Cosmology — Problem Set 10 — Solution

Problem 1 — Black-Body Radiation

$h = 6.626 \times 10^{-34}$ J·s <==== On the assignment, I wrote 6.626×10^{34} J·s. Accckkk.

$$I_\omega = \frac{\hbar\omega^3}{\pi^2 c^2 \left(e^{\hbar\omega/kT} - 1\right)}$$

(a) The units of $\hbar\omega$ are Joules, of ω are 1/s which is Hz, of $\Delta\omega$ the same as ω, and c m/s.

(b) The units of $I_\omega \Delta\omega$ are the same as the units of $\frac{\hbar\omega\,\omega^3}{c^2}$. The numerator is J/s^3. The denominator is $(m/s)^2$, so the ratio is $J/s/m^2$. That's power per unit area! Exactly what you'd expect an intensity to be.

(c) Now we are just multiplying what we got in (b) by an area, so that is J/s which is Watts.

(d) The wavelength of orange light is 600nm, so ω for orange light the answer (units are per second) is:

In[2]:=
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N[2 Pi / (600 × 10^-9 / (3 × 10^8))]
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Out[2]=
$$3.14159 \times 10^{15}$$

(e) Graph I_ω from $\omega = 0$ to ω equal to 3x whatever you got in Part d for $T = 5777$ K.

In[128]:=
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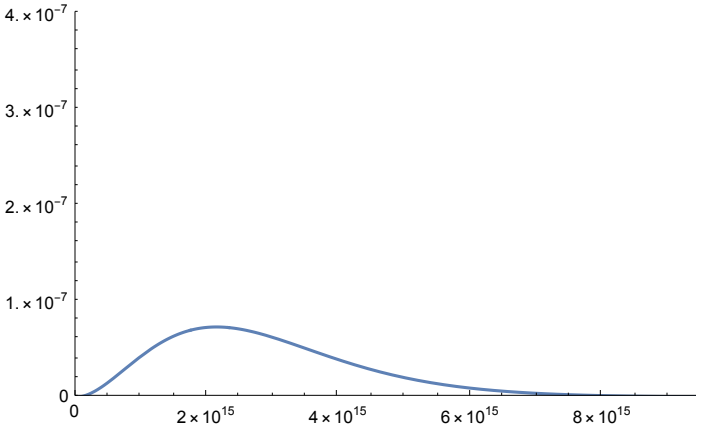
hbar = 6.626 × 10^-34 / (2 Pi);
orange = 3.14 × 10^15;
ktee = 5777 * 1.381 × 10^-23;
c = 3 × 10^8;
function[ω_] := hbar ω^3 / Pi^2 / c^2 / (Exp[hbar ω / ktee] - 1);
Plot[function[ω], {ω, 0, 3 orange}, PlotRange → {{0, 3 orange}, {0, 4 × 10^-7}}]
```

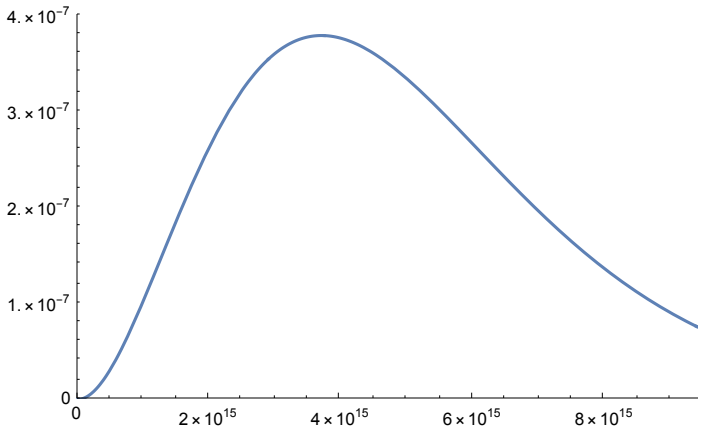
Out[133]=



(f) Repeat (e) for a star of 10000K.

In[126]:=
```
ktee = 10 000 * 1.381 × 10^-23;
Plot[function[ω], {ω, 0, 3 orange}, PlotRange → {{0, 3 orange}, {0, 4 × 10^-7}}]
```

Out[127]=



(g) Repate (e) for a star of 3000K.

In[134]:=
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ktee = 3000 * 1.381 × 10^-23;
Plot[function[ω], {ω, 0, 3 orange}, PlotRange → {{0, 3 orange}, {0, 4 × 10^-7}}]
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Out[135]=