# Drumheads Redux

## Completed and Analyzed in class, April 15, 2025

This is our nineteenth notebook. It builds on the techniques in the eighteenth notebook. The difference between a guitar string and a drumhead is just the number of dimensions. After graduating to two dimensions, it will (I hope) be fairly apparent how to go to three dimensions.

I am hopeful that we can cover both the rectangular and circular drumhead in one class. If we only did the rectangular one, that would be too modest a goal. It is hardly any different than the guitar string.

### Rectangular Drumhead — Theory

Back in the fourteenth notebook we had these acceleration formulas:

$$a_{j,k} = {v_0}^2\left(z_{j,k+1} + z_{j,k-1} + z_{j+1,k} + z_{j-1,k} - 4\,z_{j,k}\right)$$

For a guitar string, the corresponding equation is:

$$\frac{\partial^2 z}{\partial t^2} = {v_0}^2\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$$

Let's be clear about the dependent and independent variables. The time is $t$. The drumhead is stretched along the $x$- and $y$-axes. Those are the independent variables. The dependent variable is the displacement, which we are putting in the $z$-direction, so the dependent variable is $z$, and we want to find the function of three variables, $z(t, x, y)$.

#### Partial Derivatives

Now that we have three independent variables, we will have things like this:

```
In[1]:= Derivative[0, 0, 2][z][t, x, y] // TraditionalForm
```

Out[1]//TraditionalForm=

$$z^{(0,0,2)}(t, x, y)$$

### The Drumhead Differential Equation

Recopying what was above, you can give Mathematica this differential equation:

$$\frac{\partial^2 z}{\partial t^2} = {v_0}^2\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$$