

N9.18 is a crude estimate of when you would reach terminal velocity, obtained just by setting $g t = v_T$. That is the intersection of a line with slope $-g$ with the terminal velocity line that is horizontal below.

N9.19 is a plug-in, but we have gone far further and solved Problem N9D.4, which began by suggesting we have Mathematica or Wolfram Alpha do the following integral:

$$\text{In[} := \text{Integrate}\left[\frac{1}{1 - \frac{v^2}{c^2}}, \{v, 0, v_{\text{Final}}\}\right]$$

Out[]=

$$c \operatorname{ArcTanh}\left[\frac{v_{\text{Final}}}{c}\right] \text{ if } \operatorname{Re}\left[\frac{c}{v_{\text{Final}}}\right] > 1 \text{ || } \operatorname{Re}\left[\frac{c}{v_{\text{Final}}}\right] < -1 \text{ || } \frac{c}{v_{\text{Final}}} \notin \mathbb{R}$$

$$\text{In[} := c = N[200\ 000 / 3600];$$

$$g = 9.8$$

Out[]=

$$9.8$$

$$\text{In[} := c$$

Out[]=

$$55.5556$$

The tanh function is sinh over cosh, where $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$.

The arctanh relation to tanh is directly analogous to the arctan's relation to tan.

So we take Tanh of both sides of the equation that Mathematica found us and get

$$\text{In[} := v[t_] := -c \operatorname{Tanh}[g t / c]$$

Then we plot it:

$$\text{In[} := \text{Plot}[-g t, v[t], -c], \{t, 0, 20\}, \text{PlotRange} \rightarrow \{\{0, 20\}, \{0, -c\}\}]$$

Out[]=

