Manhattan Project - Term 3 Exam

November 8, 2020 — Covering the technical material in Reed Chs. 5 to 7

Do not feel time pressure. We will try to hold it to 45 minutes, but not cut people off if they need a little more time to check their math.

As with the last exam, I made the numbers round, and it shouldn't be necessary to use a calculator. A goal of the course is for you to all become good at throwing around astronomic and microscopic numbers without reaching for a calculator.

There may be enough space on the exam to do the work and answer. Take your pick on whether you'd rather work on your own paper.

Specific Heat

1. Reactor Meltdown

One day, the coolant in a reactor producing 1100 MW stops flowing. Whatever stopped the coolant also stopped many of the control rods from working, and despite yours and the other operators' best efforts, the reactor continues to produce 100 MW of heat. The temperature of the fuel pellets in the fuel rods will now start rising.

This reactor contains 18 million uranium dioxide fuel pellets. Each pellet is 1cm long and 1cm in diameter, and is therefore $\frac{\pi}{4}\,\mathrm{cm}^3$ in volume. The density of uranium dioxide is $11\,\mathrm{g}/\mathrm{cm}^3$. Let's be kind to ourselves and do some rounding. We will round each pellet's mass to a nice tidy 10 grams and we will round the 18 million pellets to 20 million. That's actually going to make our calculation a bit optimistic, but it's a crude calculation anyway.

(a) If the reactor is still producing power (heat) at the rate of 100 MW, how much is each of the 20 million pellets being heated (your answer will be in Watts, or Joules / second — please include units).

(b) The specific heat C of the uranium dioxide pellets at reasonable temperatures is (on average in the range of interest) about 0.3 Joules/gram/°K. Using just algebra rather than plugging in any numbers, and starting with the definition of specific heat,

$$C \equiv \frac{\Delta E}{m \Delta T}$$

divide numerator and denominator by Δt where Δt is an arbitrary amount of time, and then use that power, $P \equiv \frac{\Delta E}{\Delta t}$, to simplify the numerator. You now have an equation involving P, m, ΔT, C, and Δt. Solve this equation for Δt.

(c) The uranium dioxide pellets are clad in zirconium, which melts at 1855 °K. To keep making things round, let's assume the reactor started at 355°K which is a modest temperature. The uranium pellets can therefore rise by ΔT = 1500 °K before they melt the zirconium. Plug in to the formula you got in (b) to find out how long this takes. HINT: Be careful to carry through and cancel off your units as a satisfying cross-check on your algebra.

(d) You got an answer in seconds. Convert it to minutes. How long do you tell your neighbors they have to get in the car before the meltdown. Or to put it differently, how long do you and your fellow operators have to get the cooling system working?

Radiation Fatalities

2. What Were the Pilots Exposed To?

An empirical formula for the radius of radiation fatalities is

$$r_{\text{lethal}} = Y^{0.19}$$

We are going to pick the numbers so that we have a shot at getting an answer without a calculator.

First we'll round the empirical formula:

$r_{\text{plane}} = Y^{0.2} = Y^{1/5}$

In other words, we've rounded it so that r_{lethal} is the fifth root of Y.

For this formula to work, Y has to be expressed as a multiple of 2.5kt. r_{lethal} will then be in km. This is an empirical formula, not a fundamental one, and you can't make much sense of the units except to follow the recipe as given.

(a) If Y were 80kt, expressed as a multiple of 2.5kt, your value for Y is? (Just divide Y by 2.5, nothing fancy here.)

(b) Then calculate the 5th root of what you got in (a). This is the lethal radius in km.

(c) If the pilots got to 20km before the blast hit them, what multiple of the lethal radius is this?

$r_{\text{pilots}} = X\, r_{\text{lethal}}$

(d) A very pessimistic and naive view of the way radiation travels in air says that it simply weakens with distance as $\frac{1}{r^2}$. The formula in Reed says it actually weakens far faster with distance. Faster than $\frac{1}{r^7}$. Using the multiple you found in (c) and $\frac{1}{r^7}$ as the rate of weakening with distance, what fraction of the lethal radiation dose did the pilots get? The lethal dose is 600 rems. What do you estimate the their dose as?

HINT: It's *not* $\frac{1}{10}$ as much or 60 rems.

Fallout

3. Fallout After Almost One Hour

As in Problem 7.9, we are going to start with as many atoms of Barium and Krypton as we started with atoms of Uranium, and the number or Uranium atoms was about 24×10^{23} atoms. But this time we are going to wait almost an hour, after which pretty much all of the Krypton will be gone, and instead the radiation of the fallout will now be coming from the Barium.

(a) If $t_{1/2}$ for Barium-141 is 18 minutes, how many Barium atoms are there after 54 minutes?

(b) The decay time $\tau \equiv \frac{t_{1/2}}{\ln 2}$. This is about 1500 seconds. Using that the decay rate is $\frac{N}{\tau}$ what is the rate of decay for the number of Barium atoms you found in part (a).

(c) Using that 1 Curie is 3.7×10^{10} fissions per second and rounding that to 4×10^{10} to as usual make life without a calculator easier, how many Curies of radiation are there after 54 minutes?

(d) Let's say that after the 54 minutes, the fallout has had time to spread to 20 square miles, which is about 5×10^{7} square meters. How many Curies per square meter are being suffered in this region?

NOTE: It was 1 million Curies per square meter in Problem 7.9. As you can see, avoiding being exposed for an hour helps a great deal.