An Exercise for the Exploding Universe Problem — Taylor & Wheeler Problem 3-10

On the following three pages is a deceptively easy exercise, requiring only a ruler, that is designed to get you thinking what an explosion looks like in both (a) the reference frame, A, set at the center of where the explosion occurred, and (b) the reference frame B of any one of the particles in the explosion.

Certainly, in frame A, we have that d_Ci = v_Ci * t, which says that the distance d_Ci an object is from the center, C, is proportional to the time (as measured in that frame) times v_Ci the velocity measured in that frame.

We now turn to an observer riding on one of the particles….

The fascinating and useful thing, which this exercise *might* convince you of, is that in frame B, we still have d_Oi = v_Oi * t, where now d_Oi is the distance from the observer to particle i, v_Oi is the velocity of particle i as measured in the observer's frame, and t is now the time as measured in the observer's frame (not the time in the rest frame).

This is a fabulous simplification. It means that we can easily describe and easily do calculations of things like Δt_reception by doing them in the observer's rest frame, despite the fact that the observer might be moving rapidly relative to the center of the explosion.

That is the way to approach Problem 3-10. This exercise just helps you move to that mindset.

After you have done the exercise, return to the discussion below.

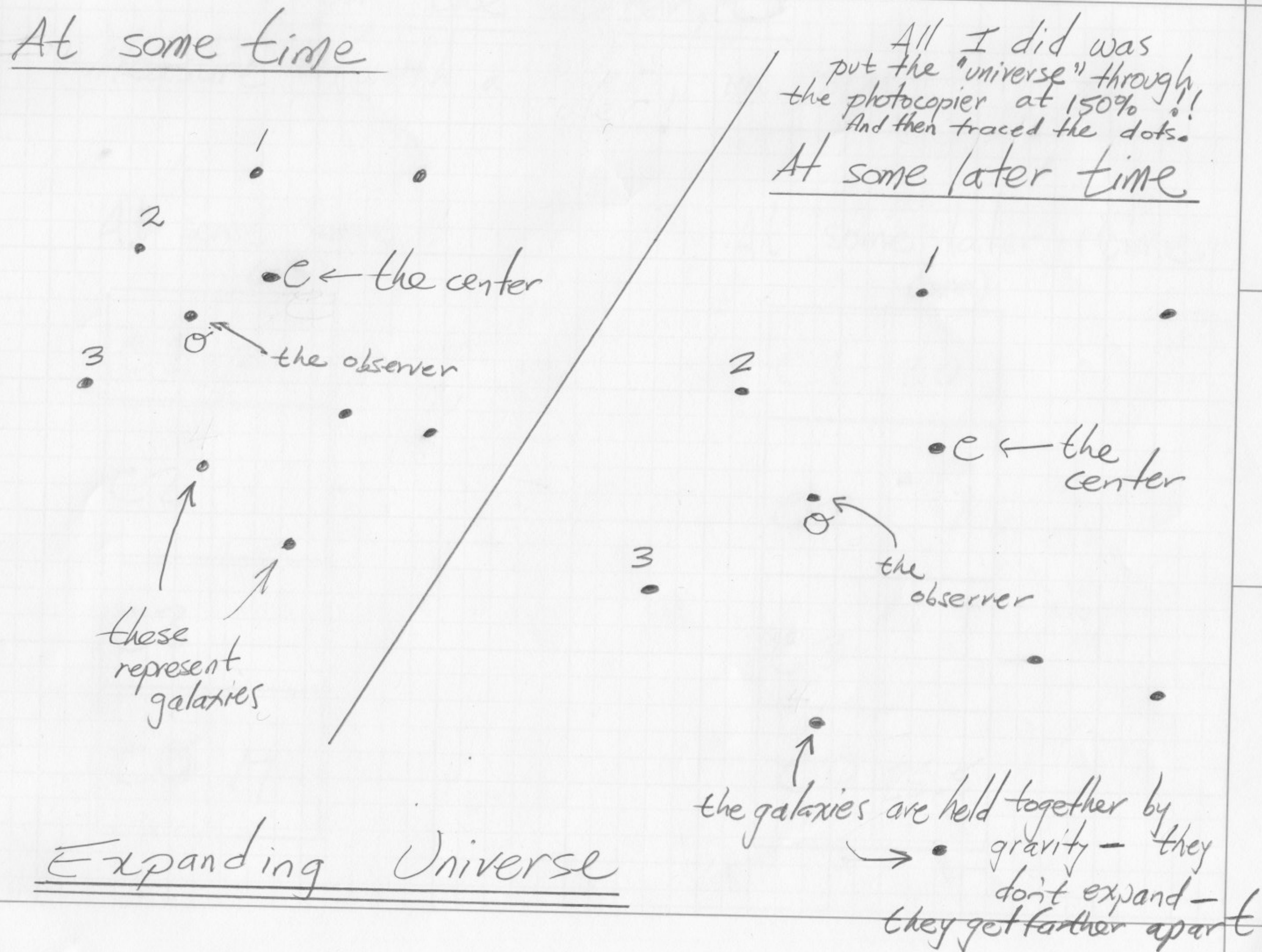
Check your numbers on the first page of tables. If you look closely, you will see a pattern, assuming you haven't made any big errors. The numbers in the second table are all 1.5x the numbers in the first table. In other words, if the later time is t_2, and the earlier time is t_1, the numbers in the second table are all t_2 / t_1 larger, and you can infer that t_2 / t_1 is 1.5.

Now look at your numbers on the second page of tables. We have not done a proper relativistic transformation between the two frames, so the drawing and the numbers would actually require some corrections to make this easy exercise more accurate. But is it not the case that the numbers in the second table on that page are all 1.5x the numbers in the first table?

If we had done a proper relativistic transformation, to see what the universe looked like in the frame of the observer, it would have pancaked (shrunk) in the direction of motion, and stayed the same in the perpendicular direction.

But would it not still be true that d_Oi = v_Oi * t, where t is now the time as measured in the observer's frame? All this depends on is that t was zero when the explosion occurred and the distances grow linearly with time (neither accelerating nor decelerating).

And that is all you need to do a great job of Problem 3-10.


At some time
1
2
c ← the center
o
the observer
3
these represent galaxies
All I did was put the "universe" through the photocopier at 150%!!
And then traced the dots.
At some later time
1
2
c ← the center
o
the observer
3
the galaxies are held together by gravity – they don't expand – they get farther apart
Expanding Universe

From the Center, C

Measure (with a ruler, in mm)

At some time

	(mm)
C1	20
C2	
C3	
C0	17

At some later time

	(mm)
C1	30
C2	
C3	
C0	25.5

From the Observer, O

Measure (with a ruler, in mm)

At some time

	(mm)
O1	30
O2	
O3	
Oe	17

At some later time

	(mm)
O1	45
O2	
O3	
Oe	25.5