Damped Pendulum — With Animated Graphics

Done in class, February 7, 2025

This is the sixth notebook for you to finish in-class.

Forced Oscillation

Angular Acceleration

In[306]:=

```
gravity = 9.80665;
(* the value of gravity in units of meters / seconds-squared *)
length = 0.24840;
(* A pendulum whose length is 9.7795 inches converted to meters  *)
(* The natural frequency of such a
 pendulum provided the swings are not large: *)
omega0 = Sqrt[gravity / length];
gamma = 0.03;
(* A real pendulum swinging in air typically has a small gamma. *)
period = 2 Pi / omega0;
(* The length was chosen so that the period is 1 second. To be *)
(* precise, 2 Pi / omega0 = 0.999989,
and 2 Pi / Sqrt[omega0^2-gamma^2] = 1.000000. *)
α[t_, theta_, omega_] := -omega0^2 Sin[theta] -2 gamma omega;
```

Simulation Parameters

In[312]:=

```
tInitial = 0.0;
tFinal = 50.0;
steps = 200000;
deltaT = (tFinal - tInitial) / steps;
```

Initial Angle and Angular Velocity

Let’s let the pendulum be initially held still at 10° and gently released:

In[316]:=

```
thetaInitial = 10 °;
omegaInitial = -gamma thetaInitial;
(* gamma is small, and this is only 0.3° / second. *)
(* Putting in the small initial velocity makes
 the approximate theoretical solution simplify. *)
initialConditions = {tInitial, thetaInitial, omegaInitial};
```

General Second-Order Runge-Kutta — Theory Recap

So you don't have to flip back to the damped pendulum theory notebook, I'll recapitulate:

$t^* = t + \lambda \Delta t$

$\theta^* = x(t_i) + v(t_i) \cdot \lambda \Delta t$

$\omega^* = \omega(t_i) + \alpha(t_i, x(t_i), v(t_i)) \cdot \lambda \Delta t$

$t_{i+1} = t_i + \Delta t$

$\omega(t_{i+1}) = \omega(t_i) + \left(\left(1 - \frac{1}{2\lambda}\right) \alpha(t_i, x(t_i), \omega(t_i)) + \frac{1}{2\lambda} \alpha(t^*, x^*, \omega^*)\right) \cdot \Delta t$

$\theta(t_{i+1}) = \theta(t_i) + (\omega(t_i) + \omega(t_{i+1})) \frac{\Delta t}{2}$

We got this by mindlessly making the replacements:

$x \to \theta$
$v \to \omega$
$a \to \alpha$

General Second-Order Runge-Kutta — Implementation

The implementation of the damped pendulum is almost the same as the damped oscillator. Figure out what needs to be changed.

In[319]:=

```
lambda = 1;
rungeKutta2[cc_] := (
  (* Extract time, angle, and angular velocity from the list *)
  curTime = cc[[1]];
  curAngle = cc[[2]];
  curAngularVelocity = cc[[3]];
  (* Compute tStar, xStar, vStar *)
  tStar = curTime + lambda deltaT;
  thetaStar = curAngle + curAngularVelocity lambda deltaT;
  angularVelocityStar =
   curAngularVelocity + α[curTime, curAngle, curAngularVelocity] lambda deltaT;
  (* Implement General Second-Order Runge-Kutta *)
  newTime = curTime + deltaT;
  newAngle = curAngle + (curAngularVelocity + newAngularVelocity) deltaT/2;
  newAngularVelocity =
   curAngularVelocity + ((1 - 1/(2 lambda)) α[curTime, curAngle, curAngularVelocity] +
       1/(2 lambda) α[tStar, thetaStar, angularVelocityStar]) deltaT;
  {newTime, newAngle, newAngularVelocity}
 )
N[rungeKutta2[initialConditions]]
(* Test the rungeKutta2 function you just wrote. *)
(* The output just below should be {0.0025,0.174498,-0.022372} *)
```

Out[321]=

```
{0.00025, 0.174539, -0.00694977}
```

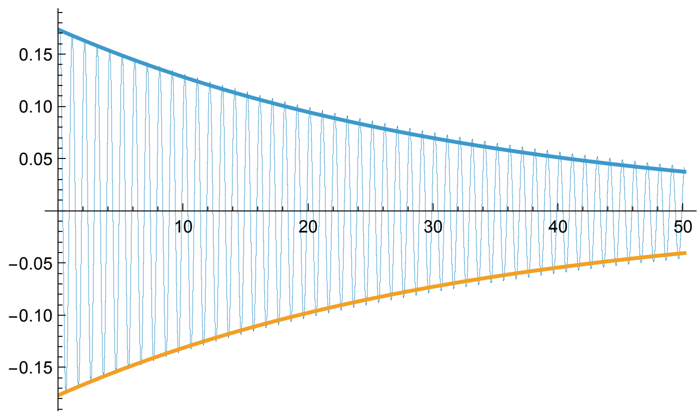
Displaying Oscillation

Nest the procedure, transpose the results, and produce a plot of the angle θ as a function of time:

In[322]:=

```
rk2Results = NestList[rungeKutta2, initialConditions, steps];
rk2ResultsTransposed = Transpose[rk2Results];
times = rk2ResultsTransposed[[1]];
thetas = rk2ResultsTransposed[[2]];
thetaPlot = ListPlot[Transpose[{times, thetas}]];
(* the theoretical solution is approximately known,
provided the angle remains small *)
(* let's plot the envelope of the theoretical solution *)
envelopeFunction[t_] := thetaInitial Exp[-gamma t]
approximateTheoreticalEnvelope =
  Plot[{envelopeFunction[t], -envelopeFunction[t]}, {t, tInitial, tFinal}];
Show[{thetaPlot, approximateTheoreticalEnvelope}]
```

Out[328]=



In the preceding plot, the theoretical solution is approximately known, provided the angle remains small, and so I added the envelope of the theoretical solution to the plot.

Displaying Theory

In the following plot, I have included the theoretical oscillation, not just the envelope (but the same approximation that the angle must remain small still applies):

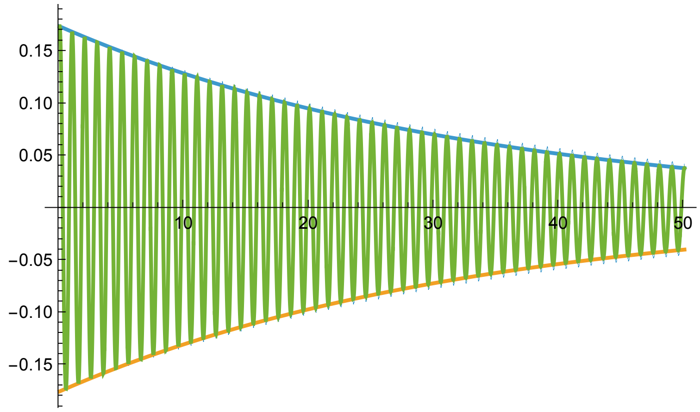
In[329]:=

```
approximateTheoreticalOscillationPlot =
  Plot[{envelopeFunction[t], -envelopeFunction[t],
    envelopeFunction[t] × Cos[Sqrt[omega0^2 - gamma^2] t]}, {t, tInitial, tFinal}];
```

In[330]:=

```
Show[{thetaPlot, approximateTheoreticalOscillationPlot}]
```

Out[330]=



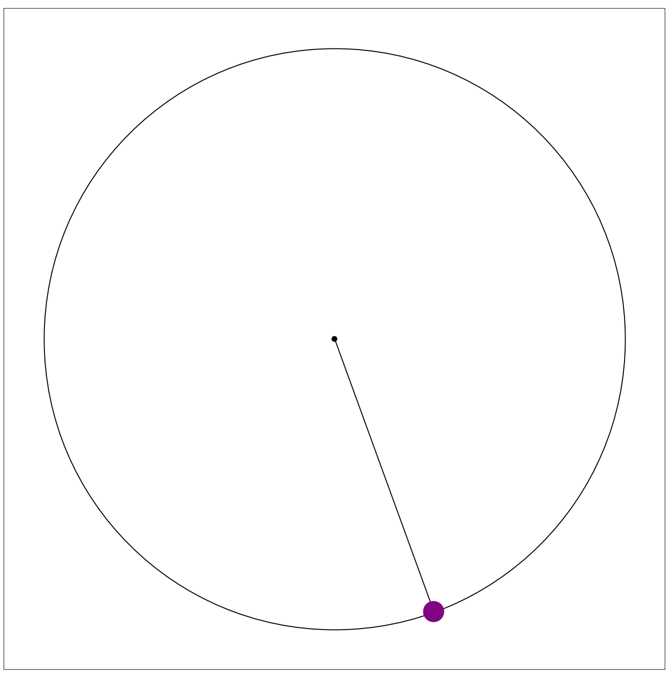
Drawing a Pendulum with Coordinates and Graphics

To do a legible job of this, you may need to review Section 14 of *EIWL3*. The goal is to finish implementing the function below so that you get a picture something like the one I have pasted in.

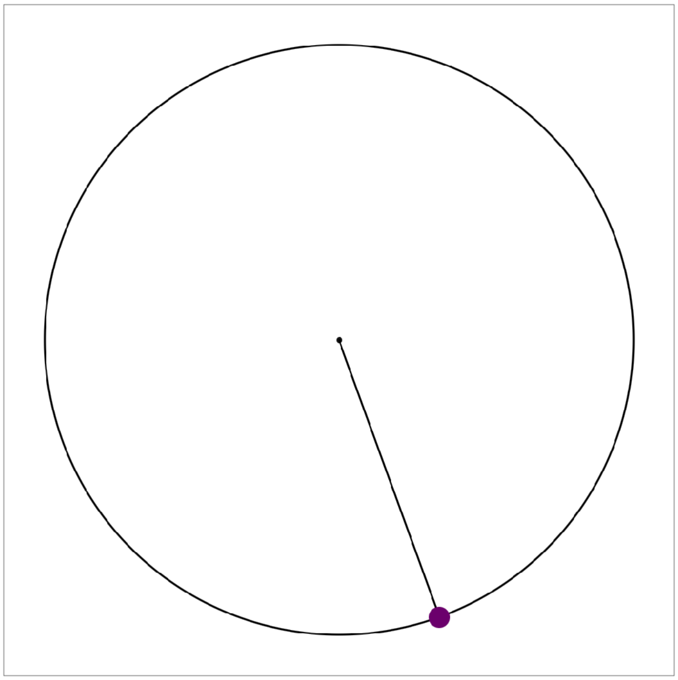
In[331]:=

```
pendulumGraphic[angle_] := Graphics[{
   EdgeForm[Thin], White,
   RegularPolygon[{0.0, 0.0}, 0.4, 4],
   Black,
   Circle[{0, 0}, length],
   (* all I left for you to add is two points and a line *)
   Point[{0, 0}],
   Line[{{0, 0}, length {Cos[angle - 90 °], Sin[angle - 90 °]}}],
   PointSize[0.03],
   Purple,
   Point[length {Cos[angle - 90 °], Sin[angle - 90 °]}]
  }]
pendulumGraphic[20 °]
```

Out[332]=



The pendulum graphic you are trying for (when the function is passed in 20° for the angle, and of course your function should do the right thing for any other angle):



Animating the Graphics

It’s also nice to have an animation, arranged so that the default duration of the animation is the actual duration of the animation:

In[333]:=

```
Animate[pendulumGraphic[thetas⟦step⟧],
 {step, 0, steps, 1}, DefaultDuration → tFinal - tInitial]
```

Out[333]=