11/8 in Class – Euler's Method and one Simulation
edited after class to fix time in Problem 2

1. Use Euler's Method to find a numerical solution for $\dot{x} = 3t^2$ given that $x(0) = 0$. Let it run from $t = 0$ to $t = 2$s. Start with at least 10 time steps, and try smaller intervals to see how your answer converges.

   (a) Do the first few by hand on the white board.

   (b) Write the equation you will use, $x(k+1) = \ldots$

   (c) Then go back and write a MatLab program to do it for you.

   (d) Graph both your calculated x and the exact x as functions of time. Compare them.

   (e) Play around. Decrease the time step and/or increase the total time. What happens?

2. Use Euler's approximation to solve this DE: $\dot{N} = -\Gamma N$ where $\Gamma = 4.552 \times 10^5 \frac{1}{s}$. At $t = 0$, $N = 1000$. For the first pass, go from t=0 to 1×10^{-5}s (fixed this number) seconds with only 10 time steps. (That's just for debugging. Up the number of time steps when you're confident it's working.)

   (a) This one is separable and so also pretty easy to solve–find the expression for the exact $N(t)$.

   (b) Write the equations you will use, $N(k+1) = \ldots$ and $t(k+1) = \ldots$

   (c) Do the first few by hand on the white board.

   (d) Then write a MatLab program to do it for you.

   (e) Graph both your calculated, approximate N and the exact N as functions of time. Compare them.

   (f) Play around. Decrease the time step by a lot. What happens?

3. Back to simulations (for the DE you just solved!) A muon is a particle that is often created during collisions between cosmic rays and the earth's atmosphere. A muon is a bit like a heavy electron (about 200 times heavier, with the same charge). Unlike an electron, a muon is unstable, meaning it decays. The muon has a mean lifetime, τ, of 2.197×10^{-6} s. The decay rate per unit time, Γ, is equal to the inverse of the mean lifetime.

$$\Gamma = \frac{1}{\tau}$$

   The probability (P) that any given muon will decay in some time interval, dt, is $P = \Gamma dt$. Imagine starting with 1000 muons. Your task is to run a simulation using the random number generator to determine if a given muon decays, and plot the number of muons as a function of time as detailed below.

   (a) Go to the board and work together to write a detailed structure plan for how you will code a simulation of the decay of 1000 muons. Be sure your structure plan answers:

      i. What do you think is a reasonable size for the time interval, dt?

ii. What do you think is a reasonable time scale for the entire simulation to run?

iii. What constants do you need?

iv. Will you use for loops or vectorize? Explain how you will implement either in words.

(b) Code it. This could be iterative. Go back to your group and your structure plan if you need a new one! Call it `muonDecay.m`

(c) Run the simulation of this case and plot the number of particles as a function of time. Use blue circles for the simulated data.

(d) Write out your data for t and N as two columns in a data file called `muonDecay.txt`

(e) From the last problem, you calculated the theoretical curve, plot this on top of your simulated data. This time make it a black line. (Which should be close to your blue circles.)