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# Coupled Harmonic Oscillators

Completed and Analyzed in class, February 18, 2025

This is the eighth notebook for you to complete.

## Coupled Oscillators — Formulas for the Accelerations

After all was said and done developing the theory, we were down to the following two simple equations for the accelerations,

$$a_1 = -\omega_0^2 x_1 + \omega_{12}^2 (x_2 - x_1)$$

$$a_2 = -\omega_0^2 x_2 - \omega_{12}^2 (x_2 - x_1)$$

where,

$$\omega_0^2 \equiv k/m$$

$$\omega_{12}^2 \equiv k_{12}/m$$

## Coupled Oscillators — Implementing the Accelerations

```
omega0 = 2 Pi; (* this value is chosen so  
that the period with no coupling is 1 second *)  
omega12 = 0.2 omega0; (* this value is chosen so that the coupling is weak *)  
  
a1[x1_, x2_] := bunny  
a2[x1_, x2_] := rabbit
```

## Initial Conditions

Let's let the spring be initially unstretched with no velocity and see what the external force does to it:

```

In[500]:=
tInitial = 0.0;
tFinal = 60.0;
initialx1 = -0.5;
initialx2 = 0.0;
initialv1 = 0.0;
initialv2 = 0.0;
(* We have to decide how we are packing the time,
the two positions, and the two *)
(* velocities into the initial conditions
list. How about this as one of two obvious *)
(* choices
(and I really can't see that one choice is any better than the other): *)
initialConditions = {tInitial, initialx1, initialx2, initialv1, initialv2};

```

## Second-Order Runge-Kutta — Formulas for Two Particles

As on Exam 1, just to cut down on the work, let's set  $\lambda = \frac{1}{2}$ , which simplifies the Second-Order Runge-Kutta formulas:

$$t_{i+1} = t_i + \Delta t$$

$$x_1^* = x_1(t_i) + v_1(t_i) \cdot \frac{\Delta t}{2}$$

$$x_2^* = x_2(t_i) + v_2(t_i) \cdot \frac{\Delta t}{2}$$

$$v_1(t_{i+1}) = v_1(t_i) + a_1(x_1^*, x_2^*) \cdot \Delta t$$

$$v_2(t_{i+1}) = v_2(t_i) + a_2(x_1^*, x_2^*) \cdot \Delta t$$

$$x_1(t_{i+1}) = x_1(t_i) + (v_1(t_i) + v_1(t_{i+1})) \frac{\Delta t}{2}$$

$$x_2(t_{i+1}) = x_2(t_i) + (v_2(t_i) + v_2(t_{i+1})) \frac{\Delta t}{2}$$

## Second-Order Runge-Kutta — Implementing the Formulas

```

steps = 12 000;
deltaT = (tFinal - tInitial) / steps;

rungeKutta2[cc_] := (
  curTime = cc[[1]];
  curx1 = squirrel;
  newTime = curTime + deltaT;
  (* Now I have left you six equations to implement *)
  chipmunk;
  chimpanzee;
  chihuahua;
  chipotle;
  macdonalds;
  wendys;
  {newTime, newx1, newx2, newv1, newv2}
)
(* Test your implementation on the initial conditions *)
rungeKutta2[initialConditions]
(* I know I am notoriously wrong on these, but I actually check *)
(* pretty carefully. Sigh. Anyway, this is what I get: *)
(* {0.005,-0.499743,-9.8696 10-6,0.102644,-0.00394784} *)

```

## Using NestList[] to Repeatedly Apply rungeKutta2[]

In[511]:=

```
rk2Results = NestList[rungeKutta2, initialConditions, steps];
```

## Transposing to Get Points We Can Put in ListLinePlot[]

```

rk2ResultsTransposed = Transpose[rk2Results];
times = rk2ResultsTransposed[[1]];
x1s = bananas;
x2s = apples;
timesAndx1s = timesAndBananas;
timesAndx2s = timesAndApples;

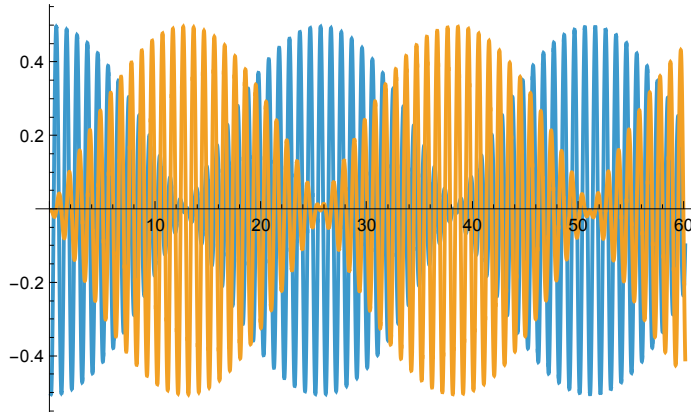
```

## Plotting The Results

In[518]:=

```
ListLinePlot[{timesAndx1s, timesAndx2s}]
```

Out[518]=

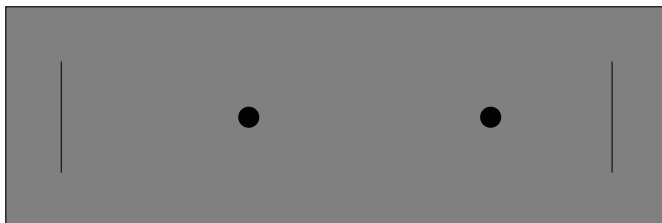


## A Simple Graphic

In[519]:=

```
coupledOscillatorGraphic[x1_, x2_] := Graphics[{
  (* the first line makes a gray rectangle *)
  {EdgeForm[Thin], Gray, Polygon[{{0, -1}, {6, -1}, {6, 1}, {0, 1}}]},
  Line[{{0.5, -0.5}, {0.5, 0.5}}],
  Line[{{5.5, -0.5}, {5.5, 0.5}}],
  Style[Point[{x1 + 2, 0}], PointSize[0.03]],
  Style[Point[{x2 + 4, 0}], PointSize[0.03]]
}]
coupledOscillatorGraphic[0.2, 0.4]
```

Out[520]=

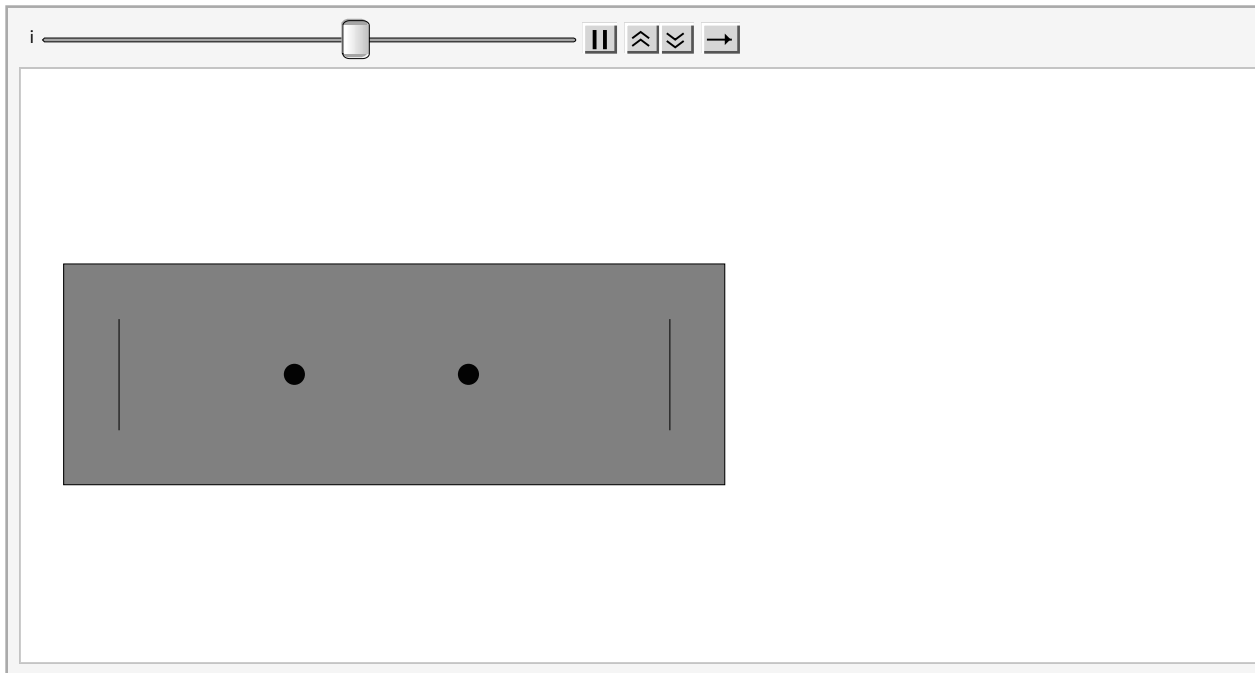


## Animating The Graphics

In[521]:=

```
Animate[coupledOscillatorGraphic[x1s[[i]], x2s[[i]],
  {i, 1, steps, 1}, DefaultDuration → tFinal - tInitial]
```

Out[521]=



## Comparing With YouTube

YouTube could be a great reference, but it has filled up with such an amazing amount of garbage, that finding a good video of coupled harmonic oscillators was surprisingly hard. I finally found a whole page of demonstrations put together by Caltech: [https://physicsdemos.caltech.edu/index\\_simple.html](https://physicsdemos.caltech.edu/index_simple.html), and one of the demonstrations was of the situation that we have simulated:

<https://youtu.be/Eoux0MsZqBY>

Just watch the first little bit. Then, if we want to understand more of what is going on later in the video, we are going to have to put in a bit of damping and a driving force, but we have already done so much of the fundamental work in building up this notebook, that would not actually be very difficult.