Hubble's Law and The FRW Metric

We have read Hubble's 1929 paper, seen what is now called a Hubble Plot, and we know the current best estimate of the expansion rate of the universe today

$$H_0 \approx \frac{70\,\text{km/sec}}{\text{Mpc}}$$

TWB use $H_0 = (73 \pm 2)\dfrac{\text{km/sec}}{\text{Mpc}}$

and in units where $c = 1$ and time is measured in meters

$$H_0 = (8.0 \pm 0.2) \times 10^{-27}\,\frac{1}{\text{meter}}$$

That's the observation. The theory is that

$$(\Delta\tau)^2 = \Delta t^2 - R^2(t)\left[(\Delta\chi)^2 + S^2(\chi)(\Delta\phi)^2\right]$$

What is the relationship between H and R?

Well, that is easy actually. $H(t)$ is the slope of the line in the Hubble Plot. H_0 is the value of that slope today.

The slope of the line is the recession velocity of a galaxy divided by the distance to the galaxy. The distance to a galaxy at χ_{galaxy} is $d(t) = R(t)\,\chi_{galaxy}$.

The recession velocity is

$$\frac{d(t+\Delta t) - d(t)}{\Delta t}$$

So

$$H(t) = \frac{\dfrac{d(t+\Delta t) - d(t)}{\Delta t}}{d(t)} = \frac{\dfrac{R(t+\Delta t) - R(t)}{\Delta t}\cancel{\chi_{galaxy}}}{R(t)\cancel{\chi_{galaxy}}}$$

$\dfrac{R(t+\Delta t) - R(t)}{\Delta t}$ is the derivative of $R(t)$ w.r.t. t, usually written $\dfrac{dR}{dt}$, but TWB have a shorthand, $\dot{R}$, so we'll use that:

$$H(t) = \frac{\dot{R}}{R}$$

This is Eq. 60