Quantum Physics, Preparation for Tuesday, Jan. 16

Study Q2.1 to Q2.3 from *Six Ideas*

For most classes, we will do an entire chapter. This is the way Moore's book is meant to be used. However, if anyone starts to feel thrown under the bus, we certainly will ease off and make time to consolidate. It is a bit much to include Q2.4 (resonance) for Tuesday, so just do Q2.1 to Q2.3.

Presentations

Let's have both presentations and problem sets for next time. I have an idea for Tuesday to have just two big presentation teams and presentations. I'm going to give each team half of a big and interesting problem.

For Problem Set 2

Energy in Coupled Oscillators

1. Go back to "The Bridge." One of the problems that was considered in the bridge was the problem of two masses coupled with three springs.

As the most simplified version of this problem, we let all three springs have the same spring constant, k, we let both masses be the same, m, and we defined $\omega_0{}^2 = k/m$.

We argued that there are two solutions, one where the masses move back and forth together. A solution like this is called a "mode." The first mode was:

$x_1(t) = A\cos\omega t$
$x_2(t) = A\cos\omega t$

and $\omega = \omega_0$. (The first mode also has the same thing but with sin instead of cos as another solution.)

The second mode had the two masses moving in opposite directions. *The second mode was:*

$x_1(t) = A\cos\omega t$
$x_2(t) = -A\cos\omega t$

and $\omega = \sqrt{3}\ \omega_0$. (The second mode also sine instead of cosine as another solution.)

In class on Friday, we studied the first mode. *For this problem use the second mode (with the cosine solution).*

(a) Using the second mode, how much is the left spring stretched at time t? The energy in a spring is $\frac{1}{2} k x^2$ where x is how much it is stretched (or compressed). What is the energy in the left spring?

(b) Repeat (a) for the right spring. You should get similar answers to (a).

(c) Repeat (a) for the center spring. This spring will be different than (a) or (b).

(d) All all of (a), (b), and (c) up and simplify. This is the potential energy stored in the springs.

(e) What is $v_1(t) = \frac{d}{dt} x_1(t)$? What is the kinetic energy of mass 1, which is $\frac{1}{2} m {v_1}^2$?

(f) Repeat (e), but for $v_2(t)$ and $\frac{1}{2} m {v_2}^2$?

(g) Now add all the energies you found in (d), (e), and (f) up. Use that for the second mode $\omega^2 = 3\, {\omega_0}^2 = 3\, k/m$ and use a standard trig identity. If you don't get a super-simple answer check your work.

This problem was designed to help you see how energy can be traded back and forth between potential energy and kinetic energy such that total energy is conserved. We could repeat it for one of our solutions to the infinite coupled harmonic oscillator problem. Alternatively, instead of going straight to the infinite case, we could do other interesting cases. For example, a super-interesting case is where the spring in the middle is weak compared to the other two springs.

Energy in Coupled Oscillators

2. Moore Q2T.3 and Q2T.4, a couple of quick superposition problems.

3. Moore Q2B.2, a more detailed superposition problem

Fundamental modes and harmonics of a string

4. Moore Q2M.2, a guitar string

5. Repeat Moore Q2M.2, but consider the “third harmonic” of the string. The third harmonic is the name for the harmonic with two nodes. There won't be a new tension! That was already determined in Q2M.2. But there will be new tones. The figures to look at are Figure Q2.7 and Figure Q2.8 with $n = 3$. What are the new frequencies corresponding to the third harmonic of E and the third harmonic of G? For the musicians only: the third harmonic is the perfect 5th but an octave up.