Let us plot the two functions we have derived. Let us define $x = r/M$.

Then our two functions are

$$\frac{dr}{dt} = -\left(1 - \frac{2}{x}\right)\sqrt{\frac{2}{x}}$$

and

$$\frac{dr_{\text{shell}}}{dt_{\text{shell}}} = -\sqrt{\frac{2}{x}}$$

When I have Mathematica plot these, I will call them bookkeeperSpeed and shellSpeed, and we'll plot them from $x = 2$ (the event horizon) out to $x = 10$, and because we are plotting speeds, we'll forego the overall minus signs.

In[34]:=

```
bookkeeperSpeed[x_] := (1 - 2/x) Sqrt[2/x];

shellSpeed[x_] := Sqrt[2/x];

Plot[{Callout[bookkeeperSpeed[x], "|dr/dt|"], Callout[shellSpeed[x], "|dr_shell/dt_shell|"]},
 {x, 2, 10}, PlotRange → {{0, 10}, {0, 1}}, Ticks → {Range[0, 10, 1], Range[0, 1, 0.1]},
 GridLines → {Range[0, 10, 1], Range[0, 1, 0.1]}, AspectRatio → 1]
```

Out[36]=