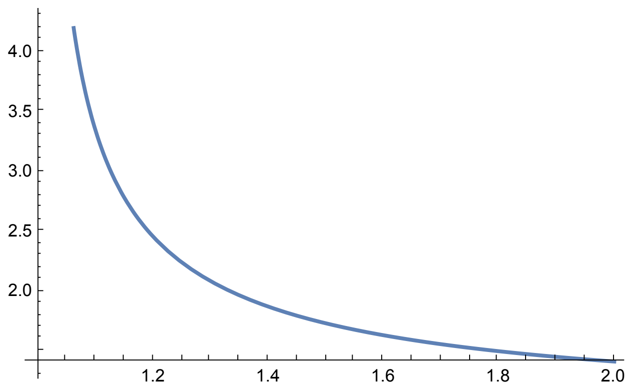
In[1]:=
```
integrand[ρ_] := 1/Sqrt[1 - 1/ρ]
```

In[2]:=
```
Plot[integrand[ρ], {ρ, 1, 2}]
```

Out[2]=



Note that this integrand blows up at $\rho = 1$. This is potentially severe trouble. But, let's rewrite the integrand:

$$\frac{1}{\sqrt{1-\frac{1}{\rho}}} = \frac{\sqrt{\rho}}{\sqrt{\rho-1}}$$

Then notice that at the troublesome point, $\rho = 1$, the numerator is completely reasonable and tends toward 1 as ρ tends toward 1. So without changing the nature of the singularity in the integrand, we'll just replace the numerator by 1.

$$\frac{1}{\sqrt{\rho-1}}$$

Let's make another change of variables $x = \rho - 1$. The lower limit of integration is now 0, and the upper limit is now 1. And the integrand is now just

$$\frac{1}{\sqrt{x}}$$

But everybody knows (he-he) that the integral of $\frac{1}{\sqrt{x}}$ is $2\sqrt{x}$. And this definite integral evaluated from 0 to 1 is just 2, because the lower limit is 0 and has no ill behavior despite the fact that the integrand blew up as $\frac{1}{\sqrt{x}}$. This is known as an integrable singularity. And the upshot is that there isn't an infinite proper distance as we measure a radial path all the way down to $r = 2M$.