LC

```
a = -i / √2;
b = -1 / √2;

Sc = ( 0  0  a  b
       0  0  b  a
       a  b  0  0
       b  a  0  0 );

MatrixForm@Sc
```

$$\begin{pmatrix} 0 & 0 & -\dfrac{i}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \\ 0 & 0 & -\dfrac{1}{\sqrt{2}} & -\dfrac{i}{\sqrt{2}} \\ -\dfrac{i}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 & 0 \\ -\dfrac{1}{\sqrt{2}} & -\dfrac{i}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

```
Ph1 = Exp[i (ϕ + θ)];
Ph2 = Exp[i (ϕ - θ)];

PH1[θ_, ϕ_] := (ϕ + θ);
PH2[θ_, ϕ_] := (ϕ - θ);
```

S31

```
S31 = ExpToTrig[Sc[[3, 1]] Ph1 Sc[[3, 1]] + Sc[[4, 1]] Ph2 Sc[[3, 2]]]
```

$$\frac{1}{2} \cos[\theta - \phi] - \frac{1}{2} \cos[\theta + \phi] - \frac{1}{2} i \sin[\theta - \phi] - \frac{1}{2} i \sin[\theta + \phi]$$

```
FullSimplify[S31]
```

$$\sin[\theta] \left(-i \cos[\phi] + \sin[\phi] \right)$$

```
FullSimplify[S31 == -i Sin[θ] Exp[i ϕ]]
```

True

S41

```
S41 = ExpToTrig[Sc[[3, 1]] Ph1 Sc[[4, 1]] + Sc[[4, 1]] Ph2 Sc[[4, 2]]]
```

$$\frac{1}{2} i \cos[\theta - \phi] + \frac{1}{2} i \cos[\theta + \phi] + \frac{1}{2} \sin[\theta - \phi] - \frac{1}{2} \sin[\theta + \phi]$$

```
FullSimplify[S41]
```

$$i \, e^{i \phi} \cos[\theta]$$

```
Simplify[S41 == i Cos[θ] Exp[i ϕ]]
```

True

S32

```
Simplify[S32 == i Cos[θ] Exp[i ϕ]]
```

$e^{i\phi} \left(\text{Cos}[\theta] - i\, \text{Sin}[\theta] \right) == 0$

```
S32 = ExpToTrig[Sc[[3, 2]] Ph1 Sc[[3, 1]] + Sc[[4, 2]] Ph2 Sc[[3, 2]]]
```

$\frac{1}{2}\, i\, \text{Cos}[\theta - \phi] + \frac{1}{2}\, i\, \text{Cos}[\theta + \phi] + \frac{1}{2}\, \text{Sin}[\theta - \phi] - \frac{1}{2}\, \text{Sin}[\theta + \phi]$

```
FullSimplify@S32
```

$i\, e^{i\phi}\, \text{Cos}[\theta]$

```
Simplify[S32 == i Cos[θ] Exp[i ϕ]]
```

True

```
S42
```

```
S42 = ExpToTrig[Sc[[3, 2]] Ph1 Sc[[4, 1]] + Sc[[4, 2]] Ph2 Sc[[4, 2]]]
```

$-\frac{1}{2}\, \text{Cos}[\theta - \phi] + \frac{1}{2}\, \text{Cos}[\theta + \phi] + \frac{1}{2}\, i\, \text{Sin}[\theta - \phi] + \frac{1}{2}\, i\, \text{Sin}[\theta + \phi]$

```
FullSimplify[S42]
```

$i\, e^{i\phi}\, \text{Sin}[\theta]$

```
Simplify[S42 == i Sin[θ] Exp[i ϕ]]
```

True

Ideal

```
a = -i / √2 ;
b = 1 / √2 ;

      ⎛ 0  0  a  b ⎞
      ⎜ 0  0  b  a ⎟
Sc =  ⎜ a  b  0  0 ⎟ ;
      ⎝ b  a  0  0 ⎠

MatrixForm@Sc
```

$$\begin{pmatrix} 0 & 0 & -\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

```
Ph1 = Exp[i (ϕ + θ)];
Ph2 = Exp[i (ϕ - θ)];

PH1[θ_, ϕ_] := (ϕ + θ);
PH2[θ_, ϕ_] := (ϕ - θ);
```

S31

S31 = ExpToTrig[Sc[[3, 1]] Ph1 Sc[[3, 1]] + Sc[[4, 1]] Ph2 Sc[[3, 2]]]

$\frac{1}{2} \cos[\theta - \phi] - \frac{1}{2} \cos[\theta + \phi] - \frac{1}{2} \dot{\imath} \sin[\theta - \phi] - \frac{1}{2} \dot{\imath} \sin[\theta + \phi]$

FullSimplify[S31]

$\sin[\theta] \left(-\dot{\imath} \cos[\phi] + \sin[\phi] \right)$

FullSimplify[S31 == -$\dot{\imath}$ Sin[θ] Exp[$\dot{\imath}$ ϕ]]

True

S41

S41 = ExpToTrig[Sc[[3, 1]] Ph1 Sc[[4, 1]] + Sc[[4, 1]] Ph2 Sc[[4, 2]]]

$-\frac{1}{2} \dot{\imath} \cos[\theta - \phi] - \frac{1}{2} \dot{\imath} \cos[\theta + \phi] - \frac{1}{2} \sin[\theta - \phi] + \frac{1}{2} \sin[\theta + \phi]$

FullSimplify[S41]

$\cos[\theta] \left(-\dot{\imath} \cos[\phi] + \sin[\phi] \right)$

Simplify[S41 == -$\dot{\imath}$ Cos[θ] Exp[$\dot{\imath}$ ϕ]]

True

S32

Simplify[S32 == $\dot{\imath}$ Cos[θ] Exp[$\dot{\imath}$ ϕ]]

$e^{\dot{\imath} \phi} \cos[\theta] == 0$

S32 = ExpToTrig[Sc[[3, 2]] Ph1 Sc[[3, 1]] + Sc[[4, 2]] Ph2 Sc[[3, 2]]]

$-\frac{1}{2} \dot{\imath} \cos[\theta - \phi] - \frac{1}{2} \dot{\imath} \cos[\theta + \phi] - \frac{1}{2} \sin[\theta - \phi] + \frac{1}{2} \sin[\theta + \phi]$

FullSimplify@S32

$\cos[\theta] \left(-\dot{\imath} \cos[\phi] + \sin[\phi] \right)$

Simplify[S32 == -$\dot{\imath}$ Cos[θ] Exp[$\dot{\imath}$ ϕ]]

True

S42

S42 = ExpToTrig[Sc[[3, 2]] Ph1 Sc[[4, 1]] + Sc[[4, 2]] Ph2 Sc[[4, 2]]]

$-\frac{1}{2} \cos[\theta - \phi] + \frac{1}{2} \cos[\theta + \phi] + \frac{1}{2} \dot{\imath} \sin[\theta - \phi] + \frac{1}{2} \dot{\imath} \sin[\theta + \phi]$

FullSimplify[S42]

$\dot{\imath} \, e^{\dot{\imath} \phi} \sin[\theta]$

```
Simplify[S42 == i Sin[θ] Exp[i ϕ]]
```

True