Manhattan Project - Term 2 Exam

October 8, 2020 — Covering the technical material in Reed Chs. 2 and 3

I was verbose in writing this exam. Sometimes trying to make it clearer actually makes it harder, because it takes time just to read the question. Do not feel time pressure. We will try to hold it to 45 minutes, but not cut people off if they need a little more time to check their math.

Do not use a calculator. I made the numbers round, and a goal of the course is for you to all become good at throwing around astronomic and microscopic numbers without reaching for a calculator.

There may be enough space on the exam to do the work and answer, but I'd recommend working on your own blank paper.

Molarity, Energy Released

1. Energy of a Critical Mass of Plutonium-239

Pure Plutonium-239, has atomic weight 239.05216 grams per mol. Round this up quite a bit to 250 grams per mol so that you don't have to use a calculator in what follows.

a. One critical mass of Plutonium-239 is about 10kg. About how many mols of Plutonium-239 are in a critical mass?

b. Knowing that Avogradro's number, N_A, is about 6×10^{23}, how many atoms of Plutonium-239 are in a critical mass?

c. If only 10% of these atoms fission during the explosion, how many of these atoms fission?

d. If each fission of Plutonium-239 releases about 3×10^{-11} J, how many Joules do you expect a Plutonium bomb to release?

Density, Critical Mass

2. Size of a Critical Mass of Plutonium-239

Let's do this one both algebraically — parts (a) and (c) — and with numbers — parts (b), (d), and (e):

a. Suppose a mass M of plutonium is the critical mass (enough to make a bomb). Suppose the density of plutonium is ρ. Using the definition of density $\rho = \frac{M}{V}$, just solve that for V. This is super-easy. Don't over-think.

b. Plug in the numbers $\rho = 20\ \mathrm{g/cm^3}$ and the critical mass of M =10kg, to find the volume. You can leave your answer in cm^3.

c. If the critical mass is shaped into a sphere, its volume, V, is given by $V = \frac{4}{3}\pi R^3$. You see from (a) you also had an expression for V. Set the two expressions for V equal to each other. Then do some algebra to get an expression for R.

d. Back to plugging in numbers. In your expression for R, the ratio $\frac{3}{4\pi}$ should have showed up. Since π is about 3, this is about $\frac{1}{4}$. Make that approximation and simplify. Report your final answer for R, the radius of a critical mass of Plutonium-239, in centimeters.

e. Diameter is twice the radius. A rough conversion factor is that 4 inches is about 10cm. What is the diameter of a critical mass of Plutonium in the familiar unit of inches?

Radioactivity, Half-Life

3. Iodine-131 in Radioactive Fallout

Iodine-131 is one of the fission products of Uranium. If ingested in radioactive fallout, it is taken up by the thyroid, because that's what the thyroid does: it takes in iodine and produces iodine-containing thyroid hormones. Unfortunately, Iodine-131 is itself radioactive. It decays by β^- decay with a half-life of 8 days. Beta rays (electrons) damage thyroid cells.

That's just context. Here is the problem:

(a) Complete the following reaction:

$$ {}^{131}_{53}\mathrm{I} \underset{\substack{\beta^- \\ \text{8 days}}}{\longrightarrow} {}^{A}_{Z}X + \text{electron} $$

I just want you to answer with the numbers Z and A.

(b) The element X above is Xenon, and the abbreviation is actually Xe. In 32 days (to make a nice round multiple of 8 days), if your thyroid started with 1600 Iodine-131 atoms, how many of those Iodine-131 atoms would be left at the end of those 32 days? Also, how many beta rays (electrons) was your thyroid bombarded with in this time?

Mass Defect, $E = mc^2$

4. Fusion in the Sun

In the core of the Sun, 600 million tons of Hydrogen are fused to Helium every second. The basic process is that four Hydrogens $^1_1$H fuse to make one Helium $^4_2$He. Two positrons are produced and annihilate with nearby electrons. That's just context. Here is the problem:

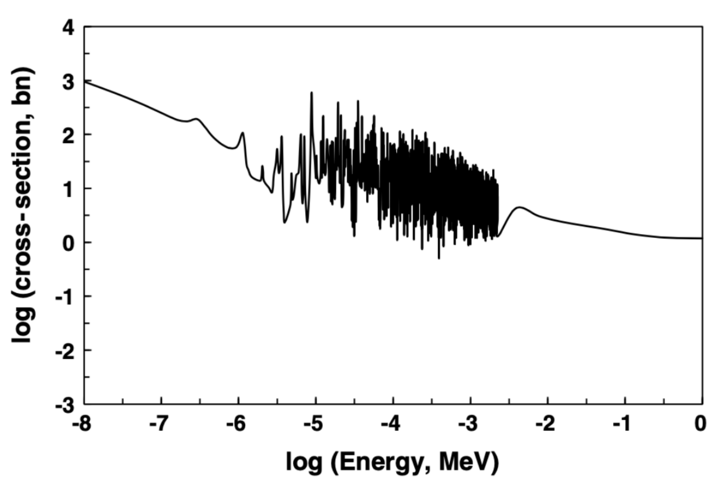
(a) Hydrogen has atomic mass of 1.008 grams/mol. Helium has atomic mass of 4.003 grams per mol. If four mols of Hydrogen fuse to make one mol of Helium, how many grams disappear into energy?

(b) Convert your answer in a from grams to kg, and while you are at it, round your result to one sig fig.

(c) Multiply your answer to (b) by the speed of light **squared** to get the energy released. The speed of light is 3×10^8 m/s. Thanks to the incredible convenience of MKS units, your result is in Joules, with no conversion factors needed.

Cross Sections, Log-Log Plots

5. Fission Cross Section for Neutrons Bombarding U-235



a. The above plot shows the fission cross-section for neutrons hitting U-235. Find 1 MeV neutrons on the plot. Holding up something straight against the plot to guide your eye, estimate the log of the cross-section in barns of these neutrons. You can be quite rough in your answer.

b. Convert the result you just got (which is a logarithm) to the actual value in barns.