Torsion Waves — Theory — The Second Derivative

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We are headed into the last of three notebooks on torsion pendula, and by the time we we are done with the last one it will become obvious that we are finally seeing waves.

Torsion Pendulum — Theory

The theory we developed for the torsion pendulum and promptly used in our eleventh notebook was:

$$\tau = -\kappa_L\, \theta - \kappa_R\, \theta$$

and

$$\tau = I\alpha$$

The formula that plays the role of $F = ma$ but involves torque is read, "torque, τ, equals the moment of inertia, I, times angular acceleration, α." I want to stress that this law involving torques is not an independent law. In a mechanics course, you would derive this law from Newton's Second Law.

We put those two equations together to get:

$$\alpha = (-\kappa_L\, \theta - \kappa_R\, \theta) / I$$

Then we defined

$$\omega_0{}^2 \equiv \frac{\kappa_L + \kappa_R}{I}$$

and our formula was elegantly written as

$$\alpha = -\omega_0{}^2\, \theta$$

Coupled Torsion Pendula — Theory

The theory we developed for coupled torsion pendula and promptly used in our twelfth notebook was:

$$\alpha_1 = -\omega_0{}^2 \theta_1 + \omega_{12}{}^2 (\theta_2 - \theta_1)$$

$\alpha_2 = -\omega_0{}^2 \theta_2 - \omega_{12}{}^2 (\theta_2 - \theta_1)$

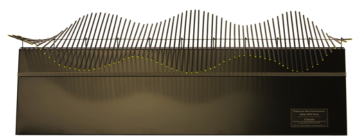
where,

$\omega_0{}^2 \equiv \frac{\kappa}{I}$

$\omega_{12}{}^2 \equiv \frac{\kappa_{12}}{I}$

Torsion Waves — Theory

Now we just need to graduate from one or two rods connected by stainless steel wires to seventy-two rods and we'll be able to build a notebook that simulates this:



Of course, seventy-two is not a magic number, and we will want to generalize to *n* rods, where is anything from from 1 to 1000, or whatever you like.

Let's write down the equations for seven rods with eight wires connecting them and to the walls. That should be enough to see where we need to go:

$\alpha_1 = -\omega_0{}^2 \theta_1 + \omega_0{}^2 (\theta_2 - \theta_1)$

$\alpha_2 = -\omega_0{}^2 (\theta_2 - \theta_1) + \omega_0{}^2 (\theta_3 - \theta_2)$

$\alpha_3 = -\omega_0{}^2 (\theta_3 - \theta_2) + \omega_0{}^2 (\theta_4 - \theta_3)$

$\alpha_4 = -\omega_0{}^2 (\theta_4 - \theta_3) + \omega_0{}^2 (\theta_5 - \theta_4)$

$\alpha_5 = -\omega_0{}^2 (\theta_5 - \theta_4) + \omega_0{}^2 (\theta_6 - \theta_5)$

$\alpha_6 = -\omega_0{}^2 (\theta_6 - \theta_5) + \omega_0{}^2 (\theta_7 - \theta_6)$

$\alpha_7 = -\omega_0{}^2 (\theta_7 - \theta_6) - \omega_0{}^2 \theta_7$