Next Up Previous Contents
Next: Introducing and **Up:** Quantum Physics 130 **Previous:** Sample Test Problems **Contents**

Harmonic Oscillator Solution using Operators

Operator methods are very useful both for solving the Harmonic Oscillator problem and for any type of computation for the HO potential. The operators we develop will also be useful in quantizing the electromagnetic field.

The Hamiltonian for the **1D Harmonic Oscillator**

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$$

looks like it could be written as the square of a operator. It can be rewritten in terms of the operator A

$$\boxed{A \equiv \left(\sqrt{\frac{m\omega}{2\hbar}}x + i\frac{p}{\sqrt{2m\hbar\omega}}\right)}$$

and its Hermitian conjugate $A^\dagger$.

$$\boxed{H = \hbar\omega\left(A^\dagger A + \frac{1}{2}\right)}$$

We will use the commutators between A, $A^\dagger$ and H to solve the HO problem.

$$\boxed{[A, A^\dagger] = 1}$$

The commutators with the Hamiltonian are easily computed.

$$\begin{aligned} [H, A] &= -\hbar\omega A \\ [H, A^\dagger] &= \hbar\omega A^\dagger \end{aligned}$$

From these commutators we can show that $A^\dagger$ is a raising operator for Harmonic Oscillator states

$$\boxed{A^\dagger u_n = \sqrt{n+1}u_{n+1}}$$

and that A is a **lowering operator**.

$$\boxed{Au_n = \sqrt{n}u_{n-1}}$$

Because the lowering must stop at a ground state with positive energy, we can show that the allowed energies are

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega.$$

The actual wavefunctions can be deduced by using the differential operators for A and $A^\dagger$, but often it is more useful to define the n^{th} eigenstate in terms of the ground state and raising operators.

$$\boxed{u_n = \frac{1}{\sqrt{n!}}(A^\dagger)^n u_0}$$

Almost **any calculation** of interest can be done without actual functions since we can express the operators for position and momentum.

$$\begin{aligned} x &= \sqrt{\frac{\hbar}{2m\omega}}(A + A^\dagger) \\ p &= -i\sqrt{\frac{m\hbar\omega}{2}}(A - A^\dagger) \end{aligned}$$

Subsections

Next Up Previous Contents

Next: Introducing and **Up:** Quantum Physics 130 **Previous:** Sample Test Problems **Contents**

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