Classical Mechanics

The Plan — including my two additional problems — for Thursday, Dec. 11

Study Chapter N10 of *Six Ideas*

I am hopeful that you will find Chapter N10 pretty easy given all that you learned heretofore. One part that gets tricky is doing the approximations to turn the simple pendulum into the harmonic oscillator. Of course, the simple pendulum is not the harmonic oscillator, but if the swinging of the pendulum is small enough that we can focus on terms of order θ and neglect terms of order θ^3 or higher, then they are approximately the same. This approximation is performed in Section N10.6.

Problems From Chapter N10

1. N10B.4 (the MKS units for the spring constant are Newtons/meter or Joules / meter^2, but in atomic physics a better unit of energy for the tiny energies involved is the eV and a better unit of distance for the tiny distances involved is the Angstrom, so convert your answer to eV / Angstrom^2)

2. N10M.7 (let's assume the person on the pogo stick is a kid weighing 40kg)

3. N10A.1 (another advanced problem and this one gives you an idea of how oscillators show up as approximations to atomic potentials that do not look at all like oscillators until you make the approximations)

Problems 4 and 5 for Problem Set 21 are on the next two pages, and the presentations for Thursday are on the last page.

Problem 4. Relating the Two Most Common Expressions for Oscillatory Motion

One common expression for the solution of oscillatory motion problem involves the sine and the cosine, and the other just involves just a cosine, but with an offset angle. Which is more convenient depends on the situation.

(a) Show that $x(t) = C\cos\omega t + D\sin\omega t$ satisfies $ma = -kx$. This works for any value of C and D, but for it to work, what must ω be?

HINT/COMMENT: Proceed in an organized fashion. First compute $v(t) \equiv \frac{dx}{dt}$. Then compute $a(t) \equiv \frac{dv}{dt} = \frac{d^2x}{dt^2}$. Then plug in what you have discovered.

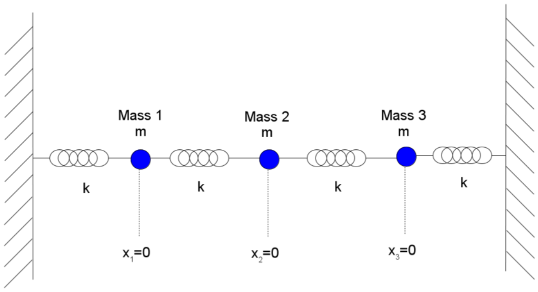
(b) Show that $x(t) = A\cos(\omega t + \theta)$ satisfies $m\frac{d^2x}{dt^2} = -kx$. This works for any value of C and D, but for it to work, what must ω be?

HINT/COMMENT: As in (a), proceed in an organized fashion.

(c) The solution in (a) involved two constants, which we called C and D. The solution in (b) also involved two constants, A and θ. In class I used the formula for $\cos(\alpha + \beta)$ to get expressions for C and D in terms of A and θ. Repeat that derivation.

(d) If θ is π and A is 1 meter, and ω is 0.5 seconds, what are $x(0)$, $v(0)$, and $a(0)$?

Problem 5. A More Sophisticated Coupled Objects Oscillation Problem



It is not our job in this course to solve the most general problem of N equal-mass objects coupled by $N+1$ equal-spring-constant springs, although it is fun and it leads you to start visualizing compression waves.

Let's nonetheless make a little progress on the problem of 3 masses coupled with a total of 4 springs, so we start to see how it is going to go.

(a) Write down Newton's Second Law for each of the three masses in the diagram above (shown at equilibrium with the springs neither stretched nor compressed). I'll write down Newton's Second Law first one to get you started, so you only have to write down the expressions for masses 2 and 3:

$m a_1 = F_{1,\text{total}} = -k x_1 + k(x_2 - x_1)$

Understand and double-check the signs!

(b) It is pretty clear that there is a solution mass 2 stays at rest, and mass 3 does the opposite of what mass 1 is doing. Can visualize that solution in your mind before proceeding with the algebra? Now proceed with the algebra by putting in $x_2(t) = 0$ and $x_3(t) = -x_1(t)$ into the equations you found in (a). The three equations are going to simplify a lot.

(c) Now that you have the three equations simplified try $x_1(t) = A\cos(\omega t + \theta)$. What must ω be in terms of k and m for this to work. HINT: It ain't what you got for ω in 4(b)!

Presentations for Thursday

Grace: Show that the solution we worked with in 4(a) satisfies energy conservation. You will need to compute the kinetic energy and the potential energy, and add them, and use trig identities to simplify.

Sasha: Show that the solution we worked with in 4(b) also satisfies energy conservation. Your proof will be very similar to Grace's, so I will give you something extra to tack on: For $\theta = 45\,°$, sketch KE and sketch PE from $t = 0$ to $t = \frac{4\pi}{\omega}$ (that's two periods of oscillation). Do they look like they add up to a constant?

Sam: Demonstrate the law of superposition for homogeneous, linear differential equations, using as an example the differential equation for the harmonic
oscillator. What if the differential equation has an inhomogeneous term?

Brian: Use Mathematica to solve and animate Problem 5 in all generality, not just the special case of $x_2(t) = 0$ and $x_3(t) = -x_1(t)$.