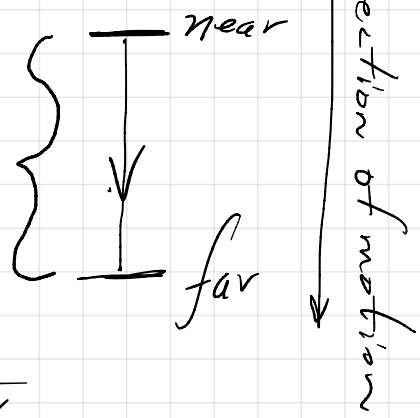
Length Contraction Derivation

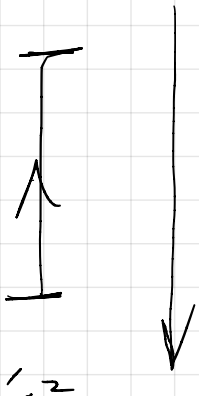
To get from the near mirror to the far mirror in time t_1, the photon must go $w + vt_1$

(v: speed mirrors are moving; w: spacing between mirrors)



So $$ct_1 = w + vt_1 \Rightarrow t_1 = \frac{w}{c-v}$$

To get from the far mirror to the near mirror in time t_2, the photon must go $w - vt_2$



So $$ct_2 = w - vt_2 \Rightarrow t_2 = \frac{w}{c+v}$$

$$T' = t_1 + t_2 = \frac{w}{c-v} + \frac{w}{c+v} = \frac{2wc}{(c-v)(c+v)} = \frac{T'c^2}{c^2-v^2}$$

So $\frac{T}{T'} = \frac{1}{1-v^2/c^2} = \gamma^2$. Oh-oh! We expected $\frac{T}{T'} = \gamma$.

The only way out of this is if w (the spacing between mirrors in the moving frame) appears to us (in the rest frame) as $\frac{w}{\gamma}$.

This shortening by γ is called "length contraction."