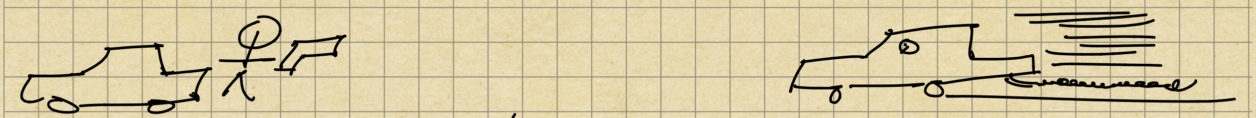
Loops and Orbits – Week 1 – Day 1 – Physics & Math

Sloppy terminology we will *not* use, but let's review it anyway.

$$s \equiv \frac{d}{t}$$ "speed is distance over time"

Notice the triple equals. That means this is the <u>definition</u> of speed

The definition of speed can be made precise. Let's look at one aspect of the definition of speed that is problematic:

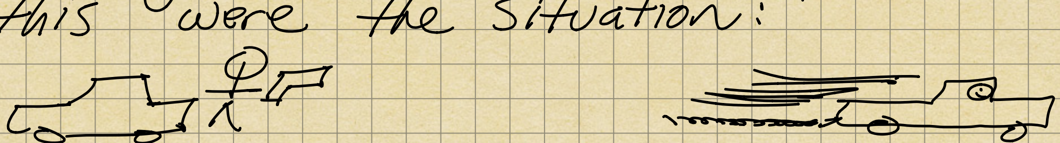


parked cop with radar gun

you, going 60 in a 55 zone

The cop says, "I got you going 60 in a 55 zone."

You say, "that would be true, officer, if this were the situation:"



"But I was going toward you, so my speed was negative 60 and negative 60 is less than 55!"

Velocity

On your day in court the judge explains that an accurate definition of speed does not depend on direction, and that you were confusing speed with velocity. Here is the definition of velocity:

$$V \equiv \frac{\Delta x}{\Delta t}$$

(After this bruising experience you decide you will always refer to and use velocity, not speed.)

The triangles are capital Greek letter deltas. We read this equation out loud or in our heads as:

"velocity is (by definition) delta x over delta t."

Two things that we'll get more precise about <u>later</u>:

(1) This is just one equation of 2 or 3 if the movement can be in 2 or 3 dimensions.

(2) If you are accelerating (speeding up, slowing down, or turning) then we have defined "average velocity" not velocity.

Two things that we'll get more precise about now:

(1) Δx read "delta x" or "the change in x" is

$$\Delta x \equiv x_{after} - x_{before}$$

x is the value on a coordinate axis, which in scientific work is usually measured in meters.

(2) $$\Delta t \equiv t_{after} - t_{before}$$

Δt is read "delta t" or "the change in t," and it is the elapsed time.

The before time and the after time can be any two times, as long as the before position and the after position are the positions that correspond to those times.

In scientific work time is almost always measured in seconds. The definitions of v, Δx, and Δt don't change, even if you are working in everyday units such as miles and hours. $\Longrightarrow$ DO LAO-1-1-WS1

Position from Velocity!?

At this point, you have definitions for Δx, Δt, and v. You've done LA0-1-1-WS1. Let's take one row out of the table you computed.

$$v_{5\to6} \equiv \frac{x_6 - x_5}{t_6 - t_5} = \frac{63m - 54m}{3744s - 3743s} = 9\frac{m}{s}$$

Let's rearrange the algebra

$$(t_6 - t_5)\, v_{5\to6} = x_6 - x_5$$

$$x_6 = x_5 + (t_6 - t_5)\, v_{5\to6}$$

That says we can get x_6 from x_5 and $v_{5\to6}$!

It of course didn't matter that it was positions 5 and 6 we were considering. We also have

$$x_5 = x_4 + (t_5 - t_4)\, v_{4\to5}$$

By substituting the circled stuff in for x_5 you can see that we can actually get x_6 from x_4!

This just keeps going. You can get x_6 from x_0 if you also know

$v_{0\to1}$, $v_{1\to2}$, $v_{2\to3}$, $v_{3\to4}$, $v_{4\to5}$, and $v_{5\to6}$

It's going to get tiring for me to write out all these examples. So we need some notation. We write

$$x_{i+1} = x_i + (t_{i+1} - t_i)\, v_{i\to i+1}$$

Put $i=5$ into this equation. Do you see that you get the equation for x_6? Put $i=4$ into the equation. Do you see that you get the equation for x_5?

$\Longrightarrow$ Do LAO-1-1-WS2

Maybe it is not a surprise that the addition and multiplication that gets us x_6 from x_5 "undoes" the subtraction and division that got us $v_{5\to6}$ from x_6 and x_5.