Quantum Physics, Preparation for Friday, Apr. 19

Study the "Magnetic Resonance I" Handout

The handout goes a little further in setting up the problem than we did in class, and you will need it to do Problem Set 19 below.

Presentation

Miles (maybe someone wants to join him!?): How are atoms or electrons or ions counted so as to make 1 mole (abbreviated mol) *in practice;* and in particular, how is the gram which is now *defined* as 1/12 of the mass of an Avogadro's number (602,214,076,000,000,000,000,000) of Carbon-12 atoms counted, *in practice?*

Exam Review

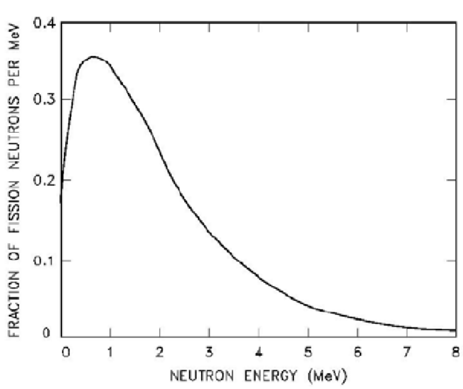
Is there anything you want to ask in preparation for Exam 4? Exam 4 covers Problem Sets 14-18 (Moore Q12.5 to end of Q15). You will need a calculator for this exam, unlike Exams 1-3.

For Problem Set 19

Fast vs. Thermal Neutrons

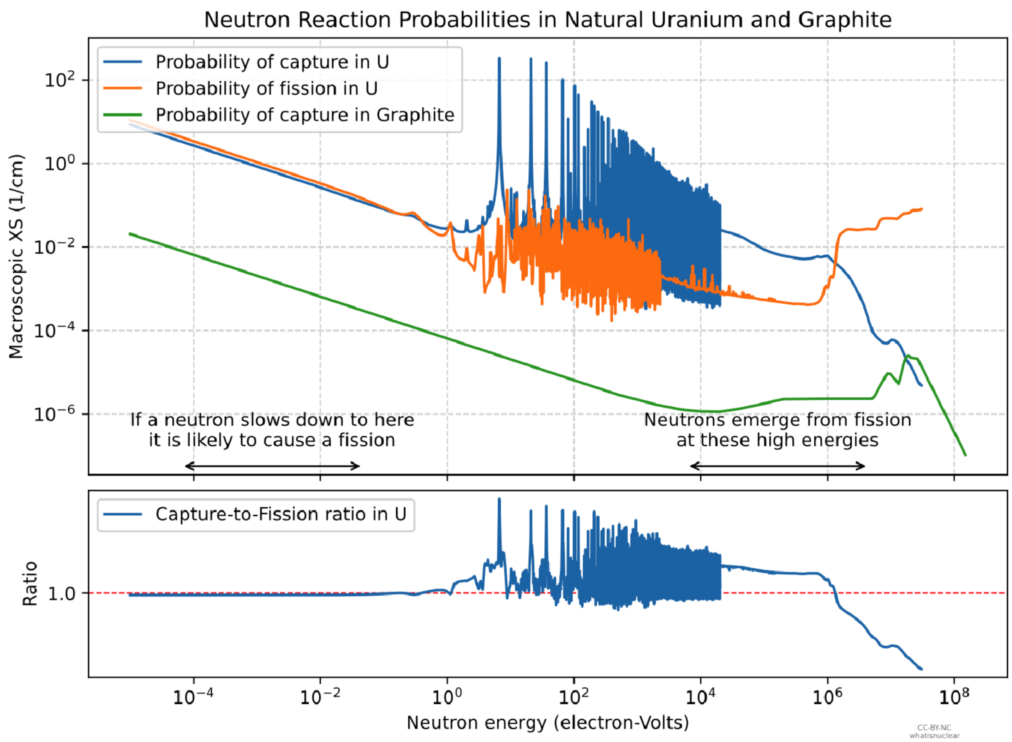
I made a mistake reading neutron reaction probabilities in class. Specifically, I started off with too high an estimate of the initial energy of the neutrons. Let's rectify this.

1. Below is the measured fission neutron spectrum for the neutrons that come out of Uranium-235 fission. A quick look at the graph shows that it peaks around 1 MeV.



1. (CONT'D)

(a) Now that we know a typical fission neutron energy, find the corresponding place on the graph below and estimate the ratio of probabilities of capture to fission. The vertical axis is steeply logarithmic. To get a good estimate, you need to estimate the position of the blue line, the position of the orange line, and then use the 10^x function on a calculator.



(b) The typical temperature inside a nuclear reactor isn't that hot. This isn't fusion in the Sun. Fission reactors just boil water to make steam to run turbines which produce electricity. The typical temperature in a boiling water reactor is about 285°C. Convert this to °K.

(c) Neutrons at temperature T have typical kinetic energy of $\frac{3}{2} k_B T$. Boltzmann's constant is 8.3×10^{-5}eV/°K. What is the typical kinetic energy of a neutron that has "thermalized" down to the temperature you found in (b)? Also, take $\log_{10}$ of this number so you can report your answer by specifying the x-value in 10^x eV.

(d) Using the answer you found in (c), what is likely to happen to a thermalized neutron? In other words, what are the odds it will cause another fission vs. the odds it will be captured.

DISCUSSION: The point of this problem was to show that it is very important to "moderate" or "thermalize" the neutrons in a reactor, otherwise they just get captured instead of having a chance to cause additional fissions.

Magnetic Resonance

In the “Magnetic Resonance I” handout, we got:

$$i\frac{d}{dt}\begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}\omega_0 & \omega_1 \cos\omega t\\ \omega_1\cos\omega t & -\omega_0\end{pmatrix}\begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix}.$$

This is a fancy shorthand for two equations. You may need to quickly review matrix and vector multiplication, because we only used it a little back in Chapters Q6 and Q7. Here is a lightning review: if you have a matrix M and a column vector w where $M = \begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}$ and $w = \begin{pmatrix}w_1\\ w_2\end{pmatrix}$, then when you see $M\,w$ or $\begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}\begin{pmatrix}w_1\\ w_2\end{pmatrix}$ in an equation, this product is by definition the column vector $\begin{pmatrix}m_{11}\,w_1 + m_{12}\,w_2\\ m_{21}\,w_1 + m_{22}\,w_2\end{pmatrix}$.

2. Use the matrix multiplication rules above to write out the two equations involving $\psi_+(t)$ and $\psi_-(t)$.

HINT/CROSS-CHECK: Make sure that if you put in $\omega_1 = 0$ into your equations that you recover the two equations at the very bottom of p. 3 of the “Magnetic Resonance I” handout.

3. The only case we can solve exactly is the case where $\omega_1 = 0$. Solve that case. You will have two unknown constants, $\psi_+(0)$ and $\psi_-(0)$, but other than those two unknowns, you will have nice tidy answers for $\psi_+(t)$ and $\psi_-(t)$.

4. Now we are going to assume that ω_1 is small but not 0. I am going to call the two functions that you got in Problem 3, $a_+(t)$ and $a_-(t)$. In problem 3 we of course called them $\psi_+(t)$ and $\psi_-(t)$, but we can’t call them that now, because they aren’t the solution any more (because ω_1 is not 0 any more). Now for the ansatz: assume that $\psi_+(t) = c_+(t)\,a_+(t)$ and that $\psi_-(t) = c_-(t)\,a_-(t)$. Stick the ansatz into the equations you got in Problem 3. Use product rule to simplify the time derivatives.

HINT/CROSS-CHECK: Make sure that if you put in $\omega_1 = 0$ into your equations for $c_+(t)$ and $c_-(t)$ that these functions can then simply be constants.

DISCUSSION (THE IDEA OF THE ANSATZ): Hopefully, if ω_1 is small but not zero, then $c_+(t)$ and $c_-(t)$ will be slowly varying functions of time. After all, they become constants if $\omega_1 = 0$.

5. Again refer to Problem 3, where you found the solutions for $\psi_+(t)$ and $\psi_-(t)$ in the case where $\omega_1 = 0$. We would be remiss if we didn’t think a little harder about what this solution means. You can reassemble your solution into a column vector $|\,\psi(t)> = \begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix}$. Then, using the methods on pp. 114-115 of Moore, calculate: (a) $|<+z\,|\,\psi(t)>|^2$, (b) $|<-z\,|\,\psi(t)>|^2$, (c) $|<+x\,|\,\psi(t)>|^2$, and (d) $|<-x\,|\,\psi(t)>|^2$.

HINT/CROSS-CHECK: Obviously the answers to (a) and (b) have to add up to 1, and to use that cross-check, you will need to use that $|\psi_+(0)|^2 + |\psi_-(0)|^2 = 1$.