# Loops and Orbits - Week 1 - Day 2 - Physics & Math

## Position from Velocity!?

At this point, you have definitions for $\Delta x$, $\Delta t$, and $v$. You've done LAO-1-1-WS1. Let's take one row out of the table you computed.

$$v_{5\to 6} \equiv \frac{x_6 - x_5}{t_6 - t_5} = \frac{63m - 54m}{3744s - 3743s} = 9\frac{m}{s}$$

Let's rearrange the algebra

$$(t_6 - t_5)\, v_{5\to 6} = x_6 - x_5$$

$$x_6 = x_5 + (t_6 - t_5)\, v_{5\to 6}$$

That says we can get $x_6$ from $x_5$ and $v_{5\to 6}$!

If of course didn't matter that it was positions 5 and 6 we were considering. We also have

$$x_5 = x_4 + (t_5 - t_4)\, v_{4\to 5}$$

By substituting the circled stuff in for $x_5$ you can see that we can actually get $x_6$ from $x_4$! This just keeps going. You can get $x_6$ from $x_0$ if you know $v_{0\to1}$, $v_{1\to2}$, $v_{2\to3}$, $v_{3\to4}$, $v_{4\to5}$, and $v_{5\to6}$

## Indices

It's going to get tiring for me to write out all these examples. So we need some notation. We write

$$x_{i+1} = x_i + (t_{i+1} - t_i) v_{i\to i+1}$$

Put $i=5$ into this equation. Do you see that you get the equation for $x_6$? Put $i=4$ into the equation. Do you see that you get the equation for $x_5$?

$\Longrightarrow$ Do LAO-1-1-WS2

Worksheet 2 passed out Week 1 Day 1

We have learned that you can get $x_{10}$ in 10 steps from $x_0$

$x_{10}$: 105m in our example

$x_0$: 27m in our example

I'm going to write out the whole formula:

$$
\begin{aligned}
x_{10} = x_0 &+ (t_1 - t_0)\, v_{0\to 1} \\
&+ (t_2 - t_1)\, v_{1\to 2} \\
&+ (t_3 - t_2)\, v_{2\to 3} \\
&+ (t_4 - t_3)\, v_{3\to 4} \\
&+ (t_5 - t_4)\, v_{4\to 5} \\
&+ (t_6 - t_5)\, v_{5\to 6} \\
&+ (t_7 - t_6)\, v_{6\to 7} \\
&+ (t_8 - t_7)\, v_{7\to 8} \\
&+ (t_9 - t_8)\, v_{8\to 9} \\
&+ (t_{10} - t_9)\, v_{9\to 10}
\end{aligned}
$$

## Sums

We need some mathematical notation so I don't have to write out stuff like that.

$$x_{10} = x_0 + \sum_{i=0}^{10} (t_{i+1} - t_i)\, v_{i \to i+1}$$

↑ capital $\Sigma$ is the summation symbol (↑ sigma)

## Looking Back

We defined velocities by taking differences of positions and then dividing by elapsed time

$$v_{5 \to 6} \equiv \frac{x_6 - x_5}{t_6 - t_5}$$

Maybe it isn't a surprise that we can undo subtraction and division by doing addition and multiplication:

$$x_6 = x_5 + (t_6 - t_5)\, v_{5 \to 6}$$

Since everyone that signed up is from STEM, maybe it won't scare you to know that the subtractions and divisions are derivatives, and the additions and multiplications are integrals.