# 賽局論 HW1

許博翔

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**Problem 2.** First, there are two lemmas:

**Lemma 2.1.** $E \subseteq F \iff \sim F \subseteq \sim E$.

**Lemma 2.2.** $PE = \sim K \sim E$.

Let's prove P2, P4 by K2, K4.
$K \sim E \overset{\text{K2}}{\subseteq} \sim E \overset{\textbf{Lemma (2.1)}}{\Rightarrow} E \subseteq \sim K \sim E \overset{\textbf{Lemma (2.2)}}{=} PE$, which is P2.
$P \sim E \overset{\text{K4}}{\subseteq} KP \sim E \overset{\textbf{Lemma (2.1)}}{\Rightarrow} PKE \overset{\textbf{Lemma (2.2)}}{=} P \sim P \sim E \overset{\textbf{Lemma (2.2)}}{=} \sim KP \sim E \subseteq \sim P \sim E \overset{\textbf{Lemma (2.2)}}{=} KE$, which is P4.
$KE \overset{\text{P2}}{\subseteq} PKE \overset{\text{K4}}{\subseteq} KPKE \overset{\text{P4}}{\subseteq} K^2E$.

**Problem 3** (12.12.4)**.** $P \sim E \overset{\text{K4}}{\subseteq} KP \sim E$
$\overset{\textbf{Lemma (2.1)}}{\Rightarrow} PKE \overset{\textbf{Lemma (2.2)}}{=} P \sim P \sim E \overset{\textbf{Lemma (2.2)}}{=} \sim KP \sim E \subseteq \sim P \sim E \overset{\textbf{Lemma (2.2)}}{=} KE$, which is P4.

**Problem 4** (12.12.8)**.**

(a) $KE \overset{\text{K3}}{\subseteq} K^2E$.

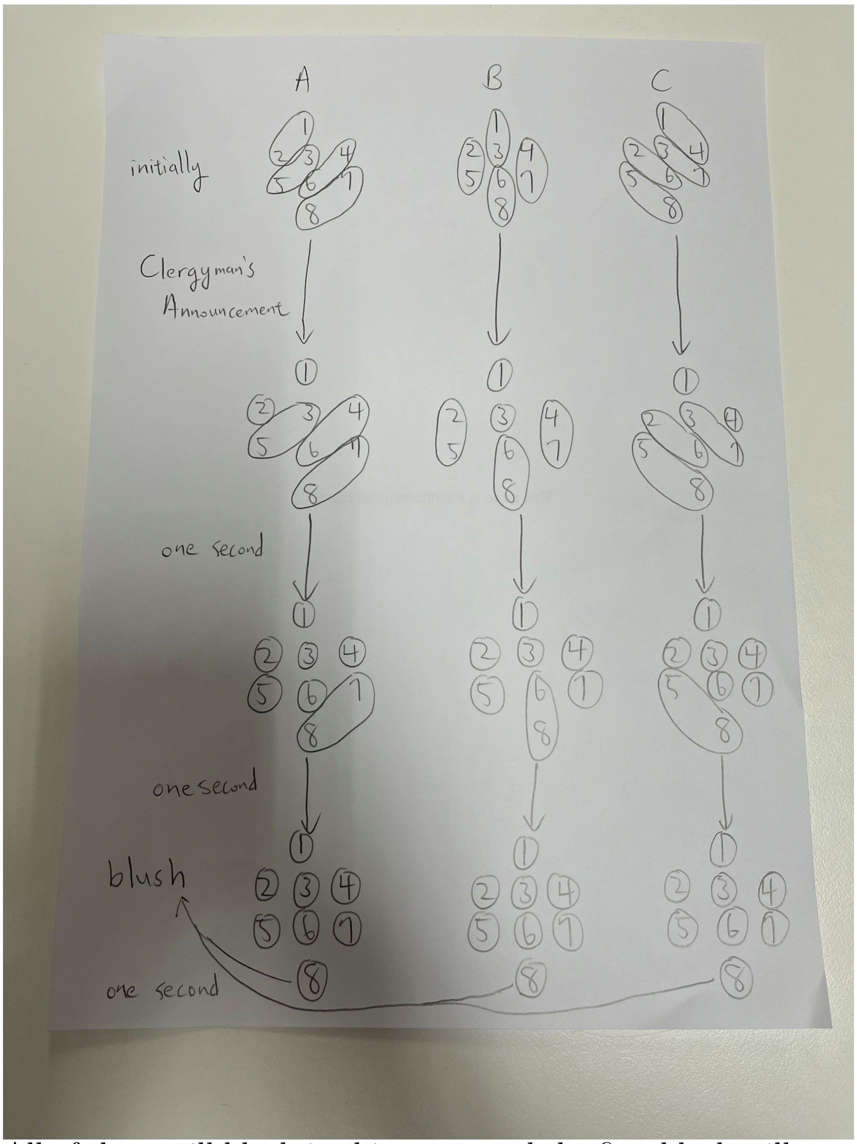
(b) $\sim KE \overset{\textbf{Lemma (2.2)}}{=} P \sim E \overset{\text{K4}}{\subseteq} KP \sim E \overset{\textbf{Lemma (2.2)}}{=} K \sim KE$.

(c) $PE \overset{\text{K4}}{\subseteq} KPE$.

(d) $\sim PE \overset{\textbf{Lemma (2.2)}}{=} K \sim E \overset{\text{K3}}{\subseteq} K^2 \sim E \overset{\textbf{Lemma (2.2)}}{=} K \sim PE$.

$\therefore KE, \sim KE, PE, \sim PE$ are truisms.

**Problem 5** (12.12.14)**.**




All of them will blush in this story, and the first blush will occur three seconds after the announcement.

**Problem 6.**

**Lemma 6.1.** $P\{w\} = P(w)$.

**Remark 1.** Lemma 2 in class proved it.

(a) $\because P_A(1) \overset{\textbf{Lemma (6.1)}}{=} P_A\{1\} \overset{\text{K4}}{\subseteq} KP_A(1)$.

$\therefore P_A(1) = \{1, 2\}$ is a truism.

(b) Clearly, $w \in P(w)$ by the definition.

By Theorem 4 in class, $P(w_1) = P(w_2) \iff P(w_1) \cap P(w_2) \neq \emptyset$.

Since $2 \in P(1) \cap P(2)$, there is $P(1) = P(2) = \{1,2\}$.

Since $4 \in P(3) \cap P(4),\ 5 \in P(3) \cap P(5)$, there is $P(3) = P(4) = P(5) = \{3,4,5\}$.

$K\{4,5\} = \sim P \sim \{4,5\} = \sim P\{1,2,3\} \overset{\text{P1}}{=} \sim (P\{1,2\} \cup P\{3\}) \overset{\text{P1}}{=} \sim (P\{1\} \cup P\{2\} \cup P\{3\}) \overset{\textbf{Lemma (6.1)}}{=} \sim (P(1) \cup P(2) \cup P(3)) = \sim (\{1,2\} \cup \{1,2\} \cup \{3,4,5\}) = \sim \{1,2,3,4,5\} = \emptyset$.

$\therefore$ in no state will $A$ know that the event $\{4,5\}$ has occurred.

(c) Telling how many elements $B$'s current possibility set contains is equivalent to tell what the current possibility set is. Let $P_A^0$ denote the original possibility sets of $A$. $P_A(1) = P_A^0(1) \cap P_B(1) = \{1,2\},\ P_A(3) = P_A^0(3) \cap P_B(3) = \{3\},\ P_A(4) = P_A^0(4) \cap P_B(4) = \{4,5\}$.

By Theorem 4 in class, the possibility sets is a partition of $\{1,2,3,4,5\}$.

$\therefore$ the possibility partition of $A$ will be changed to $\{1,2\}, \{3\}, \{4,5\}$.

**Problem 7.** $\forall \omega' \in P(\omega),\ P(\omega) \subseteq P(\omega')$.

*Proof.* The following three lemmas are from the powerpoints in class:

**Lemma 7.1.** If $E \subseteq F$, then $PE \subseteq PF$.

**Lemma 7.2.** For all states $c, d$, there is $\{c\} \subseteq P\{d\} \iff \{d\} \subseteq P\{c\}$.

**Lemma 7.3.** For all states $c, d$, there is $c \in P(d) \iff \{d\} \subseteq P\{c\}$.

Suppose $\omega' \in P(\omega)$, and we want to show that $\forall \omega'' \in P(\omega),\ \omega'' \in P(\omega')$.

Suppose that $\omega'' \in P(\omega)$.

$w' \in P(\omega) \overset{\textbf{Lemma (7.3)}}{\Rightarrow} \{\omega\} \subseteq P\{\omega'\} \overset{\textbf{Lemma (7.2)}}{\Rightarrow} \{\omega'\} \subseteq P\{\omega\} \overset{\textbf{Lemma (7.1)}}{\subseteq} PP\{\omega''\} \overset{\text{P3}}{\subseteq} P\{\omega''\}$.

$\Rightarrow \omega'' \in P(\omega')$.

$\therefore P(\omega) \subseteq P(\omega')$. ■

**Problem 8.** Consider $P = P_A$ and $\Omega = \{1,2,3,4,5\}$ in problem 6.

Consider $E = \sim \{1\}, F = \sim \{2\}$.

**Lemma 8.1.** $P \sim E \cap P \sim F \neq P(\sim E \cap \sim F)$.

*Proof.* $P \sim E = \{1,2\} = P \sim F$, but $P(\sim E \cap \sim F) = P\emptyset \overset{\text{P0}}{=} \emptyset$.
$\therefore P \sim E \cap P \sim F = \{1,2\} \neq \emptyset = P(\sim E \cap \sim F)$. ■

$KE \cup KF = \sim P \sim E \cup \sim P \sim F = \sim (P \sim E \cap P \sim F) \overset{\textbf{Lemma (8.1)}}{\neq} \sim P(\sim E \cap \sim F) = K \sim (\sim E \cap \sim F) = KE \cup KF$.

$\therefore$ the answer is false.