# 人工智慧導論 HW4

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June 5, 2024

Reference: all by myself

## 1 Hand-written Part

**Problem 1.** $\varphi'(s) = \frac{1 \cdot (1+e^{-s}) - s \cdot (-e^{-s})}{(1+e^{-s})^2} = \frac{1+(1+s)e^{-s}}{(1+e^{-s})^2}$.

**Problem 2.**

(A) $\mathbf{v}_0 = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$.

$$\mathbf{v}_1 = \begin{pmatrix} 0 & 1 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{6} \\ \frac{1}{3} \end{pmatrix}.$$

$$\mathbf{v}_2 = \begin{pmatrix} 0 & 1 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{6} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{6} \\ \frac{1}{2} \end{pmatrix}.$$

$$\mathbf{v}_3 = \begin{pmatrix} 0 & 1 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{1}{6} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{5}{12} \\ \frac{1}{4} \\ \frac{1}{3} \end{pmatrix}.$$

$$\mathbf{v}_4 = \begin{pmatrix} 0 & 1 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{5}{12} \\ \frac{1}{4} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{5}{12} \\ \frac{1}{6} \\ \frac{5}{12} \end{pmatrix}.$$

$$\mathbf{v}_5 = \begin{pmatrix} 0 & 1 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{5}{12} \\ \frac{1}{6} \\ \frac{5}{12} \end{pmatrix} = \begin{pmatrix} \frac{3}{8} \\ \frac{5}{24} \\ \frac{5}{12} \end{pmatrix}.$$

(B) Suppose that $\mathbf{v}^* = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$.

$$\mathbf{v}^* = \begin{pmatrix} 0 & 1 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} \mathbf{v}^*.$$

$$\Rightarrow \left( \begin{pmatrix} 0 & 1 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} - I \right) \mathbf{v}^* = 0.$$

$$\Rightarrow \begin{pmatrix} -1 & 1 & 0.5 \\ 0 & -1 & 0.5 \\ 1 & 0 & -1 \end{pmatrix} \mathbf{v}^* = 0.$$

$\Rightarrow -v_2 + 0.5v_3 = 0, v_1 - v_3 = 0.$

$$\Rightarrow \mathbf{v}^* = \begin{pmatrix} v_3 \\ 0.5v_3 \\ v_3 \end{pmatrix}.$$

Since $(1 + 0.5 + 1)v_3 = 1$, there is $v_3 = 0.4$.

$$\therefore \mathbf{v}^* = \begin{pmatrix} 0.4 \\ 0.2 \\ 0.4 \end{pmatrix}.$$

**Problem 3.**

(A)
- Iteration 1:

  Partition: $\{(1,2)\}, \{(3,4), (7,0), (10,2)\}$

  Centroids: $(1,2), (\frac{20}{3}, 2)$

- Iteration 2:

  Partition: $\{(1,2), (3,4)\}, \{(7,0), (10,2)\}$

  Centroids: $(2,3), (8.5,1)$

- Iteration 3:

  Partition: $\{(1,2),(3,4)\},\{(7,0),(10,2)\}$

  Centroids: $(2,3),(8.5,1)$

The convergent result:

Partition: $\{(1,2),(3,4)\},\{(7,0),(10,2)\}$

Centroids: $(2,3),(8.5,1)$

(B)
- Iteration 1:

  Partition: $\{(1,2),(3,4)\},\{(7,0),(10,2)\}$

  Centroids: $(2,3),(8.5,1)$

- Iteration 2:

  Partition: $\{(1,2),(3,4)\},\{(7,0),(10,2)\}$

  Centroids: $(2,3),(8.5,1)$

The convergent result:

Partition: $\{(1,2),(3,4)\},\{(7,0),(10,2)\}$

Centroids: $(2,3),(8.5,1)$

The result is same to (A).

(C) Consider the two different initial centroids: $\{(1,2),(3,4)\}$ and $\{(1,2),(7,0)\}$.

- The former centroids:
  - Iteration 1:

    Partition: $\{(1,2)\},\{(3,4),(5,6),(7,0),(10,2)\}$

    Centroids: $(1,2),(6.25,3)$
  - Iteration 2:

    Partition: $\{(1,2),(3,4)\},\{(5,6),(7,0),(10,2)\}$

    Centroids: $(2,3),(\frac{22}{3},\frac{7}{3})$
  - Iteration 3:

    Partition: $\{(1,2),(3,4)\},\{(5,6),(7,0),(10,2)\}$

    Centroids: $(2,3),(\frac{22}{3},\frac{7}{3})$

The convergent result:

Partition: $\{(1,2),(3,4)\},\{(5,6),(7,0),(10,2)\}$

Centroids: $(2,3),(\frac{22}{3},\frac{7}{3})$

$\mathrm{E_{in}} = \frac{53}{3}$.

- The latter centroids:
  - Iteration 1:

    Partition: $\{(1,2),(3,4),(5,6)\},\{(7,0),(10,2)\}$

    Centroids: $(3,4),(8.5,1)$
  - Iteration 2:

    Partition: $\{(1,2),(3,4),(5,6)\},\{(7,0),(10,2)\}$

    Centroids: $(3,4),(8.5,1)$

  The convergent result:

  Partition: $\{(1,2),(3,4),(5,6)\},\{(7,0),(10,2)\}$

  Centroids: $(3,4),(8.5,1)$

  $\mathrm{E_{in}} = 11.25$.

Since $11.25 \neq \frac{53}{3} \approx 17.67$, at least one of the above does not converge to the global minimum.

**Problem 4.** Let $A = \sum_{1\le n\le N, g_t(\mathbf{x}_n)\neq y_n} w_n^t, B = \sum_{1\le n\le N, g_t(\mathbf{x}_n)= y_n} w_n^t$.

$\Rightarrow \epsilon_t = \frac{A}{A+B}$.

$d_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}} = \sqrt{\frac{B}{A}}$.

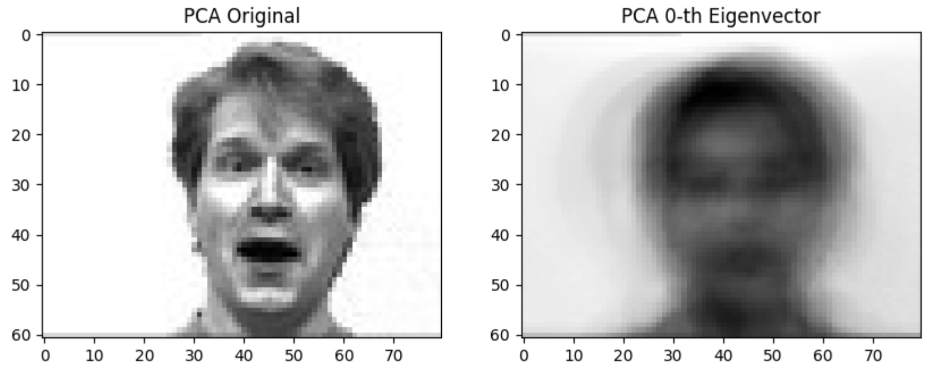
$\sum_{n=1}^{N} w_n^{t+1}\delta(g_t(\mathbf{x}_n), y_n) = \sum_{1\le n\le N, g_t(\mathbf{x}_n)\neq y_n} w_n^{t+1} = d_t \sum_{1\le n\le N, g_t(\mathbf{x}_n)\neq y_n} w_n^t = d_t A = \sqrt{\frac{B}{A}}A = \sqrt{AB}$.

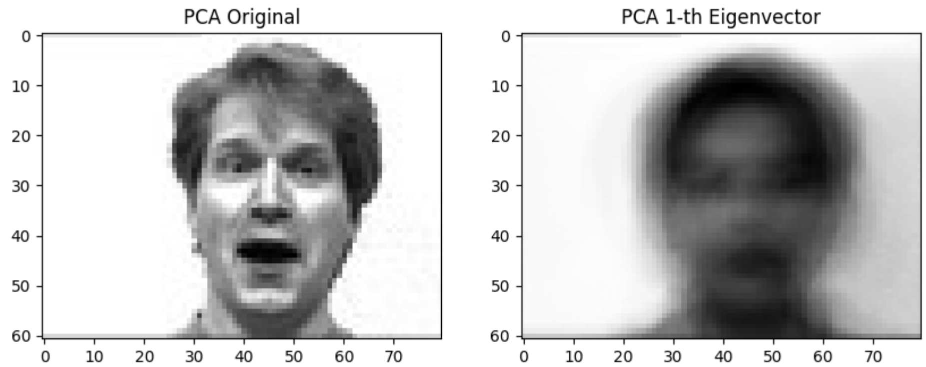
$\sum_{n=1}^{N} w_n^{t+1} = \sum_{1\le n\le N, g_t(\mathbf{x}_n)\neq y_n} w_n^{t+1} + \sum_{1\le n\le N, g_t(\mathbf{x}_n)= y_n} w_n^{t+1} = \sum_{1\le n\le N, g_t(\mathbf{x}_n)\neq y_n} d_t w_n^t + \sum_{1\le n\le N, g_t(\mathbf{x}_n)= y_n} w_n^t \frac{1}{d_t} = Ad_t + \frac{B}{d_t} = A\sqrt{\frac{B}{A}} + B\sqrt{\frac{A}{B}} = 2\sqrt{AB}$.

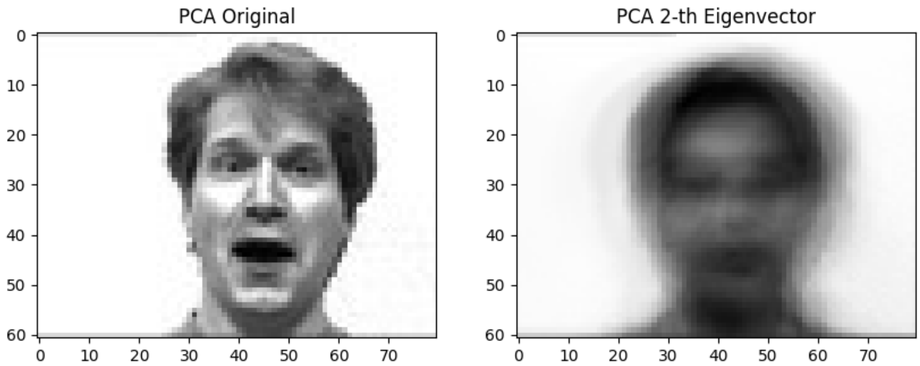
$$\therefore \frac{\sum\limits_{n=1}^{N} w_n^{t+1}\delta(g_t(\mathbf{x}_n), y_n)}{\sum\limits_{n=1}^{N} w_n^{t+1}} = \frac{\sqrt{AB}}{2\sqrt{AB}} = 0.5.$$

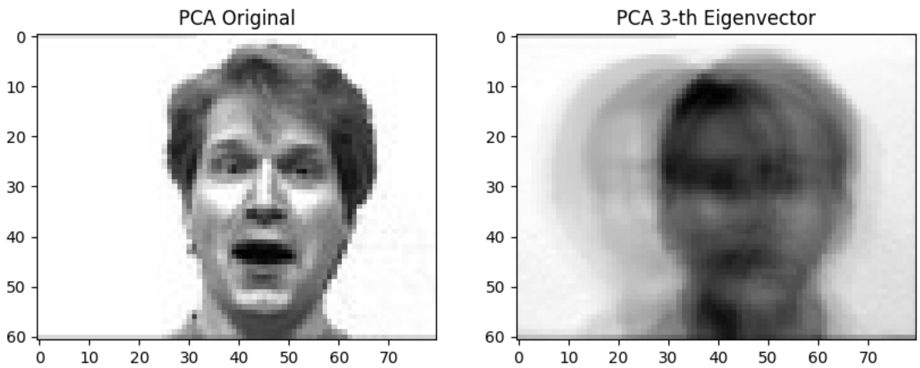
# 2 Programming Part

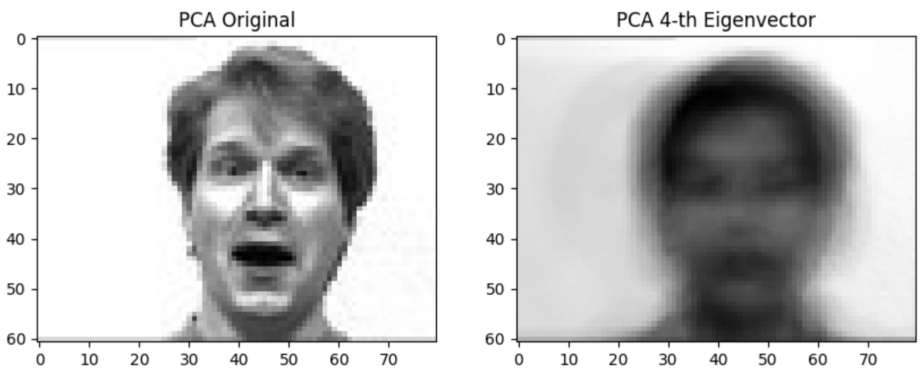
## 2.1 (a)



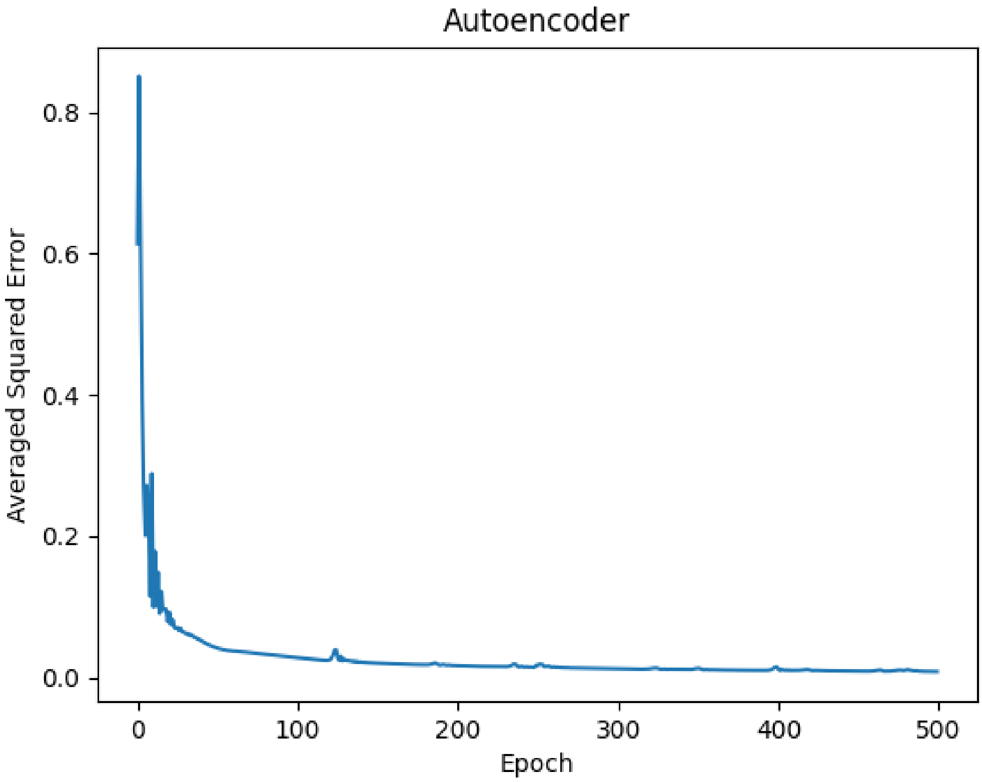




PCA Original
PCA 2-th Eigenvector

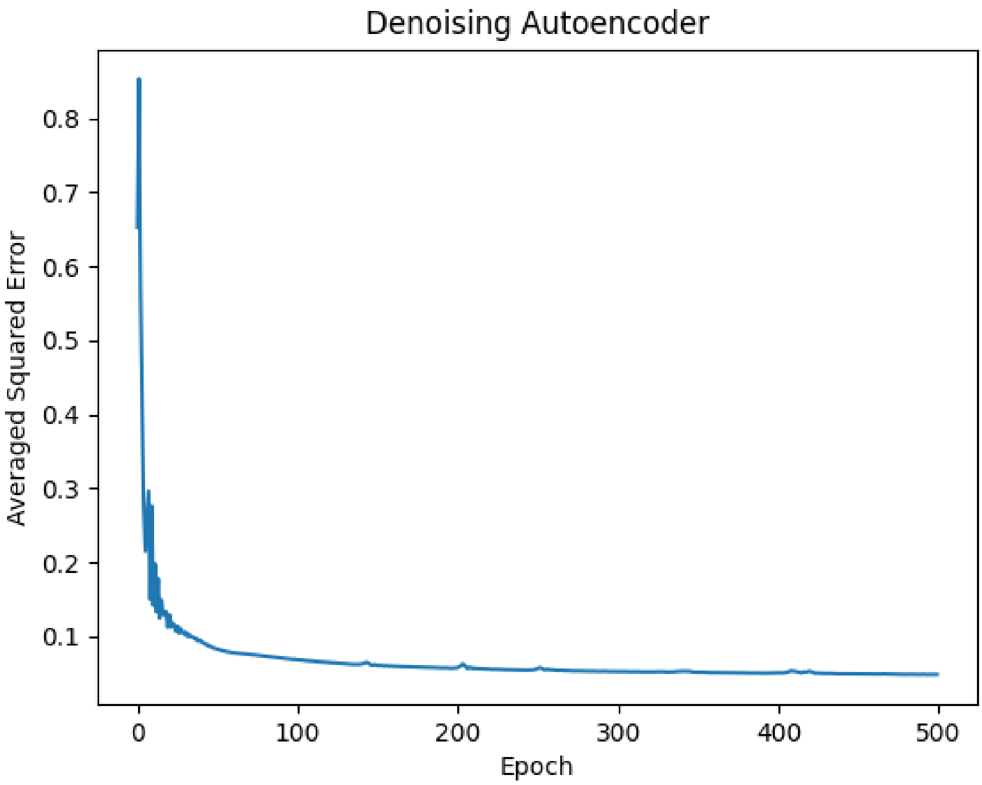

PCA Original
PCA 3-th Eigenvector


PCA Original
PCA 4-th Eigenvector

## 2.2 (b)



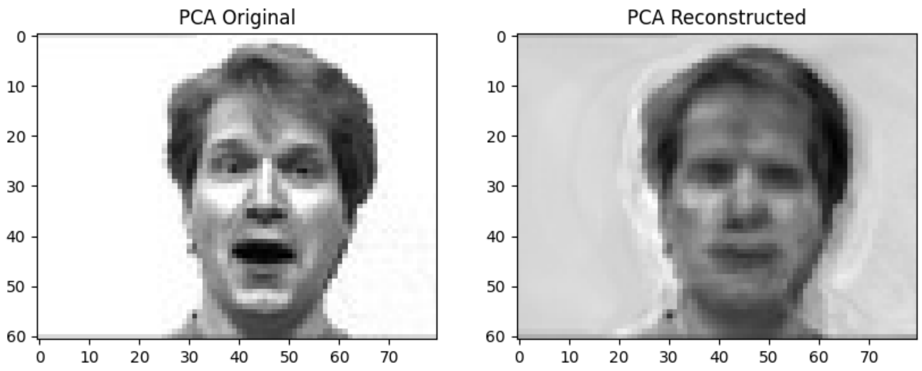
Autoencoder
Averaged Squared Error
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500
Epoch

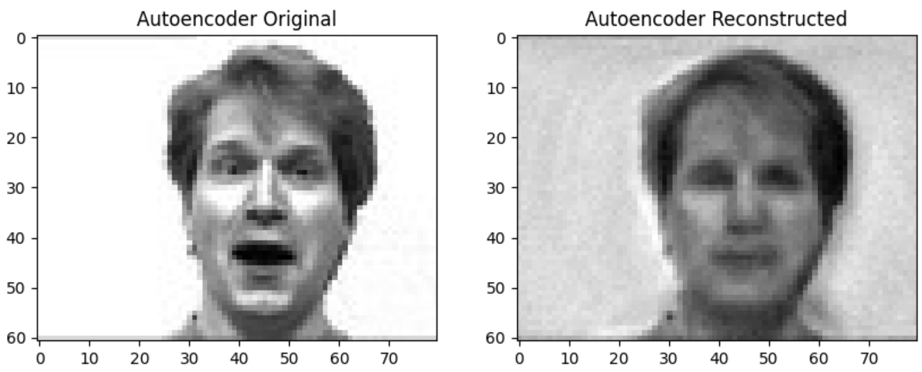
Denoising Autoencoder
Averaged Squared Error
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
Epoch

## 2.3 (c)

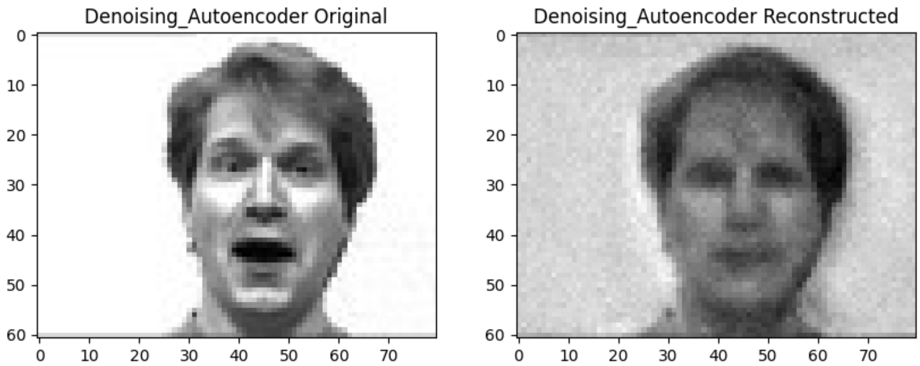
PCA mean squared error: 0.010710469688056315.




Autoencoder mean squared error: 0.012823789826138236.




DenoisingAutoencoder mean squared error: 0.01361696472773301.




## 2.4 (d)

Architecture 1:

4880 neorons → 488 neorons → 4880 neorons

Mean squared error: 0.019768476466042514.

Architecture 2:

4880 neorons → 488 neorons → 244 neorons → 488 neorons → 4880 neorons

Mean squared error: 0.013616964727733301.

Architecture 3:

4880 neorons → 128 neorons → 488 neorons → 244 neorons → 488 neorons → 128 neorons → 4880 neorons

Mean squared error: 0.01518507843373457

One can see that Architecture 2 has the best performance. The reason that Architecture 1 performs worse than Architecture 2 may be that it is not deep enough and therefore underfits the model. However, the deeper architecture doesn't imply the better performance. The reason that Architecture 3 performs worse than Architecture 2 may be that it is too deep and therefore overfits the model.