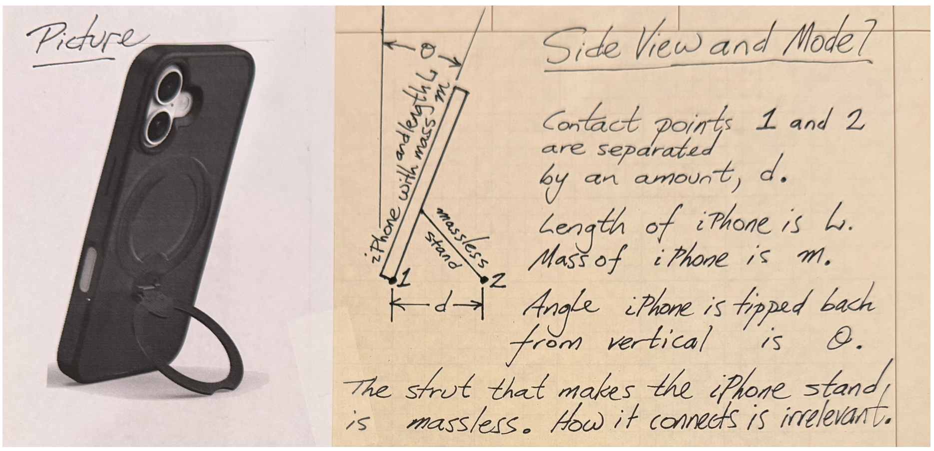
Classical Mechanics — Exam 3

Monday, Dec. 15, 2025

1. Statics — iPhone Stand



There is a directly upward (normal) force whose magnitude is $|\vec{F_1}|$ at point 1 and a directly upward (normal) force whose magnitude is $|\vec{F_2}|$ at point 2. We will apply the methods of the statics chapter to discover these forces in terms of the variables m, $|\vec{g}|$, θ, L, and d.

As noted in the drawing, how the strut that forms the stand connects to the iPhone is irrelevant. It is an idealized, massless strut.

(a) Write down an equation that expresses the fact that the upward and downward forces on the phone must add up to zero.

Now we will turn to torques. Use point 1 as the origin of the coordinate system.

(b) What is the torque on the iPhone/stand system due to $|\vec{F_2}|$? Note that in two-dimensional problems, we usually express counterclockwise torques as positive. Is this torque counterclockwise?

(c) What is the torque on the iPhone/stand system due to the downward force of gravity, whose magnitude is $m\,|\vec{g}|$. HINT: It ain’t just $m\,|\vec{g}|\,L$.

(d) The sum of the torques must be zero, so get an equation involving the two torques you found in (b) and (c).

(e) The equation you found in (d) tells you $|\vec{F_2}|$. Use that equation to get rid of $|\vec{F_2}|$ in the equation you found in (a). Solve the equation to find $|\vec{F_1}|$.

2. Coupled Objects — The Worker and the Bricks

For our N6 problem, I am just going to steal one from Moore, and I want you to do it with Newton's laws (although you could have done it with conservation of energy, like we would have done it in Unit C).

> **N6M.8** An 82-kg worker clings to a lightweight rope going over a lightweight, low-friction pulley. The other end of the rope is connected to a 67-kg barrel of bricks. If the worker is initially at rest 15 m above the ground, how fast will he or she be moving when he or she hits the ground?

Also, I don't really care that much about Moore's numbers. So how about we give the numbers names: $m_{\text{worker}} = 82$ kg, $m_{\text{bricks}} = 67$ kg, $h = 15$ m, and gravity, $|\vec{g}| = 9.8\ \text{m}/\text{s}^2$. Call the magnitude of the unknown tension in the rope $|\vec{T}|$. Have the y-axis point up.

(a) Because the worker and the bricks are coupled by a rope of constant length that passes over a pulley, what can you say about $a_{y,\text{worker}}$ and $a_{y,\text{bricks}}$? HINT: It ain't $a_{y,\text{worker}} = a_{y,\text{bricks}}$.

(b) What do Newton's laws applied to the worker say?

(c) What do Newton's laws applied to the bricks say?

(d) Subtract the equations you got in (b), and (c), to get rid of $|\vec{T}|$. Also, get rid of $a_{y,\text{bricks}}$ from the equation using the equation in (a).

(e) Solve for $a_{y,\text{worker}}$. No need to plug in yet.

(f) EXTRA CREDIT (only if time): Finish the job to get an answer for the speed of impact. Start with the usual usual pair of constant acceleration formulas: $h = -\frac{1}{2} a_{\text{worker}} t^2$ and $v = -a_{\text{worker}} t$ to first get $h = -\frac{1}{2} a_{\text{worker}} \left(\frac{-v}{a_{\text{woker}}}\right)^2 = -\frac{1}{2} \frac{v^2}{a_{\text{worker}}}$, then solve for v and plug in.

3. Circularly Constrained Motion — Angle Grinder

An angle grinder tool is shown above. It usually has a “cutoff wheel” mounted in it, as shown above.

Let’s assume that in 1.5 seconds after hitting the power switch the cutoff wheel has gone from 0 rpm to 9000 rpm.

(a) First convert 9000 rpm to revolutions per second. Then get a value for the average angular acceleration in revolutions/second^2.

(b) Multiply by 2π to get the final angular velocity, ω, and the average angular acceleration in radians per second, and α, in radians per second^2. Use π is about 3 to get some nice round numbers.

(c) Assuming steady angular acceleration, half-way through the 1.5 seconds, what was the angular velocity, $\omega_{\text{half-way}}$ in radians per second?

(d) Assuming the cutoff wheel has a radius of 8cm, what is the magnitude of the centripetal acceleration, $|\vec{a}_{\text{centripetal}}|$, at the rim of the cutoff wheel at the half-way time, in meters / second^2?

Note: I can’t resist pointing out that $|\vec{a}_{\text{centripetal}}|$ is a high acceleration, and if the cutoff wheel is damaged, the forces required to maintain these accelerations will tear the cutoff wheel apart.

(e) At this same time, what is the magnitude of the tangential acceleration, $|\vec{a}_{\text{tangential}}|$, at the rim of the cutoff wheel, also in meters / second^2?

(f) EXTRA CREDIT (only if time): Find the magnitude of the total acceleration using the pieces you found in (d) and (e)). It will be hardly different than what you got in (c).

4. Projectile Motion — Carnival Shot

This story is usually told about a monkey and a hunter, but that seems unnecessarily morbid. Imagine instead that you are aiming a carnival gun at a prize. The carnival gun is kind of weak (not very explosive), but you do not bother to compensate for the noticeable parabolic curve that the weakly-propelled projectile will follow. Instead you aim straight at the prize.

To save money, whenever you aim at an expensive prize, the carnival arranges for the prize to disconnect from its hanger and start falling exactly at the same time as you release the trigger.

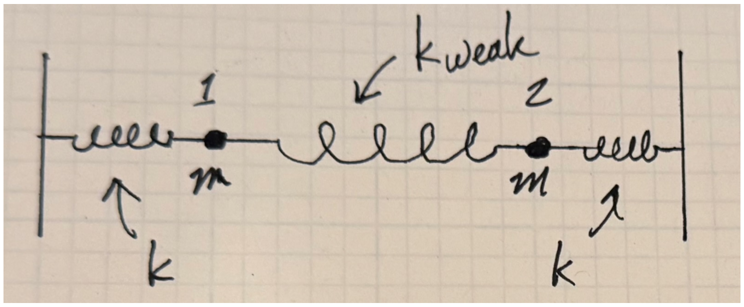
Does the projectile from the gun pass above the falling prize, below it, or does it hit the prize!?!?!

Your answer may depend on the gun angle, θ, the projectile initial speed, $|\vec{v_0}|$, gravity, $|\vec{g}|$, the vertical distance, h, and the horizontal distance, D, from the gun to the prize.

You need to set up the projectile equation for the position of the gun's projectile, and another equation for the position of the falling prize.

All you know about the variables in the initial conditions is that you have aimed at the prize, which means $\frac{h}{D} = \tan\theta$. The other variables are whatever they are.

5. Oscillatory Motion — Yet Another Coupled Oscillators Problem



This is the usual two equal masses and three springs situation, EXCEPT, the middle spring is going to be weak spring compared to the two springs that connect the masses to the walls. We'll call the spring constant of the weak middle spring, k_{weak}.

(a) Write down and expression for $m a_1$ using Newton's Laws and Hooke's Law (for springs). Your expression for the forces will involve x_1, x_2, k, and k_{weak}.

(b) Do the same thing for $m a_2$.

(c) Now assume that the two masses move together. That means $x_2(t) = x_1(t)$ (and also that $a_2(t) = a_1(t)$). Use those facts to simplify the equations in (a) and (b). That is very easy and you get a standard result. What is the oscillation frequency in terms of k and m?

(d) Now assume the two masses move oppositely. That means $x_2(t) = -x_1(t)$ (and also that $a_2(t) = -a_1(t)$. Use those facts to get a different simplification of the equations in (a) and (b).

(e) What is the oscillation frequency of the type of oscillation you found in (e) in terms of k, k_{weak}, and m? HINT: It should be slightly higher than what you found in (d).

(f) EXTRA CREDIT (only if time), when you see something like $\sqrt{k + 2\, k_{weak}}$ and you know that you can factor out a $\sqrt{k}$ to get $\sqrt{k}\ \sqrt{1 + 2\, k_{weak} / k}$. Now when you see $\sqrt{1 + 2\, k_{weak} / k}$, and know that I have often used the approximation $(1 + x)^n \approx 1 + n x$ provided x is sufficiently small, what occurs to you, *e.g.,* what is playing the role of n and of x? So what does ω become in the approximation that k_{weak} is small compared to k?

Name ____________________

1. / 4
2. / 4 + 1EC
3. / 4
4. / 4 + 1EC
5. / 4 + 1EC

GRAND TOTAL

/ 20 MAX

The exam covered the seven Chapters N4-N10, in *Six Ideas,* Unit N. There is not enough time to do seven different problems in a class period, so I chose five representative problems. THE EXAM MIGHT STILL BE KIND OF LONG. CUT YOUR LOSSES AND MOVE ON IF YOU GET STUCK ANYWHERE. THAT IS GOOD TEST-TAKING STRATEGY ON ANY EXAM IN ANY CLASS.

Chapter N4 is on statics. A typical feature of statics problems is balancing torques and forces in such a way that no object spins or accelerates.

Chapter N5 is on linearly constrained problems. A typical feature of linearly constrained problems is that a normal force keeps the object on a path, but often we do not have to know the magnitude of the normal force, because the result (that it keeps the object constrained on the path) is already known.

Chapter N6 is on coupled objects. A key feature is that you have to apply Newton's Laws to each object. Typically you start by drawing a free-body diagram for each object. Often there are known constraints just as there were in Chapter N5. For example, we might know that two objects are always connected by a taught rope.

N7 on non-uniform circular motion beefed up our circular motion understanding from what we studied in Section N1.6 (which was uniform circular motion) to include tangential acceleration. Doing vector combination of the centripetal and tangential acceleration is a common feature of non-uniform circular motion problems.

N8 on non-inertial frames we agreed to not include in the test, because I would rather you deeply understand inertial frames. N8.1, N8.2, and N8.3 on Galilean relativity were nonetheless important sections in this chapter. The non-inertial frames stuff only began in N8.4.

N9 on projectile motion is bread-and-butter freshman physics, where you solve quadratic equations in various circumstances.

N10 on oscillatory motion is a cool and important subject because oscillations are ubiquitous in the real world. When combined with the ideas of N6, you get oscillations of multiple coupled objects, and these become chemists models for molecules and engineers models for skyscrapers.