Position from Velocity — Constant Acceleration

Started in class, January 21, 2025

Finished version due before class, January 24, 2025

I was going to give you guys a finished notebook, and indeed I have a finished notebook that does a fancy job of what you are about to do below, but I decided that it is much better for you to produce the core code.

The Constant Acceleration Worksheet

In the last class, we manually applied the theory to one-dimensional motion with constant acceleration.

We used the midpoint approximation, we chose $\Delta t = 0.4$, we chose $v(t) = 6 \cdot t$, and we iterated from $t_{\text{initial}} = 0.0$ to $t_{\text{final}} = 6.0$ by applying these formulas 15 times:

$t_{i+1} = t_i + \Delta t$

$x(t_{i+1}) \approx x(t_i) + v\left(t_i + \frac{\Delta t}{2}\right) \cdot \Delta t$

The index i in our example ranged from 0 to 15 with $t_0 = t_{\text{initial}}$ and $t_{15} = t_{\text{final}}$. This gave us 15 times steps and 16 points (counting both the initial and final ones).

Here is a function and some variables that capture everything about the specific problem we are trying to solve.

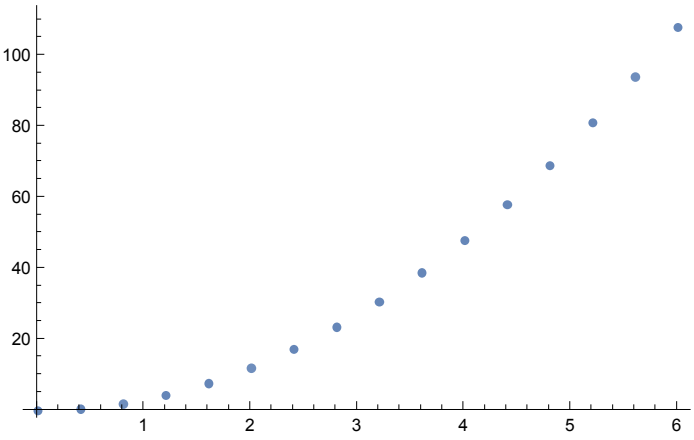
In[•]:=
```
v[t_] := 6 t;
steps = 15;
tInitial = 0.0;
tFinal = 6.0;
deltaT = (tFinal - tInitial) / steps;
```

An Inelegant Solution

The following works, but it is lousy programming (the kind of inelegant, inefficient programming most people do most of the time). I put it here so that you know what final result you are trying to get to. Don't even bother reading the lousy code at the top of the next page.

```
times = Range[tInitial, tFinal, deltaT];
midpointTimes = Drop[times, -1] + 0.2;
(* drop the last element which would be 6.2 *)
midpointVelocities = 6 midpointTimes; (* our velocity function is just 6 t *)
displacements = midpointVelocities deltaT;
(* the product of velocity and deltaT gives displacement *)
positions = Accumulate[displacements];
(* it happens that Mathematica has a function that does most of what we need *)
positions = Prepend[positions, 0.0];
(* Accumulate didn't include the initial position of 0.0,
so we need to explicitly prepend it *)
orderedPairs = Transpose[{times, positions}];
(* we have a list of times and a list of positions,
but we need ordered pairs to feed to ListPlot,
so we have to transpose the two lists *)
ListPlot[orderedPairs]
```

Out[ ]=



The Form of an Elegant Solution

I want you to do a much more elegant solution that uses the ideas I showed you in the Heads or Tails notebook. Specifically, I want you define a function that takes the current time and the current position, and then uses the theory to compute the next time and the next position. Then I want you to use NestList to get all 15 subsequent positions. You'll be building out something like this:

In[ ]:=

```
procedure[t_] := t + deltaT
results = NestList[procedure, 0, 15];
ListPlot[results];
```

This is going to be trickier than it looks, because the function **procedure** that we just defined takes one argument and returns one value and that is the only kind of function that **NestList** can handle. How on Earth are you going to supply both t_i and x_i to **procedure**? How on Earth are you going to return t_{i+1} and x_{i+1} from **procedure**?

Your Elegant Solution Goes Here

The rain in Spain falls mainly on the plane (while you are sitting on the tarmac, waiting to fly home).