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# Lasers

We would like to understand just the basics, which is: how is a material arranged so as to get a huge number of photons into a single quantum state, a teeny bit of which are bled off to make the laser beam.

## Pumping the Material

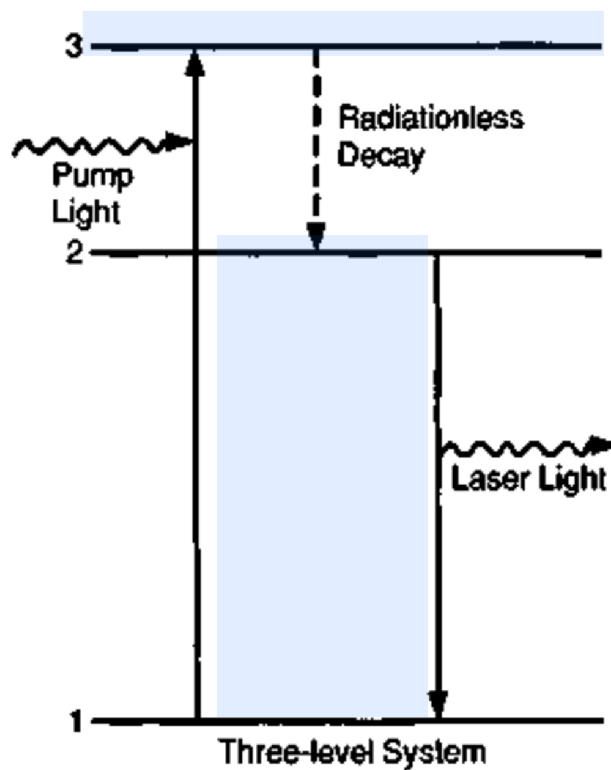
We need an “unnatural” amount of the atoms in a material to be in a state with energy  $E_2$  vs. a state with energy  $E_1$  where  $E_2$  is greater than  $E_1$ . What does “unnatural” mean. Well, the more technical term is “population inversion.”

Normally, if a material has two energy levels,  $E_2$  and  $E_1$ , then the fraction of atoms in state  $E_2$  vs. the fraction of atoms in state  $E_1$  is determined in an ironclad way by the temperature  $T$  of the material, and the ratio is:

$$e^{-(E_2-E_1)/k_B T}$$

If  $E_2 > E_1$  this ratio is clearly less than 1. So there is no way to do this if a material is in thermal equilibrium. In other words, you can’t just heat it. We want to create a material that has more atoms in state with energy  $E_2$  than in state with energy  $E_1$ .

To do this, a material that has three states is typically used, much like in a fluorescent tube. The energy-level diagram for the three states looks like:



The pump light is not laser light. It has a lot of energy in some part of its spectrum, but it has not been arranged to have all of its photons in the same state. The pump light's job is to move a lot of atoms from state 1 to state 3.

### The $E_3 \rightarrow E_2$ and $E_2 \rightarrow E_1$ Transitions

Assuming the pump light can get a lot of the atoms to go to a state with energy  $E_3$ . The second property of the material is that it is chosen so that just by bumping into each other, a transition to the state with energy  $E_2$  can occur. This transition is "radiationless." A photon is not emitted. Energy is conserved by the creation of heat as the atoms bump each other. The material is chosen so that this transition happens quickly compared to the transition from  $E_2 \rightarrow E_1$ .

Brian had a whacky analogy involving chukars. If the chukars reproduce a lot faster than the hunters cull them, the population will grow exponentially until something else comes into play (like the carrying capacity of the ecosystem), and a large population of chukars will be present in equilibrium.

It would be fairly easy to make the chukars analogy more precise using population dynamics models that are studied in classes like Andy's ecological modeling class, but it was intended just to build intuition, so probably there is no need to make it more precise.

Analogously to chukars, the material being pumped is chosen so that transitions from  $E_3$  to  $E_2$  will

occur quickly, and in particular occur much more quickly and readily than the transitions from  $E_2$  to  $E_1$ . This is critical. It means that a large population of atoms will accumulate in state  $E_2$ , just like a large number of chukars would accumulate if the hunter culling rate were low compared to the reproduction rate.

Assuming the above properties are satisfied by the material, it will have far more atoms in state  $E_2$  than in state  $E_1$ . The material is ready for lasing to occur in it.

## Starting the Lasing

In addition to choosing the material to be pumped for the above properties, the material has to be shaped into a lasing cavity.

We have been doing lots of quantons -in-a-box problems. A lasing cavity could be shaped like a rectangular or cubical box. In practice it is usually shaped as a cylinder. Also, in a lasing cavity the quantons are photons, so we can quit saying “quantons.”

The pump light comes in through the side walls of the cylinder. The end caps are silvered. This is how you make a mirrored cylinder that the photons accumulate in.

For photons in a box, or any quantons in a box, we know they have wavefunctions for each energy level. To not confuse the photons energy levels with the atomic energy levels, let's call the photon energy levels  $\epsilon_i$ .

The cavity will have some energy levels  $\epsilon_i$  that are equal to  $E_2 - E_1$ . So the many atoms in the material that are in state  $E_2$  can transition to state  $E_1$  by emitting a photon with  $\epsilon_i = E_2 - E_1$ . When they do this, the pump light will push them back up to the state with energy  $E_3$ .

*The remarkable quantum thing that makes lasing possible is that if there are  $n$  photons in state with energy  $\epsilon_i$  then the probability amplitude for another photon to be emitted into this exact same state is enhanced by a factor of  $\sqrt{n + 1}$ . This probability amplitude must be squared to get a probability, so the transition probability is enhanced by a factor of  $n + 1$ .*

This enhancement is known as “stimulated emission.” The term laser is an acronym for “Light Amplification by Stimulated Emission of Radiation.”

## A Subtlety

In quantum mechanics, there is stimulated absorption as well. In particular, the probability to excite an atom in state  $E_1$  into the state  $E_2$  by absorption of a photon also is enhanced. It is proportional to  $\sqrt{n}$  where  $n$  is the number of photons in the state with  $\epsilon_i = E_2 - E_1$ .

So now you see why the population inversion is important. If the populations of  $E_2$  and  $E_1$  were equal, you'd still have enhancement, but equally in both directions (both stimulated emission and stimulated absorption).

We need both stimulated emission and the population inversion to cause a lot of photons to end up in the same photon-in-a-box state.

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## WHY $\sqrt{n + 1}$ !?

Well, it is a bit advanced to say where this factor of  $\sqrt{n + 1}$  comes from, but in quantum mechanics it is definitely present in every transition probability amplitude you calculate for photons. Again, the  $n + 1$  in this formula is the number of photons present in the final state.

In deep contrast, for fermions, such as electrons, you get a factor that is 0 for adding another fermion if there is already one fermion in the final state. This equivalent to saying that “no two identical fermions can be in the same state.” A second one simply will not transition into it. That fact is a famously advanced one, and we'll just have to accept it as a fact. At least it is easy to precisely state the result, so the proof (which is known as the “spin-statistics theorem”) is the part that is advanced, not the result.