

## Electromagnetic Radiation: Interference

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 28 and 29.

**21.1** Interpret the following two problems in complex numbers geometrically, and show that the absolute value of  $A$  in each case is as given:

(a)

$$A = re^{i\theta/2} + re^{-i\theta/2}$$

$$|A| = 2r \cos(\theta/2)$$

(b)

$$A = \sum_{n=0}^N re^{in\theta}$$

$$|A| = r \frac{\sin\left(\frac{N+1}{2}\theta\right)}{\sin(\theta/2)}$$

**21.2** A charge  $q$  traverses a circular path of radius  $a$  at an angular velocity  $\omega$ .

- Evaluate the electric field  $\mathbf{E}(t)$  at a great distance  $R$  from the charge, at an angle  $\theta$  with respect to the axis of the circular path.
- Find the intensity of the radiation  $I(\theta)$  at a great distance  $R$  on the axis ( $\theta = 0$ ) and in the plane of the circle ( $\theta = \pi/2$ ).

Assume that  $\omega a \ll c$  and  $a \ll R$ .

**21.3** The power per unit area delivered by an electromagnetic wave is proportional to the mean-square electric field strength.

- If the total power radiated by an oscillating charge is  $P_{\text{total}}$ , how much power  $P$  falls on a unit area normal to the radius vector  $\mathbf{R}$  at an angle  $\theta$  with respect to the axis of oscillation?
- Evaluate  $P$  in  $\text{W m}^{-2}$  for a vertically oriented dipole suspended from a cosmic ray radiosonde balloon at an altitude of 25 km and at a horizontal distance of 25 km from the receiver, as shown in Fig. 21-1, if the transmitter is radiating 0.5 W total.

**21.4** Two vertical antennas are arranged as shown in Fig. 21-2 and are driven in phase. The antennas are driven so that one would, if alone, radiate a certain intensity  $I_0$  in all horizontal directions, and the other, an intensity  $2I_0$ . What should be the observed intensity  $I$  in the various directions shown in the figure?

**21.5** Four identical dipole radiators are aligned parallel to one another and are equally spaced along a line at a distance 2.50 cm apart. They are driven at a frequency of  $3.00 \times 10^9$  Hz and are phased so that, starting from one end, each successive dipole lags the preceding one by  $90^\circ$ . Find the intensity pattern of the radiation  $I(\theta)$  at a great distance in the equatorial plane (perpendicular to the dipole axes), and sketch this function in the polar coordinates shown in Fig. 21-3. Such a diagram is called the *radiation pattern* or *lobe pattern* of an antenna system.

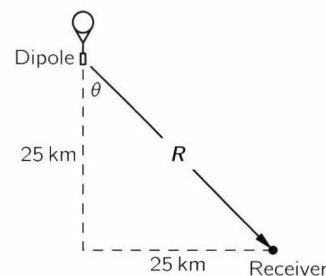


Figure 21-1

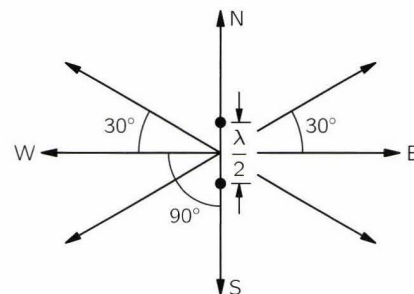


Figure 21-2

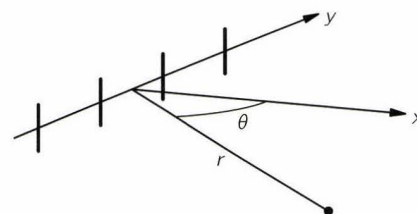


Figure 21-3

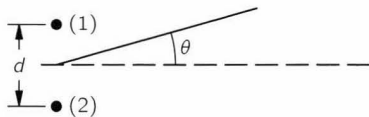


Figure 21-4

**21.6** Two parallel dipoles are situated a distance  $d = \lambda/2$  apart, and are oscillating with the same frequency and amplitude (s. Fig. 21-4).

- Find  $I = I(\theta)$  (the lobe pattern) in the equatorial plane and sketch the pattern in a polar diagram if the oscillators are in phase.
- Find by what fraction of a period oscillator (2) must lag oscillator (1) so that an observer at  $\theta = 210^\circ$  will see maximum signal. In this case, for which value of  $\theta$  will there be no signal?
- Find and sketch the lobe pattern implied in part (b) above.

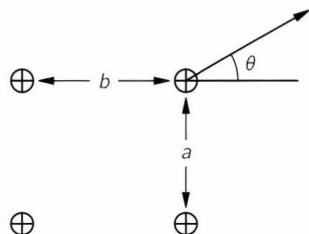


Figure 21-5

**21.7** Four vertical dipoles are located at the corners of a horizontal rectangle of sides  $a, b$  as shown in Fig. 21-5. If they are driven in phase, at wavelength  $\lambda$ , what minimum values ( $> 0$ ) should  $a, b$  have to produce maximum intensity in the direction  $\theta = 30^\circ$  far from the charges?

**21.8** An observer is at distance  $R$  from two identical charges  $q$  that are both passing through the origin at  $t = 0$ . The first charge moves only along the  $z$ -axis, and  $z(t) = d \sin \omega t$ ; the second charge moves only along the  $x$ -axis, and  $x(t) = d \sin \omega t$ . What is the magnitude of the electric field  $E(t)$  at the two following points:

- $x = R/\sqrt{2}, y = 0, z = R/\sqrt{2}$ ,
- $x = 0, y = R, z = 0$ .

Assume  $R \gg c/\omega$  and  $\omega d \ll c$ .

**21.9** A field engineer testing radiation patterns is flying in a helicopter at a ground speed of  $120 \text{ mi h}^{-1}$  at low altitude in a circular pattern of  $2.0 \text{ mi}$  radius about the mid-point between two vertical dipole transmitting antennae that lie in a north-south line. For the frequency being used for the test, the antennae are a half wavelength apart. Normally the antennae are operated in phase. However, the transmitter operator decided to play a joke on the field engineer, by changing the phase relation between the antennae at such a rate that no change in radiation intensity could be observed aboard the helicopter. If he started this change when the helicopter was due east of the antennae at what rate  $da/dt$  was he changing the phase relation when the helicopter was  $\theta^\circ$  north of east of the antennae?

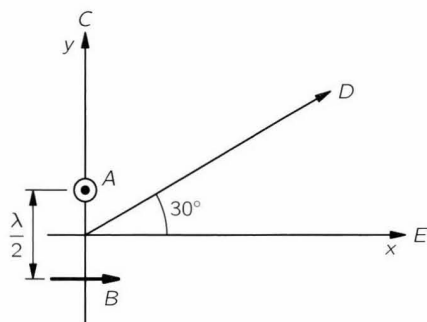


Figure 21-6

**21.10** Two electric dipoles,  $A$  and  $B$ , are situated one-half wavelength apart and are perpendicular to one another and to the line joining their centers, as shown in Fig. 21-6. They are driven at the same frequency and phase, but dipole  $B$  has twice the amplitude of dipole  $A$ . Find the intensity  $I$  and the direction of the  $\mathbf{E}$  vector of the radiation at a great distance from the dipoles in the directions  $C, D$ , and  $E$  (all in  $xy$ -plane) indicated in the figure.

**21.11** A double line of  $N$  equally spaced oscillating dipoles is situated as shown in Fig. 21-7. All dipoles in row  $A$  are driven in the same phase, and all those in row  $B$  lag  $90^\circ$  in phase behind those of row  $A$ . Sketch the radiation pattern  $I(\theta)$  in the equatorial plane at a great distance from the array.

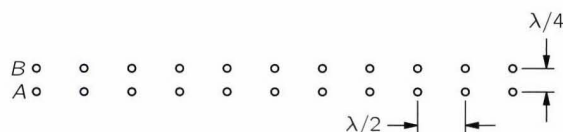


Figure 21-7

**21.12** The conduction electrons in a long, straight, fine wire of length  $L$  are all oscillating along the wire with angular frequency  $\omega$ , small amplitude  $a$ , and in the same phase. Find the magnitude of the electric field  $E(t)$  at a great distance  $R$  ( $R \gg L$ ), at an angle  $\theta$  with respect to the wire.



## Electromagnetic Radiation: Diffraction

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 30.

**22.1** Consider  $n$  equally spaced dipole radiators, each of length  $a$  and separation  $d$  along a straight line, as shown in Fig. 22-1, oscillating along the line at the same amplitude  $A$  and frequency  $\omega$ , but with successive phase shifts  $\alpha$ . Each dipole radiator consists of a large number of atomic dipoles. Show that the diffraction pattern of the intensity at large distance at an angle  $\theta$  is given by

$$I = I_0 \frac{\sin^2(\beta/2)}{(\beta/2)^2} \frac{\sin^2(n\phi/2)}{\sin^2(\phi/2)},$$

where

$$\begin{aligned}\phi &= \alpha + \frac{2\pi d}{\lambda} \sin \theta, \\ \lambda &= \frac{2\pi c}{\omega}, \\ \beta &= \frac{2\pi}{\lambda} a \sin \theta.\end{aligned}$$

and  $I_0$  is the intensity of a single dipole at the angle  $\theta$ .

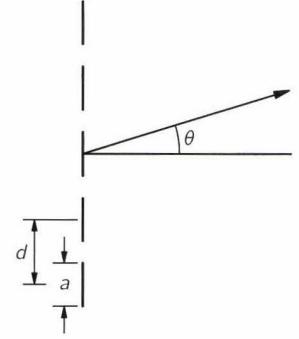


Figure 22-1

**22.2** A certain spectral line has a wavelength of  $5500 \text{ \AA}$  and a width of  $10 \text{ \AA}$ . What is the  $Q$  of the atomic oscillator?

**22.3** Under what conditions will an illuminated slit cast a “geometrical” shadow? (i.e., such that diffraction effects are negligible.)

**22.4** An automobile with the customary two headlights (considered as point sources) is approaching from a distance on a straight road. The lights on the car are situated  $120 \text{ cm}$  apart.

- How far  $d$  from an observer would the car be when he could just be sure he was seeing two lights and not one, assuming the aperture of his iris is  $0.5 \text{ cm}$  and the effective wavelength of the light is  $5500 \text{ \AA}$ ?
- Would the fact that the light is “white” (a mixture of wavelengths) make it easier or harder to resolve the two sources?

**22.5** The wavelengths of the  $D$ -lines of sodium are  $5889.95 \text{ \AA}$  and  $5895.92 \text{ \AA}$ , respectively. How long  $x$  must a grating with  $600 \text{ lines/mm}$  be in order to resolve these lines in the first order spectrum?

**22.6** A point source of light  $L$  emitting a single wavelength  $\lambda$  is situated a small distance  $d$  above an ideal plane mirror. A screen stands at the end of the mirror at distance  $D$  from  $L$ , as shown in Fig. 22-2. ( $D \gg d$ ) Find  $I(z)/I_0$ , the relative intensity of light on the screen as a function of  $z$ , the plane of the mirror being at  $z = 0$ . (Note: a mirror changes the phase of the light it reflects by  $180^\circ$ .)

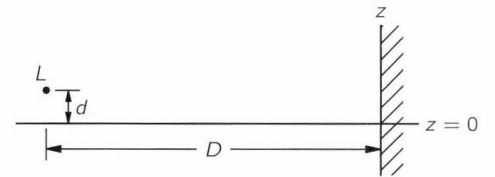


Figure 22-2

**22.7** Parallel light of wavelength  $\lambda$  is normally incident from the left on a circular hole in a screen and the resultant intensity is observed at a point on the axis of the hole at  $z = 10\lambda$  to the right.

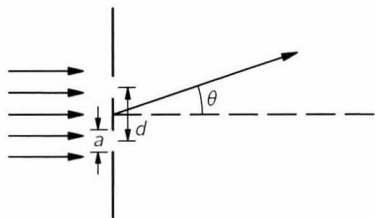


Figure 22-3

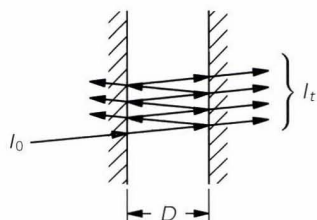


Figure 22-4

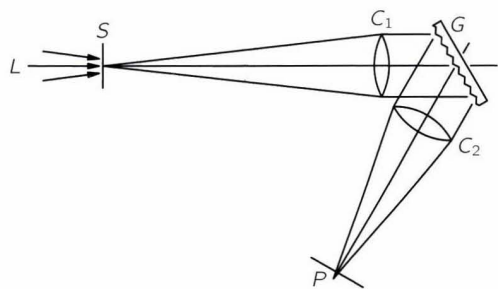


Figure 22-5

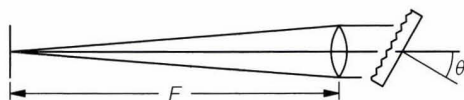


Figure 22-6

- (a) What radius  $r$  should the hole have to produce maximum intensity at the observation point?
- (b) By what fraction  $I/I_{\max}$  is the intensity reduced if the screen is removed?

**22.8** (a) Calculate the transmitted diffraction pattern  $I(\theta)/I_{\max}$  for a screen, as shown in Fig. 22-3, having two parallel slits of width  $a = 6$  cm, separated by a distance  $d = 12$  cm, which is exposed to a parallel beam of  $\lambda = 3$  cm microwave radiation.

- (b) How many orders of principal maxima are there and for which values of  $\theta$ ?
- (c) What is  $I(\theta)/I_{\max}$  at the positions of the principal maxima?

**22.9** The wavelengths of spectral lines are commonly measured to  $0.001 \text{ \AA}$  using spectrographs whose resolving power may be only  $0.010 \text{ \AA}$ . Are any basic laws of physics being violated in the process? Explain.

**22.10** A Fabry-Perot interferometer consists of a pair of very accurately flat surfaces, parallel to each other at a distance  $D$  apart. The surfaces are coated so as to reflect a fraction  $R^2$  of the intensity of the light incident on them normally, and to transmit a fraction  $T^2$  of the intensity. Light of intensity  $I_0$  and wavelength  $\lambda$  is incident upon one surface from the left (see Fig. 22-4). Part of this beam is transmitted directly through the system, but some of the light is reflected from the second surface, then from the first surface, and is thence transmitted. In general, the outgoing beam is made up of light which has been reflected 0, 2, 4, 6, ... times and transmitted through 2 films, all summed together. How should the relative transmitted intensity  $I_t/I_0$  depend upon  $D$ ,  $\lambda$ ,  $R$ , and  $T$ ?

*Note:* Narrow-band optical filters, called interference filters, operate on this same principle, but the two reflecting surfaces are made by high-vacuum coating a piece of glass with several layers, accurately controlled in thickness, or clear materials having various indexes of refraction.

**22.11** A common type of grating spectrograph is constructed as shown in Fig. 22-5. Light from a source  $L$  passes through a narrow slit  $S$ , thence through a collimator lens (or mirror)  $C_1$  which renders it parallel (so that it strikes the grating as would plane waves from infinity). This parallel light is then diffracted by the grating  $G$ ; the diffracted light which proceeds in a certain range of angular directions strikes another lens  $C_2$ , called the camera lens, and is focused in a plane  $P$ , where the spectrum appears as a band, perhaps crossed by narrow spectrum lines at various places. Let the length and width of the slit be  $h$  and  $w$ , the focal lengths of  $C_1$  and  $C_2$  be  $F_1$  and  $F_2$ , the angles between the grating normal and the axes of  $C_1$  and  $C_2$  be  $\theta_i$  and  $\theta_d$ , and the number of lines per mm on the grating be  $N$ .

- (a) How wide  $h'$  will the spectrum band appear at  $P$ ?
- (b) What wavelength(s)  $\lambda_m$  will appear on the axis of  $C_2$  at  $P$ ?
- (c) How far apart  $D$  in the focal plane at  $P$  will two spectral lines appear whose wavelength differs by  $1.00 \text{ \AA}$ ? This quantity is often called the *dispersion* of the instrument.
- (d) If the slit width  $w$  is much larger than the resolution  $1.22 \lambda F_1/A_1$  of the collimator lens, where  $A_1$  is the aperture, how wide  $w'$  should a spectral line at  $P$  be?

**22.12** The spectrograph at the 150 ft solar tower telescope of the Mt. Wilson Observatory is of the Littrow type, shown schematically in Fig. 22-6. In this arrangement, a single lens acts as both the collimator and camera lens, and  $\theta_i = -\theta_d$  (nearly). The spectrum is formed in a strip adjacent to the slit. The

focal length of the Mt. Wilson instrument is  $F = 23$  m, and the grating has a ruled area  $15\text{ cm} \times 25\text{ cm}$  with 600 lines/mm. The fifth order spectrum is commonly used.

- At what angle  $\theta$  should the grating be tilted to bring the line  $\lambda = 5250.218\text{ \AA}$  of neutral iron in coincidence with the entrance slit in the fifth order spectrum?
- What other wavelengths in the range  $3600\text{ \AA}$ – $7000\text{ \AA}$  will also be coincident with the slit?
- Suggest a simple way to remove the unwanted orders, leaving only the fifth order.
- What is the dispersion  $d$  of the instrument at fifth order  $\lambda = 5250\text{ \AA}$ ?\*
- What is the minimum  $\Delta\lambda$  which can theoretically be resolved at fifth order  $\lambda = 5250\text{ \AA}$  by this instrument?

**22.13** When the grooves of a diffraction grating are shaped in such a way as to throw most of the incident radiation into a particular direction, the grating is said to be blazed for this direction. Suppose it were possible to shape the grooves perfectly in a sawtooth shape as shown in Fig. 22-7, each groove surface being tilted at a certain angle  $\theta_b$ .

- Use the notion of the diffracted beam being the radiation emitted by oscillators in the material, which are driven in phase with the incoming radiation, to deduce in what direction the diffracted beam would be most intense if  $\theta_i = 0$ . (Assume white light.)
- Estimate the approximate angular range over which the blaze would extend.

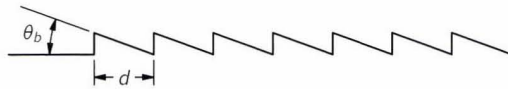


Figure 22-7

\* Note that, although  $\theta_i = -\theta_d$  at  $\lambda = 5250\text{ \AA}$ ,  $\theta_i$  is fixed while  $\theta_d$  depends upon  $\lambda$ .