Brian’s Solution — Waves Exam 3

April 29, 2025

This exam tests your fluency with the core of the Wolfram Language, as it was presented in *An Elementary Introduction to the Wolfram Language, 3rd Edition (EIWL3),* Sections 25-34 and 38-41. There is one problem with two or three parts corresponding to each section. ***Tip: all of them are meant to be quick. If you get bogged down, move on.***

Directions:

After downloading this notebook, rename it with your first name in the filename. E.g., Eli-Exam3.nb, Harper-Exam3.nb, Hexi-Exam3.nb, Jeremy-Exam3.nb, Rania-Exam3.nb, Tahm-Exam3.nb, or Walker-Exam3.nb.

Then disconnect from the wifi and work the exam. Save your notebook early and often so that you don’t lose work in progress.

Your answers always go into the Wolfram Language Input cells that begin with a comment, e.g.,

```
(* 1a *) foobar /@ Plus[Array]
```

All your answers should execute and re-execute without warnings or error messages.

You may refer to your downloaded copies of EIWL3, and anything else we developed in the course (like your cheat sheets!), but not to any web resources.

When you are done, save your notebook one last time, re-join the wifi, and then email it to me.

This exam was designed to require about 45 minutes, but if you need a full hour, that is ok. Everyone will stop at the one-hour mark.

1. Applying Functions (*EIWL3* Section 25)

(a)

Use `Map` with a *levelspec* to put a frame around each individual number in the array `Array[Plus,{10,10}]` (we don’t want frames around already-framed things — just one level of frames around the individual numbers).

In[1]:=
```
(* 1a *) Map[Framed, Array[Plus,{10,10}], {2}]
```

Out[1]=

$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	$\boxed{8}$	$\boxed{9}$	$\boxed{10}$	$\boxed{11}$
$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	$\boxed{8}$	$\boxed{9}$	$\boxed{10}$	$\boxed{11}$	$\boxed{12}$
$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	$\boxed{8}$	$\boxed{9}$	$\boxed{10}$	$\boxed{11}$	$\boxed{12}$	$\boxed{13}$
$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	$\boxed{8}$	$\boxed{9}$	$\boxed{10}$	$\boxed{11}$	$\boxed{12}$	$\boxed{13}$	$\boxed{14}$
$\boxed{6}$	$\boxed{7}$	$\boxed{8}$	$\boxed{9}$	$\boxed{10}$	$\boxed{11}$	$\boxed{12}$	$\boxed{13}$	$\boxed{14}$	$\boxed{15}$
$\boxed{7}$	$\boxed{8}$	$\boxed{9}$	$\boxed{10}$	$\boxed{11}$	$\boxed{12}$	$\boxed{13}$	$\boxed{14}$	$\boxed{15}$	$\boxed{16}$
$\boxed{8}$	$\boxed{9}$	$\boxed{10}$	$\boxed{11}$	$\boxed{12}$	$\boxed{13}$	$\boxed{14}$	$\boxed{15}$	$\boxed{16}$	$\boxed{17}$
$\boxed{9}$	$\boxed{10}$	$\boxed{11}$	$\boxed{12}$	$\boxed{13}$	$\boxed{14}$	$\boxed{15}$	$\boxed{16}$	$\boxed{17}$	$\boxed{18}$
$\boxed{10}$	$\boxed{11}$	$\boxed{12}$	$\boxed{13}$	$\boxed{14}$	$\boxed{15}$	$\boxed{16}$	$\boxed{17}$	$\boxed{18}$	$\boxed{19}$
$\boxed{11}$	$\boxed{12}$	$\boxed{13}$	$\boxed{14}$	$\boxed{15}$	$\boxed{16}$	$\boxed{17}$	$\boxed{18}$	$\boxed{19}$	$\boxed{20}$

(b)

Copy what you did in (a), but for this part, also turn the result into a grid using **Grid** and the “as an afterthought” syntax:

In[2]:=
```
(* 1b *) Map[Framed, Array[Plus,{10,10}], {2}] // Grid
```

Out[2]=

2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18
10	11	12	13	14	15	16	17	18	19
11	12	13	14	15	16	17	18	19	20

2. Pure Anonymous Functions (*EIWL3* Section 26)

(a)

Use the `#` and `&` notation to create an anonymous function that cubes whatever is given it, and then use `/@` to apply it to every member of the list `{1,2,3,4,5}`.

```
In[3]:= (* 2a *) #^3 & /@ {1,2,3,4,5}
Out[3]= {1, 8, 27, 64, 125}
```

(b)

Use the `#1`, `#2`, and `&` notation to create an anonymous function that divides its first argument by its second argument. Combine this with `Apply` and a *levelspec* to apply the function to `{{1,2},{2,3},{3,4},{4,5}}`. Once you have this right, you will get $\{\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5}\}$.

```
In[4]:= (* 2b *) Apply[#1 / #2 &, {{1,2},{2,3},{3,4},{4,5}}, 2]
```

Out[4]= $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right\}$

3. Applying Functions Repeatedly (*EIWL3* Section 27)

(a)

Use `Nest` to apply `Factorial` twice to `{1,2,3,4}`. If you have this right, 620,448,401,733,239,439,360,000 will be one of the elements of your answer.

```
In[5]:= (* 3a *) Nest[Factorial, {1,2,3,4}, 2]
Out[5]= {1, 2, 720, 620 448 401 733 239 439 360 000}
```

(b)

Use `NestList` to apply `Factorial` three times to `{1,2,3}`, as well as showing the results of doing it 0, 1, and 2 times. If you have this right, you will have an insanely large result at the third step. Do not go any higher, or I do not know what will happen to your computer.

In[6]:=
```
(* 3b *) NestList[Factorial, {1,2,3}, 3]
```

Out[6]=
```
{{1, 2, 3}, {1, 2, 6}, {1, 2, 720}, {1, 2,
  2 601 218 943 565 795 100 204 903 227 081 043 611 191 521 875 016 945 785 727 541 837 850 835 \
   631 156 947 382 240 678 577 958 130 457 082 619 920 575 892 247 259 536 641 565 162 052 015 \
   873 791 984 587 740 832 529 105 244 690 388 811 884 123 764 341 191 951 045 505 346 658 616 \
   243 271 940 197 113 909 845 536 727 278 537 099 345 629 855 586 719 369 774 070 003 700 430 \
   783 758 997 420 676 784 016 967 207 846 280 629 229 032 107 161 669 867 260 548 988 445 514 \
   257 193 985 499 448 939 594 496 064 045 132 362 140 265 986 193 073 249 369 770 477 606 067 \
   680 670 176 491 669 403 034 819 961 881 455 625 195 592 566 918 830 825 514 942 947 596 537 \
   274 845 624 628 824 234 526 597 789 737 740 896 466 553 992 435 928 786 212 515 967 483 220 \
   976 029 505 696 699 927 284 670 563 747 137 533 019 248 313 587 076 125 412 683 415 860 129 \
   447 566 011 455 420 749 589 952 563 543 068 288 634 631 084 965 650 682 771 552 996 256 790 \
   845 235 702 552 186 222 358 130 016 700 834 523 443 236 821 935 793 184 701 956 510 729 781 \
   804 354 173 890 560 727 428 048 583 995 919 729 021 726 612 291 298 420 516 067 579 036 232 \
   337 699 453 964 191 475 175 567 557 695 392 233 803 056 825 308 599 977 441 675 784 352 815 \
   913 461 340 394 604 901 269 542 028 838 347 101 363 733 824 484 506 660 093 348 484 440 711 \
   931 292 537 694 657 354 337 375 724 772 230 181 534 032 647 177 531 984 537 341 478 674 327 \
   048 457 983 786 618 703 257 405 938 924 215 709 695 994 630 557 521 063 203 263 493 209 220 \
   738 320 923 356 309 923 267 504 401 701 760 572 026 010 829 288 042 335 606 643 089 888 710 \
   297 380 797 578 013 056 049 576 342 838 683 057 190 662 205 291 174 822 510 536 697 756 603 \
   029 574 043 387 983 471 518 552 602 805 333 866 357 139 101 046 336 419 769 097 397 432 285 \
   994 219 837 046 979 109 956 303 389 604 675 889 865 795 711 176 566 670 039 156 748 153 115 \
   943 980 043 625 399 399 731 203 066 490 601 325 311 304 719 028 898 491 856 203 766 669 164 \
   468 791 125 249 193 754 425 845 895 000 311 561 682 974 304 641 142 538 074 897 281 723 375 \
   955 380 661 719 801 404 677 935 614 793 635 266 265 683 339 509 760 000 000 000 000 000 000 \
   000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 \
   000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 \
   000 000 000 000 000 000 000}}
```

4. Tests and Conditionals (*EIWL3* Section 28)

(a)

Use **`PrimeQ`** and **`/@`** to generate a **`True`** or **`False`** list that is twenty elements long expressing which numbers in **`Range[20]`** are prime:

In[7]:=
```
(* 4a *) PrimeQ /@ Range[20]
```

Out[7]=
```
{False, True, True, False, True, False, True, False, False, False,
 True, False, True, False, False, False, True, False, True, False}
```

(b)

Combine **`PrimeQ`** with **`Select`** to only list the numbers in **`Range[20]`** that are prime:

In[8]:=
```
(* 4b *) Select[Range[20], PrimeQ]
```
Out[8]=
```
{2, 3, 5, 7, 11, 13, 17, 19}
```

5. More About Pure Functions (*EIWL3* Section 29)

(a)

Accomplish exactly the same thing as `Table[n*(n-1)/2,{n,6}]` using `Array` and a pure function.

In[9]:=
```
(* 5a *) Array[# * (# - 1) / 2 &, 6]
```
Out[9]=
```
{0, 1, 3, 6, 10, 15}
```

(b)

Make some modifications to `FoldList[Plus,{1,2,3,4,5}]` so that it produces a list of the first 10 factorials. Instead of hand-coding the list up to 10, begin by first changing `{1,2,3,4,5}` to `Range[10]`.

In[10]:=
```
(* 5b *) FoldList[Times, Range[10]]
```
Out[10]=
```
{1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800}
```

6. Rearranging Lists (*EIWL3* Section 30)

(a)

Use `Transpose` and one of the *levelspec* options to turn `{{{1,uno},{2,dos},{3,tres}},{{4,cuatro},{5,cinco},{6,seis}}}` into `{{{1,2,3},{uno,dos,tres}},{{4,5,6},{cuatro,cinco,seis}}}`

In[11]:=
```
(* 6a *)
Transpose[{{{1,uno},{2,dos},{3,tres}},{{4,cuatro},{5,cinco},{6,seis}}},2↔3]
```
Out[11]=
```
{{{1, 2, 3}, {uno, dos, tres}}, {{4, 5, 6}, {cuatro, cinco, seis}}}
```

(b)

Use `Flatten` and a *levelspec* option to turn `{{{1,uno},{2,dos},{3,tres}},{{4,cuatro},{5,cinco},{6,seis}}}` into `{{1,uno},{2,dos},{3,tres},{4,cuatro},{5,cinco},{6,seis}}`

In[12]:=
```
(* 6b *)Flatten[{{{1,uno},{2,dos},{3,tres}},{{4,cuatro},{5,cinco},{6,seis}}},1]
```
Out[12]=
```
{{1, uno}, {2, dos}, {3, tres}, {4, cuatro}, {5, cinco}, {6, seis}}
```

7. Parts of Lists (*EIWL3* Section 31)

(a)

Use the magical **`All`** position (you will need to use **`All`** more than once) to turn **`{{{Eli, Lerner},{Harper,Yonago},{Hexi,Jin}},{{Jeremy,Choy},{Rania,Zaki},{Tahm,Loyd},{Walker,Harris}}}`** into **`{{Eli,Harper,Hexi},{Jeremy,Rania,Tahm,Walker}}`**

In[13]:=
```
(* 7a *) {{{Eli, Lerner},{Harper,Yonago},{Hexi,Jin}},
  {{Jeremy,Choy},{Rania,Zaki},{Tahm,Loyd},{Walker,Harris}}}[[All,All,1]]
```

Out[13]=
```
{{Eli, Harper, Hexi}, {Jeremy, Rania, Tahm, Walker}}
```

(b)

Use a magical *negative positional argument* to extract **`{Jeremy,Rania,Tahm,Walker}`** from **`{{Eli,Harper,Hexi},{Jeremy,Rania,Tahm,Walker}}`** and combine that with **`Take`** with a different magical *negative* argument to extract **`{Tahm,Walker}`**.

In[14]:=
```
(* 7b *) Take[{{Eli,Harper,Hexi},{Jeremy,Rania,Tahm,Walker}}[[-1]],-2]
```

Out[14]=
```
{Tahm, Walker}
```

8. Patterns (*EIWL3* Section 32)

(a)

Use **`Cases`** to choose the lists that begin and end with the same letter in this list of lists (but look ahead to part (b) before you solve part (a)):

```
{
 {“a”, “l”, “u”, “l”, “a”},
 {“a”, “l”, “o”, “h”, “a”},
 {“a”, “r”, “a”, “r”, “a”},
 {“b”, “o”, “n”, “u”, “s”},
 {“c”, “i”, “v”, “i”, “c”},
 {“d”, “e”, “b”, “e”, “d”},
 {“e”, “l”, “b”, “o”, “w”},
 {“z”, “a”},
 {“z”, “z”}
}
```

In[15]:=
```
(* 8a *) Cases[{
 {"a", "l", "u", "l", "a"},
 {"a", "l", "o", "h", "a"},
 {"a", "r", "a", "r", "a"},
 {"b", "o", "n", "u", "s"},
 {"c", "i", "v", "i", "c"},
 {"d", "e", "b", "e", "d"},
 {"e", "l", "b", "o", "w"},
 {"z", "a"},
 {"z", "z"}
 },
 {x_,__,x_}
 ]
```

Out[15]=
```
{{a, l, u, l, a}, {a, l, o, h, a}, {a, r, a, r, a}, {c, i, v, i, c}, {d, e, b, e, d}}
```

(b)

The pattern `BlankNullSequence` has the shorthand `___`. Use `___` to improve the pattern you used in Part (a) so that the two-letter list `{z,z}` is also included in your result.

In[16]:=
```
(* 8b *) Cases[{
 {"a", "l", "u", "l", "a"},
 {"a", "l", "o", "h", "a"},
 {"a", "r", "a", "r", "a"},
 {"b", "o", "n", "u", "s"},
 {"c", "i", "v", "i", "c"},
 {"d", "e", "b", "e", "d"},
 {"e", "l", "b", "o", "w"},
 {"z", "a"},
 {"z", "z"}
 },
 {x_,___,x_}
 ]
```

Out[16]=
```
{{a, l, u, l, a}, {a, l, o, h, a}, {a, r, a, r, a}, {c, i, v, i, c}, {d, e, b, e, d}, {z, z}}
```

9. Assigning Names to Things (*EIWL3* Section 38)

(a)

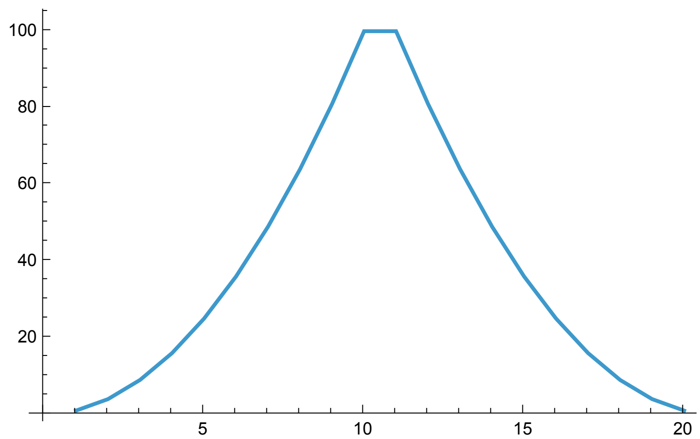
Use `Module` to compute `x=Factorial[10]`, and then produce `{x,x^2,x^3}`.

In[17]:=
```
(* 9a *) Module[{x = Factorial[10]}, {x,x^2,x^3}]
```
Out[17]=
```
{3 628 800, 13 168 189 440 000, 47 784 725 839 872 000 000}
```

(b)

Inside `Module`, let `rangeSquared=Range[10]^2`, and then produce a list line plot of `rangeSquared` joined with `Reverse[rangeSquared]`.

In[18]:=
```
(* 9b *) Module[{rangeSquared=Range[10]^2},
 ListLinePlot[Join[rangeSquared,Reverse[rangeSquared]]]]
```
Out[18]=



10. Immediate and Delayed Values (*EIWL3* Section 39)

(a)

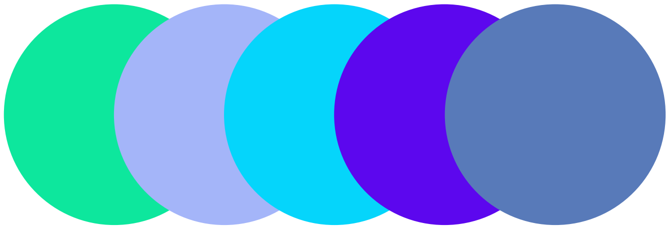
Make a one-character change to this expression, `Module[{x:=RandomInteger[10]},{x,x^2,x^3,x^4}]`, so that it produces four different powers of the same random number instead of four different powers of different random numbers.

In[19]:=
```
(* 10a *) Module[{x=RandomInteger[10]},{x,x^2,x^3,x^4}]
```
Out[19]=
```
{0, 0, 0, 0}
```

(b)

Make a one-character change to this expression, `Module[{color=RandomColor[]},Graphics[Table[Style[Disk[{i,0}],color],{i,5}]]]`, so that it produces five different-color disks.

In[20]:=
```
(* 10b *) Module[{color:=RandomColor[]},
 Graphics[Table[Style[Disk[{i,0}],color],{i,5}]]]
```
Out[20]=



11. Defining Your Own Functions (*EIWL3* Section 40)

(a) Wording on 11(a) could use improvement.

Define a function `f` that takes a list of three elements and out of them makes a list of lists that contains all six possible orderings. Using **`Permutations`** will make this easy.

Include a test of your function as `f[1,2,3]` and make sure it gets `{{1,2,3},{1,3,2},{2,1,3},{2,3,1},{3,1,2},{3,2,1}}`.

In[21]:=
```
(* 11a *) f[x_, y_, z_] := Permutations[{x, y, z}]
f[1, 2, 3]
```
Out[22]=
```
{{1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1}, {3, 1, 2}, {3, 2, 1}}
```

(b) Wording on 11(b) could also use improvement.

Define a function `g` that gives `1` for `g[0]`, and gives `n*g[n-1]` for any integer `n` greater than `0`, *but don't use an* `If` *statement!* Include a test of your function as `g[6]` and make sure it gets `720`.

In[23]:=
```
(* 11b *)g[0] = 1; g[n_] := n * g[n - 1]
g[6]
```
Out[24]=
```
720
```

12. More About Patterns (*EIWL3* Section 41)

(a)

Use the replacement rule notation — e.g., `/.` and `->` — to exchange the first and last element in any list containing two or more elements and test your replacement using the list
`{alpha, beta, gamma, delta, epsilon}`.

In[25]:=
```
(* 12a *) {alpha, beta, gamma, delta, epsilon} /. {x_, y___, z_} → {z, y, x}
```
Out[25]=
```
{epsilon, beta, gamma, delta, alpha}
```

(b)

Starting with **Characters/@RomanNumeral[Range[100]]**, select all the sequences corresponding to the Roman numerals that have XXX in them.

In[26]:=
```
(* 12b *) Cases[Characters /@ RomanNumeral[Range[100]], {___, "X", "X", "X", ___}]
```
Out[26]=
```
{{X, X, X}, {X, X, X, I}, {X, X, X, I, I}, {X, X, X, I, I, I}, {X, X, X, I, V},
 {X, X, X, V}, {X, X, X, V, I}, {X, X, X, V, I, I}, {X, X, X, V, I, I, I},
 {X, X, X, I, X}, {L, X, X, X}, {L, X, X, X, I}, {L, X, X, X, I, I},
 {L, X, X, X, I, I, I}, {L, X, X, X, I, V}, {L, X, X, X, V}, {L, X, X, X, V, I},
 {L, X, X, X, V, I, I}, {L, X, X, X, V, I, I, I}, {L, X, X, X, I, X}}
```

(c)

Use **StringJoin** to turn what you got in 12(b) into
{XXX,XXXI,XXXII,XXXIII,XXXIV,XXXV,XXXVI,XXXVII,XXXVIII,XXXIX,LXXX,LXXXI,LXXXII,LXXXIII,LXXXIV,LXXXV,LXXXVI,LXXXVII,LXXXVIII,LXXXIX}.

In[27]:=
```
(* 12c *)
StringJoin /@ Cases[Characters /@ RomanNumeral[Range[100]], {___, "X", "X", "X", ___}]
```
Out[27]=
```
{XXX, XXXI, XXXII, XXXIII, XXXIV, XXXV, XXXVI, XXXVII, XXXVIII, XXXIX, LXXX,
 LXXXI, LXXXII, LXXXIII, LXXXIV, LXXXV, LXXXVI, LXXXVII, LXXXVIII, LXXXIX}
```