Cosmology — Assignment 4 — Lorentz Transformations, The Torus
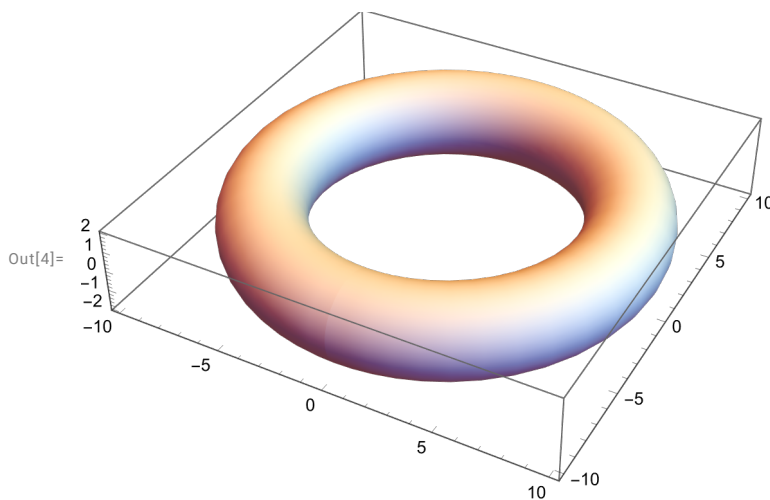
1. L-6(a) — Angled Meter Stick (write up what we started in class)

2. L-8 — Angled Velocity (use the authors' hint)

3. L-9 — The Headlight Effect

4. The Torus

In[4]:= `Graphics3D[ResourceFunction["Torus"][{0, 0, 0}, {8, 2}, {0, 0, 1}], Axes → True]`

Out[4]=

A torus has two radii. Let's call them R and r. Locating a place on the surface of the torus requires two angles, ϕ, and θ. Let's have ϕ be the angle that takes you around the big radius, R, and θ be the angle that takes you around the little radius r.

(a) It will probably be necessary to draw a cross-section of the torus to figure this out, like our authors did in Chapter 2, Figure 7

(b) How far from the axis of the torus is a point?

(c) Use your answer to (b) to determine how much distance is swept out when ϕ changes by $\Delta\phi$.

(d) How much distance is swept out when θ changes by $\Delta\theta$?

(e) Use your answers to (c) and (d) to write down the metric for this embedding of the 2-d torus in 3-d Euclidean space.