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## Manhattan Project — Assignment 3 — Half Lives and Decays

Let's recap the equations on p. 26 before doing any problems.

### Decay Rate Derivation

I derived the equation for decay rate without using any calculus. I had to use some properties of the exponential though. One of the properties was this one that you may or may not be familiar with:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

If you are curious and know factorials, the denominators are  $0!=1$ ,  $1!=1$ ,  $2!=2$ ,  $3!=6$ , etc. I will use the above formula to make an approximation below. Another property of the exponential that I used is:

$$e^{x+y} = e^x e^y$$

That's actually true whatever base is being exponentiated. For example:

$$10^{x+y} = 10^x \cdot 10^y$$

Finally, I used  $e^0 = 1$ . At least I'm not using any calculus!

So we start with the claim that

$$N(t) = N_0 e^{-\lambda t}$$

describes radioactive decay. If you have  $N_0$  atoms at time  $t = 0$ , this formula is the one that tells you how much you have at any later time. So it certainly tells you how much you have at both time  $t$  and time  $t + \Delta t$  where  $\Delta t$  is a small amount of time. We have:

$$N(t + \Delta t) = N_0 e^{-\lambda(t+\Delta t)} = N_0 e^{-\lambda t} e^{-\lambda \Delta t} = N(t) e^{-\lambda \Delta t} = N(t) \left[ 1 - \lambda \Delta t + \frac{(\lambda \Delta t)^2}{2} - \frac{(\lambda \Delta t)^3}{6} + \dots \right]$$

Here comes the tricky part! If  $\lambda \Delta t$  is small (think of something like 0.01), then every term in the infinite series is 0.01 times as small as the previous one. Let's neglect all but the first two!

$$N(t + \Delta t) = N(t)[1 - \lambda \Delta t] = N(t) - \lambda \Delta t N(t)$$

Rearrange:

$$N(t + \Delta t) - N(t) = -\lambda \Delta t N(t)$$

Rearrange more:

$$\frac{N(t+\Delta t)-N(t)}{\Delta t} = -\lambda N(t)$$

What we have on the left side is what Reed calls  $R(t)$  in equation 2.3. It is the rate that the number of particles is changing. The right-hand-side of the equation has a minus sign because the number of particles is decreasing. Let's summarize before we move on:

$$R(t) = -\lambda N(t)$$

## Relationship Between $\lambda$ and $t_{1/2}$

There is another thing we derived in class that I want to re-derive here: the relationship between  $\lambda$  and  $t_{1/2}$ .

$t_{1/2}$  is the time at which you have half as many particles. So on the left-hand side of,

$$N(t) = N_0 e^{-\lambda t}$$

we put  $N_0/2$  and for  $t$  on the right-hand side, we put  $t_{1/2}$ ,

$$N_0/2 = N_0 e^{-\lambda t_{1/2}}$$

The  $N_0$  on each side cancels, leaving:

$$1/2 = e^{-\lambda t_{1/2}}$$

Now take the reciprocal of each side of the equation:

$$2 = e^{\lambda t_{1/2}}$$

Finally take the natural log of each side. The natural log is by definition the function that undoes the exponential:

$$\ln 2 = \lambda t_{1/2}$$

We have Reed's equation 2.2:

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

### 1. Using $R(t) = -\lambda N(t)$ and $\lambda = \frac{\ln 2}{t_{1/2}}$

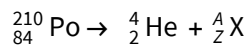
(a) Convert 138 days to seconds.

(b) If you start off with an Avogadro's number of Polonium-210 atoms ( $N_A \approx 6.02 \cdot 10^{23}$ ) and the half life of Polonium-210 is  $t_{1/2} = 138$  days, what number of atoms will be decaying per second.

(c) A Curie (abbreviated Ci) is  $3.7 \cdot 10^{10}$  decays / second. Convert your answer in (b) to Ci.

### 2. Alpha Decay

Polonium-210 alpha decays. The reaction is:

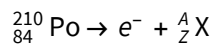


(a) What must  $A$  and  $Z$  be?

(b) Use the Table of Isotopic Masses and Natural Abundances you have. What element has the  $Z$  you found in (a)?

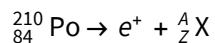
### 3. $\beta^-$ and $\beta^+$ Decay

(a) Suppose Polonium-210 did a  $\beta^-$  decay. Consult Fig. 2.12 to find out what the  $N$  and  $Z$  value of the resulting nucleus would be. ( $N$  is the number of neutrons and  $N = A - Z$ .)



In addition to reporting  $N$  and  $Z$  of the new nucleus, what is the  $A$  value of the new nucleus?

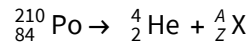
(b) Suppose Polonium-210 did a  $\beta^+$  decay. Consult Fig. 2.12 to find out what the  $N$  and  $Z$  value of the resulting nucleus would be.



In addition to reporting  $N$  and  $Z$  of the new nucleus, what is the  $A$  value of the new nucleus?

## 4. Energy Released in Fission

Returning to the actual Polonium-210 alpha decay that you found in Problem 2:



Look up the mass of each atom involved in your the Table of Isotopic Masses and Natural Abundances. Actually, Polonium-210 isn't stable enough to be in our table, so I'll just tell you that it has mass 209.98286u.

(a) What is the total mass on the left-hand side, what is the total mass on the right-hand side, and what is the difference? Keep all six decimal places.

(b) Convert the difference (this is the energy released) to MeV using the handy-dandy conversion factor between mass units and MeV in the middle of p. 34 of Reed.