4/24 in Class – Euler's Method with the Predictor-Corrector fix

The first two problems are similar–but not exact–to the previous in class work on Euler's Method. If you have old code, you should be able to quickly adapt it. If not, this is a good chance to start over.

1. Use Euler's Method to find a numerical solution for $\dot{x} = 3t^2 + 1$ where x is in meters and t in seconds. At $t = 0$, $x = 0$. Run your approximation for 3s. Start with 10 time steps to check your work, and then try smaller intervals to see how your answer converges.
   (a) This one is easy to solve–find the expression for the exact $x(t)$.
   (b) Write the equations you will use, $x(k+1) = \dots$ and $t(k+1) = \dots$
   (c) Do the first few steps of the Euler approximation by hand on the white board.
   (d) Then go back and write a MatLab program to do it for you. Check your answers.
   (e) Graph both your calculated, approximate x and the exact x as functions of time. Compare them.
   (f) Play around. Decrease the time step and/or increase the total time. What happens?

2. Use Euler's approximation to solve this DE: $\dot{N} = -\Gamma N$ where $\Gamma = 3.6 \times 10^{-4}$s. At $t = 0$, $N = 1000$. For the first pass, go from t=0 to 15,000 seconds with only 10 time steps. (That's just for debugging.)
   (a) This one is separable and so also pretty easy to solve–find the expression for the exact $N(t)$.
   (b) Write the equations you will use, $N(k+1) = \dots$ and $t(k+1) = \dots$
   (c) Do the first few by hand on the white board.
   (d) Then write a MatLab program to do it for you.
   (e) Graph both your calculated, approximate N and the exact N as functions of time. Compare them.
   (f) Play around. Decrease the time step by a lot. What happens?

3. Use Euler's approximation to solve a second order DE: solve $\ddot{y} = -g$. (This is actually a pretty bad idea since each estimate will compound the error.) Let the particle start from rest at the origin.
   (a) To do this, you will need to first find $v = \dot{y}$, then use that v to find y. Show how you will do this on the white board. You do not need to do any by hand.
   (b) Then write a MatLab program to do it for you.
   (c) Graph both your calculated v and the exact v as functions of time. Compare them.
   (d) Graph both your calculated y and the exact y as functions of time. Compare them.
   (e) Play around. Decrease the time step and/or increase the total time. What happens?

4. Repeat Problem 1 using the Predictor-Corrector fix to the Euler Method. This time put three plots on one graph:

(a) Euler method approx of x vs t

(b) Predictor-Corrector fix of the euler method x vs t

(c) Exact x vs t

(d) Find a way to label those and turn in the graph to me.

5. Repeat Problem 2 using the Predictor-Corrector fix to the Euler Method. Again, put all three plots on one graph:

   (a) Euler method approx of N vs t

   (b) Predictor-Corrector fix of the Euler method N vs t

   (c) Exact N vs t

   (d) Find a way to label those and turn in the graph to me.