

# Day 2 — Ben and Brian's Presentation of Th./Prop. IV

First, an example

An Earth-year is 365 days. A Mars-year (measured in Earth's days) is 687 days.

Earth goes around the Sun at a speed of 67,000 ~~mph~~ muff ← I CHANGED THE UNITS TO

MUFF SO THAT YOU ARE NOT  
TEMPTED TO USE 1 DAY = 24 HRS  
TO GET THE DISTANCE  
TRAVELED  $\cancel{\text{mph}} = \frac{\text{miles}}{\text{hour}} = \frac{\text{miles}}{\cancel{\text{hour}}} \cancel{24 \text{ hours}}$   
 $= \frac{\text{miles}}{\text{day}} - 24$

Mars goes around  
the Sun at  
54,000 muff

Using compound ratios, as formulated  
in Th./Prop. IV, what is the  
ratio of distance traveled by  
Earth in an Earth-year VS.  
the distance traveled by Mars in a  
Mars-year?

$$d_E : d_M :: \Delta t_E : \Delta t_M \times V_E : V_M$$

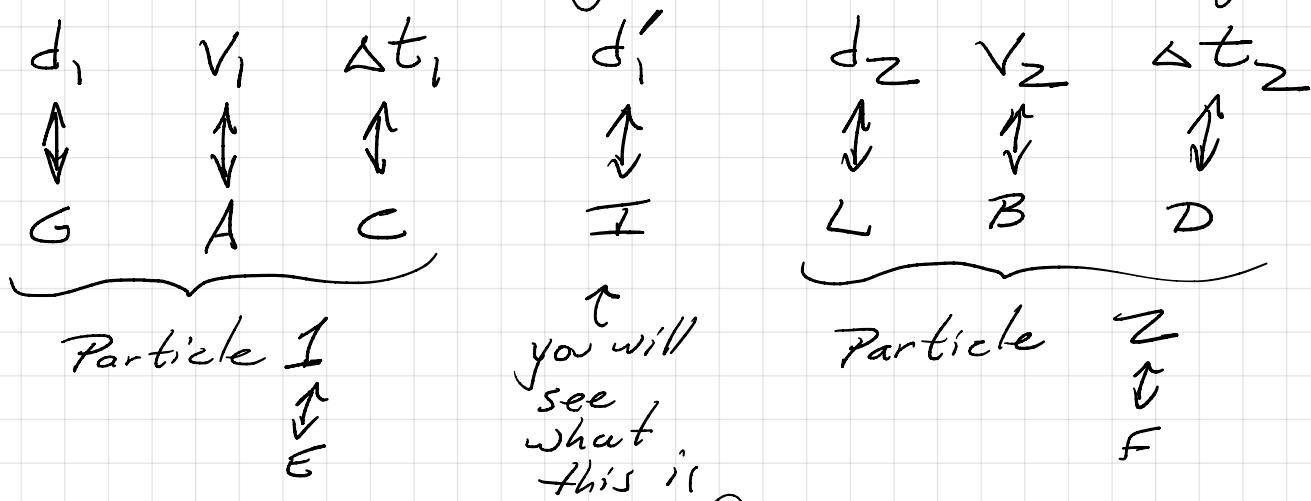
$$= 365 : 687 \times 67,000 : 54,000$$

$$\approx 2 : 3$$

NB:  
THIS EXAMPLE  
IS OF CIRCULAR  
MOTION, NOT STRAIGHT-LINE  
MOTION, SO IT ISN'T QUITE  
WHAT GALILEO  
HAS IN MIND.

# Now the Proof

Instead of Galileo's unmemorable letters, we make the following more memorable assignments



Particle 1 goes  $v_1$  for  $\Delta t_1$ , resulting in  $d_1$

Particle 2 goes  $v_2$  for  $\Delta t_2$ , resulting in  $d_2$

The proof has us consider, what if

Particle 1 went  $v_2$  for  $\Delta t_1$ , resulting in  $d'_1$ ?

In that case

$$d_1 : d'_1 :: v_1 : v_2 \leftarrow \text{because time is unchanged}$$

We also know

$$d'_1 : d_2 :: \Delta t_1 : \Delta t_2 \leftarrow \text{because in this comparison, the speed is unchanged}$$

However

$$d_1 : d_2 :: d_1 : d'_1 \times d'_1 : d_2$$

$$\therefore v_1 : v_2 \times \Delta t_1 : \Delta t_2 \quad \text{Q.E.D.}$$

Perhaps this diagram makes Galileo's intermediate step clearer

makes Galileo's  
clearer

$\Delta t_1$  C  
V<sub>1</sub> A  
SAME

d<sub>1</sub> G

$\Delta t_1$  C  
V<sub>2</sub> B  
SAME

d<sub>1</sub> T

$\Delta t_2$  D  
V<sub>2</sub> B

d<sub>2</sub> L

In other words,  
by introducing  
d<sub>1</sub> he  
is able  
to change  
just one  
quantity  
at a time