3.1

(a)

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ zx & xy & yz \end{vmatrix} = z\,\vec{a_x} + x\,\vec{a_y} + y\,\vec{a_z} \quad \#$$

(b)

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos y & -x\sin y & 0 \end{vmatrix} = 0\cdot\vec{a_x} + 0\cdot\vec{a_y} + (-\sin y + \sin y)\,\vec{a_z} = \vec{0} \quad \#$$

(c)

$$\begin{vmatrix} \frac{\vec{a_r}}{r} & \vec{a_\phi} & \frac{\vec{a_z}}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & r\cdot\frac{e^{-r^2}}{r} & 0 \end{vmatrix} = \frac{\vec{a_z}}{r}\cdot\frac{\partial}{\partial r} e^{-r^2} = -2e^{-r^2}\,\vec{a_z} \quad \#$$

(d)

$$\begin{vmatrix} \frac{\vec{a_r}}{r^2\sin\theta} & \frac{\vec{a_\theta}}{\sin\theta} & \vec{a_\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 2r\cos\theta & r\cdot r & 0 \end{vmatrix} = \vec{a_\phi}\left(2r + 2r\sin\theta\right) = 2r(1+\sin\theta)\,\vec{a_\phi} \quad \#$$

3.3

(a)

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial}{\partial t}\cdot\vec{D}$$

$$\nabla\times\vec{H} = \vec{a_y}\frac{\partial}{\partial z}H_x(z,t) - \vec{a_z}\frac{\partial}{\partial y}H_x(z,t) = \frac{\partial H_x}{\partial z}\ \vec{a_y}$$

$$\Rightarrow \frac{\partial H_x}{\partial z} = J_y + \frac{\partial D_y}{\partial t}$$

※

(b)

$$\nabla\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\nabla\times\vec{E} = \frac{\vec{a_z}}{r}\frac{\partial}{\partial r}rE_\phi(r,t) - \frac{\vec{a_r}}{r}\cdot\frac{\partial}{\partial z}rE_\phi(r,t) = \left(\frac{\partial rE_\phi}{\partial r}\right)\cdot\frac{1}{r}\vec{a_z}$$

$$\Rightarrow \frac{1}{r}\frac{\partial(rE_\phi)}{\partial r} = -\frac{\partial B_z}{\partial t}$$

※

3.5

$$\nabla\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\nabla\times\left(\frac{\vec{B}}{\mu_0}\right) = \vec{J} + \frac{\partial}{\partial t}(\varepsilon_0\vec{E}) = \frac{\partial}{\partial t}(\varepsilon_0\vec{E})$$

$$\nabla\times\vec{E} = \vec{a_y}\frac{\partial}{\partial z}E_0\cos(wt-\alpha y-\beta z)$$

$$-\vec{a_z}\cdot\frac{\partial}{\partial y}E_0\cos(wt-\alpha y-\beta z)$$

$$= \beta E_0\sin(wt-\alpha y-\beta z)\vec{a_y} - \alpha E_0\sin(wt-\alpha y-\beta z)\vec{a_z}$$

$$\Rightarrow -\frac{\partial\vec{B}}{\partial t} = E_0\sin(wt-\alpha y-\beta z)(\beta\vec{a_y}-\alpha\vec{a_z})$$

$$\Rightarrow \vec{B} = \frac{1}{w}E_0\cos(wt-\alpha y-\beta z)(\beta\vec{a_y}-\alpha\vec{a_z})$$

$$\nabla\times\left(\frac{\vec{B}}{\mu_0}\right) = \frac{E_0}{\mu_0 w}\cdot\begin{vmatrix}\vec{a_x} & \vec{a_y} & \vec{a_z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & \cos(wt-\alpha y-\beta z)\beta & -\cos(wt-\alpha y-\beta z)\alpha\end{vmatrix}$$

$$= \frac{E_0}{\mu_0 w}\vec{a_x}\left(-\alpha^2\sin(wt-\alpha y-\beta z) - \beta^2\sin(wt-\alpha y-\beta z)\right)$$

3.5

$$\Rightarrow \frac{\partial}{\partial t}(\varepsilon_0 \vec{E}) = \frac{E_0}{\mu_0 w}\sin(wt - \alpha y - \beta z)(-\alpha^2 - \beta^2)\,\vec{a_x}$$

$$\frac{\partial}{\partial t}(\varepsilon_0 \vec{E}) = \frac{\partial}{\partial t}\left(\varepsilon_0 E_0 \cos(wt - \alpha y - \beta z)\vec{a_x}\right) = -\varepsilon_0 E_0 w \sin(wt - \alpha y - \beta z)\,\vec{a_x}$$

$$\therefore \frac{-\alpha^2 - \beta^2}{\mu_0 w} = -\varepsilon_0 w$$

$$\Rightarrow \alpha^2 + \beta^2 = \mu_0 \varepsilon_0 w^2$$

\#

3.7

(a)

$$\nabla \times \vec{H} = \vec{J}$$

$\because \vec{J}$ only has x-component and independent of x, y

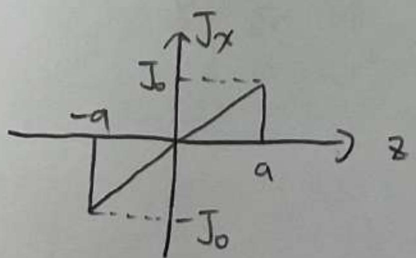
$\therefore$ the above is $-\frac{\partial H_y}{\partial z} = J_x$

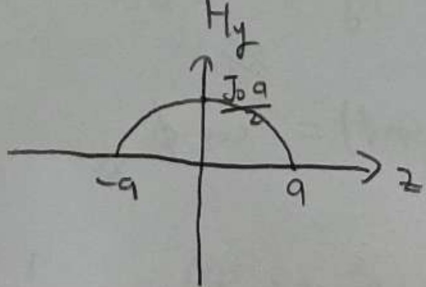
$$\Rightarrow H_y = -\int J_x\,dz = \begin{cases} -\int_{-\infty}^{z} 0\,dz & , z < -a \\ -\int_{-\infty}^{-a} 0\,dz - \int_{-a}^{z} J_0 \frac{z}{a}\,dz & , -a < z < a \\ -\int_{-\infty}^{-a} 0\,dz - \int_{-a}^{a} J_0 \frac{z}{a}\,dz - \int_{a}^{z} 0\,dz & , z > a \end{cases}$$

$$= \begin{cases} C & , z < -a \\ \frac{J_0 (a^2 - z^2)}{2a} + C & , -a < z < a \\ C & , z > a \end{cases}$$

$\because$ when $J_0 = 0$, the field is 0

$\therefore C = 0$





\#

3.1

(b)

$\nabla \times \vec{H} = \vec{J}$

$\because$ $\vec{J}$ only has ϕ-component and independent of ϕ, z

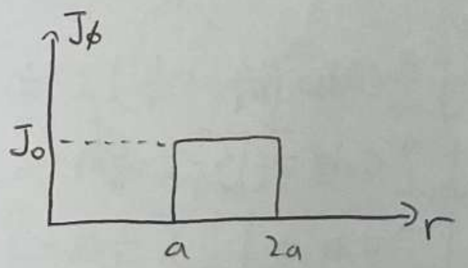
$\therefore$ it becomes $-\frac{\partial H_z}{\partial r} = J_\phi$

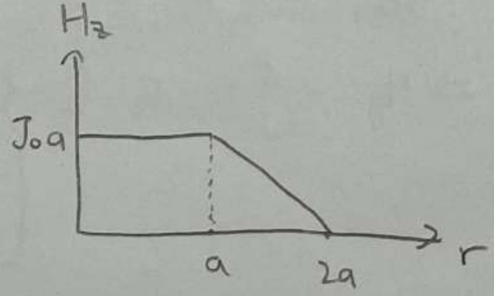
$$\Rightarrow H_z = -\int J_\phi \, dr = \begin{cases} -\int_0^r 0 \, dr & , r<a \\ -\int_0^a 0 \, dr + \int_a^r J_0 \, dr & , a<r<2a \\ -\int_0^a 0 \, dr + \int_a^{2a} J_0 \, dr + \int_{2a}^r 0 \, dr & , r>2a \end{cases}$$

$$= \begin{cases} C & ; r<a \\ -J_0 (r-a) + C & ; a<r<2a \\ -J_0 a + C & ; r>2a \end{cases}$$

$\because$ for $r \to \infty$, $H_z \to 0$

$\therefore$ $C = J_0 a$





#

3.8

(a)

$$\frac{\partial}{\partial x}(zx) + \frac{\partial}{\partial y}(xy) + \frac{\partial}{\partial z}(yz) = z + x + y$$ #

(b)

$$\frac{\partial}{\partial x}(3) + \frac{\partial}{\partial y}(y-3) + \frac{\partial}{\partial z}(2+z) = 0 + 1 + 1 = 2$$ #

(c)

$$\frac{1}{r}\frac{\partial}{\partial \phi}(r \sin\phi) = \cos\phi$$ #

(d)

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \cdot r\cos^2\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \cdot \left(-r\cos\theta\sin\theta\right)\right)$$

$$= 3\cos^2\theta + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\cos^3\theta - \cos\theta\right) = 3\cos^2\theta - 3\cos^2\theta + 1 = 1$$ #

3.10

(a)
$$\nabla\cdot\left(\left(\frac{1}{y^k}\right)\left(2x\,\vec{a_x}+y\,\vec{a_y}\right)\right)=\frac{2}{y^k}+(1-k)\cdot\frac{1}{y^k}=0$$

$$\Rightarrow\ 2+1-k=0\Rightarrow k=3 \quad ※$$

(b)
$$\nabla\cdot\left(r\left(\sin k\phi\,\vec{a_r}+\cos k\phi\,\vec{a_\phi}\right)\right)=\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r^2\sin k\phi\right)+\frac{1}{r}\frac{\partial}{\partial\phi}r\cos k\phi$$

$$=2\sin k\phi-k\sin k\phi=0$$

$$\Rightarrow\ 2-k=0\Rightarrow k=2 \quad ※$$

(c)
$$\nabla\cdot\left(\left(1+\frac{2}{r^3}\right)\cos\theta\,\vec{a_r}+k\left(1-\frac{1}{r^3}\right)\sin\theta\,\vec{a_\theta}\right)$$

$$=\frac{1}{r^2}\frac{\partial}{\partial r}\left(\left(r^2+\frac{2}{r}\right)\cos\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(k\left(1-\frac{1}{r^3}\right)\sin^2\theta\right)$$

$$=2\left(\frac{1}{r}-\frac{1}{r^4}\right)\cos\theta+\left(\frac{1}{r}-\frac{1}{r^4}\right)k\cdot 2\cos\theta=0$$

$$\Rightarrow\ 1+k=0\Rightarrow k=-1 \quad ※$$

3.13

Let the positive z-axis denote the north.

Suppose the angular velocity of the earth spinning is w.

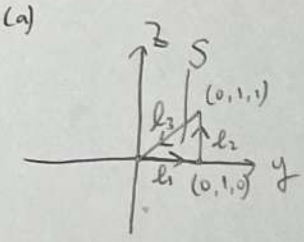
$$\vec{v}=wr\,\vec{a_\phi}$$

$$\nabla\times\vec{v}=\begin{vmatrix}\frac{\vec{a_r}}{r} & \vec{a_\phi} & \frac{\vec{a_z}}{r}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ 0 & wr^2 & 0\end{vmatrix}=2w\,\vec{a_z}$$

※

3.15

(a)



$l_1: x=z=0 \Rightarrow dx=dz=0$

$\Rightarrow d\vec{l} = dy\,\vec{a_y}$

$\int_{l_1} \vec{A}\cdot d\vec{l} = \int_0^1 xy\,dy = \frac{1}{2}x = 0$

$l_2: y=1,\ x=0 \Rightarrow dx=dy=0 \Rightarrow d\vec{l} = dz\,\vec{a_z}$

$\int_{l_2} \vec{A}\cdot d\vec{l} = \int_0^1 yz\,dz = \frac{1}{2}y = \frac{1}{2}$

$l_3: x=0,\ y=z \Rightarrow dx=0,\ dy=dz \Rightarrow d\vec{l} = dy(\vec{a_y}+\vec{a_z})$

$\int_{l_3} \vec{A}\cdot d\vec{l} = \int_1^0 (xy+yz)\,dy = \int_1^0 y^2\,dy = -\frac{1}{3}$

$\therefore$ the result $= 0 + \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

$d\vec{S} = dy\,dz\,\vec{a_x}$, $\nabla\times\vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ zx & xy & yz \end{vmatrix} = z\vec{a_x} + x\vec{a_y} + y\vec{a_z}$

$\int_S (\nabla\times\vec{A})\cdot d\vec{S} = \int_0^1 \int_0^y z\,dz\,dy = \int_0^1 \frac{y^2}{2}\,dy = \frac{1}{6}$

The 2 results are the same. #

(b)

$\oint_C \vec{A}\cdot d\vec{l} = \oint_C \vec{A}(\vec{a_x}dx + \vec{a_y}dy + \vec{a_z}dz) = \oint_C (\cos y\,dx - x\sin y\,dy)$

$= \oint_C d(x\cos y) = x\cos y \Big|_{x_0,y_0,z_0}^{x_0,y_0,z_0} = 0$, where C is any closed path.

$\nabla\times\vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos y & -x\sin y & 0 \end{vmatrix} = \vec{a_z}(-\sin y + \sin y) = 0$

$\Rightarrow \int_S (\nabla\times\vec{A})\,d\vec{S} = 0$, where S is the surface of C #