賽局論 HW2

許博翔

October 6, 2023

Problem 1.

(a)

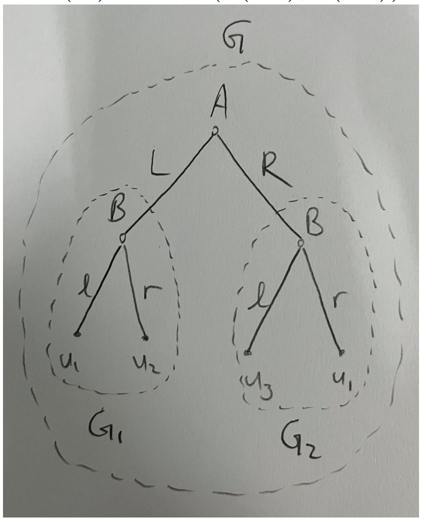
$A \setminus B$	ll	lr	rl	rr
L	u_1	u_1	u_2	u_2
R	u_3	u_1	u_3	u_1

.

(b) The saddle points are $(L, lr), (R, lr)$.

(c) $v(G_1) = \min(u_1, u_2) = u_1, v(G_2) = \min(u_3, u_1) = u_1$.

$\Rightarrow v(G) = \max(v(G_1), v(G_2)) = u_1$.



Problem 2. Let A be the first player, which is the winner since the initial Nim is unbalanced.

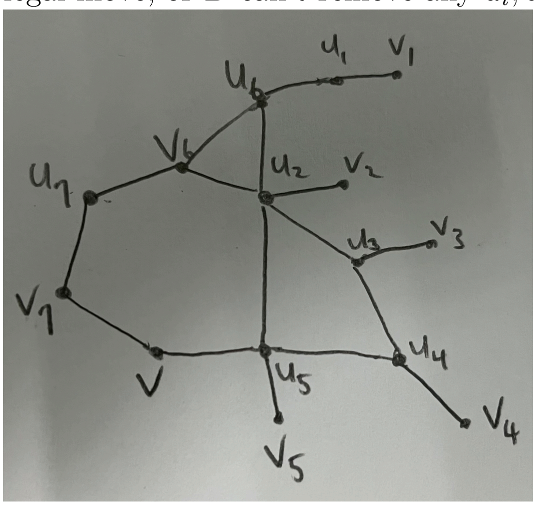
Let B be the second player.

Let a_i denote the number of stones left in the i-th piles.

(a_1, a_2, a_3, a_4) changes as follow:

$(1, 3, 7, 15) \xrightarrow{A} (1, 3, 7, 5) \xrightarrow{B} (0, 3, 7, 5) \xrightarrow{A} (0, 2, 7, 5) \xrightarrow{B} (0, 0, 7, 5) \xrightarrow{A} (0, 0, 5, 5) \xrightarrow{B} (0, 0, 0, 5) \xrightarrow{A} (0, 0, 0, 0)$, A wins.

Problem 3 (2.12.15)**.** Let A be the first player and B be the second player.
Suppose that $E = \{u_1v_1, u_2v_2, \ldots, u_kv_k\}$.
Consider the following strategy for B:
If A takes u_i in the last step, takes v_i.
If A takes v_i in the last step, takes u_i.
$\because$ by the definition of E, no two edges have an endpoint in common.
$\therefore u_1, v_1, u_2, v_2, \ldots, u_k, v_k$ are distinct.
$\therefore$ the above strategy is legal, since u_iv_i are joined by an edge, and u_i, v_i won't be taken before A takes either u_i, v_i.
This is the winning strategy for B since the game ends either all $u_1, v_1, \ldots, u_k, v_k$ are removed (that is, all vertices), after which is A's turn and she does not have a legal move, or A can't remove any u_i, v_i.
Since there are odd vertices in the graph, such E does not exist.
Consider the following strategy:
A removes the vertex v in her first step, and then after each B's step:
If B takes u_i in the last step, takes v_i.
If B takes v_i in the last step, takes u_i.
This strategy is legal, since u_iv_i are joined by an edge, and u_i, v_i won't be taken before A takes either u_i, v_i.
This is the winning strategy for A since the game ends either all $v, u_1, v_1, \ldots, u_7, v_7$ are removed (that is, all vertices), after which is B's turn and he does not have a legal move, or B can't remove any u_i, v_i.



Problem 4 (2.12.19)**.** Let A be the first player.

By the definition of saddle points, $\forall s^*, t^*$, $(s,t) \preceq_A (s^*,t), (s,t) \succeq_A (s,t^*), (s',t') \preceq_A (s^*,t'), (s',t') \succeq_A (s',t^*)$.

$\Rightarrow \forall s^*, t^*$,

$(s,t') \preceq_A (s,t) \preceq_A (s',t) \preceq_A (s',t') \preceq_A (s^*,t')$

$(s,t') \succeq_A (s',t') \succeq_A (s',t) \succeq_A (s,t) \succeq_A (s,t^*)$.

$\therefore (s,t')$ is a saddle point.

Similarly, (s',t) is a saddle point.

Problem 5 (2.12.26)**.**

(a) Everybody just votes according to their rankings:

In the first vote (Alice or Bob), Boris and Horace will choose Alice, while Maurice will choose Bob, so Alice wins.

In the second vote (Alice or nobody), Boris and Maurice will choose Alice, while Horace will choose nobody, so Alice wins.

(b) The reason why Horace should switch to voting for the candidate he likes least at the first vote:

If in the first vote, Bob wins instead of Alice, then in the second vote (Bob or nobody), Boris, Horace will choose nobody, while Maurice will choose Bob, so nobody wins instead of Alice, which is a best result for Horace.

(c) Everybody votes strategically:

In the second vote, all of them will vote according to their rankings since the second vote will determine the result.

$\therefore$ the strategy to the second vote is same as the above.

That is, if Alice wins in the first vote, then Alice will win in the second vote; if Bob wins in the first vote, then nobody will win in the second vote.

Therefore in the first vote, it is equivalent to voting for Alice or nobody to win. Boris and Maurice will choose Alice, while Horace will choose nobody, so Alice wins in the end.