the desired analytical expression.

Problem 2b

$\Sigma_{i=1}^{\infty}\pi_i = \Sigma_{i=1}^{\infty}(\theta^{i-1-\theta^i}) = (\theta^{1-1} - \theta^1) + (\theta^{2-1} - \theta^2) + (\theta^{3-1} - \theta^3) + ...$

This is a telescoping series where the first term is equal to 1 and the last term approaches θ^{∞}. Since $|\theta| < 1$ we know that the series converges and the sum is equal to 1.

$lim_{k \rightarrow \infty}(1 - \theta^k) = 1$

Problem 2c

Since $|\theta| < 1$, $\pi_i = \theta^{i-1} - \theta^i$ and $\Sigma_{i=1}^{\infty}\pi_i = 1$, we know that all the πs are less than 1 in absolute value. So Y_t is covariance stationary. Or, in other words, the coefficients are absolutely summable. I don't believe $Y_t - Y_{t-1}$ is covariance stationary since the coefficient in front of y_{t-1}, which is -1, is not absolutely summable.