

Manhattan Project - Problem Set 6 - Solution

1. Reed Problem 5.1, p. 235 - Air-Cooled Reactor

Operating at power P in a time Δt the reactor produces $Q = P\Delta t$ of heat.

The volume of air flowing through the reactor in this same time, Δt , is $V = f\Delta t$ where f is the flow rate. This air has mass $m = \rho V = \rho f \Delta t$.

(Don't confuse Δt , a time, with ΔT , a temperature rise.)

Reed gives us a thermodynamic formula:

$$Q = mc\Delta T$$

So, $P\Delta t = pf\Delta t \approx c\Delta T$

The time interval cancels out and you are only left with rates (power is a rate and air flow, f , is a rate). Solve for ΔT :

$$\begin{aligned} \Delta T &= \frac{P}{PfC} \\ &= \frac{1 \text{ MJ}}{\cancel{1 \text{ MJ}} \cdot 14.16 \frac{\text{m}^3}{\text{sec}} \cdot \cancel{1000 \text{ J}}} \\ &= \frac{1000}{14.16} \text{ K} = 70.6 \text{ K} \end{aligned}$$

$$\begin{aligned} P &= 1 \text{ MW} = \frac{1 \text{ MJ}}{\text{sec}} \\ \rho &= \frac{1 \text{ kg}}{\text{m}^3} \\ C &= \frac{1000 \text{ J}}{\text{kg} \cdot \text{K}} \\ f &= 30000 \frac{\text{ft}^3}{\text{min}} \\ &= 30000 \left(\frac{12 \text{ inches}}{1 \text{ ft}} \right)^3 \frac{0.0254 \text{ m}}{1 \text{ inch}}^3 \\ &= 849.5 \frac{\text{m}^3}{\text{min}} = 14.16 \frac{\text{m}^3}{\text{sec}} \end{aligned}$$

A change of $1K$ is the same as 1°C which is $\frac{9}{5}^\circ\text{F}$ so that is 127°F . If it started at room temp (72°F) it comes out at about 200°F . That is a very hot and large blowdryer.

2. Reed Problem 5.2 - Uranium Load

We need the mass of N channels.

Each channel is a cylinder with mass $m_{\text{channel}} = \rho V = \rho \pi r^2 L$

So the total mass is

$$m_{\text{total}} = N m_{\text{channel}}$$

$$= N \rho \pi r^2 L$$

$$= 1248 \cdot \pi \cdot (1.397 \text{ cm})^2 \cdot 731.5 \text{ cm}$$

$$= 106067129 \text{ g}$$

$\left. \begin{array}{l} N=1248 \\ \rho = 18.95 \frac{\text{g}}{\text{cm}^3} \\ r = \frac{d}{2} = \frac{1.1 \text{ inches}}{2} \cdot \frac{2.54 \text{ cm}}{\text{inch}} = 1.397 \text{ cm} \\ L = 24 \text{ ft} \cdot \frac{12 \text{ inches}}{\text{foot}} \cdot \frac{2.54 \text{ cm}}{\text{inch}} = 731.5 \text{ cm} \end{array} \right\}$

\leftarrow a truly silly way of reporting the answer ☺

How about 106 metric tons as a useful way of reporting the answer.

3. Reed Problem 5.3 - Graphite Bricks

$$m_{\text{total}} = N m_{\text{brick}}$$

$$N = 40000$$

$$m_{\text{brick}} = \rho V_{\text{brick}}$$

$$\rho = 2.15 \frac{\text{g}}{\text{cm}^3}$$

$$V_{\text{brick}} = w \cdot w \cdot l$$

$$w = 4.125 \text{ inches} \cdot \frac{2.54 \text{ cm}}{\text{inch}} = 10.48 \text{ cm}$$

$$l = 16 \text{ inches} \cdot \frac{2.54 \text{ cm}}{\text{inch}} = 41.91 \text{ cm}$$

So

$$m_{\text{total}} = N \rho w^2 l = 40,000 \cdot \frac{2.15 \text{ g}}{\text{cm}^3} \cdot (10.48 \text{ cm})^2 \cdot 41.91 \text{ cm}$$

$$\left. \begin{array}{l} \downarrow \text{silly again ☺} \\ = 395857318 \text{ g} \end{array} \right\} = 396 \text{ metric tons of graphite}$$

\leftarrow useful!

4. Reed Problem 5.4, p. 235

Uranium Enrichment

(a) Build a table

Enrichment Round	Concentration
0	0.00720
1	0.00792
2	0.00871
3	0.00958
4	0.01054
5	0.01160
6	0.01276

(b) The idea of building the table was (i) to start seeing how painfully slow repeated enrichment would be, and (ii) to see the pattern:

	f_0	$\boxed{r = 1.1}$
0	$r f_0$	$\boxed{f_0 = 0.00720}$
1	$r^2 f_0$	
2	$r^3 f_0$	
:		
i	$r^i f_0$	$\boxed{f = r^i f_0}$

(c) If you see the pattern

$$f = r^i f_0 \quad \text{or} \quad \frac{f}{f_0} = r^i$$

Now comes the fancy algebra. Take \log_{10} of both sides, and use

$$\log_{10} r^i = i \log_{10} r$$

$$\log_{10} \frac{f}{f_0} = i \log_{10} r \quad \text{or} \quad i = \frac{\log_{10} \frac{f}{f_0}}{\log_{10} r}$$

To find the needed number of enrichment rounds, put in the needed final concentration:

$$n = \frac{\log_{10} \frac{f_{\text{final}}}{f_0}}{\log_{10} r}$$

$$= \frac{\log_{10} \frac{0.9}{0.72}}{\log_{10} 1.1} = 50.65 = 51 \text{ enrichment rounds}$$

$$\boxed{\begin{aligned} f_{\text{final}} &= 0.9 \\ f_0 &= 0.0072 \\ r &= 1.1 \end{aligned}}$$

Note: It doesn't matter what logarithm base you use as long as you are consistent. You could use the natural log (\ln) if you preferred.