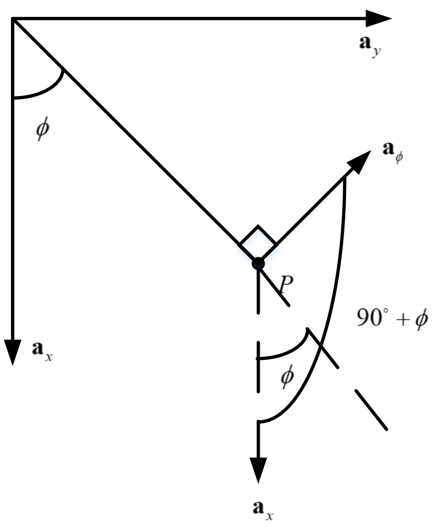
1.(C)
2. (C)

$$\mathbf{a}_x \cdot \mathbf{a}_\phi = |\mathbf{a}_x||\mathbf{a}_\phi|\cos(90^\circ + \phi) = -\sin\phi$$



3. (B)
Consider a test charge q at the point P, then the force experienced by Q is given by equation $\mathbf{F} = \dfrac{Qq}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$, which represents Coulomb's law. The electric field intensity is $\mathbf{E} = \dfrac{\mathbf{F}}{q} = \dfrac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$.
4.(A)

$$\text{Biot-Savart law}: d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l}\times\mathbf{a_R}}{R^2}$$

$$\mathbf{B} = \int d\mathbf{B} = 4\cdot\frac{\mu_0 I L}{2\pi r\sqrt{L^2+r^2}}\mathbf{a}_z = 4\cdot\frac{\mu_0 I\frac{d}{2}}{2\pi(\frac{d}{2})\sqrt{(\frac{d}{2})^2+(\frac{d}{2})^2}}\mathbf{a}_z = \frac{2\sqrt{2}\mu_0 I}{\pi d}\mathbf{a}_z$$

5.(A)

$$\mathbf{F} = m\mathbf{a} = 0 \ (\because \mathbf{a} = 0)$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = 0$$

$$\mathbf{E} = -\mathbf{v}\times\mathbf{B} = v_0(\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z)\times B_0(\mathbf{a}_x + \mathbf{a}_y + 4\mathbf{a}_z) = -v_0B_0(-5\mathbf{a}_x - 3\mathbf{a}_y + 2\mathbf{a}_z) = v_0B_0(5\mathbf{a}_x + 3\mathbf{a}_y - 2\mathbf{a}_z)$$

6. (D)

$$\int \mathbf{D}\cdot d\mathbf{S} = \mathbf{D_z}(1) = 1\varepsilon_0 E_z$$

$$I_d = \frac{d}{dt}\int \mathbf{D}\cdot d\mathbf{S} = \frac{d}{dt}\varepsilon_0 E_z = \varepsilon_0 E_0(e^{-t^2} - 2t^2 e^{-t^2})$$

When t = 0, $I_d = \varepsilon_0 E_0$
7. (C)

$$-2I+I_{23}+\frac{d}{dt}\oint_{S_2}\mathbf{D}\cdot d\mathbf{S}=0$$

$$-2I+I_{23}-2I=0$$

$$I_{23}=4I$$

8. (C)

Assume that $\mathbf{E}=\mathbf{a_R}E_R$, $d\mathbf{S}=\mathbf{a_R}dS$

$$\oint_S \mathbf{E}\cdot d\mathbf{S}=E_R\int_S \mathrm{dS}=E_R 4\pi R^2$$

For $0\le R\le b$, $[Q]_V=\int_V \rho dv=\rho_0\frac{4\pi}{3}R^3$

$$E_R=\frac{\rho_0}{3\varepsilon_0}R$$

$$\therefore \mathbf{E}=\mathbf{a_R}\frac{\rho_0}{3\varepsilon_0}R$$

For $b\le R$, $[Q]_V=\int_V \rho dv=\rho_0\frac{4\pi}{3}b^3$

$$E_R=\frac{\rho_0 b^3}{3\varepsilon_0 R^2}$$

$$\therefore \mathbf{E}=\mathbf{a_R}\frac{\rho_0 b^3}{3\varepsilon_0 R^2}$$

9.(A)

10.(A)

$$\mathbf{E}=E_0\cos 3\pi z\cos 9\pi\times 10^8 t\mathbf{a}_x$$

$$\frac{\partial\mathbf{B}}{\partial t}=-\nabla\times\mathbf{E}=-\begin{pmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ 0 & 0 & \frac{\partial}{\partial z}\\ E_x & 0 & 0\end{pmatrix}=-\frac{\partial E_x}{\partial z}\mathbf{a}_y=3\pi E_0\sin 3\pi z\cos 9\pi\times 10^8 t\mathbf{a}_y$$

$$\mathbf{B}=\frac{3\pi E_0}{9\pi\times 10^8}\sin 3\pi z\sin 9\pi\times 10^8 t\mathbf{a}_y=\frac{E_0}{3\times 10^8}\sin 3\pi z\sin 9\pi\times 10^8 t\mathbf{a}_y$$

Electromagnetics (I) Quiz#1 Grade:_______ 2023/10/24

Department:_____________ Student ID:________________ Name:_________

1. In the Cartesian coordinate system, the differential length vector $d\mathbf{l}$ along the line, equals $dx\mathbf{a}_x + dy\mathbf{a}_y + dz\mathbf{a}_z$. Find the $d\mathbf{l}$ in spherical coordinate system.

(A) $dr\mathbf{a}_r + d\theta\mathbf{a}_\theta + d\phi\mathbf{a}_\phi$ (B) $dr\mathbf{a}_r + rd\theta\mathbf{a}_\theta + r\sin\phi d\phi\mathbf{a}_\phi$

(C) $dr\mathbf{a}_r + rd\theta\mathbf{a}_\theta + r\sin\theta d\phi\mathbf{a}_\phi$ (D) $dr\mathbf{a}_r + rd\theta\mathbf{a}_\theta + \sin\theta d\phi\mathbf{a}_\phi$

2. Find the dot product of $\mathbf{a}_x$ in Cartesian coordinate system with $\mathbf{a}_\phi$ in cylindrical coordinate system, *i.e.*, $\mathbf{a}_x \cdot \mathbf{a}_\phi$.

(A) $\sin\phi$ (B) $\cos\phi$

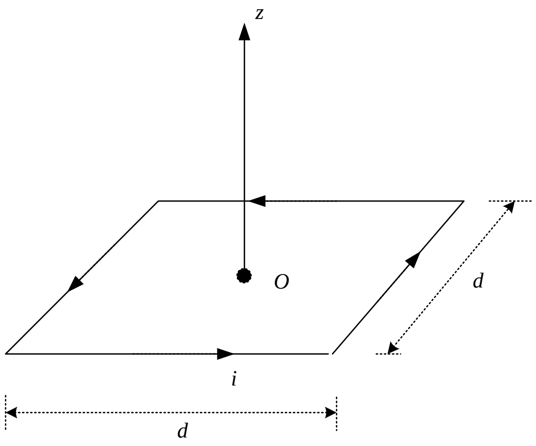
(C) $-\sin\phi$ (D) $-\cos\phi$

3. Consider the electric charge *Q* and point *P* separated *R* in free space, find the electric field intensity at point *P*.

(A) $\dfrac{Q}{2\pi\varepsilon_0 R^2}\mathbf{a}_R$ (B) $\dfrac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$

(C) $\dfrac{Q}{2\pi\varepsilon_0 R^3}\mathbf{a}_R$ (D) $\dfrac{Q}{4\pi\varepsilon_0 R^3}\mathbf{a}_R$

4. A square loop carries a current *I* as shown in the following figure. Find the magnetic flux density $\mathbf{B}$ at the center point *O* of this loop.



(A) $\dfrac{2\sqrt{2}\mu_0 I}{\pi d}\mathbf{a}_z$ (B) $\dfrac{\sqrt{2}\mu_0 I}{\pi d}\mathbf{a}_z$

(C) $\dfrac{\mu_0 I}{2\sqrt{2}\pi d}\mathbf{a}_z$ (D) $\dfrac{\mu_0 I}{\sqrt{2}\pi d}\mathbf{a}_z$

5. A test charge q moving with a velocity $\mathbf{v} = v_0(\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z)$ at a point where the magnetic field $\mathbf{B} = B_0(\mathbf{a}_x + \mathbf{a}_y + 4\mathbf{a}_z)$. Find the electric field $\mathbf{E}$ at the point for which the acceleration experienced by the test charge is zero.

(A) $v_0B_0(5\mathbf{a}_x + 3\mathbf{a}_y - 2\mathbf{a}_z)$ (B) $v_0B_0(5\mathbf{a}_x + 4\mathbf{a}_y + 3\mathbf{a}_z)$

(C) $v_0B_0(\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z)$ (D) $v_0B_0(9\mathbf{a}_x + 8\mathbf{a}_y + 7\mathbf{a}_z)$

6.For $\mathbf{E} = E_0 t e^{-t^2} \mathbf{a_z}$ in free space, which is the displacement current crossing an area of 1 m^2 in the xy-plane from the –z-side to the +z-side for t = 0?

(A) $0.5\varepsilon_0 E_0$ (B) $e^{-1/2}\varepsilon_0 E_0$

(C) $e^{-1}\varepsilon_0 E_0$ (D) $\varepsilon_0 E_0$

7. Three point charges $Q_1(t)$, $Q_2(t)$, and $Q_3(t)$ situated at the corners of an equilateral triangle of sides 1 m are connected to each other by wires along the sides of the triangle. Currents of $2I$A and $3I$A flow from Q_1 to Q_2 and Q_1 to Q_3 respectively. The displacement current emanating from a spherical surface of radius 0.1 m and centered at Q_2 is -$2I$A. What is the current flowing from Q_2 to Q_3?

(A) -4I (B) 6I

(C) 4I (D) -6I

8. Given a spherical cloud of electrons with a volume charge density $\rho = \rho_0$ for $0 \le R \le b$ and $\rho = 0$ for $b \le R$, what is the resultant electric field for $0 \le R \le b$ and $b \le R$, respectively?

(A) $\mathbf{E}=\mathbf{a_R}\dfrac{2\rho_0}{3\varepsilon_0}R$, $\mathbf{E}=\mathbf{a_R}\dfrac{2\rho_0 b^3}{3\varepsilon_0 R^2}$ (B) $\mathbf{E}=\mathbf{a_R}\dfrac{\rho_0}{3}R$, $\mathbf{E}=\mathbf{a_R}\dfrac{\rho_0 b^3}{3R^2}$

(C) $\mathbf{E}=\mathbf{a_R}\dfrac{\rho_0}{3\varepsilon_0}R$, $\mathbf{E}=\mathbf{a_R}\dfrac{\rho_0 b^3}{3\varepsilon_0 R^2}$ (D) $E = a_R \frac{\rho_0}{\varepsilon_0} R$, $E = a_R \frac{\rho_0 b^3}{\varepsilon_0 R^2}$

9.Which formula is Gauss's Law?

(A) $\oint_S D \cdot dS = Q_{in}$

(B) $\oint_C E \cdot dl = -\oint_S \frac{dB}{dt} \cdot dS$

(C) $\oint_S B \cdot dS = Q_{in}$

(D) $\oint_C H \cdot dl = \int_S J \cdot dS + \frac{d}{dt}\int_S D \cdot dS$

10.Given $\mathbf{E} = E_0 \cos 3\pi z \cos 9\pi \times 10^8 t\mathbf{a}_x$ (V/m), find a possible form of **B** (in Wb/m^2) from Faraday's law in differential form.

(A) $\dfrac{E_0}{3\times 10^8}\sin 3\pi z \sin 9\pi \times 10^8 t a_y$

(B) $-\dfrac{E_0}{3\times 10^8}\sin 3\pi z \sin 9\pi \times 10^8 t a_y$

(C) $\dfrac{E_0}{3\times 10^8}\sin 3\pi z \sin 9\pi \times 10^8 t a_x$

(D) $-\dfrac{E_0}{3\times 10^8}\sin 3\pi z \sin 9\pi \times 10^8 t a_x$