Special Relativity — Syllabus

Unofficial/Short Course Title: Relativity

Fall 2024, Deep Springs College, ***Prof. Brian Hill***

Overview

More than anyone else since Einstein, John Archibald Wheeler has influenced our understanding of spacetime. He was the person that pressed relativists to take seriously the Schwarzschild solution into the interior of the event horizon. He is a co-author (with Misner, Thorne, and Taylor, among others) on the greatest textbooks in the field and his students have gone on to greatly advance the subject, including black hole thermodynamics and quantum mechanics in curved spacetime. We will work through Taylor and Wheeler's descriptions of two things: (1) special relativity, which is how space and time are related in the absence of gravity, (2) the behavior of spacetime around a black hole. The latter is also entirely relevant for tight orbits around ordinary stars, and in fact the first evidence for general relativity listed by Einstein was the explanation of the deviation of Mercury's orbit from the ellipse expected from Newtonian gravity.

The intertwining of space and time is highly counterintuitive. The intertwined fabric is called "spacetime." Our story will begin with Galileo who formulated the version of relativity that Newton built upon, and which lasted for almost three centuries until Lorentz, Poincaré, and Einstein formulated special relativity.

Texts

We will rely on:

- ***Spacetime Physics, 2nd Edition*****, https://www.eftaylor.com/spacetimephysics/** — Softcover copies are still readily available used

Grading

- 50% assignments
- 25% (50% total) for each of two exams, dates to be determined, but coming at about the 5th and 8th week of classes
- 10% preparation for class and leadership of course

Problem Sets / Handouts / Being Neat and Organized

There will be problem sets due almost every class, limited only by how quickly I can assign, write solutions, and grade. The more problems you do the better. In addition to the problem sets and their solutions, there will be exams and exam solutions. To be organized, locate a three-ring binder and a three-ring hole punch, and file everything chronologically. Actually, reverse-chronological is the most convenient, because you then naturally open your binder to what you are currently working on. Problem sets should be *neat* and on standard 8 1/2 x 11 paper. Multi-page problem sets — and most will be multi-page — should be stapled. The nicest technical work is facilitated by engineering pads, such as these **Roaring Spring Engineering Pads at Amazon** (which are pretty expensive unless you buy by the case), and done with a mechanical pencil, a ruler, and an eraser at hand.

Absences (and late work)

The College's policies on absences (and late work) are applicable. Refer to the Deep Springs Handbook.

Week 1 — Galilean Relativity — Invariance of the Interval

- Thursday, Aug. 29 — We did a **calculus self-assessment** — We went over the **syllabus** — You read pp. 1-15 of Chapter 1 of *Spacetime Physics, 2nd Edition* — Go to the excercises at the end of the Chapter and choose an exercise to work out and present (we chose to present 1-3 and 1-4) — Also read what Galileo's character **Salvatius has to say** about Galilean relativity — I have written up and will talk you through **my own introduction** to speed, velocity, and Galilean Relativity which is less elegant than Galileo's but uses the reliable crutch of Cartesian coordinate systems

Week 2 — Free-Float Frames — Time Dilation Illustrated in Muon and Pion Decay

- Monday, Sep. 2 — Read and be prepared to discuss *Spacetime Physics* to p. 30 — Problem Set 1 to be turned in at the beginning of Monday's class is **here** — I played **Chris Hadfield's cover of David Bowie's "A Space Oddity"** as an important illustration of what it is like to be in a free-float frame — I proved **Galilean Addition of Velocities** which establishes the near impossibility that the speed of light could possibly be constant — And yet the speed of light is constant in any frame(!), and this follows easily from the invariance of the interval
- Thursday, Sep. 5 — Read and be prepared to discuss *Spacetime Physics* to the end of Chapter 2 (to p. 44) — I will use the first two terms of the Maclaurin series to present an important approximation method that you will be using a lot — Then two groups will present their detailed analyses of the falling railway coach — **Here are the variables** for analyzing the horizontal problem (Rania and Rebecca), and **here are similar variables** for analyzing the vertical problem (Walker, Will, and Eden) — Both groups should be seeking approximate expressions for $d(t)$ where initially $d(t)=d_I$, and we are really only interested in what happens in a short time Δt — **Problem Set 2** for Thursday

Week 3 — Time Dilation — The Relativity of Simultaneity — Length Contraction

- Monday, Sep. 9 — Read and be prepared to discuss *Spacetime Physics* to p. 65 — Problem Set 3 to be turned in at the beginning of Monday's class is **here** — **Maclaurin and Taylor series graphs** — Theory behind the coefficients in the Maclaurin and Taylor Series — Overview of the three biggest consequences of special relativity: (1) Time Dilation, (2) Length Contraction, (3) The Relativity of Simultaneity — Derivation of the first effect: Time Dilation (Kel and Jeremy) — The γ factor notation — Invariance of the transverse dimension via the Rockets-with-Paintbrushes thought experiment — The Relativity of Simultaneity via Einstein's Lightning-Strikes-Train thought experiment
- Thursday, Sep. 12 — Read and be prepared to discuss *Spacetime Physics* to p. 77 (the end of Chapter 3) — Problem Set 4 to be turned in at the beginning of Thursday's class is **here** — Derivation of Length Contraction by Sasha and Eli — **Length contraction board photo** — **Rania's Length Contraction handout** — The Pole-in-the-Barn Paradox — A Relativity of Simultaneity derivation using **a variant of Einstein's Lightning-Strikes-Train thought experiment**

Week 4 — Presentations of Interesting Problems — Start The Lorentz Transformation Formulas

- Monday, Sept. 16 — To give you time to consolidate the first three chapters, there is no new reading assignment due for Friday
- Thursday, Sept. 19 — Read Chapter L to p. 102, which is the derivation of the Lorentz Transformation formulas summarized in Eq. L-10a on p. 102

Week 5 — Finish The Lorentz Transformation Formulas — Exam 1

- Monday, Sept. 23 — Finish Chapter L — Problem Set 5, with due date delayed to the beginning of Monday's class, is **here**
- Thursday, Sept. 26 — **Exam 1**

Week 6 — World-Lines of Accelerating Particles — The Light-Cone and Causality

- Monday, Sept. 30 — If you feel solid about Chapters 1, 2, 3, and L, there is not much new in Chapter 4 — If you feel you need to cement in some ideas with the twin paradox as the example then Chapter 4 is great — Everyone, whether feeling solid or not, should pay close attention to pp. 127-130 of Chapter 4 — Then continue studying through Section 5.5, p. 148, of Chapter 5 — Problem Set 6, due at the beginning of Monday's class, is **here** and in your file folders along with tracing graph paper and a **hyperbola template**

- Thursday, Oct. 3 — Finish Studying Chapter 5 — Problem Set 7, due at the beginning of Thursday's class, is **here**

Week 7 — More Presentations of Interesting Problems — Non-Relativistic Momentum and Energy — $E=mc^2$

- Monday, Oct. 7 — Study all of Chapter 6 and also Sections 7.1 and 7.2 to p. 195 — Organize yourselves into three groups of three for presentations on three problems from Chapters, 5, 6, and 7, such as 5-7 (The Runner-on-The-Train Paradox), 6-6 (Relativistic Race-Walking), and 7-3 (The Kinetic Energy of a Freight Train) — Target 10 minutes plus 5 minutes for questions per presentation
- Thursday, Oct. 10 — You can continue studying Chapter 7 which is on relativistic momentum and energy — **Worksheet I**: The Runner-on-the-Train Paradox — **Worksheet II**: Relativistic Race-Walking — **Worksheet III**: $E=mc^2$ and the Energy to Raise a Freight Train a Mile High — **Non-Relativistic Momentum and Energy**

Week 8 — Relativistic Momentum and Energy

- Thursday, Oct. 31 — Continue your study of Chapter 7 of *Spacetime Physics* to the end of Section 7.5, p. 206 — Problem Set 8, with due date delayed to the beginning of this Thursday's class, is **here**

*This course would have been nothing without the clarity and brilliance of **John Archibald Wheeler** 1911-2008 and the precious textbooks in which he shared his insights.*

Einstein, Yukawa, Wheeler, and Bohr