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Numerical Examples for Day 1
Ratios and Speeds

1. An illustration of ratios (à la Euclid Book V, definition 5)

Consider four quantities

$2\sqrt{7}$	rabbits	– the first
2	rabbits	– the second
$4\sqrt{7}$	apples	– the third
4	apples	– the fourth

as modern readers we have no trouble just saying these are both ratios of $\sqrt{7}$

How does Euclid say tells us to test whether these are the same ratio.

As a hint, which continues in the spirit of definition 5, consider the equimultiple of the first and third to be $\sqrt{7}$ and the equimultiple of the second and fourth to be 2. Or take the equimultiple of the first and third to again be $\sqrt{7}$, but this time the equimultiple of the second and fourth to be 3.

Bonus consideration for modern readers: what is the interpretation of the ratio of the third to the first! I think Euclid would not want us to consider this!

2. An Application of Galileo's Theorem III, Proposition III

for modern readers, we have no trouble dividing a unit of distance by a unit of time to get a unit of speed or velocity, for example $\text{mph} \equiv \frac{\text{miles}}{\text{hour}}$. I think Galileo, like Euclid, does not want us to consider such mixed ratios. So let us introduce a new, primitive, and nondecomposable unit of speed, the muff, so we are not tempted to start using modern methods in our illustration.

Please don't be modern!

A modern reader upon encountering the example below would do

$$v = 60\,\text{mph} = 60\,\frac{\text{miles}}{\text{hour}} = \frac{60\ \text{miles}}{3600\ \text{seconds}} = \frac{1\ \text{mile}}{60\ \text{seconds}}$$

and use $t = \frac{s}{v}$

Car 1 sets off from Lone Pine to Bishop at 60 muff. Car 2 sets off at 45 muff.

What is the ratio of the speed of Car 2 to Car 1? Simplify it.

What is inverse of this ratio?

We observe that Car 1 gets to the first mile marker in 60 seconds.

According to Theorem III, Proposition III, when will we observe Car 2 reaching the first mile marker?