Lasers

We would like to understand just the basics, which is: how is a material arranged so as to get a huge number of photons into a single quantum state, a teeny bit of which are bled off to make the laser beam.

Pumping the Material

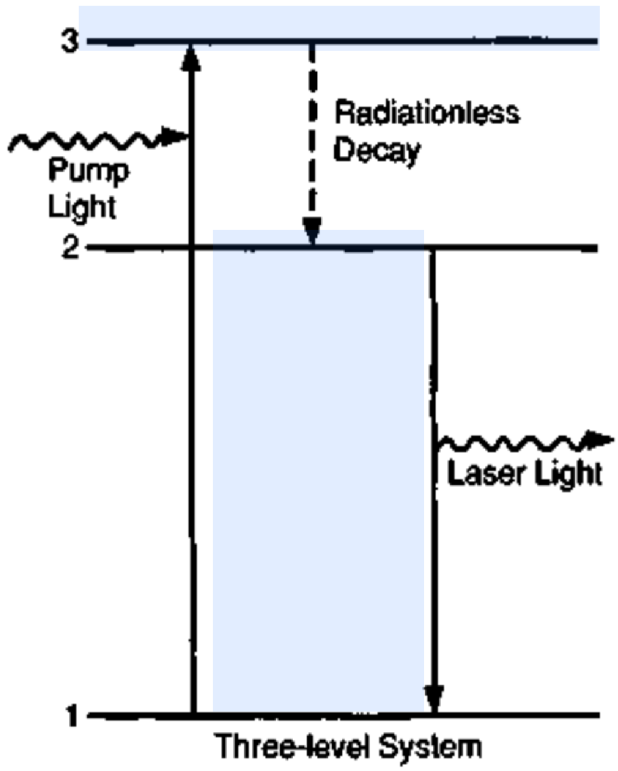
We need an “unnatural” amount of the atoms in a material to be in a state with energy E_2 vs. a state with energy E_1 where E_2 is greater than E_1. What does “unnatural” mean. Well, the more technical term is “population inversion.”

Normally, if a material has two energy levels, E_2 and E_1, then the fraction of atoms in state E_2 vs. the fraction of atoms in state E_1 is determined in an ironclad way by the temperature T of the material, and the ratio is:

$$e^{-(E_2-E_1)/k_B T}$$

If $E_2 > E_1$ this ratio is clearly less than 1. So there is no way to do this if a material is in thermal equilibrium. In other words, you can’t just heat it. We want to create a material that has more atoms in state with energy E_2 than in state with energy E_1.

To do this, a material that has three states is typically used, much like in a fluorescent tube. The energy-level diagram for the three states looks like:



The pump light is not laser light. It has a lot of energy in some part of its spectrum, but it has not been arranged to have all of its photons in the same state. The pump light's job is to move a lot of atoms from state 1 to state 3.

The $E_3 \to E_2$ and $E_2 \to E_1$ Transitions

Assuming the pump light can get a lot of the atoms to go to a state with energy E_3. The second property of the material is that it is chosen so that just by bumping into each other, a transition to the state with energy E_2 can occur. This transition is "radiationless." A photon is not emitted. Energy is conserved by the creation of heat as the atoms bump each other. The material is chosen so that this transition happens quickly compared to the transition from $E_2 \to E_1$.

Brian had a whacky analogy involving chukars. If the chukars reproduce a lot faster than the hunters cull them, the population will grow exponentially until something else comes into play (like the carrying capacity of the ecosystem), and a large population of chukars will be present in equilibrium.

Analogously to chukars, the material being pumped is chosen so that lots of transitions from E_3 to E_2 will occur, and occur much more readily than the transitions from E_2 to E_1. This is critical. It means that a large population of atoms will accumulate in state E_2.

Now we have a material with far more atoms in state E_2 than in state E_1. This material is ready for lasing to occur in it.

Starting the Lasing

In addition to choosing the material to be pumped for the above properties, the material has to be shaped into a lasing cavity.

We have been doing lots of quantons -in-a-box problems. A lasing cavity could be shaped like a box. In practice it is usually shaped as a cylinder. Also, in a lasing cavity the quantons are photons.

The pump light comes in through the side walls of the cylinder. The end caps are silvered. This is how you make a mirrored box that the photons cannot get out of.

For photons in a box, or any quantons in a box, we know they have wave functions for each energy level. To not confuse the photons energy levels with the atomic energy levels, let's call the photons' energy levels ϵ_i.

The cavity will have some energy levels ϵ_i that are at $E_2 - E_1$. So the many atoms in the material that are in state E_2 can transition to state E_1 by emitting a photon with $\epsilon_i = E_2 - E_1$.

The remarkable quantum thing that makes lasing possible is that if there are n photons in state with energy ϵ_i then the probability amplitude for another photon to be emitted into this exact same state is enhanced by a factor of $\sqrt{n+1}$. This probability amplitude must be squared to get a probability, so the transition probability is enhanced by a factor of $n + 1$.

This enhancement is known as "stimulated emission." The term laser is an acronym for "Light Amplification by Stimulated Emission of Radiation."

A Subtlety

In quantum mechanics, there is stimulated absorption as well. The probability to excite an atom in state E_1 into the state E_2 also is enhanced. It is proportional to $\sqrt{n}$ where n is the number of photons in the state with $\epsilon_i = E_2 - E_1$.

So now you see why the population inversion is important. If the populations of E_2 and E_1 were equal, you'd still have enhancement, but equally in both directions (both stimulated emission and stimulated absorption).

We need both stimulated emission and the population inversion to cause a lot of photons to end up in the same photon-in-a-box state.

WHY?!

Well, it is a bit advanced to say where this factor of $\sqrt{n + 1}$ comes from, but in quantum mechanics it is definitely present in every transition probability amplitude you calculate for photons. Again, the $n + 1$ in this formula is the number of photons present in the final state (which has one more photon in it than it started with).

In deep contrast, for fermions, such as electrons, you get a factor that is 0 if there is already one fermion in the final state. This is one way of saying that no two fermions can be in the same state. A second one simply will not transition into it. That fact is a famously advanced one, and we'll just have to accept it as a fact. At least it is easy to precisely state the result, so the proof (which is known as the "spin-statistics theorem") is the part that is advanced. The first two paragraphs of the Wikipedia entry summarize the result well.