Newton — Problem Set 2 — Solution

1. Quantity of Motion

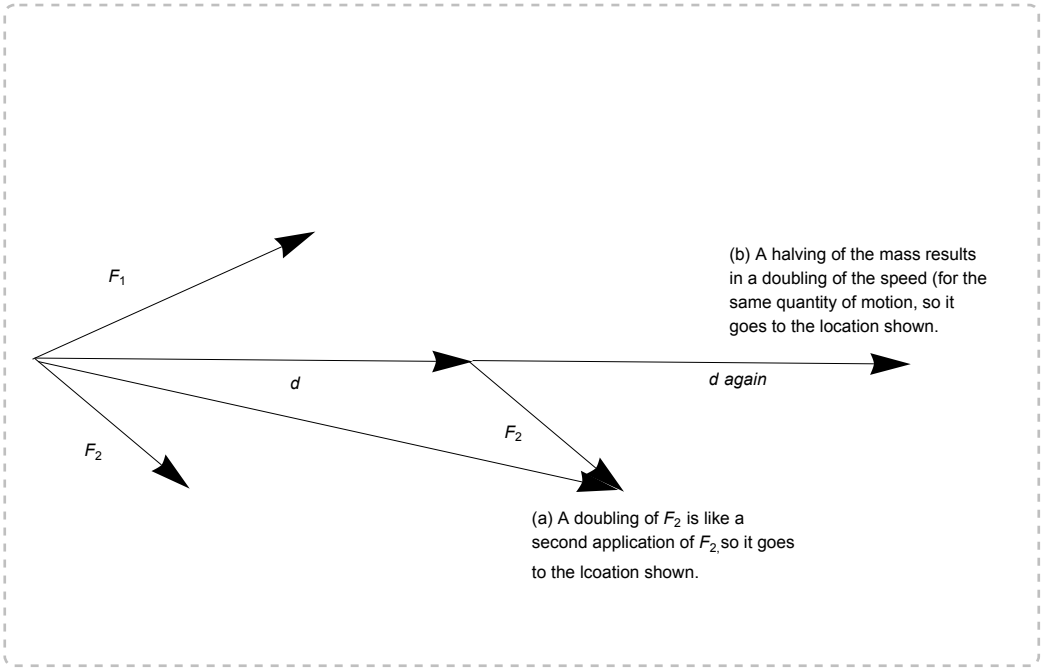
"The quantity of motion is the measure of the same arising from the velocity [or speed] and the quantity of matter [mass] conjointly."

The cloud's mass is 10x that of the droplets. The cloud's speed is 1/5 that of the droplets. Since quantity of motion is the compound of mass and speed, the cloud's quantity of motion is 2x that of droplets. They are also going different directions. The cloud is going westward. The droplets are going downward. We still don't know whether Newton (or his translator) intends to make a significant discussion between velocity and speed, so I am still using these terms interchangably.

2. The Three Laws of Motion

(a) The change in the SUV's quantity of motion in the eastward direction is its mass times 50 muff - 60 muff. So it is its mass times -10 muff.
(b) The change in the SUV's quantify of motion in the northward direction is its mass times -20 muff - 0 muff. So it is its mass times -20 muff.
(c) Change in motion is caused by equal and opposite impressed forces. The change in motion of the hay truck must be opposite of the SUV, so it is the SUV's mass times 10 muff.
direction.
(d) Similarly, it has motion northward that is the SUV's mass times 20 muff.
(e) It has gained SUV mass times 10 muff. But this is the same as its mass times 1/2 muff. However, it was going westward. So a gain in the eastward direction reduces the 60 muff speed to 59.5 muff.

3. Addition of Forces



(a) What motion will result if the impressed force F_2 is doubled in magnitude?

4. Center of Mass

(a) 0 A.U. x 2 × 10^{30} + 5.2 A.U. x 1.9 × 10^{27}kg / (2 × 10^{30}kg + 1.9 × 10^{27}kg) = 0.0049 A.U. along the line.

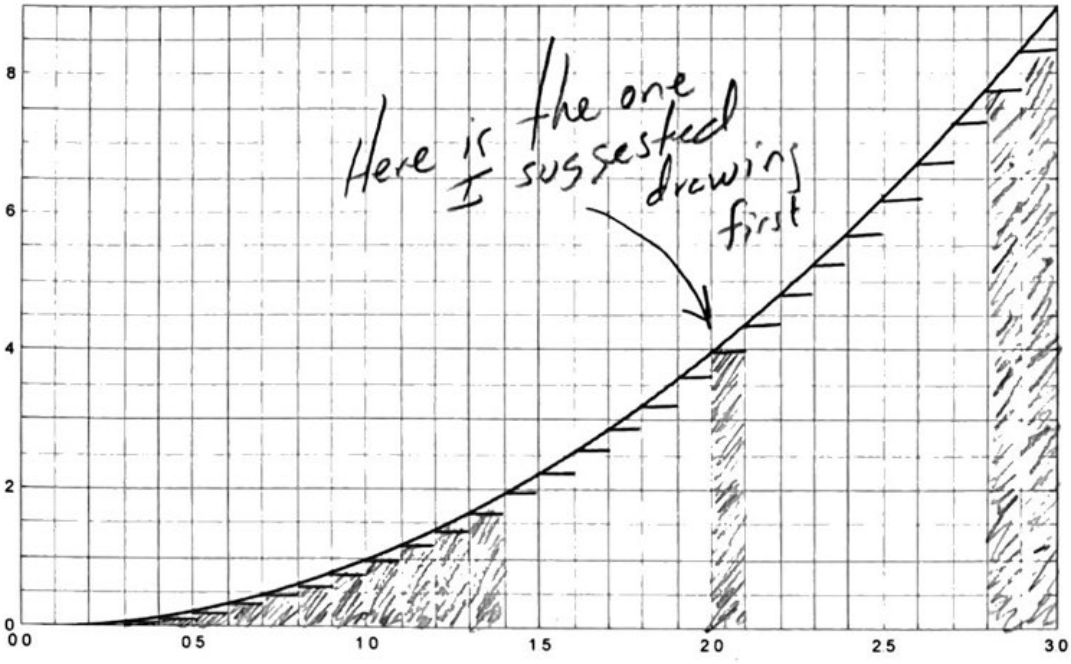
(b) Suppose the Jupiter was moving further from Sun such that after 1 year it was now at 6 A.U. During this time it goes 1/12 of the way around the Sun. Draw and describe the motion of the center of mass of Sun and the Jupiter.

5. Area Under a Curve

(a) Each little square is 0.1 wide and 0.5 tall, so each square represents 0.05 units of area. I counted something in the low 50s when I did it. I know that the right answer is 60 see part (d).

(b) Draw the quadrilaterals that fit under this curve. The first three or so will be impossibly scrunched, but after that it is doable.

(b)



(c) These have width Δx

They have height $(k\Delta x)^2$

Their area is $k^2(\Delta x)^3$

$$\sum_{k=0}^{n-1} k^2(\Delta x)^3 = (\Delta x)^3 \sum_{k=0}^{n-1} k^2$$

↑ this is a number independent of k, so it can be factored out of the sum

$$= \left(\frac{3}{n}\right)^3 \frac{(n-1)(n)(2n-1)}{6} = \frac{9}{2}\frac{1}{n^2}(2n^2-3n+1)$$

(d) $\lim_{n\to\infty}$ of this → only $\frac{9}{2}\frac{2n^2}{n^2}$ term survives

limit is 9. $\frac{9}{0.05} = 180$ ← best answer for part (a)