are replaced by rookies. The population size N therefore remains unchanged. Consider a different model in which veterans never fade away or die, but rookies continue to join the population at the same rate as before.

(a) If the population at time t is $N(t)$, why will the population at time $t+\tau$ be approximately $N(t)(1+\lambda\tau)$?
(b) Derive differential equations for the new libration.
(c) Why do these new differential equations lead to the same trajectories as the old differential equations?

10. To what does the replicator equation (9.7) reduce when the underlying game is not Chicken, but the Prisoners' Dilemma of Figure 7.3(b)? Explain why solutions to the equation have graphs like those of Figure 9.8.
11. Show that the replicator equation (9.7) is unchanged if each payoff x in the Chicken matrix of Figure 9.7(a) is replaced by $ax+b$, where $a>0$ and b are constants.
12. A general symmetric 2×2 bimatrix game is given in Figure 9.13(a). With this game, show that the replicator equations (9.7) for the fractions p_1 and p_2 of the population using strategies s_1 and s_2 respectively are:

$$p_1' = p_1(1-p_1)\{p_1(a-c)+(1-p_1)(b-d)\}$$
$$p_2' = p_2(1-p_2)\{p_2(d-b)+(1-p_2)(c-a)\}$$

	s_1	s_2
s_1	a, a	b, c
s_2	c, b	d, d

(a)

	s_1	s_2	s_3
s_1	0, 0	6, −3	−4, −1
s_2	−3, 6	0, 0	5, 3
s_3	−1, −4	3, 5	0, 0

(b)

Figure 9.13 Evolutionary stability problems.

(a) Explain why $(\tilde{p}_1,\tilde{p}_2)=(1,0)$ is always a rest point of the replicator dynamics. Show that it is an asymptotic attractor if and only if (i) $a>c$ or (ii) $a=c$ and $b>d$.
(b) Explain why $(\tilde{p}_1,\tilde{p}_2)=(0,1)$ is always a rest point of the replicator dynamics. Show that it is an asymptotic attractor if and only if (i) $d>b$ or (ii) $d=b$ and $c>a$.
(c) Explain why any pair $(\tilde{p}_1,\tilde{p}_2)$ is a rest point of the replicator dynamics if $a=c$ and $d=b$. Explain why none of these rest points is an asymptotic attractor (although they are local attractors).
(d) Suppose that $(a,d)\neq(c,b)$. Let $\tilde{p}_1=(d-b)/(a-c+d-b)$ and $\tilde{p}_2=1-\tilde{p}_1$. Explain why $(\tilde{p}_1,\tilde{p}_2)$ is a rest point of the replicator dynamics if and only if either $a\geq c$ and $d\geq b$ or else $a\leq c$ and $d\leq b$.
(e) Show that a rest point $(\tilde{p}_1,\tilde{p}_2)$ of the type considered in part (d) is an asymptotic attractor if and only if $a<c$ or $d<b$.
(f) If a rest point $(\tilde{p}_1,\tilde{p}_2)$ is completely mixed, explain why it is an asymptotic attractor if and only if $a<c$ and $d<b$. (Recall that a completely mixed strategy uses each pure strategy with positive probability.)

13. Show that any Nash equilibrium of the general symmetric 2×2 bimatrix game of Figure 9.13(a) is a rest point of its replicator dynamics. Give a counterexample to the proposition that a rest point of the replicator dynamics is always a Nash equilibrium of the game.
14. Prove the following assertions for the general symmetric 2×2 bimatrix game of Figure 9.13(a):
(a) The pure strategy s_1 is evolutionarily stable if and only if (i) $a>c$ or (ii) $a=c$ and $b>d$.
(b) The pure strategy s_2 is evolutionarily stable if and only if (i) $d>b$ or (ii) $d=b$ and $c>a$.
(c) A completely mixed strategy is evolutionarily stable if and only if $a<c$ and $d<b$.
15. Use Exercises 9.8.12 and 9.8.14 to show that a population state is an asymptotic attractor of the replicator dynamics for the general symmetric 2×2 bimatrix game of Figure 9.13(a) if and only if the corresponding strategy is evolutionarily stable.
16. Show that an evolutionarily stable strategy for the general symmetric 2×2 bimatrix game of Figure 9.13(a) fails to exist only in the case $a=c$ and $d=b$.
17. Show that the Rock-Scissors-Paper game of Figure 9.10(b) has no evolutionarily stable strategy.
18. Why can a strongly dominated strategy not be evolutionarily stable? Is it possible for a weakly dominated strategy to be evolutionarily stable?