5/3 in Class – more 4th Order Runge-Kutta and the Simple Pendulum

1. **Simple Pendulum as a Simple Harmonic Oscillator:** A simple pendulum consists of a mass on the end of a string that is free to swing back and forth. The mass of the string must be small compared to hanging mass. (If it's not, it's called a physical pendulum.) When we need numbers, let's say: $m = 1.5$kg on the end of a $\ell = .75$m string. We will start it from rest at $\theta_0 = 25$ degrees and and let it run for five full periods.

   Recall the equation for a SHO is:
   $$\ddot{\bigcirc} = \omega^2 \bigcirc$$
   where $\bigcirc$ is any position variable. For the simple pendulum, you might choose either θ or s as your position variable, where θ is the angle measured from the vertical to the string, and s is the path followed by the mass. (s would be the path of a circle if you let it swing all the way around.) Recall also that $s = \ell\theta$. So that we are all consistent, let's choose θ as our variable in the end. (You can derive in s if you prefer, then divide by ℓ.)

   (a) Go to the white board and derive the equation for $\ddot{\theta} = \cdots$. Box the exact equation and save it. (I plan to use it later.) This is not a SHO. For this–start from either Newton's Second Law or the Lagrangian for the mass.

   (b) Show that for small angles, where $\sin\theta \approx \theta$, ($\theta$ must be in radians) the simple pendulum is a SHO and find the period, τ, for it. (You can leave this in terms of symbols given–it's easy to code later.)

   (c) To use Runge-Kutta, you will need to break this second order DE into two first order DE's. Then you can do (4th order) R-K twice–once on each DE.

   (d) Still on the white board, write the equations you will use for the k's and θ and $\dot{\theta}$

   (e) Before you go back to code it–write the exact solutions for θ and $\dot{\theta}$ on the board.

   (f) Write your own 4th order Runge-Kutta approximations for $\dot{\theta}$ and θ as a functions of t.

   (g) Write two functions so that you can use MATLAB's `ode45`.

   (h) Also calculate the "exact" solution to your DE. (I put that in quotes because the exact solution to the DE for SHO is an approximation of the motion of the actual pendulum.)

   (i) Plot all three of your velocities and positions as functions of time. Keep all the velocities on one graph, and all positions on another.

   (j) How accurate are the various approximations of the position when $t = 5\tau$?

2. Do 4th order R-K on the exact second order DE of the pendulum to find the an approximation of the exact position of the mass as a function of time. (The one with the $\sin\theta$ in it.) Plot five periods of the motion for the same mass, length, θ_0 and $\dot{\theta}_0$ given above. Compare to the "exact" solution of the SHO version.

3. Using the same code, vary the starting conditions so that the mass goes all the way around–without the string going slack. (Do you remember that condition from Phys 1? If not, think about what happens when the tension force just goes to zero at the top.) You will have to give the mass an initial velocity.