Problems

1. Use the nineteen-year cycle and a pattern of alternating full and hollow months to write out the dates of the new Moons for the current year. Compare your results with the dates given by a calendar or almanac.
2. Do you see any evidence for an eight-year cycle in the list of new Moons given above?
3. The technical name for the position of a year in the nineteen-year cycle is the *golden number.* (This term originated in the Middle Ages.) The golden number may be obtained by dividing the year by 19, discarding the quotient, and adding 1 to the remainder. Thus, the golden number of 1961 is 5. (1961/19 = 103, with remainder 4. Golden number = 4 + 1.) The golden number for 1962 is 6; for 1963 it is 7, and so on. Golden numbers are usually written as Roman numerals.

   Construct a table of two columns. The first column should contain the golden numbers I through XIX. The second column should contain the date of the first new Moon of the year corresponding to each golden number.

4.9 THE THEORY OF STAR PHASES

The cycle of appearances and disappearances of the fixed stars was an important part of both early Greek and early Babylonian astronomy. As an example, take the case of the Pleiades. During the spring, the Pleiades disappeared for a month and a half when the Sun moved near them on the ecliptic. Then (in late May), the Pleiades emerged from their period of invisibility. They could be seen, for the first time in the year, rising in the east, a few minutes before dawn. This event was the *morning rising* of the Pleiades. It signaled the wheat harvest and the beginning of summer weather. In the same way, the morning rising of Arcturus was recognized everywhere in the Greek world as the beginning of autumn. The risings and settings of stars that occur just before sunrise, or just after sunset, are called *heliacal risings and settings* (because they occur in connection with the Sun). They are also called *fixed star phases.*

By the fifth century B.C., this lore was systematized into the *parapegma,* or star calendar. (The star calendar was a bit older among the Babylonians. As we have seen, the seventh-century B.C. compilation, MUL.APIN, included a star calendar.) A parapegma listed the heliacal risings and settings of the stars in chronological order. The user of the parapegma could tell the time of year by noting which stars were rising in the early morning. The parapegma served as a supplement to the chaotic civil calendars of the Greeks. Usually, but not always, the star phases were accompanied in the parapegma by weather predictions.

One could compile a list of the heliacal risings and settings of the constellations, simply by observations made at dawn and dusk over the course of a year. There is no need for any sort of theory. In this sense, the parapegma may be considered prescientific. But understanding the annual cycle of star phases was an important early goal of Greek scientific astronomy. Indeed, one of the oldest surviving works of Greek mathematical astronomy is devoted to this subject. This is the book (or really two books) written by Autolycus of Pitane around 320 B.C. and called *On Risings and Settings.* Autolycus defines the various kinds of heliacal risings and settings, then states and proves theorems concerning their sequence in time and the way the sequence depends on the star's position with respect to the ecliptic. No individual star is mentioned by name. Autolycus's goal is to provide a theory for understanding the phenomena. His style is that of Euclid.

True Star Phases

Autolycus and all the Greek scientific writers who followed him distinguished between *true* and *visible* star phases. An example of a true star phase is the *true morning rising* (TMR), which occurs when the star rises at the same moment as the Sun. At such a time the star would be invisible, owing to the brightness of the sky. The *visible morning rising* (VMR) would occur some weeks later, after the Sun had moved away from the star. The visible phases are the observable events of interest to farmers, sailors, poets, and astrologers. However, the true phases are more easily analyzed. Accordingly, Autolycus begins his treatise with a discussion of the true risings and settings. There are four true phases:

TMR	True morning rising	(Star rises at sunrise.)
TMS	True morning setting	(Star sets at sunrise.)
TER	True evening rising	(Star rises at sunset.)
TES	True evening setting	(Star sets at sunset.)

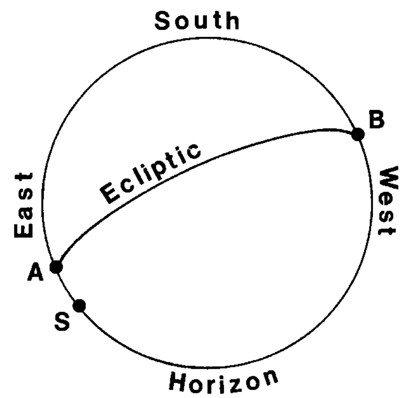

FIGURE 4.3.

Properties of True Star Phases For any star, the TMR and the TER occur half a year apart.

For any star, the TMS and the TES occur half a year apart.

These propositions are easily proved. Let star *S* be rising in the east, as in figure 4.3. Let the Sun be rising at *A*. The star is making its TMR. The TER will occur when the star is rising at *S* and the Sun is setting at *B*. The ecliptic is bisected by the horizon; thus there are six zodiac signs between *A* and *B*. If we suppose the Sun moves uniformly on the ecliptic, it will take the Sun half a year to go from *A* to *B*. Thus, the TMR and the TER occur six signs (about six months) apart in the year. The same sort of proof is easily made for the TMS and the TES.

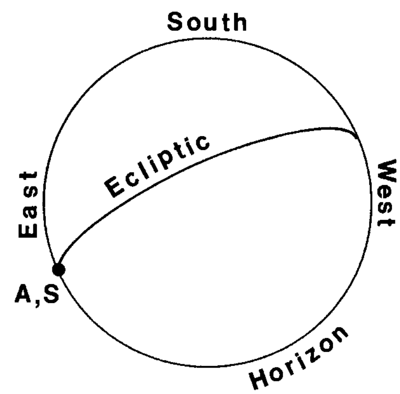

FIGURE 4.4.

The stars have their true phases in different orders according to whether they are south of the ecliptic, on the ecliptic, or north of the ecliptic.

Ecliptic Stars: If a star is exactly on the ecliptic, its TMR and TES will occur on the same day. Let star *S* be at ecliptic point *A*, as in figure 4.4. When the Sun is also at *A*, *S* and *A* rise together, thus producing the star's TMR. In the evening, *S* and *A* will set together in the west, thus producing the star's TES. (We assume that the Sun stays at the same ecliptic point for the whole day.) In the same way, one may show that for ecliptic stars, the TER and TMS occur on the same day.

Northern Stars: If a star is north of the ecliptic, the TMR will precede the TES. Let the northern star *S* be making its TMR, rising simultaneously with ecliptic point *A*, as in figure 4.3. Now, of any two points on the celestial sphere that rise simultaneously, the one that is farther north will stay up longer and set later. (We assume the observer is in the northern hemisphere.) *S* and *A* rise together. But *A* will set first. Thus, when *S* sets, the situation will resemble figure 4.5. *S* is on the western horizon. *A*, located farther south on the sphere, will already have set and will be below the horizon. The TES of star *S* occurs when the Sun is at *C*. Thus, we must wait a few weeks for the Sun to advance eastward on the ecliptic from *A* to *C*. The TES therefore follows the TMR.

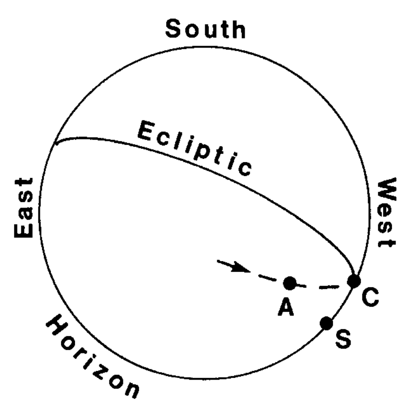

FIGURE 4.5.

Southern Stars: If a star is south of the ecliptic, the TMR will follow the TES. The proof may be made in the same way.

The proofs given above are more concise than Autolycus's proofs of the same propositions, but follow his basic method.

Example: Betelgeuse, a Southern Star Let us examine the annual cycle of a particular star, Betelgeuse, which lies in Orion's right shoulder. We will assume

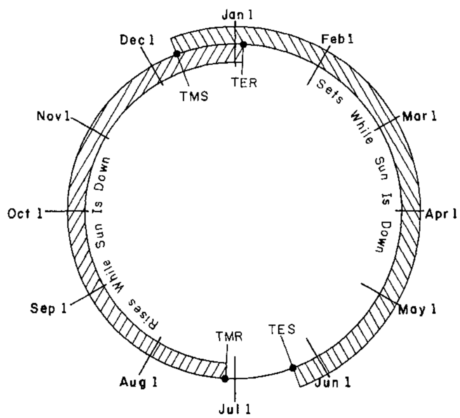

FIGURE 4.6. True phases of Betelgeuse at 40° N latitude. TMR = true morning rising; TMS = true morning setting; TER = true evening rising; TES = true evening setting.

that our observations are made at 40° N latitude. The dates of the true phases are easily read off a celestial globe. The dates below are for the twentieth century.

TMR	July 4
TMS	December 8
TER	January 2
TES	June 6

If we mark these dates on a circle which represents the year[41] and if, for simplicity, we treat the months as if they all were of the same length, we get figure 4.6. The dates of the risings and settings are particular to Betelgeuse. Further, they depend on the latitude of the observer. But several more or less general features may still be noted.

First of all, the TMR and the TER are separated by six months. Similarly, the two settings are separated by six months.

Moreover, all stars south of the ecliptic have their true phases in the same order: TMR, TMS, TER, TES (assuming an observer in the northern midlatitudes).

Example: Denebola, a Northern Star Manipulation of a celestial globe gives the following dates for the true phases of Denebola (δ Leo):

TMR	September 10
TES	October 10
TER	March 8
TMS	April 6

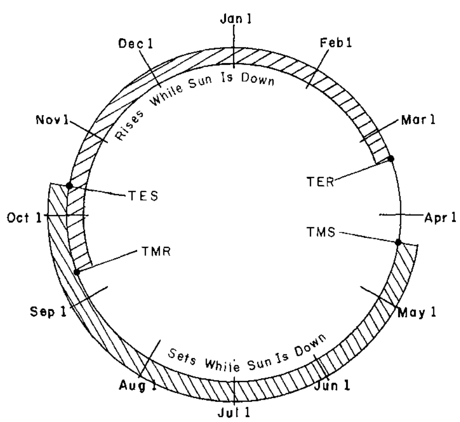

FIGURE 4.7. True phases of Denebola at 40° N latitude.

Again the TMR is separated by six months from the TER; similarly, the two settings are six months apart (see fig. 4.7). But there the similarity to the case of Betelgeuse ends, for the phases of Denebola occur in a different order.

All stars north of the ecliptic have their true phases in the same order as Denebola: TMR, TES, TER, TMS (assuming an observer in the northern midlatitudes).

Example: Regulus, an Ecliptic Star Because Regulus is located on the ecliptic, its TMR and TES occur on the same day, August 24, when the Sun is at the same point of the ecliptic as Regulus itself. Similarly, the TMS and TER occur on the same day, February 20, when the Sun is 180° away from Regulus. The time chart for the phases of Regulus therefore looks like figure 4.8. Note that there is no area of overlap; that is, there is no period of time in which the star both rises and sets while the Sun is down.

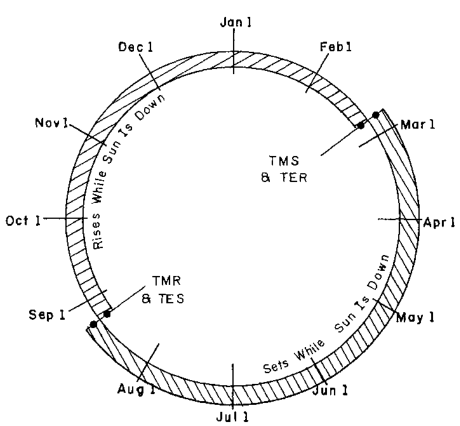

FIGURE 4.8. True phases of Regulus.

Visible Star Phases

The true star phases are unobservable. If a star crosses the horizon at the same moment as the Sun, it will be lost in the general brightness of the sky. The ancient writers therefore distinguish between the true risings and settings and the visible ones. There are four visible phases:

VMR	Visible morning rising	(Before sunrise, star is seen rising for the first time.)
VMS	Visible morning setting	(Before sunrise, star is seen setting for the first time.)
VER	Visible evening rising	(After sunset, star is seen rising for the last time.)
VES	Visible evening setting	(After sunset, star is seen setting for the last time.)

Relation between the Visible and the True Phases The visible morning phases follow the true ones. But the visible evening phases precede the true ones. The truth of these propositions follows simply from the fact that the Sun's motion on the ecliptic is from west to east, that is, opposite the diurnal revolution.

Let star *S* be rising while the Sun is rising at *A*, as in figure 4.9. This is the star's TMR, which will be invisible. But some weeks later, when the Sun has advanced from *A* to *D* on the ecliptic, the star's rising will be visible for the first time. Then the Sun will be far enough below the horizon at the rising of S for the star to be seen. Thus, when the Sun is at *D*, star *S* will make its VMR.

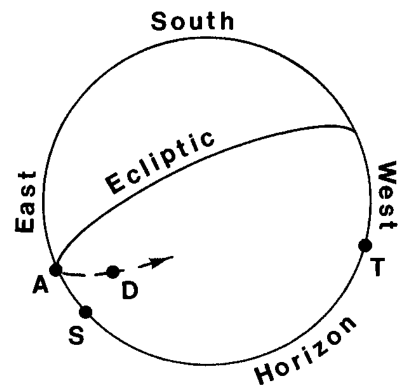

FIGURE 4.9.

Note that the same argument may be applied to the star setting at *T*. This star makes its TMS when the Sun rises at *A*. The setting will be invisible, however. The first setting of *T* to be visible will occur when the Sun has advanced to *D*.

Thus, the visible morning phases (whether risings or settings) follow the true ones. Also, the morning phases are the first events to be visible: the VMR is the first visible rising of the star in the annual cycle; the VMS is the first visible setting.

In a similar way, it may be shown that the visible evening phases (whether risings or settings) precede the true ones. Also, the evening phases are the last to be visible. That is, the VES is the last setting of the star to be visible. Similarly, the VER is the last visible rising.

A Simplifying Assumption The number of days that separate a star's visible rising or setting from its corresponding true one depends on many factors: the brightness of the star itself, the star's exact position on the horizon (the farther from the Sun's position, the better), the steepness with which the ecliptic meets the horizon when the star is is rising or setting, as well as the observer's latitude. All these factors can at least be subjected to calculation. But there are also a number of variable conditions that affect the visibility of stars—particularly stars near the horizon—such as the clarity of the atmosphere, city lights, the observer's eyesight, and so on.

A theory of visible star phases that took all these factors into account would be very complicated—too complicated, in fact, to be very useful. And the level of Greek mathematics in the fourth century B.C. would not have permitted such a treatment even if it had been desired. Autolycus was able to dispense with all these complications by means of one simplifying assumption: a star's rising or setting will be visible if the Sun is below the horizon by at least half a zodiac sign *measured along the ecliptic.*

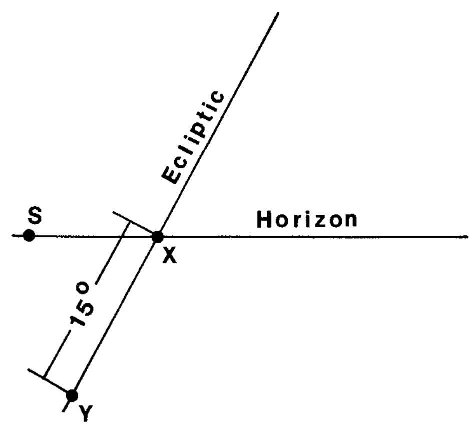

FIGURE 4.10.

In figure 4.10, let star *S* rise simultaneously with point *X* of the ecliptic. Then when the Sun is at *X*, star *S* will have its true morning rising. According to Autolycus, *S* will have its visible morning rising when the Sun reaches *Y*, which is half a zodiacal sign (15°) from *X*.

According to modern astronomers, the period of astronomical twilight extends from the Sun's setting until the time it reaches a position 18° (vertically) below the horizon. Then the sky becomes dark enough to permit observation of even the faintest stars. However, the brighter stars can be seen when the Sun is only 10° or 12° below the horizon, and these are precisely the stars that play the most prominent role in the ancient literature on star phases. Autolycus's use of 15° measured obliquely to the horizon is a pretty good approximation to 12° measured vertically, especially in the lower latitudes, where the ecliptic rises and sets fairly steeply.

Example: Betelgeuse, a Dock-Pathed Star It is easy to apply Autolycus's visibility rule to the case of Betelgeuse. Let us suppose, for simplicity, that the Sun moves 1° per day along the ecliptic. Then, by Autolycus's rule, the visible morning phases occur fifteen days after the true ones, while the visible evening phases occur fifteen days before the true ones. Making the appropriate fifteen-day adjustments to the dates of the true phases (listed above), we get

TMR July 4	+ 15 days	→	VMR July 19
TMS December 8	+ 15 days	↘	VER December 18
TER January 2	− 15 days	↗	VMS December 23
TES June 6	− 15 days	→	VES May 22

If we modify the calendar diagram of figure 4.6 to show the visible phases, we obtain figure 4.11.

Note that the order of the visible phases is different from the true phases. The change of order is due to the fact that for Betelgeuse, observed at 40° N latitude, the TMS and the TER are only 25 days apart, while this period would have to be at least 30 days to prevent a reversal of the order when the 15-day visibility rule is applied.

Betelgeuse is completely invisible between the VES and VMR. This is the time when the Sun, moving on the ecliptic, reaches the general vicinity of the star. Betelgeuse rises after dawn and sets before dusk. It is thus up only in daylight and so remains invisible for nearly two months.

The period of maximum visibility of Betelgeuse is in late December, for this date is near both the evening rising and the morning setting. That is, the star rises in the evening and sets in the morning: it crosses the sky during the night. But note in figure 4.11 that there is no overlap between the rings of visibility. During the period between the VER and the VMS, neither the star's rising nor its setting is visible. Rather, the star simply appears in the eastern sky, already above the horizon, shortly after sunset. It can then be seen during most of its transit of the sky. However, the dawn arrives and makes the star disappear shortly before it has a chance to set. In his book on star phases, *Phaseis,* Ptolemy calls this kind of star *dock-pathed.*[42] That is, the

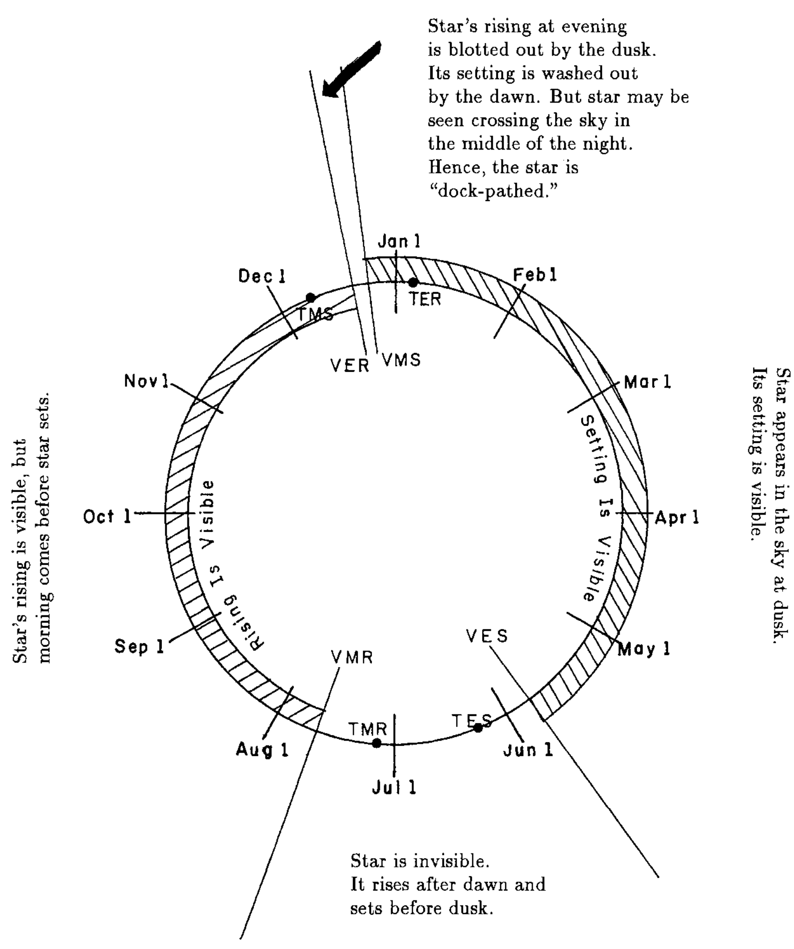

FIGURE 4.11. Visible phases of Betelgeuse at 40° N latitude. Betelgeuse, located near the ecliptic, is a "dock-pathed" star.

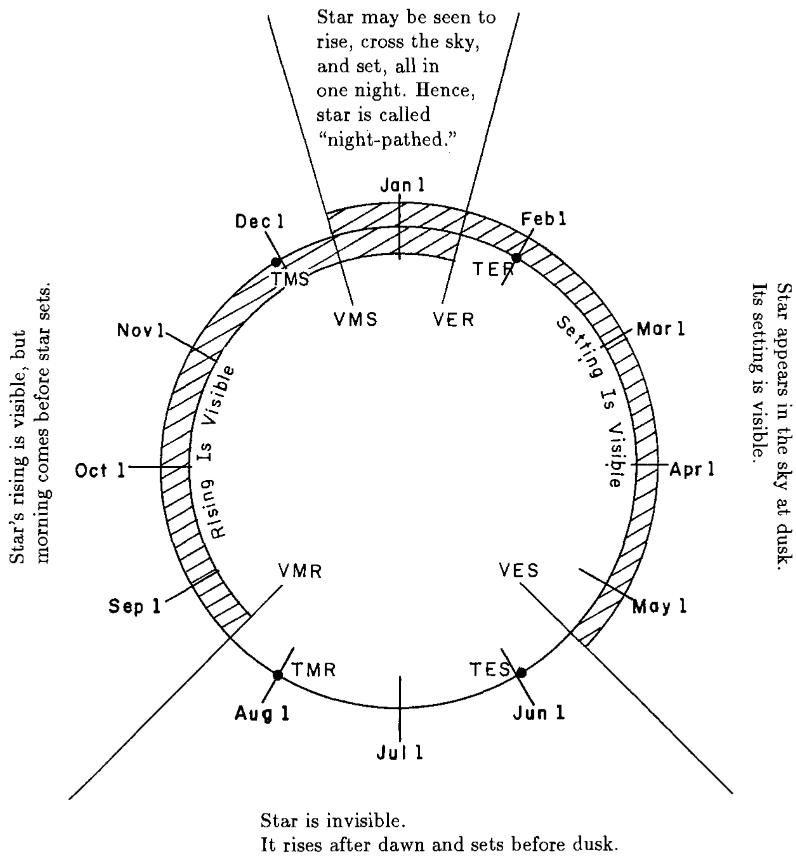

FIGURE 4.12. Visible phases of Sirius at 40° N latitude. Sirius, located well south of the ecliptic, is a "night-pathed" star.

ends of its visible path across the sky are docked, or cut off. This is a convenient term for characterizing Betelgeuse's behavior between the VER and the VMS.

All stars located on or near the ecliptic are dock-pathed and have their visible phases in the same order as Betelgeuse: VMR, VER, VMS, VES.

Example: Sirius, a Night-Pathed Star Using a celestial globe, set for latitude 40° N, we find for the true phases of Sirius:

TMR	August 2
TMS	November 30
TER	January 31
TES	May 29

Note that, as always, the TMR and TER are separated by about six months, as are the TMS and TES. Note, too, that the true phases are in the correct order for a southern star, as established above. Applying Autolycus's fifteen-day visibility rule, we obtain the following approximate dates for the visible phases of Sirius, observed at 40° N latitude:

TMR August 2	+ 15 days	→ VMR August 17
TMS November 30	+ 15 days	→ VMS December 15
TER January 31	− 15 days	→ VER January 16
TES May 29	− 15 days	→ VES May 14

The phases of Sirius are represented in figure 4.12. Note that the visible phases occur in the same order as the true ones. This is true for stars, such as Sirius, that are far enough south on the celestial sphere. The TMR and TER are far enough apart (more than 30 days) that the 15-day rule does not result in a reversal of order for the dates of the visible phases.

Sirius, like Betelgeuse, is invisible between the VES and the VMR.

The period of the Sirius's greatest visibility is between the VMS and the VER, for then Sirius rises in the evening and sets in the morning, so it crosses the sky at night. Further, as figure 4.12 shows, both the rising and setting of the star are visible. Sirius is therefore not dock-pathed. Rather, it belongs to the class of stars that Ptolemy calls *night-pathed.* At this one time of year, the whole of Sirius's transit of the visible hemisphere takes place in the night: the star's whole path, from horizon to horizon, is visible.

All stars located far enough south of the ecliptic are night-pathed and have their visible phases in the same order as Sirius: VMR, VMS, VER, VES. (We assume the observer is in the northern midlatitudes.)

Example: Arcturus, a Doubly Visible Star From a celestial globe, set for latitude 40° N, we take the dates for the true phases of Arcturus. To obtain rough dates for the visible phases, we apply Autolycus's fifteen-day visibility rule:

TMR October 7	+ 15 days	→ VMR October 22
TES December 4	− 15 days	→ VES November 19
TER April 5	− 15 days	→ VER March 21
TMS June 2	+ 15 days	→ VMS June 17

If we plot these dates on our visibility diagram, the result is figure 4.13. Note that the visible phases occur in the same order as the true ones. This is the case for stars, such as Arcturus, that are far enough north on the celestial sphere. The TMR and TES are far enough apart (more than 30 days), that the 15-day rule does not result in a reversal of order.

The behavior of Arcturus is different from that of either Betelgeuse or Sirius. The most interesting period for Arcturus is between the VMR and the VES. During this period of a bit less than a month, both the rising and the setting of the star are visible each night. But note that the star rises in the morning and sets in the evening: it crosses the sky in the daytime. In the

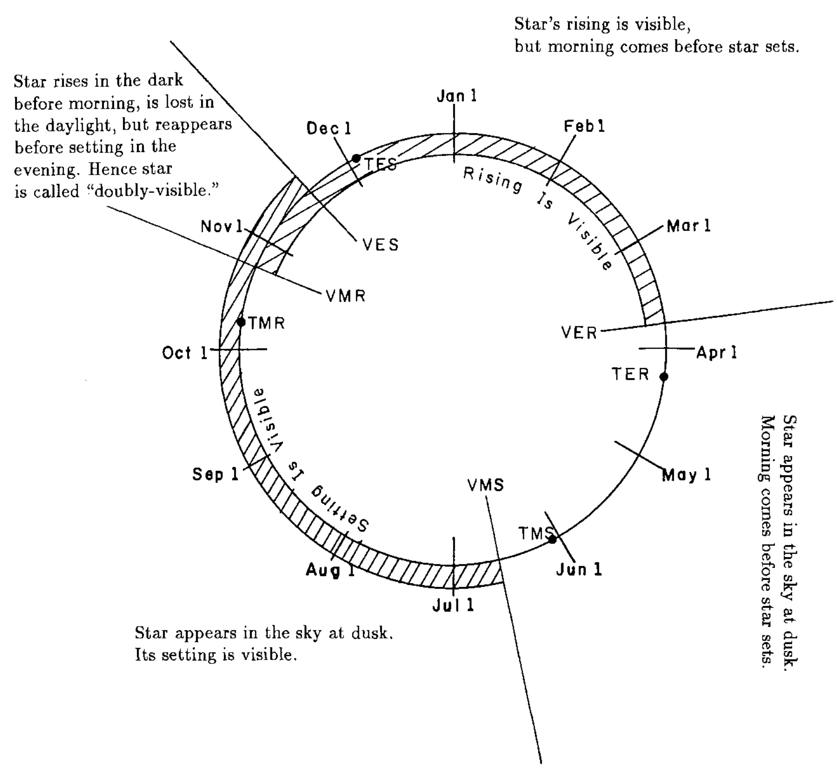

FIGURE 4.13. Visible phases of Arcturus at 40° N latutude. Arcturus, located well north of the ecliptic, is a "doubly visible" star.

early evening one therefore sees Arcturus appear well up in the western sky. Shortly afterward, it sets. But it may be seen again before the night is over, rising in the east. In the *Phaseis*, Ptolemy characterizes such a star as *doubly visible*, or *seen on both sides*. This remarkable behavior results from Arcturus's northern position on the celestial sphere: the star stays below the horizon (at latitude 40° N) for only about 9 1/2 hours each day. And so it is possible, at a particular time of year, for Arcturus to set after dark in the west and yet rise before dawn in the east.

There is another property of doubly visible stars: they never are completely obscured by the Sun. For this reason, as Ptolemy says in the *Phaseis*, they are also called *visible all year long*. These two properties are correlated: both result from the fact that the VMR precedes the VES.

All stars far enough north of the ecliptic are doubly visible and have their visible phases in the same order as Arcturus: VMR, VES, VER, VMS. (We assume the observer is in the northern midlatitudes.)

The standard terms for the phases (true morning rising, visible evening setting, etc.) were introduced by Autolycus and were universally followed. Autolycus discussed the properties of the stars that we have called dock-pathed, night-pathed, and doubly visible but did not assign these names to them. Since every star that has risings and settings may be assigned to one of these three classes, it is convenient to have names for the groups. This rigorous systemization and naming appears for the first time in Ptolemy's *Phaseis*. These terms were used by earlier writers, but less systematically—which seems to confirm the lack of standard terms for the three star classes before Ptolemy's time.

Some Inconvenient Modern Terminology In modern writing on star phases, one may encounter these terms:

Heliacal rising	= VMR
Acronychal rising	= VER
Heliacal setting	= VES
Cosmical setting	= VMS

Except for "acronychal rising," none of these terms are used by the ancient Greek astronomers. And even *acronychal* poses a problem. *Akronychos* = *akron* (tip, extremity) + *nyktos* (of the night). This adjective is used by Greek writers on star phases and, indeed, belongs to the vocabulary of everyday speech. Usually, it means *in the evening*. But Theon of Smyrna points out that the morning is also an extremity of the night, and therefore, logically enough, he applies the same word both to evening risings and to morning settings.[43] Thus, it is better and clearer to stick to Autolycus's technical vocabulary (visible evening rising, etc.), which can hardly be improved on.

Note on the Variation of Star Phases with Time

The dates of the heliacal risings and settings of stars vary slowly with time. For a century or two, the dates of the risings and settings can be taken as fixed. But if we want to compare the date of the heliacal rising of the Pleiades, for example, in Greek antiquity with the date for the same event in our own century, we must face up to the change.

The reason for the change in the dates of the heliacal risings and settings is *precession*, a slow, progressive shift in the positions of the stars on the celestial sphere. Precession is discussed in Section 6.1. For now it suffices to say that all the stars move gradually eastward, on circles parallel to the ecliptic, at the slow rate of 1° in 72 years. Suppose that the true morning rising of the Pleiades occurs on a certain day. After 72 years, the Pleiades will have shifted eastward by 1°. So, the Sun will have to run an extra degree to reach the Pleiades, and the true morning rising will occur a little later in the year.

The rate at which the star phases shift through the calendar is easily calculated. The stars' longitudes increase by 1° in 72 years. The rate of the Sun's motion on the ecliptic is 360° in 365.25 days. Thus, the time required for a one-day shift in the dates of the star phases is

$$(72 \text{ years}/^\circ) \times (360^\circ/365.25 \text{ days}) = 71 \text{ years/day}.$$

Let us apply this fact to an example. The visible morning rising of Betelgeuse (at latitude 40° N) occurs on July 19. When did the VMR of Betelgeuse occur in the first century B.C.? The first century B.C. was about 2,000 years ago. Every 71 years produced a one-day shift in the star phases. The total shift was therefore 2,000/71 = 28 days. In the past, the star phases occurred earlier in the year. Thus, the VMR of Betelgeuse should have occurred around June 21 (28 days earlier than July 19). Note that it is simplest to express all dates in terms of the Gregorian calendar.

This rough-and-ready method works well for stars near the ecliptic. For stars far from the ecliptic (such as Arcturus), the situation is more complicated and the rough-and-ready method is not usable. Instead, one should replot the stars on a celestial globe in their ancient positions and read off the dates of the star phases directly.

4.10 EXERCISE: ON STAR PHASES

1. In section 4.9, it was proved that, for a star north of the ecliptic, the TMR precedes the TES. Prove that, for a star south of the ecliptic, the TMR follows the TES. (Assume an observer in the northern hemisphere.)
2. In section 4.9, it was proved that the visible morning phases follow the true ones. Prove that the visible evening phases precede the true ones.
3. The dates of a star's phases depend on the observer's latitude.
   A. Use a globe to determine the dates of the true phases of Betelgeuse at latitudes 30° N and 30° S. Draw a calendar diagram like figure 4.6 for each of these latitudes. Note a symmetry: the four dates for 30° S are the same as the four dates for 30° N, but different phases go with the dates. Can you prove the general validity of this rule?
   B. Apply the fifteen-day visibility rule to determine approximate dates for the visible phases of Betelgeuse at latitude 60° N. Draw a calendar diagram like figure 4.11. The diagram is divided into four sections by the star's visible phases. Label each section with a brief description of the star's behavior. Pay particular attention to the section between the VMS and the VER. At 60° N, is Betelgeuse dock-pathed, night-pathed, or doubly visible?
4. The following list gives the "actual dates" of the visible phases of three stars for year −300 and latitude 38° N (Athens). The dates were calculated by Ginzel[44] and are expressed in terms of the Julian calendar.

Star	*VMR*	*VES*	*VER*	*VMS*
Pleiades	May 22	April 7	Sept 27	Nov 5
Sirius	July 28	May 4	Jan 2	Nov 23
Vega	Nov 10	Jan 23	April 20	Aug 16

   A. Use the order of the phases to place each of these stars in one of the three classes: night-pathed, dock-pathed, or doubly visible. Do your assignments make sense in view of the stars' positions on the sphere?

B. Test the rough-and-ready rule that the dates of star phases shift forward by one day every 71 years. To do this, it will be enough to work with the morning phases for these stars in the year −300.

First, express Ginzel's dates for the VMR and the VMS in terms of the Gregorian calendar (use table 4.1).

Next, use a celestial globe and the fifteen-day visibility rule to estimate the dates of the VMR and the VMS for these stars in the twentieth century.

Use the twentieth-century dates and the average shift of one day in 71 years to estimate when the VMR and the VMS of these stars occurred in the year −300. Compare your estimates with Ginzel's more elaborate calculations. For which stars does the rough-and-ready method work best?

4.11 SOME GREEK PARAPEGMATA

The Geminus Parapegma

The parapegma appended to Geminus's *Introduction to the Phenomena* is one of our most important sources for reconstructing the early history of this genre among the Greeks. The Geminus parapegma is a compilation based principally on three earlier parapegmata (now lost) by Euctemon, Eudoxus, and Callippus, but it also includes a few notices drawn from other authorities. The latest writer cited is Dositheus (ca. 230 B.C.). Thus, some historians believe that the parapegma was actually compiled not by Geminus (first century A.D.), but by some unknown person early in the second century B.C. Be that as it may, this parapegma is always found in the manuscripts appended to Geminus's *Introduction to the Phenomena.*

In the Geminus parapegma, the year is divided according to zodiac signs. Each sign begins with a statement of the number of days required for the Sun to traverse the sign. Then there follow, in order of time, the risings and settings of the principal stars and constellations, together with associated weather predictions and signs of the season. Here is the portion of the parapegma for the sign of the Virgin (an asterisk indicates that an explanatory note follows the extract):

EXTRACT FROM GEMINUS

Introduction to the Phenomena (Parapegma)

The Sun passes through the Virgin in 30 days.

On the 5th day, according to Eudoxus, a great wind blows and it thunders. According to Callippus, the shoulders of the Virgin rise;* and the etesian winds cease.*

On the 10th day, according to Euctemon, the Vintager appears,* Arcturus rises, and the Bird sets at dawn; a storm at sea; south wind. According to Eudoxus, rain, thunder; a great wind blows.

On the 17th, according to Callippus, the Virgin, half risen, brings indications; and Arcturus is visible rising.

On the 18th, according to Eudoxus, Arcturus rises in the morning; <winds> blow for the following 7 days; fair weather for the most part; at the end of this time there is wind from the east.

On the 20th, according to Euctemon, Arcturus is visible: beginning of autumn.* The Goat, great star in the Charioteer,* rises <in the evening>; and afterwards, indications;* a storm at sea.

On the 24th day, according to Callippus, the Wheat-Ear of the Virgin rises; it rains.[45]