Using superposition in conjunction with the electric field due to a point charge, we discussed the computation of the electric field due to two point charges and the computer generation of the direction lines of the electric field. We then extended the determination of electric field intensity to continuous charge distributions.

Next we introduced the magnetic field concept from considerations of Ampère's law of force, having to do with the magnetic forces between two current loops. We learned that the magnetic field exerts force only on moving charges. The magnetic force acting on a test charge q moving with a velocity $\mathbf{v}$ at a point in the field region is given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

where $\mathbf{B}$ is the magnetic flux density at that point. In terms of current flowing in a wire, the magnetic force acting on a current element of length $d\mathbf{l}$ and current I at a point in the field region is given by

$$\mathbf{F} = I\,d\mathbf{l} \times \mathbf{B}$$

The magnetic flux density due to a current element $I\,d\mathbf{l}$ in free space is given by the Biot-Savart law

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l} \times \mathbf{a}_R}{R^2}$$

where μ_0 is the permeability of free space, and R and $\mathbf{a}_R$ have the same meanings as in the expression for $\mathbf{E}$ due to a point charge. Using superposition in conjunction with the Biot-Savart law, we discussed the computation of the magnetic field due to current distributions.

Combining the electric and magnetic field concepts, we then introduced the Lorentz force equation

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

which gives the force acting on a test charge q moving with velocity $\mathbf{v}$ at a point in a region characterized by electric field of intensity $\mathbf{E}$ and magnetic field of flux density $\mathbf{B}$. We used the Lorentz force equation to discuss (1) the determination of $\mathbf{E}$ and $\mathbf{B}$ at a point from a knowledge of forces acting on a test charge at that point for three different velocities and (2) the tracing of charged particle motion in a region of crossed electric and magnetic fields.

REVIEW QUESTIONS

Q1.1. Give some examples of scalars.

Q1.2. Give some examples of vectors.

Q1.3. Is it necessary for the reference vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ to be an orthogonal set?

Q1.4. State whether $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ directed westward, northward, and downward, respectively, is a right-handed or a left-handed set.

RaoCh01v3.qxd 12/18/03 2:44 PM Page 67

Q1.5. State all conditions for which $\mathbf{A} \cdot \mathbf{B} = 0$.

Q1.6. State all conditions for which $\mathbf{A} \times \mathbf{B} = \mathbf{0}$.

Q1.7. What is the significance of $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = 0$?

Q1.8. What is the significance of $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{0}$?

Q1.9. What is the particular advantageous characteristic associated with the unit vectors in the Cartesian coordinate system?

Q1.10. What is the position vector?

Q1.11. What is the total distance around the circumference of a circle of radius 1 m? What is the total vector distance around the circle?

Q1.12. Discuss the application of differential length vectors to find a unit vector normal to a surface at a point on the surface.

Q1.13. Discuss the concept of a differential surface vector.

Q1.14. What is the total surface area of a cube of sides 1 m? Assuming the normals to the surfaces to be directed outward of the cubical volume, what is the total vector surface area of the cube?

Q1.15. Describe the three orthogonal surfaces that define the cylindrical coordinates of a point.

Q1.16. Which of the unit vectors in the cylindrical coordinate system are not uniform? Explain.

Q1.17. Discuss the conversion from the cylindrical coordinates of a point to its Cartesian coordinates, and vice versa.

Q1.18. Describe the three orthogonal surfaces that define the spherical coordinates of a point.

Q1.19. Discuss the nonuniformity of the unit vectors in the spherical coordinate system.

Q1.20. Discuss the conversion from the spherical coordinates of a point to its Cartesian coordinates, and vice versa.

Q1.21. Describe briefly your concept of a scalar field and illustrate with an example.

Q1.22. Describe briefly your concept of a vector field and illustrate with an example.

Q1.23. How do you depict pictorially the gravitational field of Earth?

Q1.24. Discuss the procedure for obtaining the equations for the direction lines of a vector field.

Q1.25. State Coulomb's law. To what law in mechanics is Coulomb's law analogous?

Q1.26. What is the value of the permittivity of free space? What are its units?

Q1.27. What is the definition of electric field intensity? What are its units?

Q1.28. Discuss two applications based on the electric force on a charged particle.

Q1.29. Describe the electric field due to a point charge.

Q1.30. Discuss the computer generation of the direction lines of the electric field due to two point charges.

Q1.31. Discuss the different types of charge distributions. How do you determine the electric field intensity due to a charge distribution?

Q1.32. Describe the electric field due to an infinitely long line charge of uniform charge density.

Q1.33. Describe the electric field due to an infinite plane sheet of uniform surface charge density.

Q1.34. State Ampère's force law as applied to current elements. Why is it not necessary for Newton's third law to hold for current elements?

Q1.35. What are the units of magnetic flux density? How is magnetic flux density defined in terms of force on a current element?

Q1.36. What is the value of the permeability of free space? What are its units?

Q1.37. Describe the magnetic field due to a current element.

Q1.38. Discuss the different types of current distributions. How do you determine the magnetic flux density due to a current distribution?

Q1.39. Describe the magnetic field due to an infinite plane sheet of uniform surface current density.

Q1.40. Discuss the analogies between the electric field due to charge distributions and the magnetic field due to current distributions.

Q1.41. How is magnetic flux density defined in terms of force on a moving charge?

Q1.42. Discuss two applications based on the magnetic force on a current-carrying wire or on a moving charge.

Q1.43. State the Lorentz force equation.

Q1.44. Discuss the determination of **E** and **B** at a point from the knowledge of forces experienced by a test charge at that point for several velocities. What is the minimum required number of forces?

Q1.45. Give some examples of devices based on charged particle motion in electric and magnetic fields.

Q1.46. Discuss the tracing of the path of a charged particle in a region of crossed electric and magnetic fields.

PROBLEMS

Section 1.1

P1.1. **Geometrical computations involving conversion from rectangular to polar coordinates.** A bug starts at a point and travels 1 m northward, s m eastward, s^2 m southward, s^3 m westward, and so on, where $s < 1$, making a 90°-turn to the right and traveling in the new direction s times the distance traveled in the previous direction. Find the value of s for each of the following cases: **(a)** the total distance traveled by the bug is 1.5 m; **(b)** the straight-line distance from the initial position to the final position of the bug is 0.8 m; and **(c)** the final position of the bug relative to its initial position is 30° east of north.

P1.2. **Solution of simultaneous vector algebraic equations.** Three vectors **A**, **B**, and **C** satisfy the equations

$$\begin{aligned} \mathbf{A} + \mathbf{B} - \mathbf{C} &= 2\mathbf{a}_1 + \mathbf{a}_2 \\ \mathbf{A} + 2\mathbf{B} + 3\mathbf{C} &= -2\mathbf{a}_1 + 5\mathbf{a}_2 + 5\mathbf{a}_3 \\ 2\mathbf{A} - \mathbf{B} + \mathbf{C} &= \mathbf{a}_1 + 5\mathbf{a}_2 \end{aligned}$$

By writing a matrix equation for the 3×3 matrix

$$\begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$$

and solving it, obtain the vectors **A**, **B**, and **C**.

P1.3. Law of cosines from dot product. Two vectors $\mathbf{A}$ and $\mathbf{B}$ originate from a common point. **(a)** If $\mathbf{C} = \mathbf{B} - \mathbf{A}$ comprises the third side of the triangle, obtain using $\mathbf{C}\cdot\mathbf{C} = (\mathbf{B} - \mathbf{A})\cdot(\mathbf{B} - \mathbf{A})$ the law of cosines relating C to A, B, and the angle α between $\mathbf{A}$ and $\mathbf{B}$. **(b)** Find the expression for the distance from the common point to the side $\mathbf{C}$, in terms of $\mathbf{A}$ and $\mathbf{B}$ only.

P1.4. Using vector algebraic operations. Four vectors drawn from a common point are given as follows:

$$\begin{aligned}
\mathbf{A} &= 2\mathbf{a}_1 - m\mathbf{a}_2 - \mathbf{a}_3 \\
\mathbf{B} &= m\mathbf{a}_1 + \mathbf{a}_2 - 2\mathbf{a}_3 \\
\mathbf{C} &= \mathbf{a}_1 + m\mathbf{a}_2 + 2\mathbf{a}_3 \\
\mathbf{D} &= m^2\mathbf{a}_1 + m\mathbf{a}_2 + \mathbf{a}_3
\end{aligned}$$

Find the value(s) of m for each of the following cases: **(a)** $\mathbf{A}$ is perpendicular to $\mathbf{B}$; **(b)** $\mathbf{B}$ is parallel to $\mathbf{C}$; **(c)** $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ lie in the same plane; and **(d)** $\mathbf{D}$ is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.

P1.5. Straight line connecting the tips of three vectors originating from a point. Show that the tips of three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ originating from a common point lie along a straight line if $\mathbf{A}\times\mathbf{B} + \mathbf{B}\times\mathbf{C} + \mathbf{C}\times\mathbf{A} = \mathbf{0}$. Provide a geometric interpretation for this result.

P1.6. Plane containing the tips of four vectors originating from a point. Show that the tips of four vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ originating from a common point lie in a plane if $(\mathbf{A} - \mathbf{B})\cdot(\mathbf{A} - \mathbf{C})\times(\mathbf{A} - \mathbf{D}) = 0$. Then determine if the tips of $\mathbf{A} = \mathbf{a}_1$, $\mathbf{B} = 2\mathbf{a}_2$, $\mathbf{C} = 2\mathbf{a}_3$, and $\mathbf{D} = \mathbf{a}_1 + 2\mathbf{a}_2 - 2\mathbf{a}_3$ lie in a plane.

P1.7. Some vector identities.

(a) Show that

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$$

(b) Using the result of part (a), show the following:

(i) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = \mathbf{0}$

(ii) $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{B}\times\mathbf{C})\times(\mathbf{C}\times\mathbf{A}) = (\mathbf{A}\times\mathbf{B}\cdot\mathbf{C})^2$

Section 1.2

P1.8. Geometrical computations in Cartesian coordinates. Three points are given by $A(12, 0, 0)$, $B(0, 15, 0)$, and $C(0, 0, -20)$. Find the following: **(a)** the distance from B to C; **(b)** the component of the vector from A to C along the vector from B to C; and **(c)** the perpendicular distance from A to the line through B and C.

P1.9. Sphere passing through four specified points in Cartesian coordinates. Consider four points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), and (x_4, y_4, z_4). Show that the center point (x_0, y_0, z_0) of the sphere passing through these points is given by the solution of the equation

$$2\begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{bmatrix}\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} x_2^2 + y_2^2 + z_2^2) - (x_1^2 + y_1^2 + z_1^2) \\ (x_3^2 + y_3^2 + z_3^2) - (x_1^2 + y_1^2 + z_1^2) \\ (x_4^2 + y_4^2 + z_4^2) - (x_1^2 + y_1^2 + z_1^2) \end{bmatrix}$$

Then find the center point of the sphere and its radius if the four points are (1, 1, 4), (3, 3, 2), (2, 3, 3), and (3, 2, 3).

RaoCh01v3.qxd 12/18/03 2:44 PM Page 70

P1.10. Plane containing two vectors originating from a common point.

(a) Two vectors **A** and **B** originate from a common point $P(x_1, y_1, z_1)$. Show that the equation for the plane in which the two vectors lie is given by

$$\mathbf{A} \times \mathbf{B} \cdot (\mathbf{r} - \mathbf{r}_1) = 0$$

where $\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$ is the position vector and $\mathbf{r}_1 = x_1\mathbf{a}_x + y_1\mathbf{a}_y + z_1\mathbf{a}_z$ is the vector from the origin to the point P.

(b) Using the result of part (a), find the equation for the plane containing the points (1, 1, 2), (2, 2, 0), and (3, 0, 1).

P1.11. Finding differential length vector tangential to a curve. Find the expression for the differential length vector tangential to the curve $x + y = 2$, $y = z^2$ at an arbitrary point on the curve and having the projection dz on the z-axis. Then obtain the differential length vectors tangential to the curve at the points **(a)** (2, 0, 0), **(b)** (1, 1, 1), and **(c)** $(-2, 4, 2)$.

P1.12. Finding unit vector normal to a curve and a line at the point of intersection. Find the expression for the unit vector normal to the curve $x = y^2 = z^3$ at the point (1, 1, 1) and having no components along the line $x = y = z$.

P1.13. Finding unit vector normal to a surface. By considering two differential length vectors tangential to the surface $x^2 + y^2 + 2z^2 = 4$ at the point (1, 1, 1), find the unit vector normal to the surface.

P1.14. Finding differential surface vector associated with a plane. Consider the differential surface lying on the plane $2x + y = 2$ and having as its projection on the xz-plane the rectangular differential surface of sides dx and dz in the x- and z-directions, respectively. Obtain the expression for the vector $d\mathbf{S}$ associated with that surface.

Section 1.3

P1.15. Vector algebraic operations with points in cylindrical coordinates. Three points are given in cylindrical coordinates by $A(2, \pi/3, 1)$, $B(2\sqrt{3}, \pi/6, -2)$, and $C(2, 5\pi/6, 0)$. **(a)** Find the volume of the parallelepiped having the lines from the origin to the three points as one set of its contiguous edges. **(b)** Determine if the point $D(\sqrt{3}, \pi/2, 2.5)$ in cylindrical coordinates lies in the plane containing A, B, and C.

P1.16. Vector algebraic operations with points in spherical coordinates. Four points are given in spherical coordinates by $A(1, \pi/2, 0)$, $B(\sqrt{8}, \pi/4, \pi/3)$, $C(1, 0, 0)$, and $D(\sqrt{12}, \pi/6, \pi/2)$. Show that these four points are situated at the corners of a parallelogram and find the area of the parallelogram.

P1.17. Vector algebraic operations for vectors specified in cylindrical coordinates. Three unit vectors are given in cylindrical coordinates as follows: $\mathbf{A} = \mathbf{a}_r$ at $(2, \pi/6, 0)$, $\mathbf{B} = \mathbf{a}_\phi$ at $(1, \pi/3, 2)$, and $\mathbf{C} = \mathbf{a}_\phi$ at $(3, 5\pi/6, 1)$. Find: **(a)** $\mathbf{A} \cdot \mathbf{B}$; **(b)** $\mathbf{B} \cdot \mathbf{C}$; and **(c)** $\mathbf{B} \times \mathbf{C}$.

P1.18. Vector algebraic operations for vectors specified in spherical coordinates. Three unit vectors are given in spherical coordinates as follows: $\mathbf{A} = \mathbf{a}_r$ at $(2, \pi/6, \pi/2)$, $\mathbf{B} = \mathbf{a}_\theta$ at $(1, \pi/3, 0)$, and $\mathbf{C} = \mathbf{a}_\phi$ at $(3, \pi/4, 3\pi/2)$. Find: **(a)** $\mathbf{A} \cdot \mathbf{B}$; **(b)** $\mathbf{A} \cdot \mathbf{C}$; **(c)** $\mathbf{B} \cdot \mathbf{C}$; and **(d)** $\mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$.

P1.19. Conversion of vector in Cartesian coordinates to one in spherical coordinates. Convert the vector $\mathbf{a}_x + \mathbf{a}_y - \sqrt{2}\,\mathbf{a}_z$ at the point $(1, 1, \sqrt{2})$ to one in spherical coordinates.

P1.20. Equality of two vectors in different coordinates. Determine if the vector $(\mathbf{a}_{rc} - \sqrt{3}\,\mathbf{a}_\phi + 3\mathbf{a}_z)$ at the point $(3, \pi/3, 5)$ in cylindrical coordinates is equal to the vector $(3\mathbf{a}_{rs} - \sqrt{3}\,\mathbf{a}_\theta - \mathbf{a}_\phi)$ at the point $(1, \pi/3, \pi/6)$ in spherical coordinates.

P1.21. Finding unit vector tangential to a curve in cylindrical coordinates. Find the expression for the unit vector tangential to the curve given in cylindrical coordinates by $r^2 \sin 2\phi = 1, z = 0$. Then obtain the unit vectors tangential to the curve at the points: **(a)** $(1, \pi/4, 0)$ and **(b)** $(\sqrt{2}, \pi/12, 0)$.

P1.22. Finding unit vector tangential to a curve in spherical coordinates. Find the expression for the unit vector tangential to the curve given in spherical coordinates by $r = 1, \phi = 2\theta, 0 \le \theta \le \pi$. Then obtain the unit vectors tangential to the curve at the points: **(a)** $(1, \pi/4, \pi/2)$ and **(b)** $(1, \pi/2, \pi)$.

Section 1.4

P1.23. Scalar field of height of a hemispherical trough in a hemispherical dome. An otherwise hemispherical dome of radius 2 m has a symmetrically situated hemispherical trough of radius 1 m, as shown by the cross-sectional view in Fig. 1.42. Assuming the origin to be at the center of the base of the dome, obtain the expression for the two-dimensional scalar field describing the height h of the dome in each of the two coordinate systems: **(a)** rectangular (x, y) and **(b)** polar (r, ϕ).

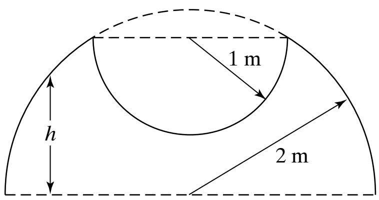


FIGURE 1.42
For Problem P1.23.

P1.24. Force field experienced by a mass in the Earth's gravitational field. Assuming the origin to be at the center of Earth and the z-axis to be passing through the poles, write vector functions for the force experienced by a mass m in the gravitational field of Earth (mass M) in each of the three coordinate systems: **(a)** Cartesian, **(b)** cylindrical, and **(c)** spherical. Describe the constant-magnitude surfaces and the direction lines.

P1.25. Field of the linear velocity of points inside the Earth. Assuming the origin to be at the center of Earth and the z-axis to be passing through the poles, write vector functions for the linear velocity of points inside Earth due to its spin motion in each of the three coordinate systems: **(a)** Cartesian; **(b)** cylindrical; and **(c)** spherical. Describe the constant-magnitude surfaces and the direction lines.

P1.26. Finding equations for the direction lines of vector fields in Cartesian coordinates. Obtain the equations for the direction lines for the following vector fields and passing through the point $(1, 2, 3)$: **(a)** $2y\mathbf{a}_x - x\mathbf{a}_y$ and **(b)** $x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$.

P1.27. Finding equation for the direction line of a vector field in cylindrical coordinates. Obtain the equation for the direction line for the vector field given in cylindrical coordinates by $(\sin\phi\,\mathbf{a}_r + \cos\phi\,\mathbf{a}_\phi)$ and passing through the point $(2, \pi/3, 1)$.

P1.28. Finding equation for the direction line of a vector field in spherical coordinates. Obtain the equation for the direction line for the vector field given in spherical coordinates by $(2\cos\theta\,\mathbf{a}_r - \sin\theta\,\mathbf{a}_\theta)$ and passing through the point $(2, \pi/4, \pi/6)$.

Section 1.5

P1.29. Electric forces on point charges. Point charges, each of value Q, are situated at the corners of a regular tetrahedron of edge length L. Find the electric force on each point charge.

P1.30. Electric force on a test charge in the field of six point charges. Six point charges, each of value Q, are situated at $(d, 0, 0)$, $(-d, 0, 0)$, $(0, d, 0)$, $(0, -d, 0)$, $(0, 0, d)$, and $(0, 0, -d)$. A test charge q located at the origin is displaced by a distance $\Delta \lll d$ along the positive x-axis. Find an approximate expression for the electric force acting on the charge.

P1.31. Finding the point charge for specified electric field intensities. For each of the following pairs of electric field intensities, find, if possible, the location and the value of a point charge that produces both fields: **(a)** $\mathbf{E}_1 = (2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$ V/m at $(2, 2, 3)$ and $\mathbf{E}_2 = (\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)$ V/m at $(-1, 0, 3)$; and **(b)** $\mathbf{E}_1 = (2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$ V/m at $(1, 1, 1)$ and $\mathbf{E}_2 = (2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)$ V/m at $(1, 2, 0)$.

P1.32. Electric field intensity due to an electric dipole. Two equal and opposite point charges Q and $-Q$ are located at $(0, 0, d/2)$ and $(0, 0, -d/2)$, respectively. Such an arrangement is known as the electric dipole. Show that the electric field intensity due to the electric dipole at very large distances from the origin compared to the spacing d is given approximately by $(Qd/4\pi\varepsilon_0 r^3)(2 \cos\theta\, \mathbf{a}_r + \sin\theta\, \mathbf{a}_\theta)$.

P1.33. Motion of a point charge along the axis of a circular ring charge. A point charge q of mass m is in static equilibrium at the origin, in the presence of a circular ring charge Q in the xy-plane and two point charges, each of value Q, on the z-axis. The ring charge is uniformly distributed, with radius a and center at the origin. The two point charges are located at $(0, 0, -a)$ and $(0, 0, a)$. All charges are of the same sign. The point charge q is constrained to move along the z-axis. It is given a slight displacement $z_0 \ll a$ and released at $t = 0$. Obtain the approximate differential equation for the motion of the charge and find the frequency of oscillation.

P1.34. Finding a circular ring charge that produces specified electric field intensities. Design an arrangement of a circular ring charge of uniform density and total charge Q equal to 1 μC that produces electric field intensities of $10^3\, \mathbf{a}_z$ V/m at the two points $(0, 0, 1)$ and $(0, 0, 2)$. If Q is not equal to 1 μC, determine, if any, the restriction on its value for a solution to exist.

P1.35. Electric field of a circular ring charge with nonuniform density. Assuming that the circular ring of Example 1.7 is coated with charge such that the charge density is given by $\rho_L = \rho_{L0} \cos\phi$ C/m, find the electric field intensity at a point on the z-axis by setting up the integral expression and evaluating it.

P1.36. Electric field of a circular disc of charge with nonuniform density. Consider a circular disc of radius a lying in the xy-plane with its center at the origin and carrying charge of density $4\pi\varepsilon_0/r$ C/m^2. Obtain the expression for the electric field intensity at the point $(0, 0, z)$ by setting up the integral and evaluating it.

P1.37. Electric field of a line charge with nonuniform density. Consider the charge distributed with density $4\pi\varepsilon_0|z|$ C/m along the line between $(0, 0, -a)$ and $(0, 0, a)$. Obtain the expression for the electric field intensity at $(r, \phi, 0)$ in cylindrical coordinates, by considering a differential length element along the line charge, setting up the field as an integral and evaluating it.

P1.38. Electric field of a slab of volume charge distributed between two planes. Consider the volume charge distributed uniformly with density ρ_0 C/m^3 between the planes $z = -a$ and $z = a$. Using superposition in conjunction with the result of

RaoCh01v3.qxd 12/18/03 2:44 PM Page 73

Example 1.9, show that the electric field intensity due to the slab of charge is given by

$$\mathbf{E} = \begin{cases} -(\rho_0 a/\varepsilon_0)\mathbf{a}_z & \text{for} \quad z < -a \\ (\rho_0 z/\varepsilon_0)\mathbf{a}_z & \text{for} \quad -a < z < a \\ (\rho_0 a/\varepsilon_0)\mathbf{a}_z & \text{for} \quad z > a \end{cases}$$

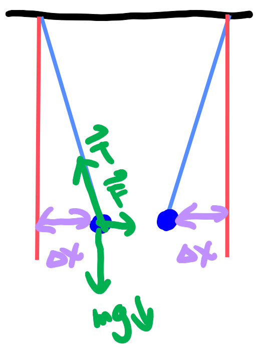
Section 1.6

P1.39. Magnetic forces on current elements. Three identical current elements $I\,dz\,\mathbf{a}_z$ A-m are located at equally spaced points on a circle of radius 1 m centered at the origin and lying on the xy-plane. The first point is $(1, 0, 0)$. Find the magnetic force on each current element.

P1.40. Magnetic flux density due to a current element. For the current element $I\,dx\,(\mathbf{a}_x + \mathbf{a}_y)$ A situated at the point $(1, -2, 2)$, find the magnetic flux densities at three points: **(a)** $(2, -1, 3)$, **(b)** $(2, -3, 4)$, and **(c)** $(3, 0, 2)$.

P1.41. Finding infinitely long wire for specified magnetic flux densities. For each of the following pairs of magnetic flux densities, find, if possible, the orientation of an infinitely long filamentary wire and the current in it required to produce both fields: **(a)** $\mathbf{B}_1 = 10^{-7}\,\mathbf{a}_y$ Wb/m^2 at $(3, 0, 0)$ and $\mathbf{B}_2 = -10^{-7}\,\mathbf{a}_x$ Wb/m^2 at $(0, 4, 0)$; and **(b)** $\mathbf{B}_1 = 10^{-7}(\mathbf{a}_y - \mathbf{a}_z)$ Wb/m^2 at $(\sqrt{2}, 0, 0)$ and $\mathbf{B}_2 = -10^{-7}\,\mathbf{a}_x$Wb/m^2 at $(0, \sqrt{2}, 0)$.

P1.42. Attraction between two long, horizontal filamentary wires. Two long identical rigid filamentary wires, each of length l and weight w, are suspended horizontally from the ceiling by long weightless threads, each of length L. The wires are arranged to be parallel and separated by a distance d, small compared to l and L. A current I is passed through both wires via flexible connections so as to cause the wires to be attracted to each other. **(a)** Should the currents be in the same sense or in opposite senses for attraction to occur? **(b)** If the current is gradually increased from zero, the wires will gradually approach each other. A condition may be reached when any further increase of current will cause the wires to swing and touch each other. Determine the critical value of I at which this happens.



P1.43. Magnetic field due to a circular loop of wire. A circular loop of wire of radius a is situated in the xy-plane with its center at the origin. It carries a current I in the clockwise sense as seen along the positive z-axis, that is, in the sense of increasing values of ϕ. Obtain the expression for $\mathbf{B}$ due to the current loop at a point on the z-axis.

P1.44. Magnetic field due to a finitely long straight wire of current. A straight wire along the z-axis carries current I in the positive z-direction. Consider the portion of the wire between $(0, 0, a_1)$ and $(0, 0, a_2)$, where $a_2 > a_1$. Show that the magnetic flux density at an arbitrary point $P(r, \phi, z)$ due to this portion of the wire is given by

$$\mathbf{B} = \frac{\mu_0 I}{4\pi r}(\cos\alpha_1 - \cos\alpha_2)\mathbf{a}_\phi$$

where α_1 and α_2 are the angles subtended by the lines from P to $(0, 0, a_1)$ and $(0, 0, a_2)$, respectively, with the z-axis. Verify your result in the limit $a_1 \to -\infty$ and $a_2 \to \infty$.

P1.45. Magnetic flux density due a square loop of wire. A square loop of wire lies in the xy-plane with its corners at $(1, 1, 0)$, $(-1, 1, 0)$, $(-1, -1, 0)$, and $(1, -1, 0)$. A

RaoCh01v3.qxd 12/18/03 2:44 PM Page 74

current of 1 A flows in the loop in the sense defined by connecting the specified points in succession. Applying the result of Problem P1.44 to each side of the loop, find the magnetic flux densities at two points: **(a)** $(0, 0, 0)$ and **(b)** $(2, 0, 0)$.

P1.46. Finding a pair of infinitely long parallel wires for specified magnetic flux densities. Design an arrangement of a pair of infinitely long, straight, filamentary wires parallel to the z-direction and in a plane parallel to the xz-plane, each carrying current I equal to 1 A but in opposite directions, which produce magnetic flux densities of $10^{-7}\,\mathbf{a}_y$ Wb/m^2 and $0.5 \times 10^{-7}\,\mathbf{a}_y$ Wb/m^2 at the points $(0, 1, 0)$ and $(0, 2, 0)$, respectively. If I is not equal to 1 A, determine, if any, the restriction on its value for a solution to exist.

P1.47. Magnetic flux density due to three plane current sheets. Three infinite plane current sheets, each of a uniform current density, exist in the coordinate planes of a Cartesian coordinate system. The magnetic flux densities due to these current sheets are given at three points as follows: at (1, 2, 3), $\mathbf{B} = 3B_0\mathbf{a}_x$; at $(7, -5, 6)$, $\mathbf{B} = B_0(-\mathbf{a}_x + 2\mathbf{a}_z)$; at $(8, 9, -4)$, $\mathbf{B} = B_0(\mathbf{a}_x + 2\mathbf{a}_y)$. Find the magnetic flux densities at the following points: **(a)** $(-6, -2, -3)$; **(b)** $(-4, -5, 7)$; and **(c)** $(6, -3, -5)$.

P1.48. Magnetic field for a current distribution between two planes. Consider current distribution with uniform density $J_0\mathbf{a}_z$ A/m^2 in the volume between the planes $y = -a$ and $y = 0$, and with uniform density $-J_0\mathbf{a}_z$ A/m^2 in the volume between the planes $y = 0$ and $y = a$. Using superposition in conjunction with the result of Example 1.12, show that the magnetic flux density due to the current distribution is given by

$$\mathbf{B} = \begin{cases} \mu_0 J_0(|y| - a)\mathbf{a}_x & \text{for} \quad |y| \le a \\ \mathbf{0} & \text{otherwise} \end{cases}$$

P1.49. Ratio of the radii of orbits of two charged particles in a uniform magnetic field. Show that the ratio of the radii of orbits of two charged particles of the same charge, but with different masses, entering a region of uniform magnetic field perpendicular to the field and with equal kinetic energies is equal to the ratio of the square roots of their masses.

Section 1.7

P1.50. Movement of a test charge in a region of crossed electric and magnetic fields. Show that in a region of uniform, crossed electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, respectively, a test charge released at a point in the field region with initial velocity $\mathbf{v} = (\mathbf{E} \times \mathbf{B})/B^2$ moves with constant velocity equal to the initial value. Compute $\mathbf{v}$ for $\mathbf{E}$ and $\mathbf{B}$ equal to $E_0(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$ and $B_0(\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)$, respectively.

P1.51. Finding magnetic field from forces experienced by a test charge. Let $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ be the forces experienced by a test charge q at a point in a region of electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, respectively, for velocities $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, respectively, of the charge. If $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are such that $\mathbf{A} = \mathbf{F}_1 \times \mathbf{F}_2 + \mathbf{F}_2 \times \mathbf{F}_3 + \mathbf{F}_3 \times \mathbf{F}_1 \neq 0$, that is, their tips do not lie on a straight line when drawn from a common point, show that

$$\mathbf{B} = \frac{1}{q}\left[\frac{\mathbf{F}_2 - \mathbf{F}_1}{(\mathbf{v}_2 - \mathbf{v}_1) \times \mathbf{A}}\right]\mathbf{A}$$

P1.52. Finding electric and magnetic fields from forces experienced by a test charge. The forces experienced by a test charge q at a point in a region of electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, respectively, are given as follows for three different

RaoCh01v3.qxd 12/18/03 2:44 PM Page 75

velocities of the test charge, where v_0 and E_0 are constants.

$$\begin{aligned}
\mathbf{F}_1 &= qE_0(\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z) && \text{for} \quad \mathbf{v}_1 = v_0\mathbf{a}_x \\
\mathbf{F}_2 &= qE_0(\mathbf{a}_x - \mathbf{a}_y - \mathbf{a}_z) && \text{for} \quad \mathbf{v}_2 = v_0\mathbf{a}_y \\
\mathbf{F}_3 &= \mathbf{0} && \text{for} \quad \mathbf{v}_3 = v_0\mathbf{a}_z
\end{aligned}$$

Find **E** and **B** at that point.

P1.53. Forces experienced by a test charge in a region of electric and magnetic fields. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ experienced by a test charge q at a point in a region of electric and magnetic fields for three different velocities of the test charge are given as follows:

$$\begin{aligned}
\mathbf{F}_1 &= \mathbf{0} && \text{for} \quad \mathbf{v}_1 = v_0\mathbf{a}_x \\
\mathbf{F}_2 &= \mathbf{0} && \text{for} \quad \mathbf{v}_2 = v_0\mathbf{a}_y \\
\mathbf{F}_3 &= qE_0\mathbf{a}_z && \text{for} \quad \mathbf{v}_3 = v_0(\mathbf{a}_x + 2\mathbf{a}_y)
\end{aligned}$$

Find the forces $\mathbf{F}_4$, $\mathbf{F}_5$, and $\mathbf{F}_6$ experienced by the test charge at that point for three other velocities: **(a)** $\mathbf{v}_4 = \mathbf{0}$, **(b)** $\mathbf{v}_5 = v_0(\mathbf{a}_x + \mathbf{a}_y)$, and **(c)** $\mathbf{v}_6 = (v_0/4)(3\mathbf{a}_x + \mathbf{a}_y)$.

P1.54. Movement of a test charge in a region of uniform electric and magnetic fields. Uniform electric and magnetic fields exist in a region of space. A test charge q released with an initial velocity $\mathbf{v}_1$ or $\mathbf{v}_2$ moves with constant velocity equal to the initial value. Show that the test charge moves with constant velocity equal to the initial value when released with an initial velocity $(m\mathbf{v}_1 + n\mathbf{v}_2)/(m + n)$ for any nonzero $(m + n)$.

REVIEW PROBLEMS

R1.1. Using vector algebraic operations and equalities. Using the equality

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

show that if $\mathbf{A} \times \mathbf{F} = \mathbf{C}$ and $\mathbf{B} \times \mathbf{F} = \mathbf{D}$, then

$$\mathbf{F} = \frac{\mathbf{C} \times \mathbf{D}}{\mathbf{A} \cdot \mathbf{D}} = -\frac{\mathbf{C} \times \mathbf{D}}{\mathbf{B} \cdot \mathbf{C}}$$

Find **F** if $\mathbf{A} = (\mathbf{a}_x + \mathbf{a}_y)$, $\mathbf{B} = (\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$, $\mathbf{C} = (\mathbf{a}_x - \mathbf{a}_y)$, and $\mathbf{D} = (6\mathbf{a}_x - 5\mathbf{a}_y - 2\mathbf{a}_z)$.

R1.2. Shortest distance from a point to a plane. The tips of three vectors **A, B**, and **C** originating from a common point determine a plane. **(a)** Show that the shortest distance from the common point to the plane is $|\mathbf{A} \cdot \mathbf{B} \times \mathbf{C}|/|\mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{C} + \mathbf{C} \times \mathbf{A}|$. **(b)** Compute its value for $\mathbf{A} = 2\mathbf{a}_z$, $\mathbf{B} = \frac{1}{2}(\mathbf{a}_{rc} - \sqrt{3}\,\mathbf{a}_\phi)$ in cylindrical coordinates, and $\mathbf{C} = \frac{1}{4}(3\mathbf{a}_{rs} + \sqrt{3}\,\mathbf{a}_\theta + 2\mathbf{a}_\phi)$ in spherical coordinates, at the point $(\sqrt{3}, 3, 2)$ in Cartesian coordinates.

R1.3. Sphere inscribed in an equilateral tetrahedron inscribed in a sphere. Find the edge length of the largest equilateral tetrahedron that can be fit inside a sphere of radius unity. Then find the radius of the largest sphere that can be fit inside that tetrahedron.

R1.4. Equation for a curve traversed on a sphere. Consider an observer always moving in the southeast direction on the surface of a spherical Earth of radius a, starting at the Greenwich meridian on the equator. **(a)** Find the equation for the curve traversed by the observer, using a spherical coordinate system with the

RaoCh01v3.qxd 12/18/03 2:44 PM Page 76

origin at the center of Earth, the North Pole at $\theta = 0$, and $\phi = 0$, corresponding to the Greenwich meridian. **(b)** Find the first two values of the south latitude when the observer is again on the Greenwich meridian. **(c)** Does the observer ever reach the South Pole?

R1.5. Three point charges on a semicircle. Consider the arrangement of three point charges Q_1, $kQ_1 (k > 0)$, and Q_2, as shown in Fig. 1.43, where Q_1 and kQ_1 are fixed and Q_2 is constrained to move on the semicircle. **(a)** Find the value of α in terms of k for which Q_2 is in equilibrium. **(b)** Find the numerical value of α for $k = 8$.

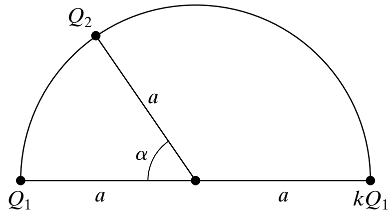


FIGURE 1.43
For Problem R1.5.

R1.6. Finitely long line charge distribution of nonuniform density. Consider a line charge distribution along the z-axis between $z = -a$ and $z = a$. **(a)** Show that, if the charge density is an even function $f_1(z)$, the electric field intensity at a point $(r, \phi, 0)$ has only an r-component, and set up the integral expression for it. **(b)** Show that, if the charge density is an odd function $f_2(z)$, the electric field intensity at $(r, \phi, 0)$ has only a z-component, and set up the integral expression for it. **(c)** Given that the charge density is

$$f(z) = \begin{cases} 4\pi\varepsilon_0(a + 2z) & \text{for} \quad -a \le z \le 0 \\ 4\pi\varepsilon_0 a & \text{for} \quad 0 \le z \le a \end{cases}$$

express $f(z)$ as the sum of even and odd functions $f_1(z)$ and $f_2(z)$, and evaluate the electric field components.

R1.7. Magnetic flux density due to a wire of current with straight and curved segments. Current I flows along a wire which is straight from ∞ to a on the x-axis, circular from $(a, 0, 0)$ to $(0, a, 0)$ and lying on the xy-plane in the sense of increasing ϕ, and then from a to ∞ on the y-axis. Find $\mathbf{B}$ at $(0, 0, a)$.

R1.8. Magnetic field due to a nonuniform current distribution between two planes. Current is distributed with density $J_0(y/a)\mathbf{a}_z$ A/m^2 in the volume between the planes $y = -a$ and $y = a$. Show that the magnetic flux density due to the current distribution is given by

$$\mathbf{B} = \begin{cases} \dfrac{\mu_0 J_0}{2a}(a^2 - y^2)\mathbf{a}_x & \text{for} \quad -a < y < a \\ \mathbf{0} & \text{otherwise} \end{cases}$$

R1.9. Movement of a test charge in a region of uniform electric and magnetic fields. Consider a test charge moving with constant velocity $\mathbf{v} = v_0(2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$ in a region of a uniform electric field of intensity $\mathbf{E} = E_0(2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)$ and a uniform magnetic field of flux density $\mathbf{B} = B_{x0}\mathbf{a}_x + B_{y0}\mathbf{a}_y + B_{z0}\mathbf{a}_z$. Is this information sufficient to find uniquely B_{x0}, B_{y0}, and B_{z0}? If not, given that $\mathbf{v}$ is perpendicular to $\mathbf{B}$, find B_{x0}, B_{y0}, and B_{z0} in terms of E_0 and v_0.