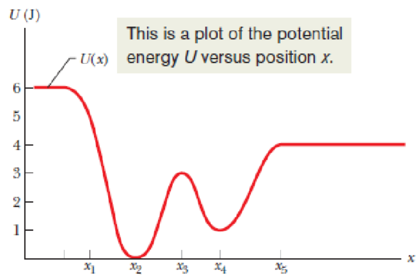
Classical Mechanics — Exam 2

Monday, Nov. 17, 2025

Covering *Six Ideas,* Unit C, Chapters C8-C11, and Unit N, Chapters N1-N3. C8 is on conservation of energy. C9 is on using potential energy graphs. C10 is on work. C11 is on rotational energy. N1 is the statement of Newton's Laws, and the introduction of motion diagrams and free-body diagrams. N2 is on getting velocity from position, and acceleration from velocity by differentiating. Finally N3 is on getting velocity from acceleration, and position from velocity by integrating. Because 7 problems would make a mighty long exam, I have combined some of the ideas from the 7 chapters to make only five problems. Of course, in real physics and engineering problems, we often have to use all of the ideas, so this was not much of a stretch.

1. Conservation of Energy — Potential Energy Graphs

The vertical axis on the graph below is in Joules.



(a) A particle is released from rest at position x_1. What is its total energy (round to nearest Joule).

(b) What will its kinetic energy be when it passes position x_4? And what will it be when it passes position x_5?

(c) If the particle has mass 0.5 kg, what will its speed be when it passes position x_4? And what will it be when it passes position x_5?

(d) If the particle was instead released from rest slightly to the right of position x_3, would it ever reach x_4? Would it ever reach x_5? How many times would it come back to its starting position (slightly to the right of position x_3), neglecting any kind of friction or other energy loss?

2. Conservation of Energy — Springs and Rotational Energy

A pinball machine has a deck that is a tilted up towards the back of the machine. At the lower right is a plunger (not shown) that stretches a plunger spring. When the plunger spring is released the energy goes into a stainless steel ball that shoots up the track at the right side of the table, and then bends around the curved part of the track at the top.

(a) If the plunger spring has spring constant $k = 3200\ \text{N}/\text{m}$ and the plunger is stretched by $\frac{1}{20}$ m, how much energy is contained in the spring, and transferred to the steel ball when the plunger is released?

(b) Use a rounded value of $g = 10\ \text{m}/\text{s}^2$ and determine how much elevation the ball could gain before running out of kinetic energy and coming to a halt.

(c) Assume the far end of the table is 8 cm higher than the end with the plunger spring and that the ball weighs 100 g. How much kinetic energy will it have when it reaches the far end of the table?

(d) Kinetic energy for a rolling steel ball is divided between c.o.m kinetic energy and rotational kinetic energy, as follows:

c.o.m kinetic energy = $\frac{1}{2} M v^2$

rot. kinetic energy = $\frac{1}{2} \alpha M R^2 \omega^2$

For a ball that is not slipping as it rolls, what is the relationship between v, R, and ω?

Use this relationship and the fact that for a spherical ball, $\alpha = \frac{2}{5}$ to get a tidy expression for the total kinetic energy of a rolling steel ball.

(e) Then combine the results of parts (c) and (d) to get the speed v at the far end of the table.

3. Work

(a) The Gentle Spring Company invents a spring whose restoring force is $-k x^3$. For cross-checking, write down the dimensions of k in the usual MKS units.

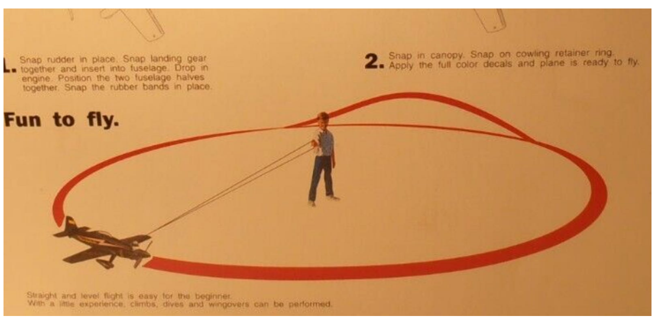
(b) If one of these gentle springs was compressed an amount d_{initial} and then is compressed to a greater amount d_{final}, how much work is done on the spring? *For this part, pretend that you are very bad at integrals, and just estimate the average force required to compress the spring as the force at the midpoint which is* $k\left(\frac{d_1+d_2}{2}\right)^3$*, and then get the work done to compress the spring from* $d_{initial}$ *to* d_{final} *using that average estimate.*

(c) Now find the work done on the spring correctly by integrating.

HINTS/CROSSCHECKING: For your own cross-checking (not for points), make sure that the dimensions of your answers to (b) and (c) are N · m which is the same as a Joule. Also check that if $d_{\text{final}} > d_{\text{initial}}$ that your answer to both (b) and (c) are positive. In other words, check that in this case (more compression at the final position than the initial position), that positive work is done on the spring.

4. Acceleration from Position — Non-Uniform(!) Circular Motion

A model airplane on a tether is accelerating its way around a circle (going faster and faster).



You model the position of the plane, starting from rest at $x = A$, $y = 0$ as

$x(t) = A \cos\theta$ and $y(t) = A \sin\theta$

$x(t) = A \cos\theta$ and $y(t) = A \sin\theta$

5. Free-Body Diagrams and Position from Acceleration

Name ____________________

1. / 4
2. / 4
3. / 5
4. / 5

GRAND TOTAL

/ 20