Guitar String

Completed and Analyzed in class, April 8, 2025

This is our eighteenth notebook. For most of your special projects, the type of work we did in the seventeenth notebook (Harmonic Oscillator Redux) is more relevant than what I am going to launch into next. Please keep referring to that notebook. It had everything you need to know about how to put differential equations into Mathematica.

In this notebook, as part of analyzing a guitar string, we are launching into putting "partial differential equations" into Mathematica.

Guitar String — Theory

Back in the thirteenth notebook titled "Torsion Waves," we got our first really good visualization of waves. Torsion waves are perhaps a little easier to visualize than the transverse waves on a guitar string, but the mathematics is completely equivalent. Here was the angular acceleration formula:

$$\alpha_j = -\omega_0{}^2(\theta_j - \theta_{j-1}) + \omega_0{}^2(\theta_{j+1} - \theta_j)$$

In a theory notebook titled "The Second Derivative," I did some casual hand-waving of the kind physicists are prone to do and that mathematicians spend their lives trying to make more rigorous, and convinced you (I hope) that what we really had was is

$$\frac{\partial^2 \theta}{\partial t^2} = v_0{}^2 \frac{\partial^2 \theta}{\partial x^2}$$

For a guitar string, in which we call the direction the string is oriented in the x-axis and for which we call the displacement $z(t, x)$, the equations are:

$$\frac{\partial^2 z}{\partial t^2} = v_0{}^2 \frac{\partial^2 z}{\partial x^2}$$

Partial Derivatives and Their Notation

Now we have to variables (t and x), and it we have to clarify for Mathematica which variable we are taking a derivative with respect to. In other words, the obvious generalization of

In[27]:=
```
Derivative[2][z][t] // TraditionalForm
```

Out[27]//TraditionalForm=

$$z''(t)$$

which would be

In[28]:=
```
Derivative[2][z][t, x] // TraditionalForm
```

Out[28]//TraditionalForm=

$$z''(t, x)$$

is ambiguous. Which variable are we taking the derivate with respect to!? Is it *t* or *x*? Mathematica specifies it this way:

In[29]:=
```
Derivative[2, 0][z][t, x] // TraditionalForm
```

Out[29]//TraditionalForm=

$$z^{(2,0)}(t, x)$$

The Guitar String Differential Equation

So our wave equation is: