General Second-Order Runge-Kutta — Forced Oscillation

Done in class, January 31, 2025

This is the fifth notebook for you to finish in-class.

Forced Oscillation

Problem Description

```
omega = 1.0;
externalForce[t_] := 100 Sin[omega t]
springConstant = 20; dampingConstant = 1;
force[t_, x_, v_] := -springConstant x - dampingConstant v + externalForce[t]
mass = 5;
a[t_, x_, v_] := force[t, x, v] / mass;
tInitial = 0;
tFinal = 100;
steps = 25 000;
deltaT = (tFinal - tInitial) / steps;
```

Initial Conditions

Let’s let the spring be initially unstretched with no velocity and see what the external force does to it:

```
In[ ]:= xInitial = 0.0;
vInitial = 0.0;
(* We also want to be able to visualize the external force,
so let's tack it on. *)
initialConditions =
  {tInitial, xInitial, vInitial, externalForce[xInitial] / springConstant};
```

General Second-Order Runge-Kutta — Implementation

The implementation will be almost the same as in the damped oscillation notebook you just completed. What small things have to be changed?

```
alpha = 1;
rungeKutta2[cc_] := (
  {newTime, newPosition, newVelocity, externalForce[newTime] / springConstant}
 )
```

Displaying Forced Oscillation

Nest the procedure and then transpose the results to produce position and velocity plots:

```
In[ ]:= rk2Results = NestList[rungeKutta2, initialConditions, steps];
rk2ResultsTransposed = Transpose[rk2Results];
positionPlot = ListPlot[Transpose[rk2ResultsTransposed[[{1, 2}]]]]
```

```
In[ ]:= positions = rk2ResultsTransposed[[2]];
forces = rk2ResultsTransposed[[4]];
```

```
In[ ]:= Animate[NumberLinePlot[{positions[[step]], forces[[step]]}, PlotRange → {-25, 25}],
 {step, 0, steps, 1}]
```

Conclusion / Commentary

Our oscillator now has the force law $F(x) = -20\,x - v$ and in addition a sinusoidal external force.

You will remember that in the conclusion of the previous notebook I defined ω_0 and γ by

```
omega0 = Sqrt[springConstant / mass]
gamma = dampingConstant / (2 mass)
```

Now you see why I put the subscript “0” on ω. We actually have three relevant frequencies in the damped, driven harmonic oscillator:

ω	(the external or driving frequency)
ω_0	(the natural frequency of the oscillator in the absence of damping)
γ	(the frequency that controls the rate of decay of the exponential)

There is enough complexity here that this system is pretty complex to play around with. Once everybody has it working, we could try adjusting the input parameters to get other values for these three frequencies.