Manhattan Project — Assignment 2 — Mass and Energy

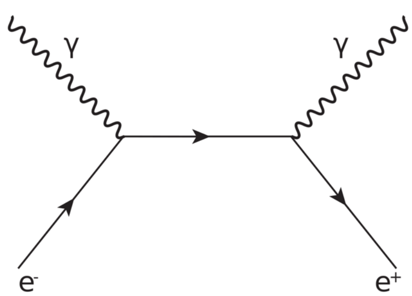
1. $E = m\,c^2$

We are going to use a lot of units of energy in this course (eV, MeV, Joule, calorie, kiloton). The most important and easiest one to work with is the Joule, or J for short. It is the easiest because in the SI (System Internationale) unit system, the Joule is defined to be 1kg * $1\ \mathrm{m}/\mathrm{s}^2$* 1 m. I demonstrated that amount of energy by dropping a 0.1kg tangerine in the gravitational field (which is about $10\ \mathrm{m}/\mathrm{s}^2$) a distance of 1m.

The speed of light is about 3×10^8m/s. If 1 gram of matter were converted entirely to energy — which you could cause to happen, for example, if you had half a gram of electrons and half a gram of positrons and you allowed them to annihilate each other and become photons (γ-rays) — how many Joules would be released, according to the equation $E = mc^2$?

2. Electron-Positron Annihilation

The electron and its anti-particle, the positron, are the simplest example of the complete conversion of mass into kinetic energy. The Feynman diagram for this process looks like this:



Time on this diagram goes upwards. An electron and a positron approach each other. They annihilate and two photons (γ-rays) depart.

Since photons are massless, this represents the complete conversion of mass into energy.

The electron and the positron *each* weigh 9.1×10^{-31} kg. The two photons each carry half of the resulting energy away. What is the energy of *each* photon?

2. The Coulomb and the electron-Volt

One Coulomb (abbreviated C) of charge passing through one Volt (abbreviated V) of electric potential either costs or releases one Joule (abbreviated J) of energy. The electron has -1.6×10^{-19}C of charge.

It requires energy if the negatively charged electron is forced down 1V. It releases energy if the negatively charged electron is allowed to go up one Volt.

(a) If one electron flew down 9V from one terminal of a 9V battery to another, how many Joules are released?

(b) If they sold 1 Volt batteries (I have never seen one), and one electron flew down 1V from one terminal to the other, how many Joules were released?

(c) The amount of energy you found in (b) comes up so often in chemistry and physics, it is given a name electron-Volt, and it is abbreviated eV.

3. Tiny Particles, Enormous Numbers, the Avogadro Constant N_A

(a) Since one electron weighs 9.1 x10^{-31}kg, how many electrons would it take to make 1kg?

(b) Since one electron has -1.6 x10^{-19}C of charge, how many electrons would it take to make -1C of charge? Mathematically, this is as easy as 3(a) even though charge is harder to visualize than mass.

(c) Find Carbon-12 on the table of isotopic masses. You will see that one Carbon-12 weighs *by the definition of the atomic mass unit*, amu, nowadays abbreviated just to u, exactly 12u. The atomic mass unit is about 1.66x10^{-24}g. How many Carbon-12 atoms would it take to make 12 grams?

DISCUSSION: Once you calculate the number in (c), it should look familiar. It is a famous number, postulated in 1909 by French physicist Jean Baptist Perrin. But we don’t call it Perrin’s number. He named it after Avogadro. It is very hard to count atoms, so as of 2019, instead of trying to count them, Avogadro’s constant is now, N_A is DEFINED to be 6.02214076 x10^{23}. So as of 2019, the kilogram is no longer a primary standard! The kg used to be a tangerine-sized piece of platinum stored in a vault in Paris. It is no longer the definition. 12 grams is now the weight of 6.02214076 x10^{23} Carbon-12 atoms, by definition, and 1kg is 1000/12 as much as 12 grams. I feel said for the piece of platinum that is no longer special.

(d) According the the discussion above, how many Carbon-12 atoms are in a kilogram?

4. Measuring mass in eV

Since mass can be converted to energy, at least in principle, we can “measure” mass by how much energy it would release if it were converted.

In problem 1, you found that a number for the amount of energy released into each of two photons when an electron and a positron became two photons. The electron and the positron weigh the same amount, and the two photons have the same energy. So what you calculated in Problem 1 was how much energy an electron could make if it could be turned into energy. (IT CAN'T all by itself. You need a positron to annihilate it with).

Anyway, imagining that the electron CAN be turned into energy all by itself, you calculated the amount of energy in Joules, that would be released.

(a) Using the conversion factor you found in 2(c) what is the amount of energy you found in 1 in eV?

(b) Using 1MeV is 10^6 eV, convert 4(a) to MeV.

DISCUSSION: It is very common to say the electron weighs that much, even though MeV is a unit of energy, not of mass. Thanks to Einstein, we know they are deeply related, and the conversion factor to Joules is just to divide by c^2. Any working particle physicist will say "an electron weighs half an MeV" without even pausing.

5. Energy Released in Fusion

There are a lot of fission examples in our reading. Just to spice things up, let's contemplate a fusion reaction.

Deuterium is the name for Hydrogen-2. Tritium is the name for Hydrogen-3. Most isotopes don't have names, but these are important enough that they do. Consider this fusion reaction:

$${}^{2}_{1}D + {}^{3}_{1}T \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}n$$

(a) Consult your table of isotopic masses. What is the total mass of the Deuterium and the Tritium. Keep six decimal places and keep your answer in amu (abbreviated u).

(b) Again consult your table of isotopic masses to which we added the mass of the neutron as 1.008665u. What is the total mass of the Helium and the neutron?

(c) Subtract (b) from (a).

(d) Using 1amu = 1.66054×10^{-27}kg, convert what you got in (c) to Joules.

(e) Using the conversion factor you found in 2(b), convert what you found in (d) to eV.

(f) Using 1MeV is 10^6 eV convert what you got in (e) to MeV.