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Polarization of sinusoidally time-varying vector fields was then considered. In the general case, the polarization is elliptical, that is, the tip of the field vector describes an ellipse with time. Linear and circular polarizations are special cases.

Finally, we learned that there is power flow and energy storage associated with the wave propagation that accounts for the work done in maintaining the current flow on the sheet. The power flow density is given by the Poynting vector

$$\mathbf{P} = \mathbf{E} \times \mathbf{H}$$

and the energy densities associated with the electric and magnetic fields are given, respectively, by

$$w_e = \frac{1}{2}\varepsilon_0 E^2$$

$$w_m = \frac{1}{2}\mu_0 H^2$$

The surface integral of the Poynting vector over a given closed surface gives the total power flow out of the volume bounded by that surface.

## REVIEW QUESTIONS

**Q3.1.** State Faraday's law in differential form for the simple case of $\mathbf{E} = E_x(z, t)\mathbf{a}_x$. How is it derived from Faraday's law in integral form?

**Q3.2.** State Faraday's law in differential form for the general case of an arbitrary electric field. How is it derived from its integral form?

**Q3.3.** What is meant by the net right-lateral differential of the $x$- and $y$-components of a vector normal to the $z$-direction? Give an example in which the net right-lateral differential of $E_x$ and $E_y$ normal to the $z$-direction is zero, although the individual derivatives are nonzero.

**Q3.4.** What is the determinant expansion for the curl of a vector in Cartesian coordinates?

**Q3.5.** State Ampère's circuital law in differential form for the general case of an arbitrary magnetic field. How is it derived from its integral form?

**Q3.6.** State Ampère's circuital law in differential form for the simple case of $\mathbf{H} = H_y(z, t)\mathbf{a}_y$. How is it derived from Ampère's circuital law in differential form for the general case?

**Q3.7.** If a pair of **E** and **B** at a point satisfies Faraday's law in differential form, does it necessarily follow that it also satisfies Ampère's circuital law in differential form, and vice versa?

**Q3.8.** State Gauss' law for the electric field in differential form. How is it derived from its integral form?

**Q3.9.** What is meant by the net longitudinal differential of the components of a vector field? Give an example in which the net longitudinal differential of the components of a vector field is zero, although the individual derivatives are nonzero.

**Q3.10.** What is the expansion for the divergence of a vector in Cartesian coordinates?

**Q3.11.** State Gauss' law for the magnetic field in differential form. How is it derived from its integral form?

**Q3.12.** How can you determine if a given vector can represent a magnetic field?

**Q3.13.** State the continuity equation and discuss its physical interpretation.

**Q3.14.** Summarize Maxwell's equations in differential form.

**Q3.15.** State and briefly discuss the basic definition of the curl of a vector.

**Q3.16.** What is a curl meter? How does it help visualize the behavior of the curl of a vector field?

**Q3.17.** Provide two examples of physical phenomena in which the curl of a vector field is nonzero.

**Q3.18.** State and briefly discuss the basic definition of the divergence of a vector.

**Q3.19.** What is a divergence meter? How does it help visualize the behavior of the divergence of a vector field?

**Q3.20.** Provide two examples of physical phenomena in which the divergence of a vector field is nonzero.

**Q3.21.** State Stokes' theorem and discuss its application.

**Q3.22.** State the divergence theorem and discuss its application.

**Q3.23.** What is the divergence of the curl of a vector?

**Q3.24.** What is a uniform plane wave? Why is the study of uniform plane waves important?

**Q3.25.** Outline the procedure for obtaining from the two Maxwell's curl equations the particular differential equation for the special case of $\mathbf{J} = J_x(z)\mathbf{a}_x$.

**Q3.26.** State the wave equation for the case of $\mathbf{E} = E_x(z, t)\mathbf{a}_x$. Describe the procedure for its solution.

**Q3.27.** Discuss by means of an example how a function $f(t - z\sqrt{\mu_0\varepsilon_0})$ represents a traveling wave propagating in the positive $z$-direction.

**Q3.28.** Discuss by means of an example how a function $g(t + z\sqrt{\mu_0\varepsilon_0})$ represents a traveling wave propagating in the negative $z$-direction.

**Q3.29.** What is the significance of the intrinsic impedance of free space? What is its value?

**Q3.30.** Summarize the procedure for obtaining the solution for the electromagnetic field due to the infinite plane sheet of uniform time-varying current density.

**Q3.31.** State and discuss the solution for the electromagnetic field due to the infinite plane sheet of current density $\mathbf{J}_S(t) = -J_S(t)\mathbf{a}_x$ for $z = 0$.

**Q3.32.** Discuss the parameters $\omega$, $\beta$, and $v_p$ associated with sinusoidally time-varying uniform plane waves.

**Q3.33.** Define wavelength. What is the relationship among wavelength, frequency, and phase velocity?

**Q3.34.** Discuss the classification of waves according to frequency, giving examples of their application in the different frequency ranges.

**Q3.35.** How is the direction of propagation of a uniform plane wave related to the directions of its fields?

**Q3.36.** Discuss the principle of an antenna array with the aid of an example.

**Q3.37.** A sinusoidally time-varying vector is expressed in terms of its components along the $x$-, $y$-, and $z$-axes. What is the polarization of each of the components?

**Q3.38.** What are the conditions for the sum of two linearly polarized sinusoidally time-varying vectors to be circularly polarized?

**Q3.39.** What is the polarization for the general case of the sum of two sinusoidally time-varying linearly polarized vectors having arbitrary amplitudes, phase angles, and directions?

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**Q3.40.** Discuss the relevance of polarization in the reception of radio waves.

**Q3.41.** Discuss right-handed and left-handed circular polarizations associated with sinusoidally time-varying uniform plane waves.

**Q3.42.** What is the Poynting vector? What is the physical interpretation of the Poynting vector over a closed surface?

**Q3.43.** Discuss how the fields far from a physical antenna vary inversely with the distance from the antenna.

**Q3.44.** Discuss the interpretation of energy storage in the electric and magnetic fields of a uniform plane wave. What are the energy densities associated with the electric and magnetic fields?

**Q3.45.** State Poynting's theorem. How is it derived from Maxwell's curl equations?

**Q3.46.** What is the time-average Poynting vector? How is it expressed in terms of the complex electric and magnetic fields?

## PROBLEMS

### Section 3.1

**P3.1.** **Evaluating curls of vector fields.** Find the curls of the following vector fields:

**(a)** $zx\mathbf{a}_x + xy\mathbf{a}_y + yz\,\mathbf{a}_z$

**(b)** $\cos y\mathbf{a}_x - x \sin y\, \mathbf{a}_y$

**(c)** $(e^{-r^2}/r)\mathbf{a}_\phi$ in cylindrical coordinates

**(d)** $2r \cos\theta\, \mathbf{a}_r + r\mathbf{a}_\theta$ in spherical coordinates

**P3.2.** **Finding B for a given E from Faraday's law in differential form.** For each of the following electric fields, find **B** that satisfies Faraday's law in differential form:

**(a)** $\mathbf{E} = E_0 \cos 3\pi z \cos 9\pi \times 10^8 t\, \mathbf{a}_x$

**(b)** $\mathbf{E} = E_0\mathbf{a}_y \cos [3\pi \times 10^8 t + 0.2\pi(4x + 3z)]$

**P3.3.** **Simplified forms of Maxwell's curl equations for special cases.** Obtain the simplified differential equations for the following cases: (**a**) Ampère's circuital law for $\mathbf{H} = H_x(z, t)\mathbf{a}_x$ and (**b**) Faraday's law for $\mathbf{E} = E_\phi(r, t)\mathbf{a}_\phi$ in cylindrical coordinates.

**P3.4.** **Simultaneous satisfaction of Faraday's and Ampere's circuital laws by E and B.** For the electric field $\mathbf{E} = E_0 e^{-\alpha z} \cos \omega t\, \mathbf{a}_x$ in free space ($\mathbf{J} = \mathbf{0}$), find **B** that satisfies Faraday's law in differential form and then determine if the pair of **E** and **B** satisfy Ampère's circuital law in differential form.

**P3.5.** **Satisfaction of Maxwell's curl equations for a specified electric field.** For the electric field $\mathbf{E} = E_0 \cos (\omega t - \alpha y - \beta z)\, \mathbf{a}_x$ in free space ($\mathbf{J} = \mathbf{0}$), find the necessary condition relating $\alpha, \beta, \omega, \mu_0$, and $\varepsilon_0$ for the field to satisfy both of Maxwell's curl equations.

**P3.6.** **Satisfaction of Maxwell's curl equations for a specified electric field.** For the electric field $\mathbf{E} = E_0 e^{-kx} \cos (2 \times 10^8 t - y)\, \mathbf{a}_z$ in free space ($\mathbf{J} = \mathbf{0}$), find the value(s) of $k$ for which the field satisfies both of Maxwell's curl equations.

**P3.7.** **Magnetic fields of current distributions from Ampere's circuital law in differential form.** For each of the following current distributions, find the corresponding magnetic field intensity using Ampère's circuital law in differential form without the displacement current density term, and plot both the current density and the magnetic field intensity components versus the appropriate coordinate:

**(a)** $\mathbf{J} = \begin{cases} J_0 \dfrac{z}{a} \mathbf{a}_x & \text{for} \quad -a < z < a \\ \mathbf{0} & \text{otherwise} \end{cases}$

**(b)** $\mathbf{J} = \begin{cases} J_0 \mathbf{a}_\phi & \text{for} \quad a < r < 2a \\ \mathbf{0} & \text{otherwise} \end{cases}$

in cylindrical coordinates

## Section 3.2

**P3.8.** **Evaluating divergences of vector fields.** Find the divergences of the following vector fields:

**(a)** $zx\mathbf{a}_x + xy\mathbf{a}_y + yz\mathbf{a}_z$

**(b)** $3\mathbf{a}_x + (y - 3)\mathbf{a}_y + (2 + z)\mathbf{a}_z$

**(c)** $r \sin \phi \, \mathbf{a}_\phi$ in cylindrical coordinates

**(d)** $r \cos \theta \, (\cos \theta \, \mathbf{a}_r - \sin \theta \, \mathbf{a}_\theta)$ in spherical coordinates

**P3.9.** **Electric fields of charge distributions from Gauss' law in differential form.** For each of the following charge distributions, find the corresponding displacement flux density using Gauss' law for the electric field in differential form, and plot both the charge density and the displacement flux density component versus the appropriate coordinate:

**(a)** $\rho = \begin{cases} \rho_0 \dfrac{x}{a} & \text{for} \quad -a < x < a \\ 0 & \text{otherwise} \end{cases}$

**(b)** $\rho = \begin{cases} \rho_0 \dfrac{r}{a} & \text{for} \quad 0 < r < a \\ 0 & \text{otherwise} \end{cases}$

in cylindrical coordinates

**P3.10.** **Realizability of vector fields as certain types of fields.** For each of the following vector fields, find the value of the constant $k$ for which the vector field can be realized as a magnetic field or as a current density in the absence of charge accumulation (or depletion):

**(a)** $(1/y^k)(2x\mathbf{a}_x + y\mathbf{a}_y)$

**(b)** $r(\sin k\phi \, \mathbf{a}_r + \cos k\phi \, \mathbf{a}_\phi)$ in cylindrical coordinates

**(c)** $[1 + (2/r^3)]\cos \theta \, \mathbf{a}_r + k[1 - (1/r^3)] \sin \theta \, \mathbf{a}_\theta$ in spherical coordinates

**P3.11.** **Realizability of vector fields as certain types of fields.** Determine which of the following static fields can be realized both as an electric field in a charge-free region and a magnetic field in a current-free region:

**(a)** $y\mathbf{a}_x + x\mathbf{a}_y$

**(b)** $[1 + (1/r^2)] \cos \phi \, \mathbf{a}_r - [1 - (1/r^2)] \sin \phi \, \mathbf{a}_\phi$ in cylindrical coordinates

**(c)** $r \sin \theta \, \mathbf{a}_\theta$ in spherical coordinates

## Section 3.3

**P3.12. Interpretation of curl with the aid of curl meter and by expansion.** With the aid of the curl meter and also by expansion in the Cartesian coordinate system, discuss the curl of the velocity vector field associated with the flow of water in the stream of Fig. 3.8(a), except that the velocity $v_z$ varies in a nonlinear manner from zero at the banks to a maximum of $v_0$ at the center given by

$$\mathbf{v} = \frac{4v_0}{a^2}(ax - x^2)\mathbf{a}_z$$

**P3.13. Interpretation of curl with the aid of curl meter and by expansion.** With the aid of the curl meter and also by expansion in the cylindrical coordinate system, discuss the curl of the linear velocity vector field associated with points inside Earth due to its spin motion.

**P3.14. Interpretation of divergence with the aid of divergence meter and by expansion.** Discuss the divergences of the following vector fields with the aid of the divergence meter and also by expansion in the appropriate coordinate system: **(a)** the position vector field associated with points in three-dimensional space and **(b)** the linear velocity vector field associated with points inside Earth due to its spin motion.

**P3.15. Verification of Stokes' theorem.** Verify Stokes' theorem for the following cases: **(a)** the vector field $\mathbf{A} = zx\mathbf{a}_x + xy\mathbf{a}_y + yz\mathbf{a}_z$ and the closed path comprising the straight lines from (0, 0, 0) to (0, 1, 0), from (0, 1, 0) to (0, 1, 1), and from (0, 1, 1) to (0, 0, 0) and **(b)** the vector field $\mathbf{A} = \cos y\,\mathbf{a}_x - x \sin y\,\mathbf{a}_y$ independent of a closed path.

**P3.16. Verification of the divergence theorem.** Verify the divergence theorem for the following cases: **(a)** the vector field $xy\mathbf{a}_x + yz\mathbf{a}_y + zx\mathbf{a}_z$ and the cubical box bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0,$ and $z = 1$ and **(b)** the vector field $y^2\mathbf{a}_y - 2yz\mathbf{a}_z$ and the closed surface bounded by the planes $x = 0, y = 0, z = 0, z = 1$ and $x + y = 1$.

## Section 3.4

**P3.17. Simplified forms of Maxwell's curl equations for special case of J.** From Maxwell's curl equations, obtain the particular differential equations for the case of $\mathbf{J} = J_z(y, t)\mathbf{a}_z$.

**P3.18. Plotting of functions of time and distance.** For each of the following functions, plot the value of the function versus $z$ for the two specified values of time and discuss the traveling-wave nature of the function.

**(a)** $e^{-|t-z|}$; $t = 0, t = 1$ s

**(b)** $(2 \times 10^8 t + z)[u(2 \times 10^8 t + z) - u(2 \times 10^8 t + z - 3)]$;
$t = 0, t = 10^{-8}$ s

**P3.19. Writing traveling wave functions for specified time and distance variations.** Write expressions for traveling-wave functions corresponding to the following cases: **(a)** time variation at $x = 0$ in the manner $10u(t)$ and propagating in the $-x$-direction with velocity 0.5 m/s; **(b)** time variation at $y = 0$ in the manner $t \sin 20t$ and propagating in the $+y$-direction with velocity 4 m/s; and **(c)** distance variation at $t = 0$ in the manner $5z^3e^{-z^2}$ and propagating in the $-z$-direction with velocity 2 m/s.

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**P3.20. Plotting time and distance variations of a traveling wave.** The variation with $z$ for $t = 0$ of a function $f(z, t)$ representing a traveling wave propagating in the $+z$-direction with velocity 100 m/s is shown in Fig. 3.34. Find and sketch: **(a)** $f$ versus $z$ for $t = 1$ s; **(b)** $f$ versus $t$ for $z = 0$; and **(c)** $f$ versus $t$ for $z = 200$ m.

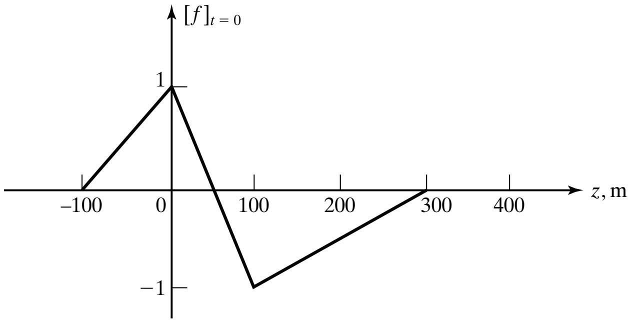


FIGURE 3.34
For Problems 3.20 and 3.21

**P3.21. Plotting time and distance variations of a traveling wave.** Repeat Problem P3.20 if the function $f$ represents a traveling wave propagating in the $-z$-direction with velocity 100 m/s.

**P3.22. Plotting field variations for a specified infinite plane-sheet current source.** An infinite plane sheet lying in the $z = 0$ plane in free space carries a surface current of density $\mathbf{J}_S = -J_S(t)\mathbf{a}_x$, where $J_S(t)$ is as shown in Fig. 3.35. Find and sketch **(a)** $E_x$ versus $t$ in the $z = 300$ m plane; **(b)** $E_x$ versus $z$ for $t = 2\ \mu$s; and **(c)** $H_y$ versus $z$ for $t = 4\ \mu$s.

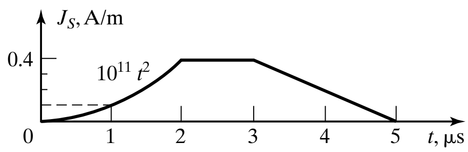


FIGURE 3.35
For Problem 3.22

**P3.23. Plotting field variations for a specified infinite plane-sheet current source.** An infinite plane sheet of current density $\mathbf{J}_S = -J_S(t)\mathbf{a}_x$ A/m where $J_S(t)$ is as shown in Fig. 3.36, lies in the $z = 0$ plane in free space. Find and sketch: **(a)** $E_x$ versus $t$ in the $z = 300$ m plane; **(b)** $H_y$ versus $t$ in the $z = -600$ m plane; **(c)** $E_x$ versus $z$ for $t = 3\ \mu$s; and **(d)** $H_y$ versus $z$ for $t = 4\ \mu$s.

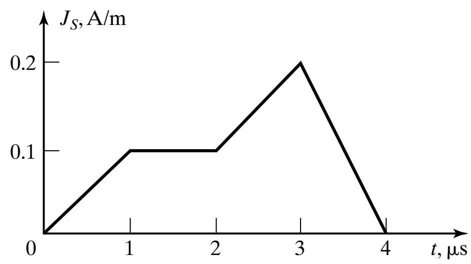


FIGURE 3.36
For Problem P3.23

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**P3.24. Source and more field variations from a given field variation of a uniform plane wave.** The time variation of the electric-field intensity $E_x$ in the $z = 300$ m plane of a uniform plane wave propagating away from an infinite plane current sheet of current density $\mathbf{J}_S(t) = -J_S(t)\mathbf{a}_x$ lying in the $z = 0$ plane in free space is given by the periodic function shown in Fig. 3.37. Find and sketch **(a)** $J_S$ versus $t$; **(b)** $E_x$ versus $t$ in $z = -600$ m plane; **(c)** $E_x$ versus $z$ for $t = 2\ \mu$s; and **(d)** $H_y$ versus $z$ for $t = 3\ \mu$s.

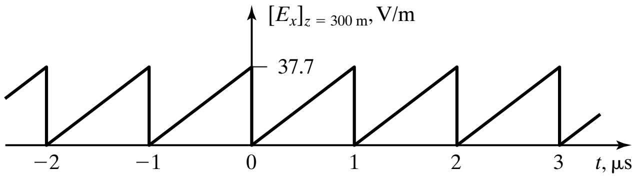


FIGURE 3.37
For Problem P3.24

## Section 3.5

**P3.25. Finding parameters for a specified sinusoidal uniform plane-wave electric field.** The electric-field intensity of a uniform plane wave propagating in free space is given by

$$\mathbf{E} = 37.7 \cos (9\pi \times 10^7 t + 0.3\pi y)\, \mathbf{a}_x \text{ V/m}$$

Find: **(a)** the frequency; **(b)** the wavelength; **(c)** the direction of propagation of the wave; and **(d)** the associated magnetic-field intensity vector **H**.

**P3.26. Writing field expressions for an infinite plane current sheet source.** Given $\mathbf{J}_S = 0.2(\sqrt{3}\mathbf{a}_x + \mathbf{a}_y) \cos 6\pi \times 10^9 t$ A/m in the $z = 0$ plane in free space, find **E** and **H** for $z \lessgtr 0$. Use the following three steps, which are generalizations of the solution to the electromagnetic field due to the infinite plane current sheet in the $z = 0$ plane:

1. Write the expression for **H** on the sheet and on either side of it, by noting that $[\mathbf{H}]_{z=0\pm} = \frac{1}{2}\mathbf{J}_S \times (\pm\mathbf{a}_z) = \frac{1}{2}\mathbf{J}_s \times \mathbf{a}_n$, where $\mathbf{a}_n$ is the unit vector normal to the sheet and directed toward the side of interest.
2. Extend the result of step 1 to write the expression for **H** everywhere, that is, for $z \lessgtr 0$, considering the traveling wave character of the fields.
3. Write the solution for **E** everywhere by noting that **(a)** the amplitude of $\mathbf{E} = \eta_0 \times$ the amplitude of **H**, and **(b)** the direction of **E**, the direction of **H**, and the direction of propagation constitute a right-handed orthogonal set, so that $\mathbf{E} = \eta_0 \mathbf{H} \times \mathbf{a}_n$.

**P3.27. Writing field expressions for an infinite plane current sheet source.** Given $\mathbf{J}_S = 0.2 \sin 15\pi \times 10^7 t\, \mathbf{a}_y$ A/m in the $x = 0$ plane in free space, find **E** and **H** for $x \lessgtr 0$. Use the three steps outlined in Problem P3.26, except that the current sheet is in the $x = 0$ plane.

**P3.28. Electric field due to an array of two infinite plane current sheets.** The current densities of two infinite, plane, parallel current sheets are given by

$$\begin{aligned} \mathbf{J}_{S1} &= -J_{S0} \cos \omega t\, \mathbf{a}_x && \text{in the } z = 0 \text{ plane} \\ \mathbf{J}_{S2} &= -kJ_{S0} \cos \omega t\, \mathbf{a}_x && \text{in the } z = \lambda/2 \text{ plane} \end{aligned}$$

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Find the electric-field intensities in the three regions: **(a)** $z < 0$; **(b)** $0 < z < \lambda/2$; and **(c)** $z > \lambda/2$.

**P3.29. An array of three infinite plane current sheets.** The current densities of three infinite plane, parallel, current sheets are given by

$$\begin{aligned}
\mathbf{J}_{S1} &= -J_{S0}\cos\omega t\,\mathbf{a}_x \quad && \text{in the } z = 0 \text{ plane}\\
\mathbf{J}_{S2} &= -kJ_{S0}\sin\omega t\,\mathbf{a}_x \quad && \text{in the } z = \lambda/4 \text{ plane}\\
\mathbf{J}_{S3} &= -2kJ_{s0}\cos\omega t\,\mathbf{a}_x \quad && \text{in the } z = \lambda/2 \text{ plane}
\end{aligned}$$

Obtain the expression for the ratio of the amplitude of the electric field in the region $z > \lambda/2$ to the amplitude of the electric field in the region $z < 0$. Then find the ratio for each of the following values of $k$: **(a)** $k = -1$, **(b)** $k = 1/2$, and **(c)** $k = 1$. Find the value(s) of $k$ for each of the following values of the ratio: **(a)** 1/3 and **(b)** 3.

## Section 3.6

**P3.30. Determination of polarization for combinations of linearly polarized vectors.** Three sinusoidally time-varying linearly polarized vector fields are given at a point by

$$\begin{aligned}
\mathbf{F}_1 &= \sqrt{3}\,\mathbf{a}_x \cos(2\pi \times 10^6 t + 30^\circ)\\
\mathbf{F}_2 &= \mathbf{a}_z \cos(2\pi \times 10^6 t + 30^\circ)\\
\mathbf{F}_3 &= \left(\frac{1}{2}\mathbf{a}_x + \sqrt{3}\,\mathbf{a}_y + \frac{\sqrt{3}}{2}\mathbf{a}_z\right)\cos(2\pi \times 10^6 t - 60^\circ)
\end{aligned}$$

Determine the polarizations of the following: **(a)** $\mathbf{F}_1 + \mathbf{F}_2$; **(b)** $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$; and **(c)** $\mathbf{F}_1 - \mathbf{F}_2 + \mathbf{F}_3$.

**P3.31. Polarization of sum of two linearly polarized vector fields.** Two sinusoidally time-varying, linearly polarized vector fields are given at a point by

$$\begin{aligned}
\mathbf{F}_1 &= (C\mathbf{a}_x + C\mathbf{a}_y + \mathbf{a}_z)\cos 2\pi \times 10^6 t\\
\mathbf{F}_2 &= (C\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)\sin 2\pi \times 10^6 t
\end{aligned}$$

where $C$ is a constant. **(a)** Determine the polarization of the vector $\mathbf{F}_1 + \mathbf{F}_2$ for $C = 2$. **(b)** Find the value(s) of $C$ for which the tip of the vector $\mathbf{F}_1 + \mathbf{F}_2$ traces a circle with time.

**P3.32. Unit vectors along the hour and minute hands of an analog watch.** Consider an analog watch that keeps accurate time and assume the origin to be at the center of the dial, the $x$-axis passing through the 12 mark, and the $y$-axis passing through the 3 mark. **(a)** Write the expression for the time-varying unit vector directed along the hour hand of the watch. **(b)** Write the expression for the time-varying unit vector directed along the minute hand of the watch. **(c)** Obtain the specific expression for these unit vectors when the hour hand and the minute hand are aligned exactly and between the 5 and 6 marks.

**P3.33. Field expressions for sinusoidal uniform plane wave for specified characteristics.** Write the expressions for the electric- and magnetic-field intensities of a sinusoidally time-varying uniform plane wave propagating in free space and having the following characteristics: **(a)** $f = 100$ MHz; **(b)** direction of propagation is the $+z$-direction; and **(c)** polarization is right circular with the electric field in the $z = 0$ plane at $t = 0$ having an $x$-component equal to $E_0$ and a $y$-component equal to $0.75E_0$.

**P3.34. Determination of sense of polarization for several cases of sinusoidal traveling waves.** For each of the following fields, determine if the polarization is right- or left-circular or elliptical.

**(a)** $E_0 \cos(\omega t - \beta y)\,\mathbf{a}_z + E_0 \sin(\omega t - \beta y)\,\mathbf{a}_x$

**(b)** $E_0 \cos(\omega t + \beta x)\,\mathbf{a}_y + E_0 \sin(\omega t + \beta x)\,\mathbf{a}_z$

**(c)** $E_0 \cos(\omega t + \beta y)\,\mathbf{a}_x - 2E_0 \sin(\omega t + \beta y)\,\mathbf{a}_z$

**(d)** $E_0 \cos(\omega t - \beta x)\,\mathbf{a}_z - E_0 \sin(\omega t - \beta x + \pi/4)\,\mathbf{a}_y$

**P3.35. Uniform plane-wave field in terms of right and left circularly polarized components.** Express each of the following uniform plane wave electric fields as a superposition of right- and left-circularly polarized fields:

**(a)** $E_0\mathbf{a}_x \cos(\omega t + \beta z)$

**(b)** $E_0\mathbf{a}_x \cos(\omega t - \beta z + \pi/3) - E_0\mathbf{a}_y \cos(\omega t - \beta z + \pi/6)$

## Section 3.7.

**P3.36. Instantaneous and time-average Poynting vectors for specified electric fields.** For each of the following electric-field intensities for a uniform plane wave in free space, find the instantaneous and time-average Poynting vectors:

**(a)** $\mathbf{E} = E_0 \cos(\omega t - \beta z)\,\mathbf{a}_x + 2E_0 \cos(\omega t - \beta z)\,\mathbf{a}_y$

**(b)** $\mathbf{E} = E_0 \cos(\omega t - \beta z)\,\mathbf{a}_x - E_0 \sin(\omega t - \beta z)\,\mathbf{a}_y$

**(c)** $\mathbf{E} = E_0 \cos(\omega t - \beta z)\,\mathbf{a}_x + 2E_0 \sin(\omega t - \beta z)\,\mathbf{a}_y$

**P3.37. Poynting vector and power flow for a coaxial cable.** The electric and magnetic fields in a coaxial cable, an arrangement of two coaxial perfectly conducting cylinders of radii $a$ and $b$ ($>a$), are given by

$$\mathbf{E} = \frac{V_0}{r \ln(b/a)} \cos\omega(t - z\sqrt{\mu_0\varepsilon_0})\,\mathbf{a}_r \qquad \text{for} \quad a < r < b$$

$$\mathbf{H} = \frac{I_0}{2\pi r} \cos\omega(t - z\sqrt{\mu_0\varepsilon_0})\,\mathbf{a}_\theta \qquad \text{for} \quad a < r < b$$

where $V_0$ and $I_0$ are constants and the axis of the cylinders is the $z$-axis. **(a)** Find the instantaneous and time-average Poynting vectors associated with the fields. **(b)** Find the time-average power flow along the coaxial cable.

**P3.38. Power radiated for specified radiation fields of an antenna.** The electric- and magnetic-field intensities in the radiation field of an antenna located at the origin are given in spherical coordinates by

$$\mathbf{E} = E_0 \frac{\sin\theta}{r} \cos\omega(t - r\sqrt{\mu_0\varepsilon_0})\,\mathbf{a}_\theta \text{ V/m}$$

$$\mathbf{H} = \frac{E_0}{\sqrt{\mu_0/\varepsilon_0}} \frac{\sin\theta}{r} \cos\omega(t - r\sqrt{\mu_0\varepsilon_0})\,\mathbf{a}_\theta \text{ A/m}$$

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Find: **(a)** the instantaneous Poynting vector; **(b)** the instantaneous power radiated by the antenna by evaluating the surface integral of the instantaneous Poynting vector over a spherical surface of radius $r$ centered at the antenna and enclosing the antenna; and **(c)** the time-average power radiated by the antenna.

**P3.39. Energy storage associated with a charge distribution.** A volume charge distribution is given in spherical coordinates by

$$\rho = \begin{cases} \rho_0 (r/a)^2 & \text{for} \quad r < a \\ 0 & \text{for} \quad r > a \end{cases}$$

**(a)** Find the energy stored in the electric field of the charge distribution. **(b)** Find the work required to rearrange the charge distribution with uniform density in the region $r < a$.

**P3.40. Energy storage associated with a current distribution.** A current distribution is given in cylindrical coordinates by

$$\mathbf{J} = \begin{cases} J_0 \mathbf{a}_z & \text{for} \quad r < 30a \\ -J_0 \mathbf{a}_z & \text{for} \quad 4a < r < 5a \end{cases}$$

Find the energy stored in the magnetic field of the current distribution per unit length in the $z$-direction.

## REVIEW PROBLEMS

**R3.1. Satisfaction of Maxwell's curl equations for a specified electric field.** Find the numerical value(s) of $k$, if any, such that the electric field in free space ($\mathbf{J} = \mathbf{0}$) given by

$$\mathbf{E} = E_0 \sin 6x \sin (3 \times 10^9 t - kz)\, \mathbf{a}_y$$

satisfies both of Maxwell's curl equations.

**R3.2. Satisfaction of Maxwell's curl equations for fields in a rectangular cavity resonator.** The rectangular cavity resonator is a box comprising the region $0 < x < a$, $0 < y < b$, and $0 < z < d$, and bounded by metallic walls on all of its six sides. The time-varying electric and magnetic fields inside the resonator are given by

$$\mathbf{E} = E_0 \sin \frac{\pi x}{a} \sin \frac{\pi z}{d} \cos \omega t\, \mathbf{a}_y$$
$$\mathbf{H} = H_{01} \sin \frac{\pi x}{a} \cos \frac{\pi z}{d} \sin \omega t\, \mathbf{a}_x - H_{02} \cos \frac{\pi x}{a} \sin \frac{\pi z}{d} \sin \omega t\, \mathbf{a}_z$$

where $E_0$, $H_{01}$, and $H_{02}$ are constants and $\omega$ is the radian frequency of oscillation. Find the value of $\omega$ that satisfies both of Maxwell's curl equations. The medium inside the resonator is free space.

**R3.3. Electric field of a charge distribution from Gauss' law in differential form.** The $x$-variation of charge density independent of $y$ and $z$ is shown in Fig. 3.38. Find and sketch the resulting $D_x$ versus $x$.

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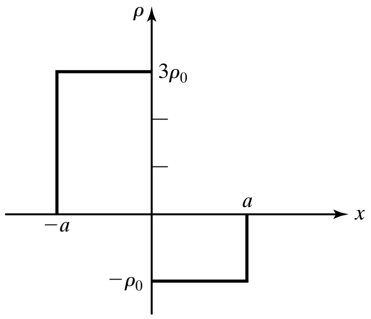


FIGURE 3.38

For Problem R3.3

**R3.4. Classification of static vector fields.** With respect to the properties of physical realizability as electric and magnetic fields, static vector fields can be classified into four groups: **(i)** electric field only; **(ii)** magnetic field only; **(iii)** electric field in a charge-free region or a magnetic field in a current-free region; and **(iv)** none of the preceding three. For each of the following fields, determine the group to which it belongs:

**(a)** $x\mathbf{a}_x + y\mathbf{a}_y$

**(b)** $(x^2 - y^2)\mathbf{a}_x - 2xy\mathbf{a}_y + 4\mathbf{a}_z$

**(c)** $\dfrac{e^{-r}}{r}\mathbf{a}_\phi$ in cylindrical coordinates

**(d)** $\dfrac{1}{r}(\cos\phi\,\mathbf{a}_r + \sin\phi\,\mathbf{a}_\phi)$ in cylindrical coordinates

**R3.5. Finding traveling-wave functions from specified sums of the functions.** Figures 3.39(a) and (b) show the distance variations at $t = 0$ and $t = 1$ s, respectively, of the sum of two functions $f(z, t)$ and $g(z, t)$, each of duration not exceeding 3 s, and representing traveling waves propagating in the $+z$- and $-z$-directions, respectively, with velocity 100 m/s. Find and sketch $f$ and $g$ versus $t$ for $z = 0$.

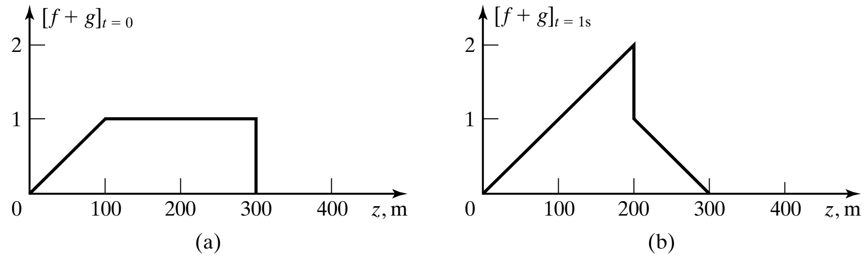


FIGURE 3.39

For Problem R3.5

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**R3.6. Plotting more field variations from a given field variation of a uniform plane wave.** The time-variation of the electric field $E_x$ in the $z = 600$ m plane of a uniform plane wave propagating away from an infinite plane current sheet lying in the $z = 0$ plane is given by the periodic function shown in Fig. 3.40. Find and sketch the following: (a) $E_x$ versus $t$ in the $z = 200$ m plane; (b) $E_x$ versus $z$ for $t = 0$; and (c) $H_y$ versus $z$ for $t = 1/3\ \mu$s.

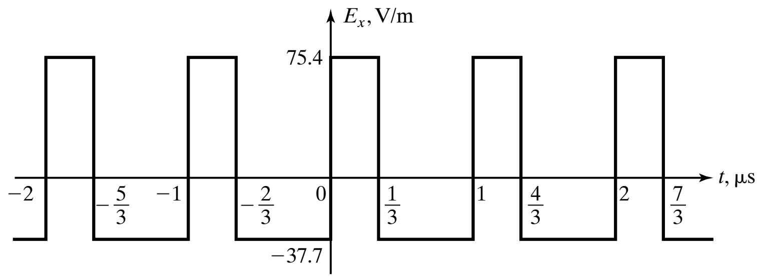


FIGURE 3.40

For Problem R3.6.

**R3.7. An array of two infinite plane current sheets.** For the array of two infinite plane current sheets of Example 3.12, assume that

$$\mathbf{J}_{S2} = -J_{S0} \sin(\omega t + \alpha)\, \mathbf{a}_x \text{ for } z = \lambda/4$$

Obtain the expression for the ratio of the amplitude of the electric field in the region $z > \lambda/4$ to the amplitude of the electric field in the region $z < 0$. Then find the following: **(a)** the value of the ratio for $\alpha = \pi/4$; and **(b)** the value of $\alpha$ for $0 < \alpha < \pi/2$, for the ratio to be equal to 2.

**R3.8. A superposition of two infinite plane current sheets.** Given $\mathbf{J}_{S1} = 0.2 \cos 6\pi \times 10^8 t\, \mathbf{a}_x$ A/m in the $y = 0$ plane and $\mathbf{J}_{S2} = 0.2 \cos 6\pi \times 10^8 t\, \mathbf{a}_z$ A/m in the $y = 0.25$ m plane, find **E** and **H** in the two regions $y < 0$ and $y > 0.25$ m. Discuss the polarizations of the fields and the time-average power flow in both regions. Note that the two current densities are directed perpendicular to each other.

**R3.9. Elliptical polarization.** The components of a sinusoidally time-varying vector field are given at a point by

$$F_x = 1 \cos \omega t$$
$$F_y = 1 \cos(\omega t + 60°)$$

Show that the field is elliptically polarized in the $xy$-plane with the equation of the ellipse given by $x^2 - xy + y^2 = 3/4$. Further show that the axial ratio (ratio of the major axis to the minor axis) of the ellipse is $\sqrt{3}$ and the tilt angle (angle made by the major axis with the $x$-axis) is 45°.

**R3.10. Work associated with rearranging a charge distribution.** Find the amount of work required for rearranging a uniformly distributed surface charge $Q$ of radius $a$ into a uniformly distributed volume charge of radius $a$.