Cosmology — Problem Set 10 — Solution

Problem 1 — Black-Body Radiation

$h = 6.626 \times 10^{-34}$ J·s <==== On the assignment, I wrote 6.626×10^{34} J·s. Accckkk.

$$I_\omega = \frac{\hbar\omega^3}{\pi^2 c^2 \left(e^{\hbar\omega/kT} - 1\right)}$$

(a) The units of $\hbar\omega$ are Joules, of ω are 1/s which is Hz, of $\Delta\omega$ the same as ω, and c m/s.

(b) The units of $I_\omega \, \Delta\omega$ are the same as the units of $\frac{\hbar\omega\ \omega^3}{c^2}$. The numerator is J/s^3. The denominator is $(\mathrm{m}/\mathrm{s})^2$, so the ratio is $\mathrm{J/s}/\mathrm{m}^2$. That's power per unit area! Exactly what you'd expect an intensity to be.

(c) Now we are just multiplying what we got in (b) by an area, so that is J/s which is Watts.

(d) The wavelength of orange light is 600nm, so ω for orange light the answer (units are per second) is:

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In[1]:= N[2 Pi / (600×10^-9 / (3×10^8))]

Out[1]= 3.14159 × 10^15
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(e) Graph I_ω from $\omega = 0$ to ω equal to 3x whatever you got in Part d for $T = 5777$ K.

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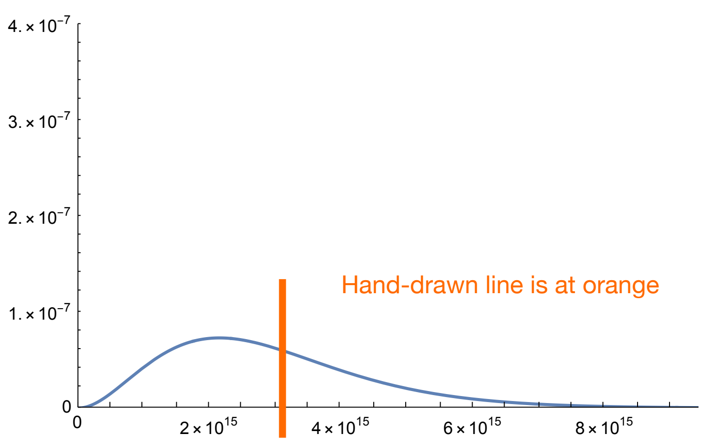
In[2]:= hbar = 6.626×10^-34 / (2 Pi);
orange = 3.14×10^15;
ktee = 5777 * 1.381×10^-23;
c = 3×10^8;
function[ω_] := hbar ω^3 / Pi^2 / c^2 / (Exp[hbar ω / ktee] - 1);
Plot[function[ω], {ω, 0, 3 orange}, PlotRange → {{0, 3 orange}, {0, 4×10^-7}}]
```

Out[7]=



(f) Repeat (e) for a star of 10000K.

In[8]:=
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ktee = 10 000 * 1.381 × 10^-23;
Plot[function[ω], {ω, 0, 3 orange}, PlotRange → {{0, 3 orange}, {0, 4 × 10^-7}}]
```
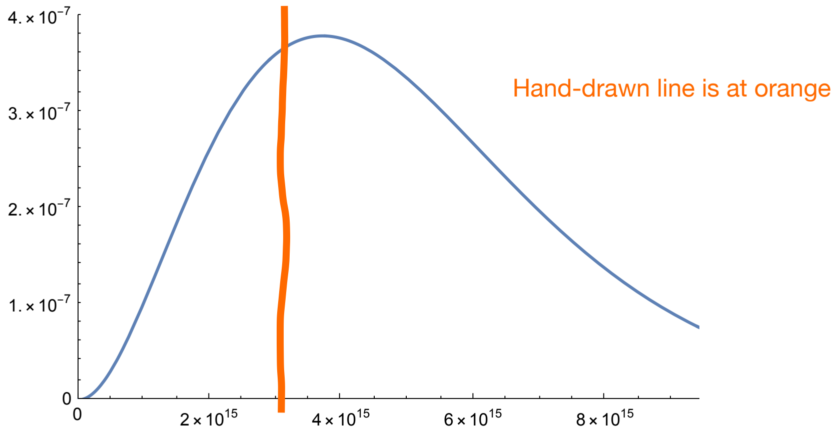

Out[9]=



(g) Repeat (e) for a star of 3000K.

In[10]:=
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ktee = 3000 * 1.381 × 10^-23;
Plot[function[ω], {ω, 0, 3 orange}, PlotRange → {{0, 3 orange}, {0, 4 × 10^-7}}]
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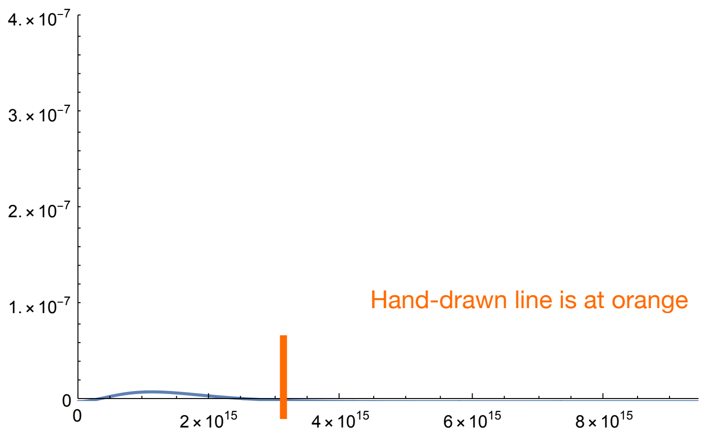
Out[11]=



Problem Set 10 Solution

Problem 2

(a) $\tau_A^2 = (t_2-t_1)^2 - R^2(t_A)\left[(\chi_2-\chi_1)^2 + S^2(\chi_A)(\phi_2-\phi_1)^2\right]$

So $\tau_A = \sqrt{J_A - R^2(t_A)S^2(\chi_A)(\phi_2-\phi_1)^2}$

and similarly

$$\tau_B = \sqrt{J_B - R^2(t_B)S^2(\chi_B)(\phi_3-\phi_2)^2}$$

(b) In the above,

J_A = "junk not involving ϕ_2"

$= (t_2-t_1)^2 - R^2(t_A)(\chi_2-\chi_1)^2$

and similarly

$$J_B = (t_3-t_2)^2 - R^2(t_B)(\chi_3-\chi_2)^2$$

(c) $0 \overset{!}{=} \frac{d}{d\phi_2}(\tau_A+\tau_B) = \frac{1}{2}\frac{1}{\tau_A}\left(-R^2(t_A)S^2(\chi_A)\right)2(\phi_2-\phi_1)$
$+\frac{1}{2}\frac{1}{\tau_B}\left(-R^2(t_B)S^2(\chi_B)\right)2(\phi_3-\phi_2)(-1)$

$$\Longrightarrow \frac{1}{\tau_A}R^2(t_A)S^2(\chi_A)(\phi_2-\phi_1) = \frac{1}{\tau_B}R^2(t_B)S^2(\chi_B)(\phi_3-\phi_2)$$

From here, it is just a bit of rewriting to get to Eq. 41 on TWB p. 14-16.

Problem 3

(a) If we quardruple L and double d_L in $\frac{L}{4\pi d_L^2}$, there is no change! So we have $m = -5\log_{100}\frac{I_{vega}}{I_{vega}} = -5\log_{100} 1 = 0.$

$$m = -5\log_{100}\frac{3.4\, I_{vega}}{I_{vega}} = -5\log_{100} 3.4$$

$$= -\frac{5}{2}\log_{10} 3.4 = -1.33$$

This is the magnitude of Sirius, the brightest star in the sky.

(b) $m - M = -5\log_{100}\frac{L/(4\pi d_L^2)}{I_{vega}}$
$-\left(-5\log_{100}\frac{L/(4\pi(10pc)^2)}{I_{vega}}\right)$

$= -5\log_{100}\frac{1/d_L^2}{1/(10pc)^2}$ ← here I used $\log a - \log b = \log\frac{a}{b}$ and did a lot of cancellation

$= 5\log_{100}\frac{d_L^2}{(10pc)^2}$

$= \frac{5}{2}\log_{10}\frac{d_L^2}{(10pc)^2}$

$= 5\log_{10}\frac{d_L}{10pc}$ ← This is Eq. 42 on TWB p. 14-18.

Problem 3 (cont'd)

(c) $$\frac{m-M}{5} = \log_{10} \frac{d_L}{10pc}$$

$$10^{\frac{m-M}{5}} = \frac{d_L}{10pc} \Rightarrow d_L = 10pc \cdot 10^{\frac{m-M}{5}}$$

(d) $$1pc = 149{,}597{,}870.7km \cdot \frac{360}{2\pi} \cdot 60 \cdot 60$$

$$= 3.0857 \times 10^{13} km$$

(e)

$$1pc = 3.0857 \times 10^{13} \cancel{km} \cdot \frac{1 \cancel{sec}}{299792458 \cancel{m}} \cdot \frac{1000 \cancel{m}}{\cancel{km}}$$

$$\frac{1 \text{ year}}{365.25 \cdot 24 \cdot 3600 \cancel{sec}}$$

$$= 3.2616 \text{ light-years}$$

There were two typos!

In Problem 1, missing minus sign

$$h = 6.626 \times 10^{-34} s$$

In Problem 3(b) missing 4π

$$M = -5 \log_{100} \frac{L/(4\pi(10pc)^2)}{I_{Vega}}$$