

N9.18 is a crude estimate of when you would reach terminal velocity, obtained just by setting $g t = v_T$. That is the intersection of a line with slope $-g$ with the terminal velocity line that is horizontal below.

N9.19 is a plug-in, but we have gone far further and solved Problem N9D.4, which began by suggesting we have Mathematica or Wolfram Alpha do the following integral:

```
In[ ]:= Integrate[ $\frac{1}{1 - \frac{v^2}{c^2}}$ , {v, 0, vFinal}]
```

```
Out[ ]:=
```

```
c ArcTanh[ $\frac{vFinal}{c}$ ] if  $\text{Re}\left[\frac{c}{vFinal}\right] > 1 \mid \mid \text{Re}\left[\frac{c}{vFinal}\right] < -1 \mid \mid \frac{c}{vFinal} \notin \mathbb{R}$ 
```

```
In[ ]:= c = N[200 000 / 3600];
```

```
g = 9.8
```

```
Out[ ]:=
```

```
9.8
```

```
In[ ]:= c
```

```
Out[ ]:=
```

```
55.5556
```

The tanh function is sinh over cosh, where $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$.

The arctanh relation to tanh is directly analogous to the arctan's relation to tan.

So we take Tanh of both sides of the equation that Mathematica found us and get

```
In[ ]:= v[t_] := -c Tanh[g t / c]
```

Then we plot it:

```
In[ ]:= Plot[{-g t, v[t], -c}, {t, 0, 20}, PlotRange -> {{0, 20}, {0, -c}}]
```

```
Out[ ]:=
```

