With Polarization

$$\nabla^2 \mathbb{E} - \epsilon_0 \mu_0 \frac{\partial^2 \mathbb{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbb{P}}{\partial t^2} = 0$$

Try $\quad \mathbb{E} = \mathbb{E}_0 e^{i(k \cdot r - \omega t)}$

complex

Pressure linear so

$$\mathbb{P} = \chi \epsilon_0 \mathbb{E}$$

complex unitless constant

"susceptibility"

Substitution

$$-k^2 \mathbb{E} + \omega^2 \epsilon_0 \mu_0 (1 + \chi) \mathbb{E} = 0$$

OR $\quad k^2 = \frac{\omega^2}{c^2}(1 + \chi)$

or $\quad k = \frac{\omega}{c}\sqrt{1 + \chi}$

$\mathcal{N} \equiv \sqrt{1 + \chi}$ complex index of refraction

$$\mathcal{N} = n + i\kappa$$

so

$$k = \frac{\omega}{c}(n + i\kappa)$$

thus suppose $\hat{k} = \hat{z}$ then

$$\mathbb{E} = \mathbb{E}_0 e^{i(kz - \omega t)}$$

$$= \mathbb{E}_0 e^{ii\frac{\omega}{c}\kappa z} e^{i(\frac{\omega}{c}nz - \omega t)}$$

$$= \mathbb{E}_0 e^{-\frac{\omega}{c}\kappa z} e^{i(\frac{\omega}{c/n}z - \omega t)}$$

$c' = c/n$

$$\frac{2\pi}{\lambda'} = k' = \frac{\omega}{c}n = k_0 n$$

$$\lambda' = \frac{1}{n}\frac{c}{f} = \frac{1}{n}\lambda_0$$

Lorentz Model

$\mathbf{r}$ in same direction as $\mathbf{E}$

$$m_e \ddot{r} = q_e E - m_e \gamma \dot{r} - k_s r$$

$$k_s \rightarrow m_e \omega_0^2$$

try solution

$$r = Ae^{-i\omega t}$$

$$\dot{r} = -i\omega r$$

$$\ddot{r} = (-i\omega)^2 r = -\omega^2 r$$

so

$$-\omega^2 m_e r = q_e E - m_e \gamma (-i\omega) r - k_s r$$

$$(k_s - \omega^2 m_e - i m_e \gamma \omega) r = q_e E$$

$$r = \frac{q_e}{k_s - \omega^2 m_e - i m_e \gamma \omega} E$$

let $\omega_0^2 = \frac{k_s}{m_e} \Rightarrow k_s = m_e \omega_0^2$

$$r = \frac{q_e / m_e}{\omega_0^2 - i\omega\gamma - \omega^2} E$$

$$p = q_e r$$

$$P = N q_e r$$

N ← # per volume

$$P = N \frac{q_e^2}{m_e} \frac{1}{\omega_0^2 - i\omega\gamma - \omega^2} E$$

so $$\epsilon_0 \chi = \frac{N q_e^2}{m_e} \frac{1}{\omega_0^2 - i\omega\gamma - \omega^2}$$

dfn $$\omega_p = \sqrt{\frac{N q_e^2}{\epsilon_0 m_e}}$$ plasma frequency

then

$$\chi = \frac{\omega_p^2}{\omega_0^2 - i\omega\gamma - \omega^2}$$