2.1

(a)

$$d\vec{l} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$$

$$x=y=z \Rightarrow dx=dy=dz$$

$$\Rightarrow d\vec{l} = dx\,(\vec{a_x} + \vec{a_y} + \vec{a_z})$$

$$\int_{(0,0,0)}^{(1,1,1)} \vec{F}\cdot d\vec{l} = \int_0^1 (y\,\vec{a_x} - z\,\vec{a_y} + x\,\vec{a_z})\cdot dx\,(\vec{a_x} + \vec{a_y} + \vec{a_z})$$

$$= \int_0^1 (y-z+x)\,dx = \int_0^1 x\,dx = \frac{1}{2}x^2\Big|_0^1 = \frac{1}{2}$$

※

(b)

$$x=y=z^3 \Rightarrow dx=dy=3z^2dz$$

$$\Rightarrow d\vec{l} = dz\,(3z^2\,\vec{a_x} + 3z^2\,\vec{a_y} + \vec{a_z})$$

$$\int_{(0,0,0)}^{(1,1,1)} \vec{F}\cdot d\vec{l} = \int_0^1 (y\,\vec{a_x} - z\,\vec{a_y} + x\,\vec{a_z})\cdot dz\,(3z^2\,\vec{a_x} + 3z^2\,\vec{a_y} + \vec{a_z})$$

$$= \int_0^1 (3z^2y - 3z^3 + x)\,dz = \int_0^1 (3z^5 - 3z^3 + z^3)\,dz$$

$$= \frac{1}{2}z^6 - \frac{1}{2}z^4\Big|_0^1 = 0$$

※

2.3

(a)

Straight line $\Rightarrow x = \frac{y}{2\pi} = z \Rightarrow dx = \frac{dy}{2\pi} = dz$

$$\Rightarrow d\vec{l} = dx\,(\vec{a_x} + 2\pi\,\vec{a_y} + \vec{a_z})$$

$$\int_{(0,0,0)}^{(1,2\pi,1)} \vec{F}\cdot d\vec{l} = \int_0^1 (\cos y\,\vec{a_x} - x\sin y\,\vec{a_y})\cdot dx\,(\vec{a_x} + 2\pi\,\vec{a_y} + \vec{a_z})$$

$$= \int_0^1 (\cos y - 2\pi\,x\sin y)\,dx = \int_0^1 \cos(2\pi x) - 2\pi x\sin(2\pi x)\,dx$$

$$= \left(\frac{1}{2\pi}\sin(2\pi x) + x\cos(2\pi x) - \frac{1}{2\pi}\sin(2\pi x)\right)\Big|_0^1 = 1$$

※

2.3 (b)

$x = z = \sin\left(\frac{y}{4}\right) \Rightarrow dx = dz = \frac{1}{4}\cos\left(\frac{y}{4}\right) dy$

$\Rightarrow d\vec{l} = dy\left(\frac{1}{4}\cos\left(\frac{y}{4}\right)\vec{a_x} + \vec{a_y} + \frac{1}{4}\cos\left(\frac{y}{4}\right)\vec{a_z}\right)$

$\int_{(0,0,0)}^{(1,2\pi,1)} \vec{F}\cdot d\vec{l} = \int_0^{2\pi}\left(\cos y\,\vec{a_x} - x\sin y\,\vec{a_y}\right)\cdot dy\left(\frac{1}{4}\cos\left(\frac{y}{4}\right)\vec{a_x} + \vec{a_y} + \frac{1}{4}\cos\left(\frac{y}{4}\right)\vec{a_z}\right)$

$= \int_0^{2\pi}\left(\frac{1}{4}\cos y\cos\left(\frac{y}{4}\right) - x\sin y\right)dy = \int_0^{2\pi}\left(\frac{1}{4}\left(\cos y\cos\left(\frac{y}{4}\right)\right) - \sin\left(\frac{y}{4}\right)\sin y\right)dy$

$= \cos y \sin\frac{y}{4}\Big|_0^{2\pi} = 1$

※

(c)

$\vec{F}\cdot d\vec{l} = \cos y\,dx - x\sin y\,dy = d(x\cos y)$

$\int_{(0,0,0)}^{(1,2\pi,1)} \vec{F}\cdot d\vec{l} = \int_{(0,0,0)}^{(1,2\pi,1)} d(x\cos y) = x\cos y\Big|_{(0,0,0)}^{(1,2\pi,1)} = 1$

∵ the result is independent of path

∴ it is conservative

※

2.5 $d\vec{l} = dr\,\vec{a_r} + r\,d\theta\,\vec{a_\theta} + r\sin\theta\,d\phi\,\vec{a_\phi}$

(a) the path: $\theta = \phi = 0 \Rightarrow d\theta = d\phi = 0$

$\Rightarrow \vec{A} = e^{-r}\vec{a_r}, \quad d\vec{l} = dr\,\vec{a_r}$

$\Rightarrow \int_{(0,0,0)}^{(2,0,0)} \vec{A}\cdot d\vec{l} = \int_0^2 e^{-r}dr = -e^{-r}\Big|_0^2 = 1 - e^{-2}$

※

2.5 (b)

the path: $r=2,\ \phi=\frac{\pi}{4} \Rightarrow dr=d\phi=0$

$\Rightarrow \vec{A}= e^{-2}(\cos\theta\,\vec{a_r}+\sin\theta\,\vec{a_\theta})+2\sin\theta\,\vec{a_\phi}$,

$d\vec{l}=2\,\vec{a_\theta}\,d\theta$

$\Rightarrow \int_{(2,0,\frac{\pi}{4})}^{(2,\frac{\pi}{2},\frac{\pi}{4})} \vec{A}\cdot d\vec{l}=\int_0^{\frac{\pi}{2}} e^{-2}\cdot 2\sin\theta\,d\theta=-e^{-2}\cdot 2\cos\theta\Big|_0^{\frac{\pi}{2}}$

$=\frac{2}{e^2}$ ※

(c) the path: $r=2,\ \theta=\frac{\pi}{6} \Rightarrow dr=d\theta=0$

$\Rightarrow \vec{A}=e^{-2}(\frac{\sqrt{3}}{2}\vec{a_r}+\frac{1}{2}\vec{a_\theta})+2\cdot\frac{1}{2}\vec{a_\phi}$, $d\vec{l}=d\phi\,\vec{a_\phi}$

$\Rightarrow \int_{(2,\frac{\pi}{6},0)}^{(2,\frac{\pi}{6},\frac{\pi}{2})} \vec{A}\cdot d\vec{l}=\int_0^{\frac{\pi}{2}} d\phi=\frac{\pi}{2}$ ※

2.6

For S^* are $x=0,\ y=0,\ z=0$

$\vec{A}=0$

$\Rightarrow \int \vec{A}\cdot d\vec{S^*}=0$

For S^* is $x=1$, $\vec{A}=yz\,\vec{a_x}+y^2z\,\vec{a_y}+yz^2\,\vec{a_z}$

$d\vec{S^*}=dy\,dz\,\vec{a_x} \Rightarrow \vec{A}\cdot d\vec{S}=yz\,dy\,dz$

$\Rightarrow \int \vec{A}\cdot d\vec{S^*}=\int_0^1\int_0^1 yz\,dy\,dz=\int_0^1 \frac{1}{2}z\,dz=\frac{1}{4}$

Since $\vec{A}$ and the surface is symmetric on x, y, z, for S^* are $y=1$, $z=1$,

$\int \vec{A}\cdot d\vec{S^*}=\frac{1}{4}$.

$\therefore$ the result $=0+0+0+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$ ※

2.8

For S^* is $\phi=0$, $\vec{A} = r\underset{\shortparallel}{\cos\phi}\,\vec{a_r}$, $d\vec{S^*} = -dr\,dz\,\vec{a_\phi}$

$\Rightarrow \vec{A}\cdot d\vec{S^*} = 0 \Rightarrow \int \vec{A}\cdot d\vec{S^*} = 0$

For S^* is $\phi=\frac{\pi}{2}$, $\vec{A} = -r\underset{\shortparallel}{\sin\phi}\,\vec{a_\phi}$, $d\vec{S^*} = dr\,dz\,\vec{a_\phi}$

$\Rightarrow \vec{A}\cdot d\vec{S^*} = -r\,dr\,dz$

$\Rightarrow \int \vec{A}\cdot d\vec{S^*} = -\int_0^1\int_0^2 r\,dr\,dz = -\int_0^1 2\,dz = -2$

For S^* is $z=0$, $d\vec{S^*} = -r\,dr\,d\phi\,\vec{a_z} \Rightarrow \vec{A}\cdot d\vec{S^*} = 0 \Rightarrow \int \vec{A}\cdot d\vec{S^*} = 0$

〃 $z=1$, $d\vec{S^*} = r\,dr\,d\phi\,\vec{a_z} \Rightarrow$ 〃 $\Rightarrow$ 〃 $=0$

For S^* is $r=2$, $\vec{A} = 2\cos\phi\,\vec{a_r} - 2\sin\phi\,\vec{a_\phi}$, $d\vec{S^*} = 2\,d\phi\,dz\,\vec{a_r}$

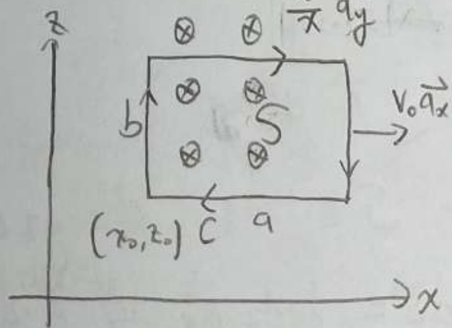
$\Rightarrow \vec{A}\cdot d\vec{S^*} = 4\cos\phi\,d\phi\,dz$

$\Rightarrow \int \vec{A}\cdot d\vec{S^*} = \int_0^1\int_0^{\frac{\pi}{2}} 4\cos\phi\,d\phi\,dz = \int_0^1 4\sin\phi\Big|_0^{\frac{\pi}{2}}\,dz = 4$

$\therefore \oint_S \vec{A}\cdot d\vec{S} = 0-2+0+0+4 = 2$ ※

2.11

$d\vec{S} = dx\,dz\,\vec{a_y}$



$$\int_S \vec{B}\cdot d\vec{S} = \int_{z_0}^{z_0+b}\int_{x_0}^{x_0+a} \frac{B_0}{x}\,dx\,dz$$

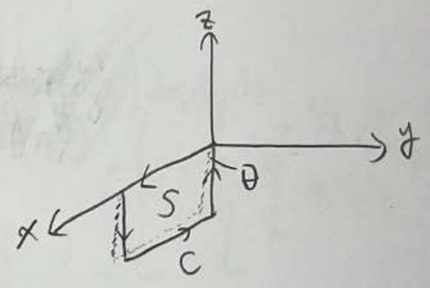
$$= \int_{z_0}^{z_0+b} B_0 \ln\left(\frac{x_0+a}{x_0}\right) dz$$

$$= b B_0 \ln\left(\frac{x_0+a}{x_0}\right)$$

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S} = -\frac{d}{dt}\, b B_0 \ln\left(\frac{x_0+a+v_0t}{x_0+v_0t}\right)$$

$$= \frac{bB_0v_0}{x_0+v_0t} - \frac{bB_0v_0}{x_0+v_0t+a} = bB_0v_0\left(\frac{1}{x_0+v_0t} - \frac{1}{x_0+v_0t+a}\right)$$ ※

2.13



$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}$$

$$= -\frac{d}{dt} B_0 ab \sin\theta$$

$$= -\frac{d\theta}{dt} B_0 ab \cos\theta$$

$$= w B_0 ab \cos\theta$$

$\because \theta \approx 0 \Rightarrow \cos\theta \approx 1$

$\therefore$ emf $\approx B_0 ab w$.

Since the emf above causes the current to go around like C, and $\vec{B}$ is in the direction $\vec{a_z}$, which slows down the loop.
$\therefore$ the loop will swing slower. ※

2.15

(a) $\vec{B}\cdot d\vec{S} = B_0\, dx\, dz$

$$\Rightarrow \int_S \vec{B}\cdot d\vec{S} = \int_0^h \int_0^{b\cos\phi} B_0\, dx\, dz = hbB_0\cos\phi$$

$$\text{emf} = -\frac{d}{dt} hbB_0\cos\phi = \frac{d\phi}{dt} hbB_0\sin\phi = whbB_0\sin\phi$$ ※

(b) $\vec{B}\cdot d\vec{S} = B_0 y(-dy\,dz) - B_0 x\, dx\, dz$

$$\Rightarrow \int_S \vec{B}\cdot d\vec{S} = \int_0^h \left(\left(\int_0^{b\sin\phi} -B_0 y\, dy\right) + \left(\int_0^{b\cos\phi} -B_0 x\, dx\right)\right) dz$$

$$= \int_0^h -B_0\left(\frac{b^2\sin^2\phi}{2} + \frac{b^2\cos^2\phi}{2}\right) dz = -\frac{1}{2} hB_0 b^2$$

$$\text{emf} = -\frac{d}{dt}\cdot\left(-\frac{1}{2} hB_0 b^2\right) = 0$$ ※

2.18

(a)

$$\oint_S \vec{J}\cdot d\vec{S} = \int_{-2}^{2}\int_{-2}^{2} (-2)\,dy\,dz + 2(-dy\,dz)$$
$$+ \int_{-2}^{2}\int_{-2}^{2} (-2)\,dx\,dz + 2(-dx\,dz)$$
$$+ \int_{-2}^{2}\int_{-2}^{2} (-4)\,dx\,dy + (4)(-dx\,dy)$$

$$= -64 - 64 + 0 = -128$$

$$\therefore \frac{d}{dt}\oint_S \vec{D}\cdot d\vec{S} = -\oint_S \vec{J}\cdot d\vec{S} = 128\ (A)$$

(b)

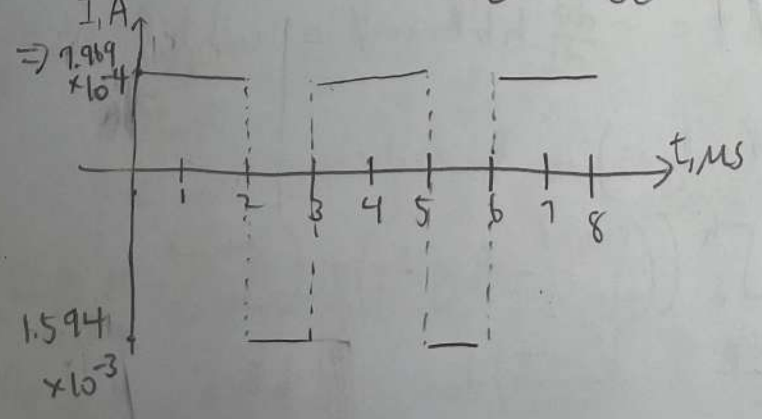
$$\vec{J} = -r\vec{a_r} - z^2\vec{a_z}$$

$$\oint_S \vec{J}\cdot d\vec{S} = \int_0^2\int_0^{2\pi} (-1)\,d\phi\,dz + \int_0^{2\pi}\int_0^1 (-4)r\,dr\,d\phi$$
$$= -4\pi - 2\int_0^{2\pi} d\phi = -8\pi$$

$$\therefore \frac{d}{dt}\oint_S \vec{D}\cdot d\vec{S} = -\oint_S \vec{J}\cdot d\vec{S} = 8\pi\ (A)$$

2.20

$$I = \frac{d}{dt}\varepsilon_0 E \approx 8.854\times10^{-12}\frac{dE}{dt},\quad \frac{dE}{dt} = 9\times10^{7}\ V/m\cdot t$$

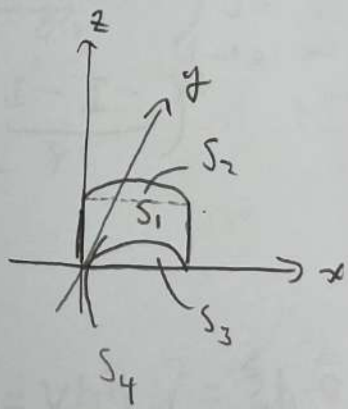


$$\text{root-mean-square} = \sqrt{\frac{2(7.969\times10^{-4})^2 + (1.594\times10^{-3})^2}{3}} \approx 11.27\times10^{-4} = 1.127\times10^{-3}\ A$$

2.21 (a)
$$\oint_S \vec{D}\cdot d\vec{S} = \int_V \rho\, dV = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \rho_0\left(3-x^2-y^2-z^2\right)dx\,dy\,dz$$
$$= \int_{-1}^{1}\int_{-1}^{1}\rho_0\left(6-\frac{2}{3}-2y^2-2z^2\right)dy\,dz = \int_{-1}^{1}\rho_0\left(12-\frac{4}{3}-\frac{4}{3}-4z^2\right)dz$$
$$= \rho_0\left(24-\frac{8}{3}-\frac{8}{3}-\frac{8}{3}\right) = 16\rho_0$$ ※

(b)
$d\sin\theta$
$$\oint_S \vec{D}\cdot d\vec{S} = \int_V \rho\, dv = \int_0^1\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \rho_0\left(r\cos\phi\sin\theta\ r\sin\phi\sin\theta\ r\cos\theta\right)r^2\sin\theta\, d\theta\, d\phi\, dr$$
$$= \int_0^1\int_0^{\frac{\pi}{2}} \frac{1}{4}\rho_0 r^5\cos\phi\sin\phi\, d\phi\, dr$$
$$= \int_0^1 \frac{1}{8}\rho_0 r^5\, dr = \frac{1}{48}\rho_0$$ ※

2.23
$\because \oint_S \vec{B}\cdot d\vec{S} = 0$
$$\oint_{S_1}\vec{B}\cdot d\vec{S}_1 = -\oint_{S_2}\vec{B}\cdot d\vec{S}_2 - \oint_{S_3}\vec{B}\cdot d\vec{S}_3 - \oint_{S_4}\vec{B}\cdot d\vec{S}_4$$
$$= -0-0-\int_0^1\int_0^{\pi} B_0 x\left(-dx\,dz\right)$$
$$= \frac{1}{2}\pi^2 B_0 \ (\mathrm{Wb})$$ ※



2.24

$\because -\frac{d}{dt}\int_V \rho\, dv = \oint_S \vec{J}\cdot d\vec{S}$

$\therefore$ We can find $\oint_S \vec{J}\cdot d\vec{S}$ in the following problems instead.

(a)
$$\oint_S \vec{J}\cdot d\vec{S} = \overset{(x=0,\,x=1,\,y=0,\,y=1,\,z=0,\,z=1)}{0+1+0+1+0+1} = 3 \ (\mathrm{A})$$ ※

(b)
$$x\vec{a}_x + y\vec{a}_y + z\vec{a}_z = r\vec{a}_r + z\vec{a}_z$$
$$\oint_S \vec{J}\cdot d\vec{S} = \overset{z=0}{0} + \overset{z=1}{(4\pi-\pi)\cdot 1} + \overset{r=1}{2\pi\cdot(-1)} + \overset{r=2}{4\pi\cdot 2} = 9\pi \ (\mathrm{A})$$ ※

2.24

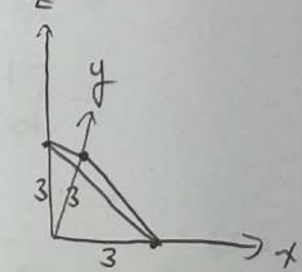
(c)

$$\vec{J} = x\vec{a_x} + y\vec{a_y} + z\vec{a_z} = R\vec{a_r}$$

$$\oint_S \vec{J}\cdot d\vec{S} = \underbrace{\int_0^{\frac{\pi}{3}}\int_0^{2\pi} 8\sin\theta\, d\phi\, d\theta}_{r=2} + \underbrace{\int_0^{\frac{\pi}{3}}\int_0^{2\pi} \sin\theta(-d\phi\, d\theta)}_{r=1} + \underbrace{0}_{\theta=\frac{\pi}{3}}$$

$$= 14\pi \int_0^{\frac{\pi}{3}} \sin\theta\, d\theta = 14\pi\,(-\cos\theta)\Big|_0^{\frac{\pi}{3}} = 7\pi\ (A)$$ ※

2.26



$$\oint_C \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{S} + \frac{d}{dt}\int_S \vec{D}\cdot d\vec{S}$$

$$= I + \frac{d}{dt}\left(\frac{Q_1(t) - Q_2(t)}{8}\right)$$

(∵ (1,1,1) is the center)

$$= I + \left(\frac{-I-I}{8}\right) = \frac{3}{4}I$$ ※

2.27

(a)

$$\oint_S \vec{D}\cdot d\vec{S} = \int_V \rho\, dV = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} (3-x^2-y^2-z^2)\,dx\,dy\,dz$$

$$= \int_{-1}^{1}\int_{-1}^{1} \left(6-\frac{2}{3}-2y^2-2z^2\right) dy\,dz$$

$$= \int_{-1}^{1} \left(12-\frac{4}{3}-\frac{4}{3}-4z^2\right) dz = 24-\frac{8}{3}-\frac{8}{3}-\frac{8}{3} = 16$$

Since each side are symmetric, the answer $= \frac{16}{6} = \frac{8}{3}$ (C) ※

(b) $$\oint_S \vec{D}\cdot d\vec{S} = \int_V \rho\, dV = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \sqrt{|xyz|}\,dx\,dy\,dz = 8\int_0^1\int_0^1\int_0^1 \sqrt{xyz}\,dx\,dy\,dz$$

$$= 8\times\frac{2}{3}\int_0^1\int_0^1 \sqrt{yz}\,dy\,dz = 8\times\frac{4}{9}\int_0^1 \sqrt{z}\,dz = 8\times\frac{8}{27} = \frac{64}{27}$$

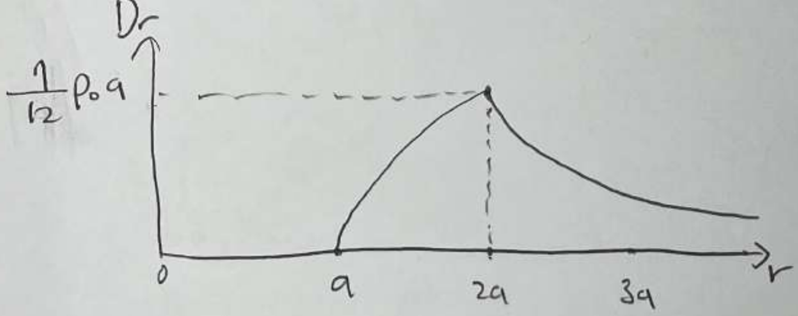
Since each side are symmetric, the answer $= \frac{1}{6}\times\frac{64}{27} = \frac{32}{81}$ (C) ※

2.29

$\because \oint_S \vec{D}\cdot d\vec{S} = \int_V \rho\, dv$

$$\therefore 4\pi r^2 D_r = \begin{cases} 0 & \text{if } r<a \\ (\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3)\rho_0 & \text{if } a<r<2a \\ (\frac{4}{3}\pi (2a)^3 - \frac{4}{3}\pi a^3)\rho_0 & \text{if } r>2a \end{cases}$$

$$\Rightarrow D_r = \begin{cases} 0 & \text{if } r<a \\ \frac{\rho_0(r^3-a^3)}{3r^2} & \text{if } a<r<2a \\ \frac{7a^3\rho_0}{3r^2} & \text{if } r>2a \end{cases}, \text{ and } \vec{D} = D_r \vec{a_R}$$



※

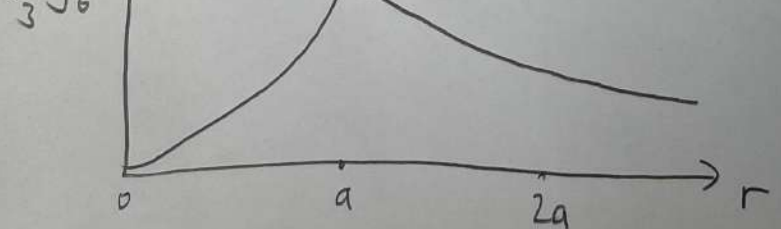
2.31

$\because \oint_C \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{S}$

$$\therefore 2\pi r H_\phi = \int_0^{\min(r,a)} \int_0^{2\pi} J_0 \left(\frac{r^*}{a}\right)\cdot r^* d\phi dr^*$$

$$= \int_0^{\min(r,a)} 2\pi \frac{J_0}{a} (r^*)^2 dr^* = \frac{2}{3}\pi \frac{J_0}{a} \min(r^3, a^3)$$

$$\Rightarrow H_\phi = \frac{\min(r^3,a^3)J_0}{3ar}, \text{ and } \vec{H} = H_\phi \vec{a_\phi}$$



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