Internal Rate of Return (IRR)
內部報酬率

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Internal Rate of Return (IRR)

- Definition
  - The rate at which an investment plan breaks even

- Also known as
  - Effective interest rate
  - Annualized effective compounded return rate
  - 年化報酬率
  - …

- Application
  - Comparison of 2 investment plans

Quiz:
Definition of IRR in plain texts!

The term "internal" indicates it does not take environmental factors (e.g., inflation) into consideration.

IRR Comparisons

○ Is this investment plan good? Profit=250?

時間(年)	0	1	2	3	4	5
收支	-2000	450	450	450	450	450

- Comparison with bank interest rate 2%

n年	0	1	2	3	4	5
收支	-2000	450	450	450	450	450
$(1+Rate)^n$	1.00	1.02	1.04	1.06	1.08	1.10
現值	-2000	441.18	432.53	424.05	415.73	407.58

NPV: Net present value

➔ NPV=121.07

- Comparison with bank interest rate 5%

n年	0	1	2	3	4	5
收支	-2000	450	450	450	450	450
$(1+Rate)^n$	1.00	1.05	1.10	1.16	1.22	1.28
現值	-2000	428.57	408.16	388.73	370.22	352.59

➔ NPV=-51.73

○ What is the equivalent interest rate?

IRR Computation

○ Net present value (NPV) = 0   ○ Net future value (NFV) = 0

$$NPV = \sum_{i=0}^{n} \frac{c_i}{(1+r)^i} = 0$$

$$NFV = \sum_{i=0}^{n} c_i(1+r)^{n-i} = 0$$

Cash flow on time line

Cash flow at period i

c_0 c_1 c_2 ... c_i ... c_n

Present value at period 0

$$\frac{c_i}{(1+r)^i}$$

$$c_i(1+r)^{n-i}$$

Future value at period n
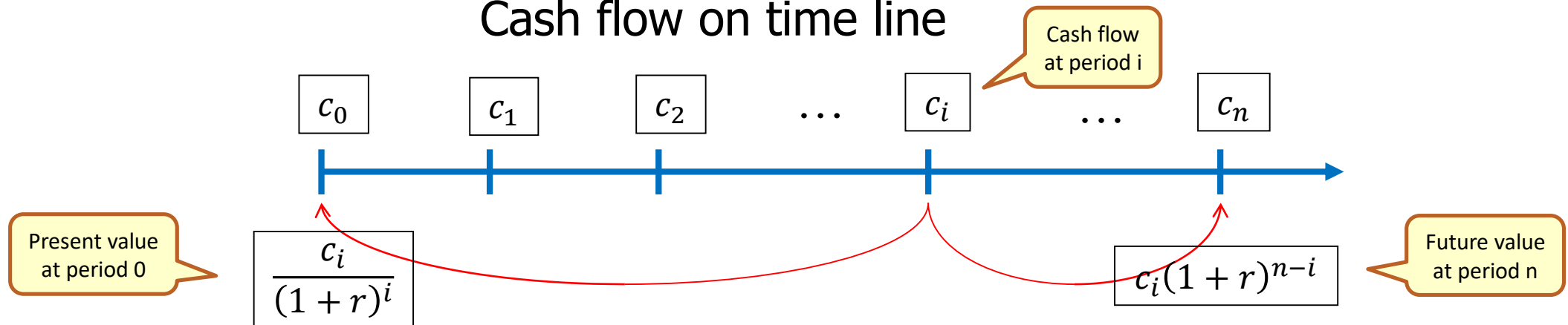
○ **Cash flow table**

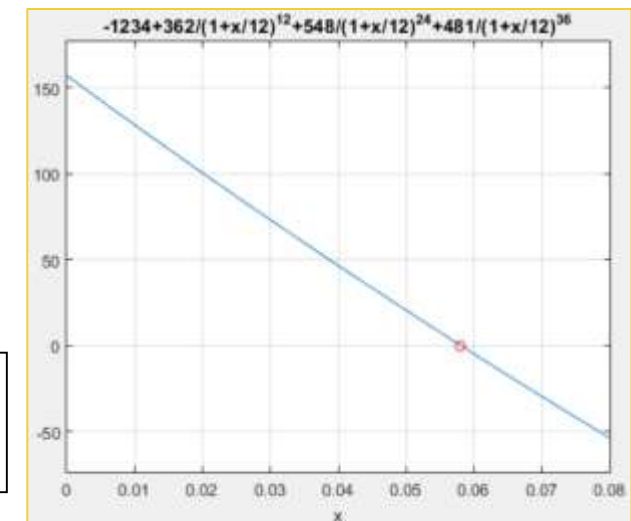| Year (i)	Cash flow (ci)
0	-1234
1	362
2	548
3	481

-1234 362 548 481

○ **Equations**

- Yearly compounding

$$NPV = -1234 + \frac{362}{(1+r)^1} + \frac{548}{(1+r)^2} + \frac{481}{(1+r)^3} = 0 \Rightarrow r = 5.96\%$$

- Monthly compounding

$$NPV = -1234 + \frac{362}{(1+r/12)^{12}} + \frac{548}{(1+r/12)^{24}} + \frac{481}{(1+r/12)^{36}} = 0 \Rightarrow r = 5.80\%$$

$-1234+362/(1+x/12)^{12}+548/(1+x/12)^{24}+481/(1+x/12)^{36}$

儲蓄險比較

郵局六年期吉利保險

30歲男性繳 6 年：IRR 值 0.839%

6 年期儲蓄險 - 郵局六年期吉利保險			
年度	保費	領回	小計
0	-16320		-16320
1	-16157		-16157
2	-16157		-16157
3	-16157		-16157
4	-16157		-16157
5	-16157		-16157
6		100000	100000
		IRR	0.839%

Based on yearly compounding

遠雄好鑽養老保險

繳6年領回38萬：IRR 2.093%

六年期儲蓄險 - 遠雄好鑽養老 6 年期			
年度	保費	領回	小計
0	-59356		-59356
1	-58762		-58762
2	-58762		-58762
3	-58762		-58762
4	-58762		-58762
5	-58762		-58762
6		380,000	380,000
		IRR	2.093%

Based on yearly compounding

投資方案比較

- 三種投資方案
  - 方案一： 2年賺20%　　　➔ irr=9.54% (年複利) or 9.15% (月複利)
  - 方案二： 5年賺50%　　　➔ irr=8.45% (年複利) or 8.14% (月複利)
  - 方案三： 10年賺100%　　➔ irr=7.18% (年複利) or 6.95% (月複利)
- Exercise
  - Write a Python function to execute the above computation.

References

- IRR
  - 綠角財經筆記 ← A very good example of IRR
  - Wiki
- 儲蓄險比較
  - IRR值EXCEL教學：儲蓄險比較利潤請看IRR值算年利率，而非報酬率
  - 六年儲蓄險台幣IRR分析-郵局六年期吉利保險(2012年)附DM
  - 六年儲蓄險台幣 IRR 分析 -遠雄好鑽養老保險 (2012附DM)