Manhattan Project - Term 2 Sample Exam

October 4, 2020 — Covering the technical material in Reed Chs. 2 and 3

This might seem hard or long, but the many parts in each problem are designed to break them down into doable steps, so don't be intimidated, and you'll get through. I welcome feedback on the appropriateness of the length or difficulty. Sometimes these things are hard to gauge.

Energy, Molarity, Density

1. Uranium Oxide

The chemical formula for the most common oxide of Uranium is U_3O_8, and the molecular weight is 286.027 grams per mol. This is called yellowcake. Rounding the molecular weight to only one sig fig, 300 g/mol, and rounding Avogadro's number, N_A to 6×10^{23} atoms/mol, so that you can readily do the problem without a calculator, answer:

a. How many molecules of uranium oxide are in a 1 kg sample of yellowcake?

b. How many atoms of uranium are in this sample?

c. If, upon fission, each atom yields 200 MeV, how many MeV could be released if all these Uranium atoms were fissioned?

d. Convert this answer to Joules. The charge on an electron is 1.6×10^{-19}C. That should be enough information to allow you to complete the conversion.

2. Density

Imagine that U_3O_8 is laid out as a cubic lattice with lattice spacing L. Each molecule therefore occupies L^3 of space.

a. Write a formula giving the mass M, of the sample in terms of the number of mols in the sample, n, the number of atoms per mol, N_A, and the molecular weight, m_W (usually in grams / mol, but since you aren't going to plug in numbers, the units don't matter much).

b. Write a formula giving the volume V of the sample in terms of, n, N_A, and L.

c. Use your results from (a) and (b) and the definition of density, $\rho \equiv \frac{M}{V}$, to write down an equation for

ρ. Solve the equation for L. Congratulations, you have a formula for estimating the size of the $U_3 O_8$ molecule!

Radioactivity, Coulomb Barrier, Cross Sections

3. Natural Radioactivity and Half-Life

Carbon-14 is rare and is produced by cosmic ray showers. When something is buried, its Carbon-14 content is not replenished and decays away. The symbol for Carbon-14 is ${}^{14}_{6}C$.

a. How many protons, Z, how many neutrons, N, and how many nucleons, A, does Carbon-14 have?

b. Carbon-14 decays with β^- decay (it emits an electron). How many protons, Z, how many neutrons, N, and how many nucleons, A, does the resulting nucleus have after Carbon-14 beta decays.

c. The half-life for Carbon-14 beta decay is 5730 years. If you start with 1000 Carbon-14 atoms, after 17,190 years, on average, how many will be left? (HINT: 3 * 5730 = 17,190).

4. Coulomb Barrier

Assume the radius of a nucleus with Z protons (charge Ze) and A nucleons is given by $a_0 A^{1/3}$.

a. Use Coulomb's formula for the energy between two charges that have been brought to a distance d from each other

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d}$$

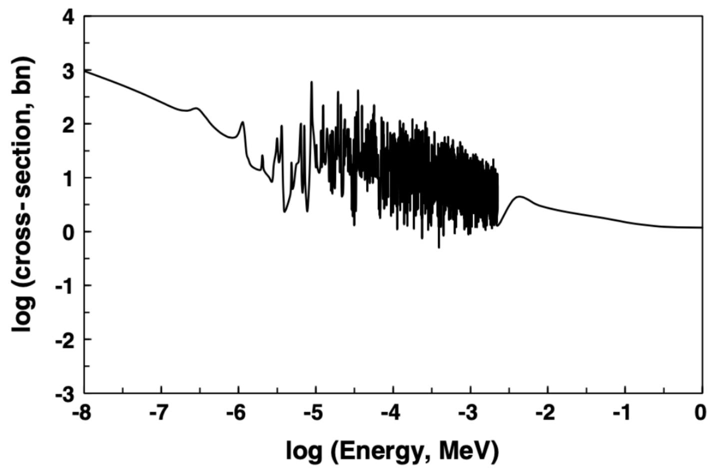
and write down an expression for the energy required to bring a single proton with charge e to the surface of this nucleus.

b. For part b., I might have you plug in some numbers to what you got in a., but the exam already feels kind of long to me, so maybe we'll skip that.

c. What is the Coulomb energy required to bring a single neutron to the surface of this nucleus?

5. Cross Sections, Log-Log Plots

Consult the following graph showing the fission cross-section for neutrons hitting U-235:



a. For 0.1 eV neutrons (careful, not for 0.1 MeV neutrons), what is the log of the energy in MeV?

b. About what would you estimate the log of the cross-section in barns as? (E.g., 0.5, 1.0, 1.5, 2.0, 2.5, etc.)

c. Using your estimate in b., what is this close to? (E.g., 1 barn, 3 barns, 10 barns, 30 barns, 100 barns, 300 barns, etc.)

CONGRATS — Looking backward

Show this practice exam to anyone that is not in our class. Ask them if they are impressed by the science you have gotten used to working with in the prior 5 weeks. Unless they are daffy, they will be thoroughly impressed, and I hope you can see it through their eyes and realize how far our class has come in becoming familiar with the science the Manhattan Project is founded on.