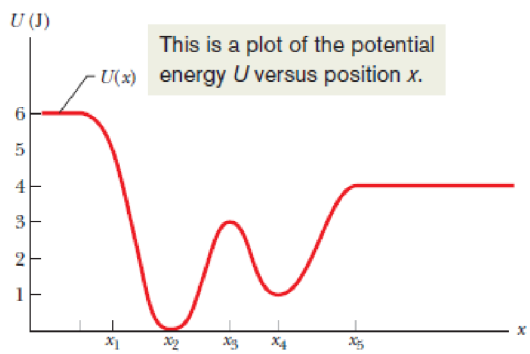
Classical Mechanics — Exam 2

Monday, Nov. 17, 2025

1. Conservation of Energy — Potential Energy Graphs

The vertical axis on the graph below is in Joules.



(a) A particle is released from rest at position x_1. What is its total energy (round to nearest Joule)? What will its kinetic energy be when it passes position x_4? And what will its kinetic energy be when it passes position x_5?

(b) If the particle has mass 0.5 kg, what will its speed be when it passes position x_4? And what will its speed be when it passes position x_5?

(c) If the particle was instead released from rest slightly to the right of position x_3, would it ever reach x_4? Would it ever reach x_5? How many times would it come back to its starting position (slightly to the right of position x_3), neglecting any kind of friction or other energy loss?

PLEASE USE YOUR OWN PAPER, DESPITE THE WHITE SPACE LEFT HERE.

2. Conservation of Energy — Springs and Rotational Energy

A pinball machine has a deck that is a tilted up towards the back of the machine. At the lower right is a plunger (not shown) that stretches a spring. When the spring is released the energy goes into a stainless steel ball that shoots up the track at the right side of the table, and then bends around the curved part of the track at the top.

(a) If the spring has spring constant $k = 3200\ \mathrm{N/m}$ and the plunger is stretched by $\frac{1}{20}$ m, how much energy is contained in the spring, and transferred to the steel ball when the plunger is released?

(b) Use a rounded value of $g = 10\ \mathrm{m/s^2}$ and determine how much elevation the ball could gain before running out of kinetic energy and coming to a halt.

(c) Assume the far end of the table is 8 cm higher than the end with the plunger spring and that the ball weighs 100 g. How much kinetic energy will it have when it reaches the far end of the table?

(d) Kinetic energy for a rolling steel ball is divided between c.o.m. kinetic energy and rotational kinetic energy, as follows:

c.o.m kinetic energy = $\frac{1}{2} M v^2$

rotational kinetic energy = $\frac{1}{2} \alpha M R^2 \omega^2$

For a ball that is not slipping as it rolls, what is the relationship between v, R, and ω? Use this relationship and the fact that for a spherical ball, $\alpha = \frac{2}{5}$ to get a tidy expression for the total kinetic energy of a rolling steel ball that does not involve ω.

(e) Combine the results of parts (c) and (d) to get the speed v at the far end of the table.

3. Work

(a) The Gentle Spring Company invents a spring whose restoring force is $-k x^3$. For later cross-checking, write down the dimensions of k in the usual MKS units.

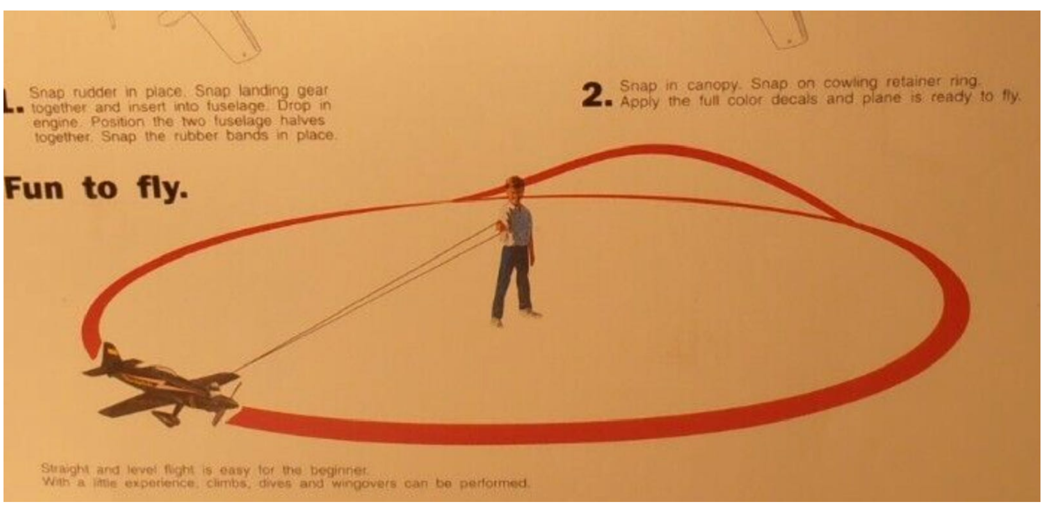
(b) If one of these gentle springs was initially compressed an amount d_{initial} and then further compressed to a greater amount d_{final}, how much work is done on the spring? *For this part, pretend that you are very bad at integrals, and just estimate the average force required to compress the spring as the force at the midpoint which is $k\left(\frac{d_1+d_2}{2}\right)^3$, and then get the work required to compress the spring from $d_{initial}$ to d_{final} using that average estimate.*

(c) Now find the work done on the spring correctly by integrating.

CROSS-CHECKING: Make sure that the dimensions of your answers to (b) and (c) are N $\cdot$ m which is the same as a Joule. Also check that if $d_{\text{final}} > d_{\text{initial}}$ that your answer to both (b) and (c) are positive. In other words, check that in this case — more compression at the final position than the initial position — that positive work is done on the spring.

4. Acceleration from Position — Non-Uniform Circular Motion

A model airplane on a tether is accelerating its way around a circle (going faster and faster). Here is a diagram that I got off the internet and might well have used when I was a kid:



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4. Acceleration from Position — CONT'D

You model the position of the plane, starting from rest at $x = A$, $y = 0$ as:

$x(t) = A \cos \theta$

$y(t) = A \sin \theta$

Your model for θ, which is a function of time with $\theta(0) = 0$, is:

$\theta(t) = \frac{1}{2} \alpha t^2$

Of course, the speed of the plane tops out after a while, so this accelerating θ is only realistic for a small amount of time after the plane is released, but we'll use it to calculate some things.

(a) Use differentiation and the chain rule to find $v_x(t)$ and $v_y(t)$.

(b) Differentiate again to find $a_x(t)$ and $a_y(t)$.

(c) Simplify your answer to (b) for the special case that $t = \sqrt{\frac{\pi}{\alpha}}$. This is when the plane has gotten 90° of the way around its first circle.

CROSSCHECKING: You expect some acceleration in the $-x$ direction, and the $-y$ direction, when the plane is 90° around the circle. The propeller is causing the acceleration in the $-x$ direction, and the tether is causing the acceleration in the $-y$ direction.

(d) EXTRA CREDIT: Write down an expression for $v_x\left(\sqrt{\frac{\pi}{\alpha}}\right)$. Can you rewrite what you found in (c) for $a_y\left(\sqrt{\frac{\pi}{\alpha}}\right)$ to see that this component of the acceleration is in perfect agreement with uniform circular motion?

5. Free-Body Diagrams and Velocity from Force and Acceleration

A hay bale of mass m is initially at rest on a ramp at $\theta = 30°$ to the horizontal leading into the back of a pickup truck. The coefficient of kinetic friction is $\mu_k = 0.3$. A feed person applies F_{feed} to the bottom end of the hay bale, attempting to push it up the ramp and into the bed of the pickup.

(a) Draw a free body diagram for the hay bale, labeling all forces with magnitudes and directions. Assume that F_{feed} is big enough to get the hay bale sliding so F_{kinetic} is going to be relevant, not F_{static}

HINT/HELP: There are three forces on the hay bale: F_{normal}, F_{gravity}, F_{kinetic}. Don't resolve the force of gravity into components yet. That will come shortly.

(b) Using a coordinate system with the x-direction parallel to the ramp, and the y-direction normal to the ramp, resolve the force of gravity into components $F_{\text{gravity},\,x}$ and $F_{\text{gravity},\,y}$. Also use $F_{\text{gravity}} = mg$ to rewrite the answer.

(c) Knowing that $a_y = 0$, get a formula for F_{normal}. Also, knowing that $F_{\text{kinetic}} = \mu_k \, F_{\text{normal}}$, get an expression for F_{kinetic}.

(d) You should have enough to compute $a_x(t)$ now.

(e) What is $v_x(t)$, assuming that $v_x(0) = 0$?

(f) Extra credit: Using $m = 20$ kg, $g = 10\,\text{m}/\text{s}^2$, $\mu_k = 0.3$, $\sin 30° = 1/2$, and $\cos 30° = 7/8$ (it is actually 0.866, but 7/8 is close enough), find out how long it takes to push the hay bale $d = 2$ m up the ramp.

Name ____________________

1. / 2
2. / 4
3. / 4
4. / 5 + 1EC
5. / 5 + 1EC

GRAND TOTAL

/ 20 MAX

The exam covered *Six Ideas,* Unit C, Chapters C8-C11, and Unit N, Chapters N1-N3.

C8 was on conservation of energy. C9 was on using potential energy graphs. C10 was on work. C11 was on rotational energy.

N1 is the statement of Newton's Laws, and the introduction of motion diagrams and free-body diagrams. N2 is on getting velocity from position, and acceleration from velocity by differentiating. Finally N3 is on getting velocity from acceleration, and position from velocity by integrating.

Because 7 problems would have made too long of an exam, I combined some of the ideas from the 7 chapters to make only 5 problems. Of course, in real physics and engineering problems, we usually have to use all of the ideas, so combining was not a stretch.

There was no problem using motion diagrams. Perhaps there will be space on Exam 3 to have one of those, but life is short, motion diagrams are to help you build intuition, not to actually solve problems, and there will be plenty of other interesting problems in N4-N10 to put on Exam 3, so perhaps not.