SECOND DEFINITIONS

9. Let the midpoint of the diameter of both the hyperbola and the ellipse be called the center of the section, and let the straight line drawn from the center to meet the section be called the radius of the section.

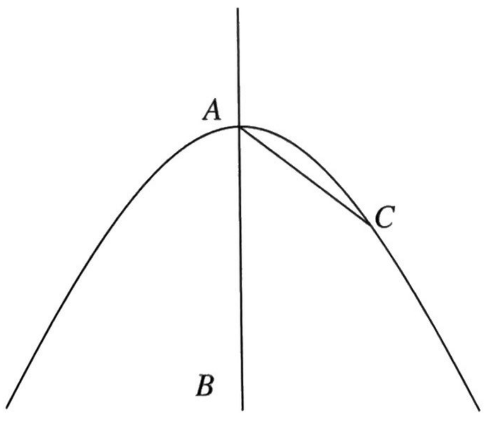
10. And likewise let the midpoint of the transverse side of the opposite sections be called the center.

11. And let the straight line drawn from the center parallel to an ordinate, being a mean proportional to the sides of the figure (εἶδος) and bisected by the center, be called the second diameter.

PROPOSITION 17

If in a section of a cone a straight line is drawn from the vertex of the line, and parallel to an ordinate, it will fall outside the section (cf. Eucl. III. 16).

Let there be a section of a cone, whose diameter is the straight line *AB*.

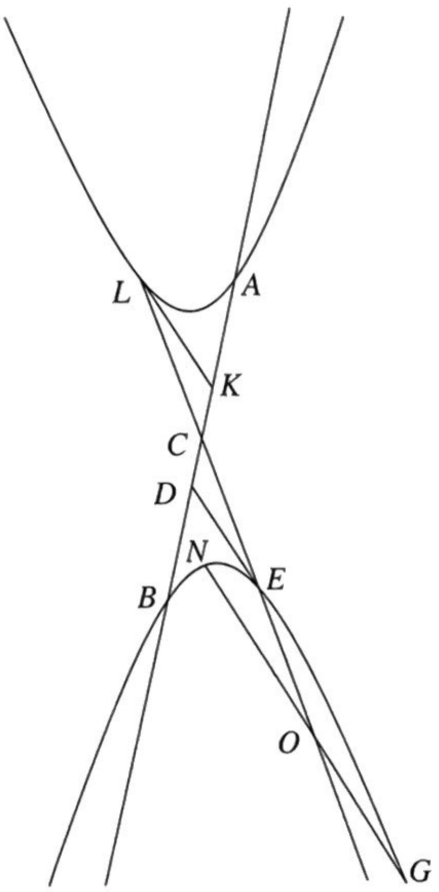


I say that the straight line drawn from the vertex, that is, from the point *A*, parallel to an ordinate, will fall outside the section.

For if possible, let it fall within as *AC*. Since then a point *C* has been taken at random on a section of a cone, therefore the straight line drawn from the point *C* within the section parallel to an ordinate will meet the diameter *AB* and will be bisected by it (I. 7). Therefore the straight line *AC* produced will be bisected by the straight line *AB*. And this is absurd. For the straight line *AC*, if produced, will fall outside the section (I. 10). Therefore the straight line drawn from the point *A* parallel to an ordinate will not fall within the line; therefore it will fall outside; and therefore it is tangent to the section.

PROPOSITION 48

If a straight line touching one of the opposite sections meets the diameter, and if through the point of contact and the center a straight line produced cuts the other section, then whatever line is drawn in the other section parallel to the tangent, will be bisected by the straight line produced.



Let there be opposite sections whose diameter is the straight line AB and center C, and let the straight line KL touch the section A, and let the straight line LC be joined and produced (I. 29), and let some point N be taken on the section B, and through N let the straight line NG be drawn parallel to the straight line LK.

I say that

$$NO = OG.$$

For let the straight line ED be drawn through E tangent to the section; therefore ED is parallel to LK (I. 44, note). And so also to NG. Since then BNG is an hyperbola whose center is C and tangent DE, and since CE has been joined and a point N has been taken on the section and through it NG has been drawn parallel to DE, by a theorem already shown (I. 47) for the hyperbola

$$NO = OG.$$

PROPOSITION 49

If a straight line touching a parabola meets the diameter, and if through the point of contact a parallel to the diameter is drawn, and if from the vertex a straight line is drawn parallel to an ordinate, and if it is contrived that as the segment of the tangent between the [ordinatewise] erected straight line

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and the point of contact is to the segment of the parallel between the point of contact and the [ordinatewise] erected straight line, so is some [found] straight line to the double of the tangent, then whatever straight line is drawn [parallel to the tangent] from the section to the straight line drawn through the point of contact parallel to the diameter, will equal in square the rectangle contained by the straight line found [i.e., the upright side] and by the straight line cut off by it [i.e., the line parallel to the tangent] from the point of contact.

Let there be a parabola whose diameter is the straight line *MBC*, and *CD* its tangent, and through *D* let the straight line *FDN* be drawn parallel to the straight line *BC*, and let the straight line *FB* be erected ordinatewise (I. 17), and let it be contrived that

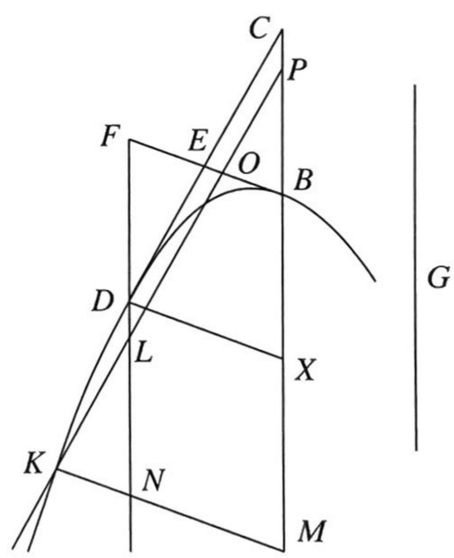
$$ED : DF :: \text{some straight line } G : 2CD,$$

and let some point *K* be taken on the section, and let the straight line *KLP* be drawn through *K* parallel to *CD*.

I say that

$$\text{sq. KL} = \text{rect. } G, DL;$$

that is that, with the straight line *DL* as diameter, the straight line *G* is the upright side.



For let the straight lines *DX* and *KNM* be dropped ordinatewise. And since the straight line *CD* touches the section, and the straight line *DX* has been dropped ordinatewise, then

$$CB = BX \text{ (I. 35).}$$

But

$$BX = FD$$

And therefore

$$CB = FD.$$

And so also

$$\text{trgl. } ECB = \text{trgl. } EFD.$$

Let the common figure *DEBMN* be added; therefore

$$\text{quadr. } DCMN = \text{pllg. } FM$$
$$= \text{trgl. } KPM \text{ (I. 42).}$$

Let the common quadrilateral *LPMN* be subtracted; therefore the remainders

$$\text{trgl. } KLN = \text{pllg. } LC.$$

And

$$\text{angle } DLP = \text{angle } KLN;$$

therefore

$$\text{rect. } KL, LN = 2 \text{ rect. } LD, DC.^*$$

And since

$$ED : DF :: G : 2CD,$$

and

$$ED : DF :: KL : LN,$$

therefore also

$$G : 2CD :: KL : LN.$$

But

$$KL : LN :: \text{sq. } KL : \text{rect. } KL, LN,$$

and

$$G : 2CD :: \text{rect. } G, DL : 2 \text{ rect. } LD, DC;$$

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* Eutocius, commenting, says: "For let the triangle KLN and the parallelogram $DLPC$ be set out. And since

$$\text{trgl. } KLN = \text{pllg. } DP,$$

let the straight line NR be drawn through N parallel to LK, and through K, KR parallel to LN; therefore LR is a parallelogram and

$$\text{pllg. } LR = 2 \text{ trgl. } KLN;$$

and so also

$$\text{pllg. } LR = 2 \text{ pllg. } DP.$$

Then let the straight lines DC and LP be produced to S and T, and let CS be made equal to DC, and PT to LP, and let ST be joined; therefore

$$\text{pllg. } DT = 2 \text{ pllg. } DP;$$

and so

$$\text{pllg. } LR = \text{pllg. } LS.$$

But it is also equiangular with it because of the angles at L being vertical; but in equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional; therefore

$$KL : LT \text{ or } DS :: DL : LN,$$

and

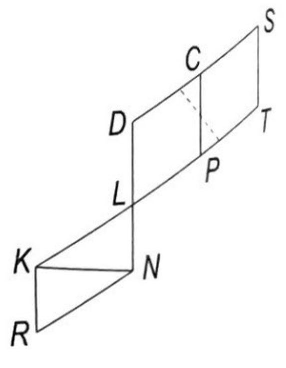
$$\text{rect. } KL, LN = \text{rect. } LD, DS.$$

And since

$$DS = 2DC,$$

hence

$$\text{rect. } KL, LN = 2 \text{ rect. } LD, DC.$$



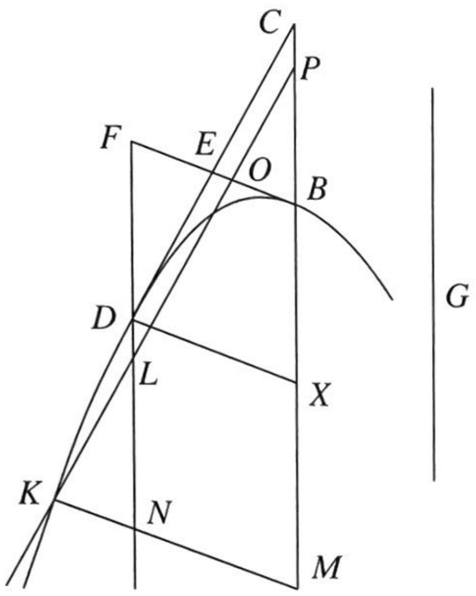
"And if DC is parallel to LP, and CP is not parallel to LD, it is clear $DCPL$ is a trapezoid, and so I say that

$$\text{rect. } KL, LN = \text{rect. } DL, (CD + LP).$$

For if LP is filled out, as we have said before, and the straight lines DC and LP are produced, and CS is made equal to LP, and PT to DC, and the straight line ST is joined, then

$$\text{pllg. } DT = 2DP,$$

and the same demonstration will fit. And this will be useful in what follows (I. 50)." (Tr.)



therefore

$$\text{sq. } KL : \text{rect. } KL, LN :: \text{rect. } G, DL : 2 \text{ rect. } CD, DL.$$

And alternately; but

$$\text{rect. } KL, LN = 2 \text{ rect. } CD, DL;$$

therefore also

$$\text{sq. } KL = \text{rect. } G, DL.$$

PROPOSITION 50

If a straight line touching an hyperbola or ellipse or circumference of a circle meets the diameter, and if a straight line is produced through the point of contact and the center, and if from the vertex a straight line erected parallel to an ordinate meets the straight line drawn through the point of contact and the center, and if it is contrived that as the segment of the tangent between the point of contact and the straight line erected [ordinatewise from the vertex] is to the segment of the straight line drawn through the point of contact and the center—[the segment] between the point of contact and the straight line erected [ordinatewise from the vertex]—so some [found] straight line is to the double of the tangent, then any straight line parallel to the tangent and drawn from the section to the straight line drawn through the point of contact and the center will equal in square a rectangular area applied to the straight line found, having as breadth the straight line cut off [of the diameter] by it [i.e., the ordinatewise line] from the point of contact,