Ch.15 Knowing what to believe

November 17, 2023

A Poker Game

Bayes-Nash Equilibrium

A Poker Game

- A deck of cards containing only the king, queen and jack of hearts.
- Before the deal, A and B must put an ante of $1 into a pot.
- The top card is then dealt to A and the second card to B.
- A moves first, and he decides whether to check or raise.
  - Check: They show their cards and the player with a larger card takes away $2 in the pot.
  - Raise: A puts another $1 into the pot. B has then to decide whether to fold or to call.
    - Fold: Game over. A takes away $3 in the pot. A wins $1, and B loses $1.
    - Call: B puts another $1 into the pot, and then they compare their cars. The player with a larger card take away $4 in the pot.
- Each player is risk-neutral.

Types

- Should A raise?
  - It depends on A's card.
- If A raises, should B call?
  - From B's point of view, A has 3 different personalities: A_K, A_Q and A_J. Or, A has 3 types.
  - A_K maximizes his expected payoff without caring about A_J's payoff.
  - B has to calculate the probability distribution of the opponent's type.
    - This distribution depends on B's cards.
    - B has 3 types as well.

Incomplete Information

- Information is complete when everything needed to specify a game is common knowledge among the players, including the preferences and beliefs of the other players.
- In the poker game, a player's type is unknown to his opponent.

Formulation of a Game with Incomplete Information

- Consider a game with n players. Let t_i denote the type of player i.
- The joint probability $p(t_1, ..., t_n)$ is common knowledge.
- Player i knows his own type t_i. He uses Bayes rule to calcualte the posterior probability about t_{-i} is $p_i(t_{-i}|t_i)$.
- Player i is payoff is $\pi^i(a_1, ..., a_n; t_1, ..., t_n)$ where a_j is player j's action including a mixed strategy.

Bayes-Nash Equilibrium

- A BNE is $\{a_i^*(t_i)\}_{i=1}^n$, where $\forall i$,$\forall t_i$,
  $a_i^*(t_i) \max \sum_{t_{-i}} p_i(t_{-i}|t_i)\pi^i(a_1^*(t_1), ..., a_i, ...a_n^*(t_n); t_1, ..., t_n)$.
- For the Poker game, we need to specify what A_K, A_Q, A_J, B_K, B_Q and B_J will do.

BNE in the Poker Game

- $$A_K \Rightarrow \quad \text{raise} \quad B\begin{cases} Q & ? \\ J & \text{fold} \quad +1 \end{cases}$$
- $$\begin{array}{lll} A_Q \Rightarrow & \text{raise} & B\begin{cases} K & \text{call} \quad -2 \\ J & \text{fold} \quad +1 \end{cases} \\ & \text{check} & B\begin{cases} K & \quad\quad -1 \\ J & \quad\quad +1 \end{cases} \end{array}$$
- $$\begin{array}{lll} A_J \Rightarrow & \text{raise} & B\begin{cases} K & \text{call} \quad -2 \\ Q & ? \end{cases} \\ & \text{check} & \quad -1 \end{array}$$
- What will A_J and B_Q do?

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- Suppose B_Q folds w.p.1.
  - $$A_J \Rightarrow \begin{array}{ccc} \text{raise} & B\left\{\begin{array}{ccc} K & \text{call} & -2 \\ Q & \text{fold} & +1 \end{array}\right. \\ \text{check} & \quad -1 \end{array}$$

    0.5*(-2) + 0.5*1=-0.5 >-1, A_J raises.
  - When A raises, if B_Q folds, B_Q loses \$1.

    If B_Q calls, B_Q's payoff = 0.5(-2)+0.5(2)=0. B_Q calls!
- Suppose B_Q calls w.p.1. If A_J raises, he'll lose \$2. So, A_j checks.
  - When A raises, B_Q should fold.
- B_Q calls with probability c.
- Similarly, A_J raises with probability r.

A_J's Mixed Strategy

- A_J raises with probability r.
- For B_Q, the probability that A raises is: $0.5 * 1 + 0.5 * r$.
- For B_Q, when A raises, the probability that it's raised by A_K (A_J) is $1/(1+r)$ $\;(r/(1+r))$.
- r is chosen to make B_Q indifferent between calling and folding.
- If B_Q calls, his expected payoff is $(-2)/(1+r) + 2r/(1+r) = -1$. $r = 1/3$.

B_Q's Mixed Strategy

- B_Q calls with probability c.
- c is chosen to make A_J indifferent between raising and checking.
- A_J raises, his expected payoff is
  $(-2) * (0.5 + 0.5c) + 1 * 0.5(1 - c) = -1.$ $c = 1/3.$