Mathematical Analysis

The Foundation of Derivative and Integral Calculus

Unofficial/Short Course Title: Calculus I

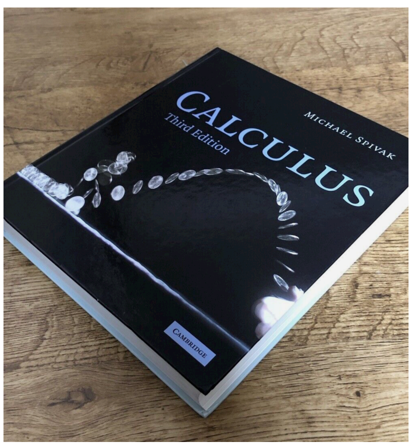
Syllabus

Spring 2025, Deep Springs College, ***Prof. Brian Hill***

Required Text

Michael Spivak, Calculus, 3rd Edition, Cambridge University Press, 2006 (Amazon link)

Because there are a lot of editions, reprints, and knock-offs, to confirm that you are buying the correct edition from Cambridge University Press, check that what you are buying looks like:



The Mathematical Association of America review begins, "This is the best Calculus textbook ever written." Need more be said?

We will cover the first 14 chapters, up to and including "The Fundamental Theorem of Calculus."

Overview

There is a wealth of material that logically fits in between standard high school mathematics and calculus. This material has been deemed too hard by most schools — both those with college prep tracks and with colleges themselves. It is simply skipped! Only those that continue on to the junior level as mathematics majors get to see it, and they see it in a course that is often titled "Introduction to Analysis" or "Introduction to Mathematical Reasoning." It is perfect for the Deep Springs curriculum insofar as it requires no prerequisites beyond very good high school mathematics, and yet it is both more advanced and provocative than what is generally found in lower-division math courses. The material is called "mathematical analysis" or "real analysis" and it includes many theorems that are absolutely essential to calculus but are typically used as if they are obvious and without proof.

After having built the foundation of calculus, we will then proceed to the more conventional material of a first-semester calculus course — the derivative, the integral, and as a culmination, their deep relationship via "The Fundamental Theorem of Calculus."

One might think that with only so much time to either obtain and apply practical results or to do mathematical proofs, that one should spend one's time on pedestrian problems. Michael Spivak, renowned for textbooks in mathematics ranging from the introductory level to textbooks used by mathematics graduate students, disagrees: “calculus ought to be the place in which to expect, rather than avoid, the strengthening of insight with logic.” He goes on: “precision and rigor are neither deterrents to intuition, nor ends in themselves, but the natural medium in which to formulate and think about mathematical questions.”

A course in the topics of mathematical analysis that undergird calculus prepares one to think rigorously, to understand real analysis the way mathematicians do, and to understand fundamental theorems that are beautiful and significant in their own right. As a treat we will even go beyond the confines of the real numbers and in our final unit introduce imaginary numbers (also known as complex numbers). This will allow us to conclude with the 1799 proof by Gauss of The Fundamental Theorem of Algebra.

Grading

- 40% assignments
- 50% total for three exams, coming in the 5th, 10th, and 15th week of the semester
- 10% preparation for class and leadership of course

Problem Sets / Handouts / Exams

There will be problem sets due at minimum every week, and usually every class, limited only by how quickly I can assign, write solutions, and grade. The more problems you do the better.

In addition to the problem sets and their solutions, there will be handouts, exams, and exam solutions to file. Locate a three-ring binder and a three-ring hole punch, and file everything chronologically. Actually, reverse-chronological is the most convenient, because you then naturally open your binder to what you are currently working on.

Problem sets should be *neat* and on standard 8 1/2 x 11 paper. Multi-page problem set submissions should be stapled. The nicest technical work is facilitated by engineering pads, such as these **Roaring Spring Engineering Pads at Amazon** (which are pretty expensive unless you buy by the case), and done with a mechanical pencil, a ruler, and an eraser at hand.

Absences (and late work)

The College's policies on absences (and late work) are applicable. Refer to the Academic Year 2024-2025 Deep Springs Handbook.

Daily Schedules

Week 1 — Chapter 1 — Properties of the Real Numbers (P1-P12)

- Monday, Jan. 13 — Reading: Chapter 1, pp. 3-10, postulates (P1)-(P12) for the real numbers — How to read mathematics: (1) when the author “leaves something for the reader” stop and do it, and (2) keep a constant eye on what has so far been postulated (as an axiom), defined, or proven (in a theorem, a lemma, or a corollary) and be very careful not to use things that seem obvious but have not yet been postulated, defined, or proven — In-class: we got started on the end-of-chapter problems
- Thursday, Jan. 16 — Problem Set 1: Write up Problems 1-3 (which have many subparts) — For all of the first three problems, it would be best to work in a two-column format where you do the work in the left column, and enumerate which postulates you used in the right column — To keep the verbosity manageable, let's agree that we don't have to repeatedly note that $a-b$ means $a+(-b)$, that a/b means $a\cdot b^{-1}$, and that $-(a\cdot b)=(-a)\cdot b=a\cdot(-b)$ — Second Reading: Finish Chapter 1, and read the first three pages of Chapter 2 — In-class: scrutinizing what is meant by equality, proving $(-(-a))=a$, and proving that the additive inverse is unique

Week 2 — The Natural Numbers — Induction — Σ Notation

- Monday, Jan. 20 — Problem Set 2: Let's do Problems 5-7 (still in Chapter 1) — NB: to keep the verbosity of your proofs manageable, you may start using everything you have previously proven (but be sure you aren't using things we haven't proven!) — As an example, you don't have to keep re-proving $(-(-a))=a$ every time you use it, or that the multiplicative inverse (when it exists) and additive inverse are unique, because you know how that goes now — Reading: Finish Chapter 2 — In-class: We did inequality proofs
- Thursday, Jan. 23 — No new reading — Problem Set 3: Just Problems 1 and 2 from Chapter 2 — In-class: How about we do a selection of the parts from Problem 3 and 4? — Are there other end-of-chapter problems that particularly interest you? — I find lots of them interesting-looking, such as 13, 14, and 15 — Avoid problems marked with an asterisk unless you are finding the others to be easy

Week 3 — Functions as Sets — Addition, Multiplication, and Division of Functions — Composition of Functions

- Monday, Jan. 27 — Reading: First half of Chapter 3 to p. 44 (ending with commutativity of addition for functions and of multiplication for functions "should also present no difficulty") — Problem Set 4: Problems 1-3 of Chapter 3
- Thursday, Jan. 30 — Problem Set 5: Problems 5 and 6 (still from Chapter 3) — Reading: Finish Chapter 3 (but skip the Chapter 3 Appendix) and then continue through to p. 60 of Chapter 4 — In-class: **Examples of Lagrange Interpolation**, open and closed intervals, even and odd functions

Week 4 — Graphing in Cartesian and Polar Coordinates — Vectors

- Monday, Feb. 3 — Reading: Finish Chapter 4 — Problem Set 6: Problems 3, 4, 5 and 9 from Chapter 4 — In-class: We did some more strange functions (stair step and saw tooth), and then started into into Chapter 4 Appendices 1 and 3 (pp. 84-89) on Vectors and Polar Coordinates
- Thursday, Feb. 6 — Reading: Appendices 1 and 3 of Chapter 4 — Problem Set 7: Problems 18(v) and 21(a) of Chapter 4 (on pp. 72-73), Problems 1, 2, and 3 of Appendix 1 (on p. 77-78), and Problems 6 and 9(i) of Appendix 3 (on pp. 88-89) — In-class: As review before the exam, we will do more problems from Chapters 1 to 4

Week 5 — Exam 1 — Start Limits

- Monday, Feb. 10 — **Exam 1** on Spivak Chapters 1 to 4, including Appendices 1 and 3 of Chapter 4
- Thursday, Feb. 13 — Reading: Study Chapter 5 to the bottom of p. 100 — In-class: **Mapping the square function** and **Cube root limits**

Week 6 — Finish Limits — Start Continuous Functions

- Monday, Feb. 17 — Reading: Finish Chapter 5 — Problem Set 8: Chapter 5, Problems 1-4, 7, and 8 — In-class: Focus on the limit of the inverse function (Theorem 2, Part 3, p. 102) — **Illustration of g Inverse**
- Thursday, Feb. 20 — Reading: Chapter 6, p. 113-115, including the statement of Theorem 2, but not the proof — Problem Set 9: Still from Chapter 5, do Problems 6, 9, 10, and 11 — In-class: lots of time going over problems 6 and 10, limits at infinity, **including 5-39(v)**, and a look ahead at Problem 2 of Chapter 6

Week 7 — Finish Continuous Functions

- Monday, Feb. 24 — Reading: Finish Chapter 6 — Problem Set 10: Chapter 6, Problems 1-5, but for Problem 2 just do 4-17's functions — In-class: Preview the first eight theorems of Chapter 7

Week 8 — Properties of Continuous Functions — Start Least Upper Bounds

- Monday, March 17 — Reading: Study the first eight theorems in Chapter 7 (note that the proofs of the first three theorems are deferred until Chapter 8) — Problem Set 11: Chapter 7, Problems 1-3
- Thursday, March 20 — Reading: Finish studying Chapter 7 and read Chapter 8 through the proof of Theorem 1 — Problem Set 12: Chapter 7, Problems 10-12, and Chapter 8, Problems 1 and 3

Week 9 — Finish Least Upper Bounds — Uniform Continuity — Start Derivatives

- Monday, March 24 — Reading: Finish Chapter 8 and study the Appendix to Chapter 8 — Presentations: Rania and Will, Problem 1, p. 144; **Brian and Hexi, Problem 2, p. 144**; Brendan and Jeremy, Problem 10, p. 139 — Surprise: **Continuity Implies Uniform Continuity** (only on a closed interval)
- Thursday, March 27 — Start Chapter 9, derivatives (which are not on Exam 2!) through to the middle of p. 154 — Problem Set 13: Chapter 9, Problems 1-6 — Brian and Brendan: **Newton and Liebniz**

Week 10 — Exam 2 — Finish Derivatives

- Monday, March 31 — **Exam 2** on Spivak Chapters 5 to 8, including the Appendix to Chapter 8
- Thursday, April 3 — Reading: Finish Chapter 9, derivatives — Problem Set 14: Chapter 9: Problems 7-11 — Two groups presented Problems 12 and 14? — I presented a better solution to the last part of Exam 2 Problem 7

Week 11 — Differentiation (Widely Used Properties of Derivatives) — Start Applications of Derivatives

- Monday, April 7 — Reading: Study Chapter 10 to the bottom of p. 173 — Problem Set 15: Problems 1 to 7, but for Problems 1, 2, 4, 5, and 6, just do the first half of the many parts (e.g., if there 18 parts, only do the first 9)
- Thursday, April 10 — Reading: Finish studying Chapter 10 and continue studying in Chapter 11 to p. 192 — Chain Rule (Derivation and Application) — Problem Set 16: Chapter 10, Problem 17, and Chapter 11, Problems 1-3 — Presentations: Rania presented Chapter 10, Problem 22; Hexi and Jeremy presented applications of the Mean Value Theorem

Week 12 — Finish the Significance of the Derivative — Inverse Functions

- Monday, April 14 — Reading: Finish Chapter 11 (skip the Chapter 11 Appendix) — Problem Set 17: Chapter 11, Problems 6 to 12 (these are typical applications to real-world problems of the extrema-finding techniques in Chapter 11, albeit somewhat contrived)
- Thursday, April 17 — Reading: Only one class devoted to Chapter 12 — Problem Set 18: Chapter 12, Problems 4 to 7 — Discuss the very useful results arising from combining the inverse function idea with the chain rule theorem (Theorem 5, pp. 234-235) — Begin discussing partitions and integrability

Week 13 — Integrals

- Monday, April 21 — Reading: Start Chapter 13 through p. 260 — Problem Set 19: Chapter 13, Problems 1, 2, and 7(i)-7(iv) — You should be able to check (by counting squares on the graph paper) that your analytical results are consistent with the graphs
- Thursday, April 24 — Reading: Finish Chapter 13 — Problem Set 20: Chapter 13, Problems 8, 12, and 15 — Also, problems 18 and 19 are at our current ability level and instructive — — After we are satisfied with Chapter 13, and especially Theorems 3 and 8 which prepare us for the Appendix, we will preview the two versions of "The Fundamental Theorem of Calculus"

Week 14 — The Fundamental Theorem of Calculus — Exam 3

- Monday, April 28 — Reading: Finish Chapter 14 — Problem Set 21: Problems 1 (parts (i)-(iv) only), 4 (take derivatives to show x-independence), and 7 (could be tricky!)
- Thursday, May 1 — **Exam 3** on Spivak Chapters 9 to 14