Oscillations and Waves in Mathematica

*An Introduction to Programming, Numerical Methods, and the Physics of Oscillations and Waves

Syllabus

Unofficial/Short Course Title: Oscillations and Waves

Spring 2025, Deep Springs College, ***Prof. Brian Hill***

Required Resources

Computing Resources

- Mathematica license: Desktop and Cloud is $75/year for a student. There is also a semester (six-month) plan for $50, and even a monthly plan, for $10/month, which would work fine and save you another $10 if you only used it for the four months from early-January to early-May.
- You need a laptop on which you can install of Mathematica. Disk space is critical: 18 GB is required if you install the local documentation. 9 GB is tight but doable without local documentation.

Texts

The text we will start off with is freely available on-line, so you do not need a physical copy:

- Stephen Wolfram, *An Elementary Introduction to the Wolfram Language, 3rd Edition,* Wolfram Media, Inc., 2023 (**Online version**), hereafter abbreviated *EIWL3*.

If you want a physical copy, get the 3rd edition (**Amazon link**).

For the numerical methods and the physics theory, I will prepare handouts and Mathematica notebooks for you to download and modify.

Overview

The First Course within the Course

In order to do mathematical modeling, we need to learn a serious programming language, and we will use Mathematica for this purpose. Mathematica is a program that you interact with using the "Wolfram Language." We will learn the language by studying the entirety of *EIWL3.* The printed edition is over 300 pages divided into 48 sections. We will do 2-3 sections per class, and by the end of the book, you will be in an extremely good position to apply Mathematica to any problem that interests you. In parallel with learning the language we will be learning oscillatory motion and then waves and the numerical methods required to model them on a computer. That is the second course within the course, and where things get most interesting.

The Second Course within the Course

What we will do with Mathematica in this course is fundamental physics that all theoretical physicists know very well: oscillatory motion and waves. After we deal with constant motion and constant acceleration, we will animate simple oscillation of a single particle. The classic physical system that exhibits simple oscillation is **a mass hanging from a spring**. There is a significant increase in complexity when you graduate from an idealized spring obeying Hooke's Law to putting the mass on the end of a pendulum rod: the simple system becomes non-linear.

The next level of complexity is to step it up to two particles. If the two particles are connected, even weakly connected, this leads to surprising behavior (resonant exchange of energy) that was not present for either particle separately. The most common example is known as the coupled pendulum. Since Kel asked about chaos (not something I have much familiarity with), I will see if we can code up some chaotic motion using the compound pendulum (and note that the compound pendulum is not the same as the coupled pendulum).

Yet another level of complexity is to step it up to from two to *N* particles. After we get quite familiar with the behavior of *N* particles, where *N* is a modest number like 10 or 32, we will take the limit that *N* goes to infinity! Waves appear! They appear completely naturally from laws governing a finite but ever larger number of ever more closely spaced and ever smaller particles.

Waves first show up in a single dimension, such as waves on a musical instrument string, where each element of the string has a position, but the string itself is treated as a one-dimensional object (specified by a mass per unit length and a tension), that can vibrate in the two directions perpendicular to its length. Then we can step up the complexity yet again and consider waves in two dimensions, such as waves on square and round drumheads.

Finally the highest level of complexity we can hope to get to in a one-semester course, starting with no significant prerequisites, is to model quantum-mechanical waves in one and more dimensions. The study of three-dimensional electron waves around an atomic nucleus requires some decently advanced manipulation of differential equations, but by the end of our course, you will start having sufficient intuition about such equations and their solutions that we can make some headway and by the final class, explore the s, p, d, and f Hydrogen wave functions, and even the modulation of spherical harmonics by radial wave functions.

Grading

- 45% assignments
- 37.5% exams 15% for each of the first two exams (on physical modeling and numerical methods), and 7.5% for a third (shorter) exam focused on the Wolfram language itself
- 7.5% final project presentations during Weeks 12 and 13 of the semester.
- 10% preparation for class and leadership of course

Problem Sets / Handouts / Exams

There will be assignments in *EIWL3* due almost every class, limited only by how quickly I can plan and grade them. The more problems you do, the more fluent you will be with the Wolfram language. In addition to the problem sets and their solutions, there will be handouts on numerical methods and physics modeling, and the exams which will be delivered as Mathematica notebooks.

Problem sets will generally be turned in by email using Mathematica notebooks. The last step in your process before you submit a notebook as your solution is to quit the Mathematica kernel, and then re-execute your entire notebook with a fresh kernel. Assuming your notebook executes reproducibly and without error with a fresh kernel, you have something you can turn in. If it doesn't execute reproducibly and without error, find and fix the problems, and repeat this process until it does.

Absences (and late work)

The College's policies on absences (and late work) are applicable. Refer to the Academic Year 2024-2025 Deep Springs Handbook.

Daily Schedules

Week 1 — Lightning Introduction to Mathematica Notebooks — Velocity of a Particle

- Tuesday, Jan. 14 — Preparation for class: Study Sections 1-3 of ***An Elementary Introduction to the Wolfram Language, 3rd Edition***, hereafter abbreviated *EIWL3* — Expressions, operator precedence, symbolic manipulation, built-in functions, and user-defined functions in Mathematica — We will work through a little demonstration called ***Heads or Tails*** which demonstrates user-defined functions and nested functions

- Friday, Jan. 17 — Problem Set 1: Solve all Sections 1-4 of *EIWL3* — ***PS1 Solution*** — We learned four more notebook features that greatly enhance readability: plain text cells, section cells, subsection cells, and comments in code — In class, we worked through a small but critical bit of physics, ***Position from Velocity — Theory*** — We manually filled out a ***Position from Velocity - Constant Acceleration*** worksheet that demonstrates the theory we have developed so far for the case of linearly increasing velocity (aka constant acceleration or uniform acceleration)

Week 2 — Acceleration of a Particle — Newton's 2nd Law — Interesting Complications

- Tuesday, Jan. 21 — Problem Set 2: Solve all of the exercises from Sections 5-8 of *EIWL3* — ***PS2 Solution*** — Look ahead to Section 9 of *EIWL3* — In class: We completed the ***Position from Velocity - Constant Acceleration*** notebook
- Friday, Jan. 24 — Problem Set 3: Solve all the exercises from Sections 9, 10, and the first half of Section 11 (Exs. 11.1 to 11.15) — ***PS3 Solution*** — In-class: We will develop ***Velocity from Acceleration - Theory*** and combine this with the theory we already developed for obtaining position from velocity — In other words, we will transitively obtain velocity from acceleration and then position from velocity — We will start to see how interesting complications can arise — Finally, in class, we will complete the ***Velocity from Acceleration - Sinusoidal Force*** notebook

Week 3 — Second-Order Runge-Kutta — Mass on a Spring — Damped Oscillation

- Wednesday, Jan. 29 — Problem Set 4: Solve the rest of the exercises from Section 11 (Exs. 11.16 to 11.31), Section 12, and Section 13 — ***PS4 Solution*** — In-class: Develop ***Second-Order Runge-Kutta - Theory*** and then complete the ***Second-Order Runge-Kutta - Mass on a Spring*** notebook — Period (T), frequency $(f=1/T)$, and angular frequency $(\omega=2\pi f)$
- Friday, Jan. 31 — No new assignment in *EIWL3* — In-class: Develop ***General Second-Order Runge-Kutta - Theory*** and then complete the ***General Second-Order Runge-Kutta - Damped Oscillation*** notebook

Week 4 — Forced Oscillation — Resonance — Damped Pendulum — Animated Graphics

- Tuesday, Feb. 4 — Problem Set 5: Do the exercises from Sections 14 and 17 — ***PS5 Solution*** — Additionally, instead of turning in the exercises from Sections 15 and 16, explore Mathematica and choose any topic on which to give a presentation — In-class: We will flesh out a little more of the **General Second-Order Runge-Kutta - Best Estimate of Average** and then proceed to your presentations
- Friday, Feb. 7 — Instead of a new Problem Set: Before class, have the ***General Second-Order Runge-Kutta - Forced Oscillation*** notebook working — In-class: We will use the notebook you finished before class to analyze resonance — Develop theory to transition from the damped oscillator to the damped pendulum — ***Damped Oscillator to Damped Pendulum - Theory*** which introduces Angle, Angular Velocity, Angular Acceleration, and the Approximate Theoretical Solution — In-class: Start completing ***Damped Pendulum - With Animated Graphics*** notebook

Week 5 — Forced Pendulum — Phase Space — Chaos — Exam 1

- Tuesday, Feb. 11 — Problem Set 6: Re-download ***Damped Pendulum - With Animated Graphics*** notebook that we started in the last class and finish it — ***PS6 Solution*** — Problem Set 7: Do the exercises from *EIWL3* Sections 18 and 19 — ***PS7 Solution*** — In-class: Examine ***Forced Pendulum - Phase Space - Chaos*** notebook
- Friday, Feb. 14 — ***Exam 1 - Naval Battle*** — ***Exam 1 Solution***

Week 6 — Systems with Two Masses — Systems with Many Masses

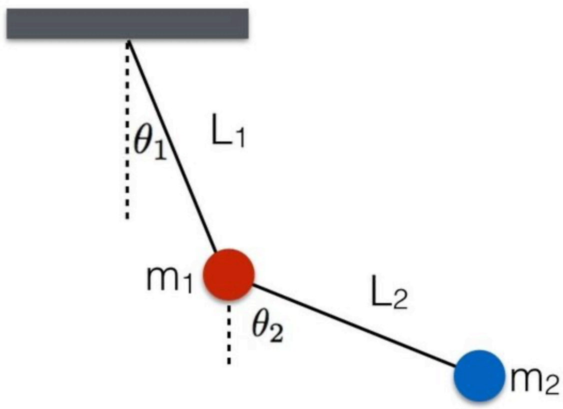
- Tuesday, Feb. 18 — Problem Set 8: Do the exercises from *EIWL3* Sections 20, 21, and 22 — In-class: Develop ***Coupled Harmonic Oscillators - Theory*** and then complete the ***Coupled Harmonic Oscillators*** notebook

EIWL3 Sections 25-34, and Sections 38-41 are when we get to the grammar and a significantly more advanced understanding of the Wolfram Language. You already learned about options in Section 20. Now pay particular attention to Sections 25 and 26, which I deliberately spread across two problem sets so that they would have more time to sink in. These two sections start exposing the heart of functional programming.

- Friday, Feb. 21 — Problem Set 9: Do the exercises from *EIWL3* Sections 23, 24, and 25 — In-class: Complete the ***Many Harmonic Oscillators*** notebook

Week 7 — Double Pendulum

- Tuesday, Feb. 25 — Problem Set 10: Do the exercises from *EIWL3* Sections 26, 27, and 28 — In-class: Present ***Double Pendulum - Theory*** and then complete the ***Double Pendulum*** notebook



Double Pendulum

Daily Schedule Term 5

See also **Daily Schedule Term 4**

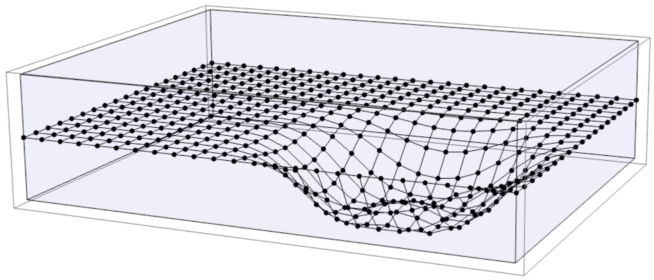
Week 8 — Torsion Pendulum — Coupled Torsion Pendula — The Second Derivative — Torsion Waves

As a way of refreshing on material that is three weeks old due to the Term 4-5 break, our eleventh and twelfth notebooks (Torsion Pendulum and Coupled Torsion Pendula) are straightforward recapitulations of our third and eighth notebooks (Mass on a Spring and Coupled Harmonic Oscillators). Similarly, our thirteenth notebook (Torsion Waves) will be a straightforward recapitulation of our ninth notebook (Many Harmonic Oscillators), but we will press on the initial conditions harder and get obvious traveling waves to appear. Also, we are about to start making a connection to an important calculus idea, "The Second Derivative." Waves typically show up in systems for which Newton's Laws involve second derivatives with respect to both space and time.

- Tuesday, March 18 — Problem Set 11: Do the Exercises from *EIWL3* Sections 29 and 30 — In-Class: Complete the ***Torsion Pendulum*** and ***Coupled Torsion Pendula*** notebooks
- Friday, March 21 — Problem Set 12: Do the Exercises from *EIWL3* Sections 31 and 32 — In-Class: Develop ***Torsion Waves - Theory***, complete the ***Torsion Waves*** notebook, and then develop ***The Second Derivative***

Week 9 — Drumheads — Two-Dimensional Grids of Masses

- Tuesday, March 25 — Problem Set 13: Do the Exercises from *EIWL3* Sections 33 and 34 — In-Class: Complete the **Rectangular Drumhead** notebook which models a drumhead with a rectangular grid of masses

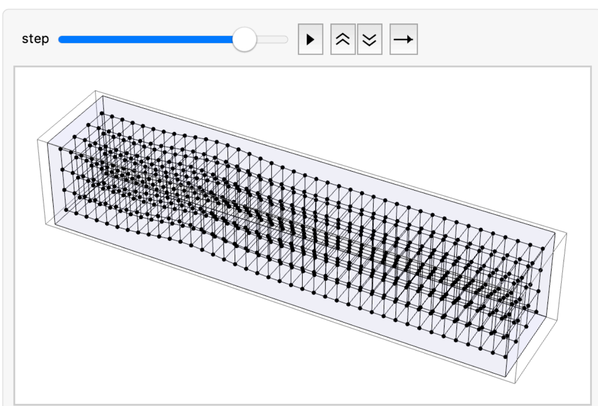


Rectangular Drumhead Mallet Strike

- Friday, March 28 — Problem Set 14: Do the Exercises from *EIWL3* Sections 35 and 36 — and then complete the **Circular Drumhead** notebook which models a drumhead with a circular grid of masses

Week 10 — Three-Dimensional Grid of Masses — Exam 2

- Tuesday, April 1 — Problem Set 15: Do the Exercises from *EIWL3* Sections 37 and 38 — In-Class: Let's attempt to consolidate our understanding or at least record in one place the **Drumheads — Theory** that we have been using, which now includes the ideas of longitudinal and transverse waves and then complete the **Cubical Grid of Masses - Transverse Waves**, and finally discuss what Friday's exam will cover (obviously we have covered so much since Exam 1, it would be unfair and impossible to comprehensively test it)



Transverse Wave Transiting With Periodic Boundary Conditions

- Friday, April 4 — ***Exam 2 - Longitudinal (Compression) Waves*** — ***Exam 2 Solution*** (much of the exam will involve things similar to what you did in the transverse waves notebook) — Check out the next problem set and do it before the exam (even though it isn't officially due until April 8)

We now have a little less than one-third of the course left, and we are going to blast into the stratosphere: instead of doing large numbers of particles — like 72 rods or a grid of 18 by 24=432 masses — we are going to take the limit that n→∞ and start specifying problems to Mathematica using the notation of derivatives. You might reasonably ask that if it is too time-consuming for Mathematica to do, say, a thousand particles for ten-thousand time steps, how is Mathematica going to cope with continuous systems which have an infinite number of particles? The answer is that deep under the hood, it breaks continuous systems up into little chunks and little time steps just as we have been chunking up the world with grids and time steps. In other words, Mathematica also turns continuous problems back into problems with a large but finite number of chunks and time steps. However, from now on in this course, Mathematica is going to hide the need to break continuous systems up into little chunks from us.

Mathematica draws graphs and creates animations by choosing the chunks and the time steps to be so small that we don't generally perceive them unless we blow up the graphics to a large size. Sometimes, the chunkiness leaks through enough to be obvious, but if that happens and we are not satisfied, we can order Mathematica to use finer chunks and finer time steps. Of course, the processor in your laptop may not appreciate this, but if you are patient with your computer's processor, and also if your computer has enough memory to hold the intermediate results, there is no limit in principle to how accurately computers can do simulations — the only exceptions being simulations with chaos or singularities, and even in such situations, simulations are often informative and indicative of what happens in the real world. All that said, the limits of computer power are real, and are part of the reason why we do not yet have compelling simulations for things like the formation of the solar system, even using the largest supercomputers available.

Week 11 — Continuous Systems from the $n \to \infty$ Limit — Oscillators and Guitar Strings

- Tuesday, April 8 — Problem Set 16: Make a Wolfram language notebook containing a compilation of fundamental, low-level syntax and functions (such as @@, @@@, /@ ./, Table, Array, Module, etc.) and email it to me before or after Friday's exam — Everyone's cheat cheets (**Brian's**, **Eli's**, **Harper's**, **Hexi's**, **Jeremy's**, **Rania's**, **Tahm's**, and **Walker's** — In-class: Present the **Harmonic Oscillator Redux** notebook (this time letting Mathematica do all the hard work)

- Friday, April 11 — Problem Set 17: Do the Exercises from *EIWL3* Sections 39 and 40 — In-class: Complete the **Guitar String** notebook (the math is a lot like torsion waves, but the new thing is that again we are going to be letting Mathematica do all the hard work)

- Also Friday, April 11 — Begin **Special Projects**

Week 12 — Drumheads — Diffusion of Heat

- Tuesday, April 15 — Problem Set 18: Do the exercises from *EIWL3* Sections 41 and 42 *(NB: due date extended to Friday along with Problem Set 19 which is also due on Friday)* — Complete the **Rectangular Drumhead Redux** notebook — Presentation: Jeremy and Eli on the **Lotke-Volterra Equations**
- Friday, April 18 — Problem Set 19: Do the exercises from *EIWL3* Sections 43 and 44 — **Diffusion in One Dimension** and **Steady State Diffusion in Two Dimensions** (some of the same mathematics appears in diffusion as in waves!) — Presentation: Rania on Bohr and Wheeler's **Liquid Drop Model of Fission**

Week 13 — Diffusion of Pollutants in Three Dimensions — Schrodinger's Equation in One Dimension — Harmonic Oscillator Wave Functions

- Tuesday, April 22 — Problem Set 20: Do the exercises from *EIWL3* Sections 45 and 46 — **Diffusion in Three Dimensions - Monte Carlo** — Presentation: Hexi and Walker on the **Black-Scholes** formula
- Friday, April 25 — Problem Set 21: Do the exercises from *EIWL3* Sections 47 and 48 — In-Class: Develop ***Schrodinger Equation - Theory**, and then explore the **Schrodinger Equation - One Dimension** and **Schrodinger Equation - Interpretation and Time-Dependence** notebooks which use the harmonic oscillator as our first quantum-mechanical example — Presentation: Harper and Tahm on **External Rifle Ballistics**

Week 14 — Exam 3 — Schrodinger's Equation in Two and Three Dimension — Hydrogen Wave Functions

- Tuesday, April 29 — The **Exam 3** problems are intended to be like exercises from *EIWL3*, especially those that use your understanding of the core of the Wolfram Language (Map, Apply, anonymous functions, *etc.*), rather than your understanding of specialized library functions (like GeoGraphics, WikipediaData, etc.)
- Friday, May 2 — In-Class: Develop **Schrodinger Equation - More Theory**, and then explore the **Schrodinger Equation - Hydrogen Wave Functions** notebook — For what is perhaps the most important application of the theory in the last notebook, please see this high-quality University of Central Florida writeup of **The Exclusion Principle and the Periodic Table** by Ling, Sanny, and Moebs — Another very important application is Hydrogen atom spectroscopy