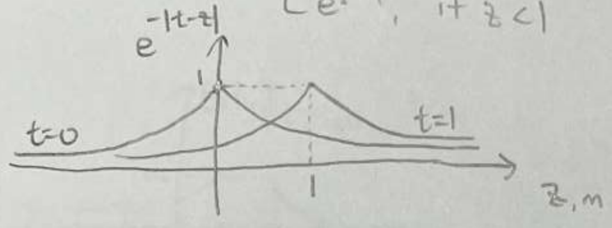
3.18

(a)

$t=0,\ e^{-|t-z|} = \begin{cases} e^{-z}, & \text{if } z>0 \\ e^{z}, & \text{if } z<0 \end{cases}$

$t=1,\ e^{-|t-z|} = \begin{cases} e^{-z+1}, & \text{if } z>1 \\ e^{z-1}, & \text{if } z<1 \end{cases}$



It's a traveling wave with velocity 1 on +z.

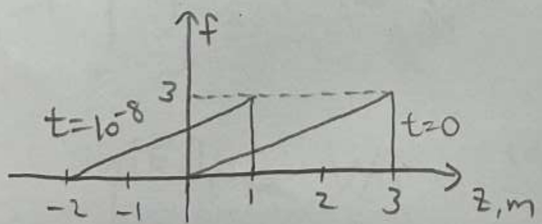
※

(b)

Let f be the function.

$t=0,\ f= z(u(z)-u(z-3))$

$t=10^{-8},\ f= (z+2)(u(z+2)-u(z-1))$

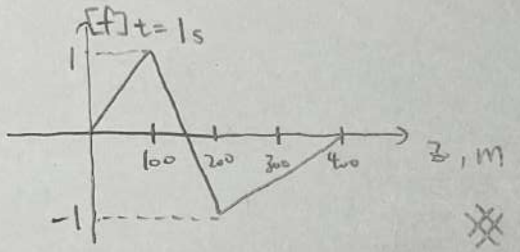


It's a traveling wave with velocity 2×10^8 on −z.

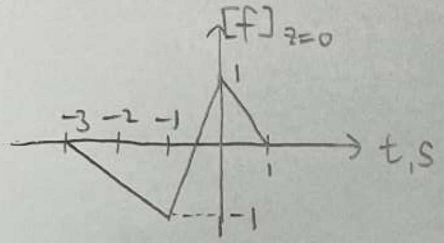
※

3.20

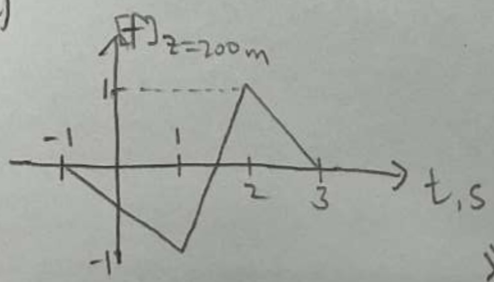
(a)


[f] t= 1s
1
-1
100
200
300
400
z, m

(b)

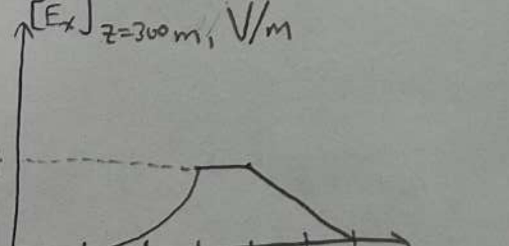

[f] z=0
1
-3 -2 -1
1
-1
t, s

(c)


[f] z=200m
1
-1
1
2
3
-1
t, s

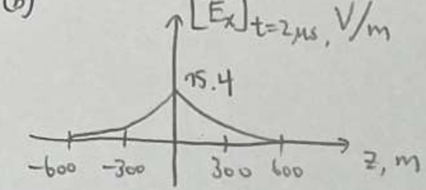
3.22

(a)

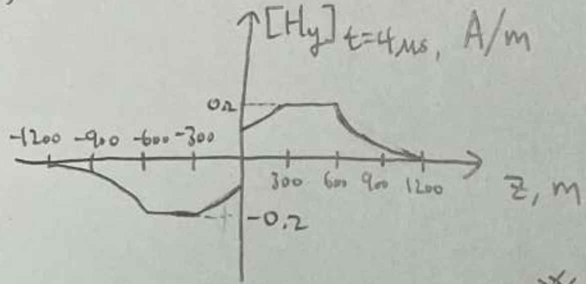

[Ex] z=300m, V/m
15.4
1 2 3 4 5 6
t, μs

3.22

(b)

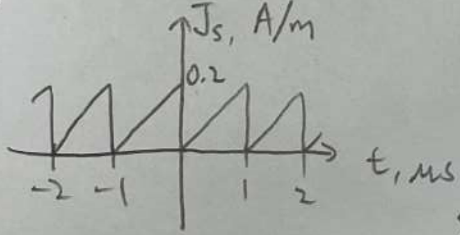


(c)

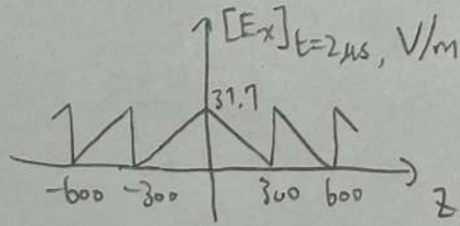


3.24

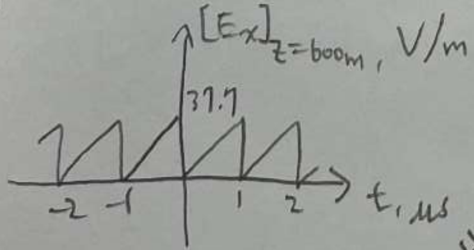
(a)



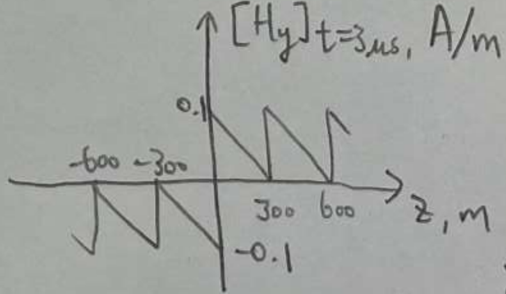
(c)



(b)



(d)



3.25

(a)

$$\frac{9\pi\times10^7}{2\pi} = 4.5\times10^7\ Hz$$

※

(b)

$$\frac{2\pi}{0.3\pi} = \frac{20}{3}\ m$$

※

(c)

$$-\frac{9\pi\times10^7}{0.3\pi} < 0$$

$\therefore$ direction is $-\vec{a_y}$

※

(d)

Direction of $\vec{E}$: $\vec{a_x}$

$\vec{E}\times\vec{H}$: $-\vec{a_y}$

$\therefore$ direction of $\vec{H}$: $\vec{a_z}$

$$\therefore H = \frac{37.7}{\eta_0}\cos\left(9\pi\times10^7 t + 0.3\pi y\right)\vec{a_z} \doteqdot 0.1\cos\left(9\pi\times10^7 t + 0.3\pi y\right)\vec{a_z}$$

A/m ※

3.27

1.

$$[\vec{H}]_{z=0\pm} = \frac{1}{2}\vec{J_s}\times(\pm\vec{a_x}) = 0.1\sin(15\pi\times10^7 t)\vec{a_y}\times(\pm\vec{a_x})$$

$$= \mp 0.1\sin(15\pi\times10^7 t)\,\vec{a_z}\ \ A/m$$

2.

$$\frac{\omega}{v} = \frac{15\pi\times10^7}{3\times10^8} = 0.5\pi$$

$$[\vec{H}]_{x\gtrless0} = \mp 0.1\sin\left(15\pi\times10^7 t \mp 0.5\pi x\right)\vec{a_z}\ \ A/m$$

3.

$$[\vec{E}]_{x\gtrless0} = \eta_0[\vec{H}]_{x\gtrless0}\times(\pm\vec{a_x}) \doteqdot \mp 37.7\sin\left(15\pi\times10^7 t \mp 0.5\pi x\right)\vec{a_z}\times(\pm\vec{a_x})$$

$$= -37.7\sin\left(15\pi\times10^7 t \mp 0.5\pi x\right)\vec{a_y}\ \ V/m$$

※

3.29

$z<0$: $\vec{E} = \frac{\eta_0 J_{s0}}{2}\left(\cos(wt+\beta z) + k\sin(wt+\beta(z-\frac{\lambda}{4})) + 2k\cos(wt+\beta(z-\frac{\lambda}{2}))\right)\vec{a_x}$

$= \frac{\eta_0 J_{s0}}{2}\left(\cos(wt+\beta z) - k\cos(wt+\beta z) - 2k\cos(wt+\beta z)\right)\vec{a_x}$

$= \frac{\eta_0 J_{s0}}{2}(1-3k)\cos(wt+\beta z)\vec{a_x}$

$z>\frac{\lambda}{2}$: $\vec{E} = \frac{\eta_0 J_{s0}}{2}\left(\cos(wt-\beta z) + k\sin(wt-\beta(z-\frac{\lambda}{4})) + 2k\cos(wt-\beta(z-\frac{\lambda}{2}))\right)\vec{a_x}$

$= \frac{\eta_0 J_{s0}}{2}\left(\cos(wt-\beta z) + k\cos(wt-\beta z) - 2k\cos(wt-\beta z)\right)\vec{a_x}$

$= \frac{\eta_0 J_{s0}}{2}(1-k)\cos(wt-\beta z)\vec{a_x}$

$\therefore$ the ratio of the amplitude $= \frac{|1-k|}{|1-3k|}$

(a) $\frac{|1-(-1)|}{|1-3(-1)|} = \frac{1}{2}$ ※ (b) $\frac{|1-\frac{1}{2}|}{|1-\frac{3}{2}|} = 1$ ※ (c) $\frac{|1-1|}{|1-3|} = 0$ ※

(a)

$\frac{|1-k|}{|1-3k|} = \frac{1}{3} \Rightarrow 3(1-k) = \pm(1-3k)$ $\begin{cases} 3(1-k) = 1-3k \quad \times \\ 3(1-k) = -1+3k \Rightarrow k = \frac{2}{3} \end{cases}$ ※

(b)

$\frac{|1-k|}{|1-3k|} = 3 \Rightarrow 1-k = \pm 3(1-3k)$ $\begin{cases} 1-k = 3-9k \Rightarrow k = \frac{1}{4} \\ 1-k = -3+9k \Rightarrow k = \frac{2}{5} \end{cases}$

$\Rightarrow k = \frac{1}{4}$ or $\frac{2}{5}$ ※

3.30

(a) $\vec{F_1}, \vec{F_2}$ have the same phase

$\therefore \vec{F_1}+\vec{F_2}$ is linearly polarized. ※

(b) $\vec{F_1}+\vec{F_2}, \vec{F_3}$ have the same amplitude, phases differ by $90°$,

and $(\vec{F_1}+\vec{F_2})\cdot\vec{F_3} = \frac{1}{2}\sqrt{3}+0+\frac{1}{2}\sqrt{3} \neq 0 \Rightarrow \vec{F_1}+\vec{F_2}, \vec{F_3}$ are not orthogonal

$\therefore \vec{F_1}+\vec{F_2}+\vec{F_3}$ is elliptically polarized. ※

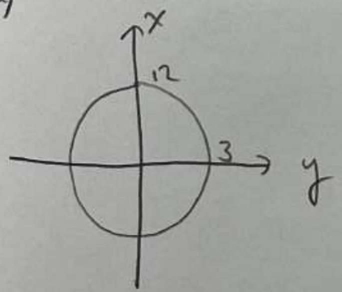
(c) $\vec{F_1}-\vec{F_2}, \vec{F_3}$ have the same amplitude, phases differ by $90°$,

and $(\vec{F_1}-\vec{F_2})\cdot\vec{F_3} = \frac{1}{2}\sqrt{3}+0-\frac{1}{2}\sqrt{3} = 0 \Rightarrow \vec{F_1}-\vec{F_2}, \vec{F_3}$ are orthogonal

$\therefore \vec{F_1}-\vec{F_2}+\vec{F_3}$ is circularly polarized. ※

3.32

(a)



$$\omega = \frac{2\pi}{60\times60\times12} = \frac{\pi}{21600}$$

ans: $\left(\cos\frac{\pi t}{21600}\right)\vec{a_x} + \left(\sin\frac{\pi t}{21600}\right)\vec{a_y}$ ※

(b)

$$\omega = \frac{2\pi}{60\times60} = \frac{\pi}{1800}$$

ans: $\left(\cos\frac{\pi t}{1800}\right)\vec{a_x} + \left(\sin\frac{\pi t}{1800}\right)\vec{a_y}$ ※

(c)

$$\left(\cos\frac{\pi t}{21600}\right)\vec{a_x} + \left(\sin\frac{\pi t}{21600}\right)\vec{a_y} = \left(\cos\frac{\pi t}{1800}\right)\vec{a_x} + \left(\sin\frac{\pi t}{1800}\right)\vec{a_y}$$

$$\Rightarrow \cos\frac{\pi t}{21600} = \cos\frac{\pi t}{1800},\ \sin\frac{\pi t}{21600} = \sin\frac{\pi t}{1800}$$

$$\Rightarrow \frac{\pi t}{21600} = \frac{\pi t}{1800} - 2\pi n \Rightarrow \left(\frac{1}{3600} - \frac{1}{43200}\right)t = n$$

$$\Rightarrow t = \frac{43200}{11}n$$

(下頁)

$\because 3600 \times 5 < \frac{43200}{11} n = t < 3600 \times 6$

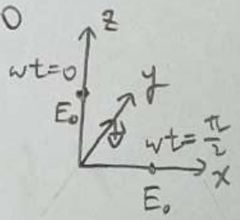
$\therefore n = 5$

$\therefore$ ans: $\left(\cos \frac{10}{11}\pi\right)\vec{a_x} + \left(\sin \frac{10}{11}\pi\right)\vec{a_y}$ ※

3.34

(a)

$y=0$

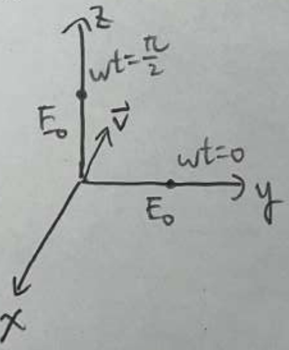


$\frac{\beta}{\omega} > 0 \Rightarrow$ direction: $+y$

$\therefore$ right-circular ※

(b)

$x=0$

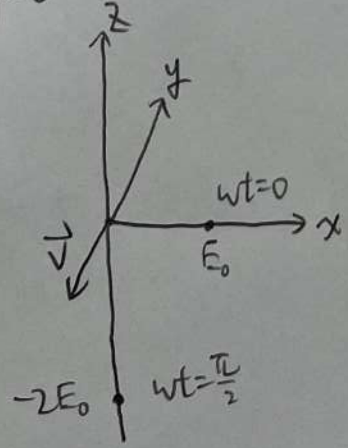


$-\frac{\beta}{\omega} < 0 \Rightarrow$ direction: $-x$

$\therefore$ left-circular ※

(c)

$y=0$



$-\frac{\beta}{\omega} < 0 \Rightarrow$ direction: $-y$

$\therefore$ left-elliptical ※

3.34

(d)

$x=0$

$\omega t=0 \Rightarrow E_0\left(\vec{a_z} - \frac{\sqrt{2}}{2}\vec{a_y}\right)$

$\omega t=\frac{\pi}{2} \Rightarrow E_0\left(-\frac{\sqrt{2}}{2}\vec{a_y}\right)$

$\left\|\vec{a_z} - \frac{\sqrt{2}}{2}\vec{a_x}\right\| \neq \left\|\frac{\sqrt{2}}{2}\vec{a_x}\right\| \Rightarrow$ elliptical

$\frac{\beta}{\omega} > 0 \Rightarrow$ direction: $+x$

$\therefore$ right-elliptical ※

3.37

(a)

$$\langle\vec{P}\rangle = \langle\vec{E}\times\vec{H}\rangle = \left\langle \frac{V_0 I_0}{2\pi r^2 \ln\left(\frac{b}{a}\right)} \cos^2\left(\omega\left(t - z\sqrt{\mu_0\varepsilon_0}\right)\right)\vec{a_z}\right\rangle$$

$$= \frac{V_0 I_0}{2\pi r^2 \ln\left(\frac{b}{a}\right)} \left\langle \frac{\cos\left(2\omega\left(t - z\sqrt{\mu_0\varepsilon_0}\right)\right)+1}{2}\right\rangle \vec{a_z}$$

$$= \frac{V_0 I_0}{4\pi r^2 \ln\left(\frac{b}{a}\right)} \vec{a_z}$$

※

(b)

$$\langle\vec{P}\rangle = \int_{r=a}^{b}\int_{\phi=0}^{2\pi} \frac{V_0 I_0}{4\pi r^2 \ln\left(\frac{b}{a}\right)} r\,d\phi\,dr$$

$$= \int_{r=a}^{b} \frac{V_0 I_0}{2r \ln\left(\frac{b}{a}\right)}\,dr = \frac{V_0 I_0}{2}$$

※

3.39

(a)

$$\text{Gauss' law: } 4\pi\varepsilon_0 r^2 D_r = \begin{cases} \int_0^r\int_0^\pi\int_0^{2\pi} \rho_0\left(\frac{r}{a}\right)^2 r^2\sin\theta\,d\phi\,d\theta\,dr, & \text{if } r<a \\ \int_0^a\int_0^\pi\int_0^{2\pi} \rho_0\left(\frac{r}{a}\right)^2 r^2\sin\theta\,d\phi\,d\theta\,dr, & \text{if } r>a \end{cases}$$

$$= \begin{cases} \frac{4\pi\rho_0 r^5}{5a^2}, & \text{if } r<a \\ \frac{4\pi\rho_0 a^3}{5}, & \text{if } r>a \end{cases}$$

$$\Rightarrow E_r = \begin{cases} \frac{\rho_0 r^3}{5\varepsilon_0 a^2}, & \text{if } r<a \\ \frac{\rho_0 a^3}{5\varepsilon_0 r^2}, & \text{if } r>a \end{cases}$$

(下頁)

$$W_e = \frac{1}{2}\int \varepsilon_0 E^2 dv = \int_0^a\int_0^\pi\int_0^{2\pi} \frac{\rho_0^2 r^6}{50\varepsilon_0 a^4} r^2 \sin\theta \, d\phi \, d\theta \, dr$$

$$+ \int_a^\infty\int_0^\pi\int_0^{2\pi} \frac{\rho_0^2 a^6}{50\varepsilon_0 r^4} r^2 \sin\theta \, d\phi \, d\theta \, dr$$

$$= \int_0^a \frac{\rho_0^2 r^8 \cdot 2\pi \cdot 2}{50\varepsilon_0 a^4} dr + \int_a^\infty \frac{\rho_0^2 a^6 \cdot 2\pi \cdot 2}{50\varepsilon_0 r^2} dr$$

$$= \frac{2\rho_0^2 \pi a^5}{225\varepsilon_0} + \frac{\rho_0^2 a^5 \cdot 2\pi}{25\varepsilon_0} = \frac{2\times 10}{225}\pi \frac{\rho_0^2 a^5}{\varepsilon_0} = \frac{4}{45}\pi \frac{\rho_0^2 a^5}{\varepsilon_0}$$ ※

(b)

Total charge $= \frac{4\pi\rho_0 a^3}{5}$

uniform charge density $\rho' = \frac{\frac{4\pi\rho_0 a^3}{5}}{\frac{4}{3}\pi a^3} = \frac{3}{5}\rho_0$

$$W_e = \frac{4}{15}\pi \frac{\left(\frac{3}{5}\rho_0\right)^2 a^5}{\varepsilon_0} = \frac{12}{125}\cdot\frac{\pi\rho_0^2 a^5}{\varepsilon_0}$$

work required $= \left(\frac{12}{125} - \frac{4}{45}\right)\pi\frac{\rho_0^2 a^5}{\varepsilon_0} = \frac{108-100}{1125}\pi\frac{\rho_0^2 a^5}{\varepsilon_0} = \frac{8\pi}{1125}\cdot\frac{\rho_0^2 a^5}{\varepsilon_0}$ ※