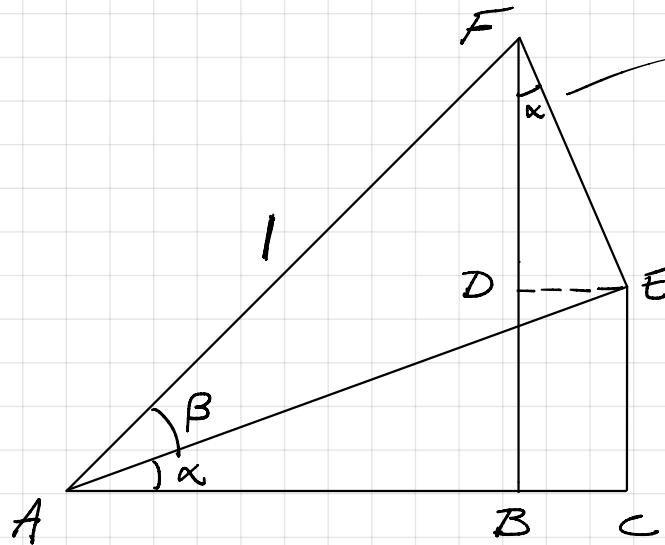


Numerical Analysis

Sep. 6, 2022

Solution to Problem Set 1

1.



one of the first things you have to do is convince yourself that this angle is also α

Use $\angle CAE = \angle AED$
and then $\angle DEF = 90^\circ - \alpha$
 $\Rightarrow \angle DFE = \alpha$

$$\overline{AB} = \cos(\alpha + \beta)$$

because it is adjacent to $\alpha + \beta$ on a triangle with hypotenuse 1

Here is the big thing:

$$\overline{AB} = \overline{\overline{AC}} - \overline{\overline{BC}}$$

$$= \overline{\overline{AC}} - \overline{\overline{DE}}$$

Then

$$\overline{FE} = \sin \beta \text{ so } \overline{DE} = \sin \alpha \sin \beta$$

$$\overline{AE} = \cos \beta \text{ so } \overline{AC} = \cos \alpha \cos \beta$$

So,

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$



$$2. A = \frac{1}{2} b^2 \frac{\sin \theta \sin \varphi}{\sin(\theta + \varphi)}$$

$$\theta = 30^\circ$$

$$\varphi = 45^\circ$$

$$b = 4 \text{ cm}$$

My HP-25 app gives

$$A = 2.93 \text{ cm}^2$$

3. The Owner's Handbook recommends starting with the "innermost" quantity first. I guess I'll start with $\theta + \varphi$.

$\begin{matrix} 3 \\ 0 \\ \boxed{\text{ENTER}} \end{matrix}$	$\begin{matrix} 75 \\ 0.97 \\ 1.04 \end{matrix}$
$\begin{matrix} 4 \\ \boxed{+} \\ \boxed{\text{F1 sin}} \end{matrix}$	0.52
$\begin{matrix} 5 \\ \boxed{\text{G1/x}} \\ 3 \\ 0 \\ \boxed{\text{F1 sin}} \end{matrix}$	0.37
$\begin{matrix} 4 \\ \boxed{\text{G1/x}} \\ 2 \\ \boxed{-} \end{matrix}$	2.93

4. and 5. Were problems of your choice. In class I showed two formulae for the volume of the upper reservoir.

4. A naive way of getting the volume of the upper reservoir

The volume of a pyramid is $\frac{1}{3} l \cdot w \cdot h$.
 Think of the reservoir as a truncated pyramid



result is in cubic feet

It is $\frac{1}{3} l \cdot w \cdot h - \frac{1}{3} l' \cdot w' \cdot h' = 4000 \text{ ft}^3$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $100 \quad 75 \quad 24 \quad 75 \quad 50 \quad 16$

Units are in feet

5. A much more sophisticated estimate.
 This formula was easy to get using calculus (an integral).

$$lw' \cdot h + l' \cdot s \cdot h + w \cdot s \cdot h + \frac{4}{3} s^2 \cdot h$$

$$l' = 75 \text{ ft} \quad w' = 50 \text{ ft} \quad s = 12.5 \text{ ft} \quad h = 8 \text{ ft}$$

Result is

$$44166.67 \text{ ft}^3$$

or about 44000 ft^3

This formula also works if the reservoir water height, h , is less than 8ft.