Cosmology — Problem Set 5 — Exploring the Black Hole

1. Taylor-Wheeler-Bertschinger (TWB) Problem 3.1 on p. 3-35

2. TWB Problem 3.2 on p. 3-35

3. Inward Falling Light — Outside the Event Horizon

We haven't derived Eq. 26 on p. 3-25 yet, but that doesn't stop us from becoming familiar with it.

(a) In Eq.26 on p. 3-25, take $t_1 = 0$ and $r_1 = 3\,M$. What does the equation simplify to? The interpretation of this is that at global coordinate $t = 0$ a light flash is directed either outward or inward from a flashlight at global coordinate $r = 3\,M$.
(b) There is a ± sign in your answer to Part (a). If the flashlight is directed inward then increasing t—which since we started with $t_1 = 0$ means positive t—must correspond to decreasing r. So which sign is going to give you positive times for decreasing r? Simplify the formula to have just that sign.
(c) Double check that plugging in $r = 2.5\,M$ indeed gives a positive t value in your answer to Part (b). What is the t value?
(d) You will notice that the t value in Part (c) has a factor of M in it. It is going to make it hard to graph t unless we give a value for M. How about instead, we graph t/M on the vertical axis. Also, we will graph r/M on the horizontal axis. If we do that, we don't have to give a value for M. Using the function in Part (b) graph t/M as a function of r/M in the region $2 < r/M < 3$.

4. Inward Falling Light — Inside the Event Horizon

At $r = 2\,M$ is the event horizon, and our coordinates behave poorly there. So we will skip over that zone and use the same procedure again, but this time using $t_1 = 0$ and $r_1 = 1.5\,M$.

(a) Repeat Problem 3(a) but this time with $t_1 = 0$ and $r_1 = 1.5\,M$.
(b) Same as 3(b), but in this new region.
(c) Same as 3(c), except your double-check needs to involve putting in some r that is *smaller* than $r_1 = 1.5\,M$. How about we choose $r = 0.5\,M$ to compute the t value.
(d) Same as 3(d), except the region to make the graph in will be $0 < r/M < 1.5$.

DISCUSSION: Problems 3 and 4 are another way of understanding what was graphed in Figure 8. Although we had to divide the analysis into two parts, and we were not able to cross the event horizon, and so it seems that something very unusual happens to the inward falling light at the event horizon, it in fact crosses the event horizon without issue. The apparent inability to cross the event horizon is a problem with our global coordinates at $r = 2\,M$ that we will eventually be able to resolve.

5. TWB Problem 3.5 on p. 3-37

HINT: Refresh your memory on how you did Problem 2-8(a) on Problem Set 1. You need $\Delta g / \Delta r$.