Cosmology — Problem Set 8 — Stones and Raindrops

All four problems on this problem set emphasize that map speed $|dr/dt|$ is very different from the speed that any observer sees. That is one of the largest points the authors are making in Chapter 6. Map speed does converge with observed speed at ∞.

Similarly, map energy is not observable. However, late in the chapter the authors argue that there is an important use of map energy. They argue that when a spherical shall having map energy E falls into a black hole, the resulting black hole ends up with mass $M + E$.

Problem 1 — TWB Problem 6-2

This is a comfort problem wherein you just have to plug in values to formulae.

Problem 2 — TWB Problem 6-4

I overlooked that this problem had a Parts D and E. I did not mean to assign those (at least not without explaining some of the physics ideas of a gas of hot electrons that the authors are utilizing!).

Problem 3 — TWB Problem 6-7

On both this problem and the next problem the part of the problem where you find the maximum of $|dr/dt|$ was not particularly instructive given the amount of algebra involved. I told some people they should skip that if pressed for time. If you correctly found the maximum, I will make that extra credit.

Problem 4 — TWB Problem 6-8

Can you approach this problem by utilizing a lot of the work you did for previous problem?

Having done problems 3 and 4, and the raindrop case that is done in the text, all of the three most important cases of a falling stone have now been covered. The three cases are:

(a) It can fall from infinity starting from rest.
(b) It can fall from less than infinity starting from rest. It will not be falling as fast as in case (a) at any given radius.
(c) It can fall from infinity starting with some inward velocity. It will be falling faster than in case (a) at any given radius.

Newtonian mechanics has exactly these three cases as well. If you fire a stone outward, case (a) is the case that determines “escape velocity.” Case (b) is less velocity than escape velocity. Case (c) is more velocity than escape velocity.