Manhattan Project — Term 2 Exam Solution

1a. $\frac{10\,kg}{250\,g/mol} = \frac{10\,\cancel{kg}}{1/4\,\cancel{kg}/mol} = 40\ mol$

b. $40\,\cancel{mol} \times 6\times10^{23}\,\frac{atoms}{\cancel{mol}} = 240\times10^{23}\,atoms$

$= 2.4\times10^{25}$ atoms of Plutonium

c. $0.1\times2.4\times10^{25}\,atoms = 2.4\times10^{24}\,atoms$

d. $2.4\times10^{24}\,atoms \cdot 3\times10^{-11}$ Joules released/atom

$= 7.2\times10^{13}$ Joules released

2 a. $\rho \equiv M/V \Rightarrow V = \frac{M}{\rho}$

b. $V = \frac{10\,kg}{20\,g/cm^3} = \frac{10\,\cancel{kg}}{\frac{1}{50}\cancel{kg}/cm^3} = 500\,cm^3$

c. $V = \frac{4}{3}\pi R^3 = \frac{M}{\rho} \Rightarrow R^3 = \frac{3}{4\pi}\frac{M}{\rho}$

$\Rightarrow R = \left(\frac{3}{4\pi}\frac{M}{\rho}\right)^{1/3}$

d. $R \approx \left(\frac{1}{4}\frac{M}{\rho}\right)^{1/3}$ ← using $\pi \approx 3$

in fact $\pi^{1/3}$ is about $3^{1/3}$ to better than 2% so this particular use of $\pi \approx 3$ is pretty innocuous

e. $d = 2R = 2\left(\frac{1}{4}\frac{M}{\rho}\right)^{1/3}$

$= 2\left(\frac{1}{4}500\,cm^3\right)^{1/3}$ from (b)

$= 2(125\,cm^3)^{1/3}$

$= 2\cdot5\,cm = 10\,cm \approx 4\,in$

"briefcase" bomb

3 a. Charge conservation tells you that if an electron is produced (and flies away as a β^- ray) then a neutron must also turn into a proton

charge $-e$ produced $\longleftrightarrow$ produced ... proton $\longleftrightarrow$ charge $+e$ produced

two results balance/cancel / no net change in charge

I'm not expecting you to repeat all that verbiage in your answer. Just reminding you how we know what nuclear change accompanies the emission of a β^- ray.

$Z \rightarrow Z+1$ nucleon number unchanged
$A \rightarrow A$

$A = 131$
$Z = 54$ $\longleftarrow$ all I was looking for

b. Start with

$1600\ {}^{131}_{53}I$

After four half lives have

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 1600 = 100$ left

Must have produced $1600 - 100 = 1500$ of ${}^{131}_{54}Xe$ and your thyroid gets bombarded with $1500\ \beta^-$ rays.

4 a. Four mols of Hydrogen weighs

$4 \cdot 1.008 = 4.032$ grams

One mol of Helium weighs

4.003 grams

Subtract 0.029 grams of rest mass "disappear" $\leftarrow$ actually, become energy

b. $0.029\, g = 2.9 \times 10^{-2} g = 2.9 \times 10^{-5} kg$

$\approx 3 \times 10^{-5} kg$

c. multiply by $c^2 = \left(3 \times 10^8 \frac{m}{s}\right)^2$

$= 9 \times 10^{16} \frac{m^2}{s^2}$

Get $27 \times 10^{11} \frac{kg\, m^2}{s^2}$ $\leftarrow$ this is a J

$= 2.7 \times 10^{12}$ J

these things just work out with no conversion factors in the MKS system, but feel free to ask me why this is not — for example — a Watt or a Pascal or some other MKS unit

5a. $1 MeV = 10^0 MeV$, so you go to the rightmost part of the graph. Reading across the "rough" answer I was looking for was log of cross-section in barns is just 0. If you got 0.1, then part b will be harder.

b. $10^0 = 1$ so answer is 1 barn.

If you got 0.1 for 5a, then $10^{0.1} = 10^{1/10}$ which is the 10th root of 10 and isn't something any of us can likely do without a calculator. With a calculator, I get $\sqrt[10]{10} \approx$