# Manhattan Project — Assignment 7 — Hanford Reactors — Solution

## 1. Reed Problem 6.1, p. 269

My hint was to use that the area of a cylinder is:

$A = 2\,\pi R L$

An annulus has an inner radius, $R_i$ and an outer radius, $R_l$, but if those are so close in value to each other that we can treat the annulus as if it were a sheet of aluminum foil, that can be unrolled off the roll and laid out flat, then the volume of the annulus is:

$V = 2\,\pi R L (R_l - R_i)$

Which of $R_l$ or $R_i$ do you put in for $R$? It doesn't matter (at least not much), if the difference between them is small. In fact, let's compute the difference (using $R_l$ for $R$ vs. $R_i$ for $R$):

difference in $V$ calculated two ways $= 2\,\pi R_l L (R_l - R_i) - 2\,\pi R_i\, L (R_l - R_i) = 2\,\pi (R_l - R_i)\, R L (R_l - R_i)$

Ah! The difference in the two ways differs only by the something proportional to the thickness squared! What if we did the exact calculation? How much does that differ from $2\,\pi R_i\, L (R_l - R_i)$? The exact calculation would be:

$\pi R_l^2\, L - \pi R_i^2\, L = 2\,\pi L\, \frac{1}{2}\left(R_l^2 - R_i^2\right) = 2\,\pi L\, \frac{1}{2}(R_l + R_i)(R_l - R_i)$

Ah! So the exact answer is obtained by putting $R = \frac{1}{2}(R_l + R_i)$

## 2. Reed Problem 6.2

The definition of specific heat, $C$, is

$C \Delta T = E / M$

The specific heat depends on the material and on the temperature, and when a material goes through a phase transition (like ice to water at 32°F), the specific heat can't be defined at that temperature.

Anyway, we rearrange the definition to solve for the temperature rise and get:

$\Delta T = \frac{E}{C\,M}$

But the energy and mass are not given as amounts. They are given as rates. In a time $\Delta t$, the mass is $M = \text{r}\Delta t$. Similarly the energy is $E = P\Delta t$.

$$\Delta T = \frac{P\Delta t}{C\,r\Delta t}$$

That's nice! The $\Delta t$'s cancel (as they must), leaving:

$$\Delta T = \frac{P}{C\,r}$$

Now we can plug in:

$$\Delta T = \frac{250\text{ MJ / second}}{4187\text{ J/kg/K}\cdot 30{,}000\text{ gallons / minute}}$$

Oh my, there are lots of units conversions needed. For example, 1 gallon = 3.786 liters and 1 liter of water weighs 1 kg. Also there are sixty seconds in a minute, so

$$\Delta T = \frac{250\text{ MJ}\cdot 60}{4187\text{ J/K}\cdot 30{,}000\cdot 3.786} = \frac{250\cdot 10^6\cdot 60}{4187\cdot 3\cdot 10^4\cdot 3.786}\text{ K} = \frac{1\,500\,000}{4187\cdot 3\cdot 3.786} = 31.54\text{K}$$

This is a temperature rise. Multiply by 9/5 if you would rather know the temperature rise in Fahrenheit.

## 3. Reed Problem 6.3

The production rate of plutonium in a full load $64\,000\cdot 4\text{ kg} = 256\,000\text{ kg}$ of Uranium. In 100 days, the reactor produces 25000MW·days of energy. (There is no need to convert this to some more familiar unit like kWh.) We are also told that 0.76g/(MW·day) of Plutonium is produced. So we get 25000*0.76g=25*0.76kg=19kg of plutonium in 100 days.

So we got

$$\frac{19\text{ kg of Plutonium}}{256\,000\text{ kg of Uranium}}$$

Plutonium-239 and Uranium-238 (which is mostly what is in the reactor) weigh almost the same, but if we really want the ratio of the number of atoms, we should divide by the atomic weight of Plutonium and multiply by the atomic weight of Uranium:

$$\frac{19}{256\,000}\cdot\frac{238.05}{239.05} = 0.000074$$

Expressed as a ratio, this is about $\frac{1}{13\,530}$.