Perfect conductors. These are idealizations of good conductors in the limit $\sigma \to \infty$. For $\sigma = \infty$, the skin depth, that is, the distance in which the fields inside a conductor are attenuated by a factor e^{-1}, is zero. Hence, there can be no penetration of fields into a perfect conductor.

As a prelude to the consideration of problems involving more than one medium, we derived the boundary conditions resulting from the application of Maxwell's equations in integral form to closed paths and closed surfaces encompassing the boundary between two media, and in the limits that the areas enclosed by the closed paths and the volumes bounded by the closed surfaces go to zero. These boundary conditions are given by

$$\begin{aligned}
\mathbf{a}_n \times (\mathbf{E}_1 - \mathbf{E}_2) &= \mathbf{0} \\
\mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2) &= \mathbf{J}_S \\
\mathbf{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2) &= \rho_S \\
\mathbf{a}_n \cdot (\mathbf{B}_1 - \mathbf{B}_2) &= 0
\end{aligned}$$

where the subscripts 1 and 2 refer to media 1 and 2, respectively, and $\mathbf{a}_n$ is unit vector normal to the boundary at the point under consideration and directed into medium 1. In words, the boundary conditions state that at a point on the boundary, the tangential components of **E** and the normal components of **B** are continuous, whereas the tangential components of **H** are discontinuous by the amount equal to J_S at that point, and the normal components of **D** are discontinuous by the amount equal to ρ_S at that point.

Two important special cases of boundary conditions are as follows: **(a)** At the boundary between two perfect dielectrics, the tangential components of **E** and **H** and the normal components of **D** and **B** are continuous. **(b)** On the surface of a perfect conductor, the tangential component of **E** and the normal component of **B** are zero, whereas the normal component of **D** is equal to the surface charge density, and the tangential component of **H** is equal in magnitude to the surface current density.

Finally, we considered uniform plane waves incident normally onto a plane boundary between two media, and we learned how to compute the reflected and transmitted wave fields for a given incident wave field.

REVIEW QUESTIONS

Q4.1. Distinguish between bound electrons and free electrons in an atom and briefly describe the phenomenon of conduction.

Q4.2. Discuss the classification of a material as a conductor, semiconductor, or dielectric with the aid of energy band diagrams.

Q4.3. What is mobility? Give typical values of mobilities for electrons and holes.

Q4.4. State Ohm's law valid at a point, defining the conductivities for conductors and semiconductors.

Q4.5. Explain how conduction current in a material is taken into consideration in Maxwell's equations.

Q4.6. Discuss the formation of surface charge at the boundaries of a conductor placed in a static electric field.

Q4.7. Discuss the derivation of Ohm's law in circuit theory from the Ohm's law valid at a point.

Q4.8. Discuss the Hall effect.

Q4.9. Briefly describe the phenomenon of polarization in a dielectric material. What are the different kinds of polarization?

Q4.10. What is an electric dipole? How is its strength defined?

Q4.11. What is a polarization vector? How is it related to the electric field intensity?

Q4.12. Discuss the effect of polarization in a dielectric material using the example of polarization surface charge.

Q4.13. Discuss how polarization current arises in a dielectric material. How is it taken into account in Maxwell's equations?

Q4.14. Discuss the revised definition of displacement flux density and the permittivity concept.

Q4.15. What is an anisotropic dielectric material? When can an effective permittivity be defined for an anisotropic dielectric material?

Q4.16. Briefly describe the phenomenon of magnetization in a magnetic material. What are the different kinds of magnetic materials?

Q4.17. What is a magnetic dipole? How is its strength defined?

Q4.18. What is a magnetization vector? How is it related to the magnetic flux density?

Q4.19. Discuss the effect of magnetization in a magnetic material using the example of magnetization surface current.

Q4.20. Discuss how magnetization current arises in a magnetic material. How is it taken into account in Maxwell's equations?

Q4.21. Discuss the revised definition of magnetic field intensity and the permeability concept.

Q4.22. Discuss the phenomenon of hysteresis associated with ferromagnetic materials.

Q4.23. Discuss the principles behind storing data on a floppy disk and retrieving the data from it.

Q4.24. State the constitutive relations for a material medium.

Q4.25. Discuss the determination of the electromagnetic field due to an infinite plane current sheet of sinusoidally time-varying current density embedded in a material medium, explaining how it is made convenient by using the phasor technique.

Q4.26. What is the propagation constant for a material medium? Discuss the significance of its real and imaginary parts.

Q4.27. What is the intrinsic impedance for a material medium? What is the consequence of its complex nature?

Q4.28. What is loss tangent? Discuss its significance.

Q4.29. Discuss the consequence of the frequency dependence of the phase velocity of a wave in a material medium.

Q4.30. How would you obtain the electromagnetic field due to a current sheet of nonsinusoidally time-varying current density embedded in a material medium?

Q4.31. State Poynting's theorem for a material medium.

Q4.32. What are the power dissipation density, the electric stored energy density, and the magnetic stored energy density associated with an electromagnetic field in a material medium?

Q4.33. What is the condition for a medium to be a perfect dielectric? How do the characteristics of wave propagation in a perfect dielectric medium differ from those of wave propagation in free space?

Q4.34. What is the criterion for a material to be an imperfect dielectric? What is the significant feature of wave propagation in an imperfect dielectric as compared to that in a perfect dielectric?

Q4.35. What is the criterion for a material to be a good conductor? Give two examples of materials that behave as good conductors for frequencies of up to several gigahertz.

Q4.36. What is skin effect? Discuss skin depth, giving some numerical values.

Q4.37. Why are low-frequency waves more suitable than high-frequency waves for communication with underwater objects?

Q4.38. Discuss the consequence of the low intrinsic impedance of a good conductor as compared to that of a dielectric medium having the same ε and μ.

Q4.39. Why can there be no fields inside a perfect conductor?

Q4.40. What is a boundary condition? How do boundary conditions arise and how are they derived?

Q4.41. Summarize the boundary conditions for the general case of a boundary between two arbitrary media, indicating correspondingly the Maxwell's equations in integral form from which they are derived.

Q4.42. Discuss the boundary conditions on the surface of a perfect conductor.

Q4.43. Discuss the boundary conditions at the interface between two perfect dielectric media.

Q4.44. Discuss the determination of the reflected and transitted wave fields from the fields of a wave incident normally onto a plane boundary between two material media.

Q4.45. What is the consequence of a wave incident on a perfect conductor?

PROBLEMS

Section 4.1

P4.1. **Kinetic energy of electron motion under thermal agitation.** Consider two electrons moving under thermal agitation with velocities equal in magnitude and opposite in direction. A uniform electric field is applied along the direction of motion of one of the electrons. Show that the gain in kinetic energy by the accelerating electron is greater than the loss in kinetic energy by the decelerating electron.

P4.2. **Drift velocity of electron motion in a conductor for a sinusoidal electric field.**
(a) For a sinusoidally time-varying electric field $\mathbf{E} = \mathbf{E}_0 \cos \omega t$, where $\mathbf{E}_0$ is a constant, show that the steady-state solution to (4.2) is given by

$$\mathbf{v}_d = \frac{\tau e}{m\sqrt{1 + \omega^2\tau^2}}\mathbf{E}_0 \cos\left(\omega t - \tan^{-1}\omega\tau\right)$$

(b) Based on the assumption of one free electron per atom, the free electron density N_e in silver is $5.86 \times 10^{28}\ \text{m}^{-3}$. Using the conductivity for silver given in Table 4.1, find the frequency at which the drift velocity lags the applied field by

$\pi/4$. What is the ratio of the mobility at this frequency to the mobility at zero frequency?

P4.3. Surface charge densities for plane conducting slabs with net surface charge densities. (a) An infinite plane conducting slab carries uniformly distributed surface charges on both of its surfaces. If the net surface charge density, that is, the sum of the surface charge densities on the two surfaces, is ρ_{S0} C/m^2, find the surface charge densities on the two surfaces. **(b)** Two infinite plane parallel conducting slabs 1 and 2 carry uniformly distributed surface charges on all four of their surfaces. If the net surface charge densities are ρ_{S1} and ρ_{S2} C/m^2, respectively, for the slabs 1 and 2, find the surface charge densities on all four surfaces.

P4.4. Line charge in the presence of a plane conductor. The region $x < 0$ is occupied by a conductor. An infinitely long line charge of uniform density ρ_{L0} is situated along the line passing through $(d, 0, 0)$ and parallel to the z-axis, where $d > 0$. From the secondary field required to make the total electric field inside the conductor equal to zero and from symmetry considerations, as shown by the cross-sectional view in Fig. 4.28, show that the field outside the conductor is the same as the field due to the line charge passing through $(d, 0, 0)$ and a parallel "image" line charge of uniform density $-\rho_{L0}$ along the line passing through $(-d, 0, 0)$. Find the expression for the electric field outside the conductor. *Hint*: Use the expression for the electric field intensity due to an infinitely long line charge of uniform density ρ_{L0} along the z-axis given by $(\rho_{L0}/2\pi\varepsilon_0 r)\mathbf{a}_r$.

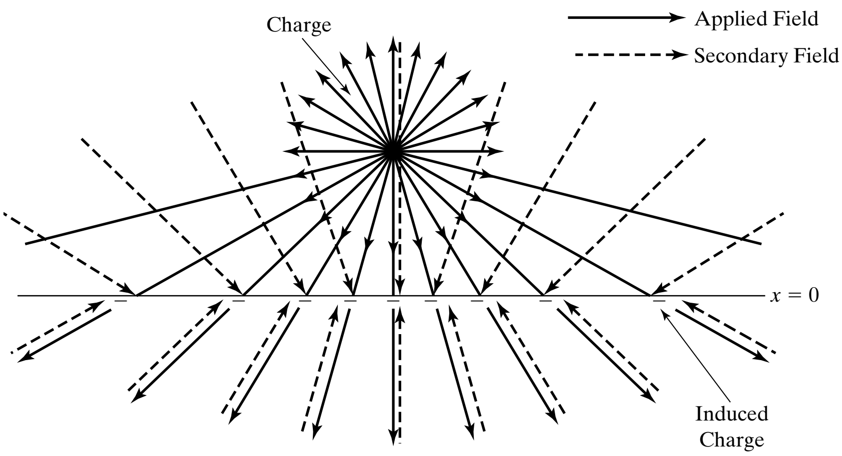

FIGURE 4.28

For Problem P4.4.

Section 4.2

P4.5. Torque on an electric dipole in an applied electric field. Show that the torque acting on an electric dipole of moment $\mathbf{p}$ due to an applied electric field $\mathbf{E}$ is $\mathbf{p} \times \mathbf{E}$. Compute the torque for a dipole consisting of $1\,\mu$C of charge at $(0, 0, 10^{-3})$ and $-1\,\mu$C of charge at $(0, 0, -10^{-3})$ in an electric field $\mathbf{E} = 10^3\,(2\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z)$ V/m.

P4.6. Point charge surrounded by a spherical dielectric shell. A point charge Q is situated at the origin surrounded by a spherical dielectric shell of uniform permittivity $4\varepsilon_0$ and having inner and outer radii a and b, respectively. Find the following: **(a)** the **D** and **E** fields in the three regions $0 < r < a$, $a < r < b$, and $r > b$ and **(b)** the polarization vector inside the dielectric shell.

P4.7. Characteristics of an anisotropic dielectric material. An anisotropic dielectric material is characterized by the **D** to **E** relationship

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

(a) Find **D** for $\mathbf{E} = E_0(\mathbf{a}_x + \mathbf{a}_y)$. **(b)** Find **D** for $\mathbf{E} = E_0(\mathbf{a}_x - \mathbf{a}_y)$. **(c)** Find **E** for $\mathbf{D} = D_0(\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)$. Comment on your result for each case.

P4.8. Characteristic polarizations and effective permittivities for an anisotropic dielectric. An anisotropic dielectric material is characterized by the **D** to **E** relationship

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

For $\mathbf{E} = E_x\mathbf{a}_x + E_y\mathbf{a}_y$, find the value(s) of E_y/E_x for which **D** is parallel to **E**. Find the effective permittivity for each case.

Section 4.3

P4.9. Magnetic dipole moment of a charged rotating disk of uniform charge density. Charge Q is distributed with uniform density on a circular disk of radius a lying in the xy-plane and rotating around the z-axis with angular velocity ω in the sense of increasing ϕ. Find the magnetic dipole moment of the rotating charge.

P4.10. Torque on a magnetic dipole in an applied magnetic field. Considering for simplicity a rectangular current loop in the xy-plane, show that the torque acting on a magnetic dipole of moment **m** due to an applied magnetic field **B** is $\mathbf{m} \times \mathbf{B}$. Then find the torque acting on a circular current loop of radius 1 mm, in the xy-plane, centered at the origin and with current 0.1 A flowing in the sense of increasing ϕ in a magnetic field $\mathbf{B} = 10^{-5}(2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$ Wb/m^2.

P4.11. Finding the parameters of a ferromagnetic material. A portion of the B–H curve for a ferromagnetic material can be approximated by the analytical expression

$$\mathbf{B} = \mu_0 kH\mathbf{H}$$

where k is a constant having units of meter per ampere. Find μ, μ_r, χ_m, and **M**.

P4.12. Finding effective permeability for an anisotropic magnetic material. An anisotropic magnetic material is characterized by the **B** to **H** relationship

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = k\mu_0 \begin{bmatrix} 7 & 6 & 0 \\ 6 & 12 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}$$

where k is a constant. Find the effective permeability for $\mathbf{H} = H_0(3\mathbf{a}_x - 2\mathbf{a}_y)$.

Section 4.4

P4.13. Finding fields for a plane-sheet sinusoidal current source in a material medium. An infinite plane sheet in the $z = 0$ plane carries a surface current of density

$$\mathbf{J}_s = -0.2 \cos 2\pi \times 10^6 t\, \mathbf{a}_x \text{ A/m}$$

The medium on either side of the sheet is characterized by $\sigma = 10^{-3}$ S/m, $\varepsilon = 6\varepsilon_0$, and $\mu = \mu_0$. Find **E** and **H** on either side of the current sheet.

P4.14. An array of two infinite plane current sheets in a material medium. Consider an array of two infinite plane, parallel, current sheets of uniform densities given by

$$\mathbf{J}_{S1} = -J_{S0} \cos 2\pi \times 10^6 t\, \mathbf{a}_x \quad \text{in the } z = 0 \text{ plane}$$
$$\mathbf{J}_{S2} = -kJ_{S0} \sin 2\pi \times 10^6 t\, \mathbf{a}_x \quad \text{in the } z = d \text{ plane}$$

situated in a medium characterized by $\sigma = 10^{-3}$ S/m, $\varepsilon = 6\varepsilon_0$, and $\mu = \mu_0$. **(a)** Find the minimum value of $d(>0)$ and the corresponding value of k for which the fields in the region $z < 0$ are zero. **(b)** For the values of d and k found in (a), obtain the electric-field intensity in the region $z > d$.

P4.15. Finding material parameters of a medium from propagation characteristics. A uniform plane wave of frequency 5×10^5 Hz propagating in a material medium has the following characteristics. **(i)** The fields are attenuated by the factor e^{-1} in a distance of 28.65 m. **(ii)** The fields undergo a change in phase by 2π in a distance of 111.2 m. **(iii)** The ratio of the amplitudes of the electric- and magnetic-field intensities at a point in the medium is 59.4. **(a)** What is the value of $\bar{\gamma}$? **(b)** What is the value of $\bar{\eta}$? **(c)** Find σ, ε, and μ of the medium.

P4.16. Finding fields for a plane-sheet nonsinusoidal current source in a material medium. Repeat Problem P4.13 for the surface current of density

$$\mathbf{J}_s = -0.2 \cos 2\pi \times 10^6 t \cos 4\pi \times 10^6 t\, \mathbf{a}_x \text{ A/m}$$

P4.17. Power flow and dissipation in a material medium. The magnetic field of a uniform plane wave propagating in a nonmagnetic ($\mu = \mu_0$) material medium is given by

$$\mathbf{H} = H_0 e^{-z} \cos (2\pi \times 10^6 t - 2z)\, \mathbf{a}_x \text{ A/m}$$

Find: **(a)** the time-average power flow per unit area normal to the z-direction and **(b)** the time-average power dissipated in the volume bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.

Section 4.5

P4.18. Finding parameters for a uniform plane-wave electric field in a perfect dielectric. The electric field of a uniform plane wave propagating in a perfect dielectric medium having $\mu = \mu_0$ is given by

$$\mathbf{E} = 10 \cos (3\pi \times 10^7 t - 0.2\pi x)\, \mathbf{a}_z$$

Find: **(a)** the frequency; **(b)** the wavelength; **(c)** the phase velocity; **(d)** the relative permittivity of the medium; and **(e)** the associated magnetic-field vector **H**.

P4.19. Plotting field variations for a nonsinusoidal current source in a perfect dielectric. An infinite plane sheet lying in the $z = 0$ plane carries a surface current of density $\mathbf{J}_S = -J_S(t)\mathbf{a}_x$ A/m, where $J_S(t)$ is as shown in Fig. 4.29. The medium on either side of the current sheet is a perfect dielectric of $\varepsilon = 2.25\varepsilon_0$ and $\mu = \mu_0$.

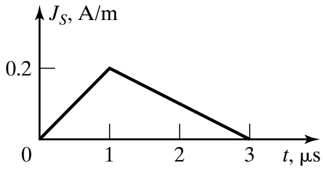

FIGURE 4.29
For Problem P4.19

Find and sketch **(a)** E_x versus t for $z = 200$ m; **(b)** H_y versus t for $z = -300$ m; **(c)** E_x versus z for $t = 2\ \mu$s; and **(d)** H_y versus z for $t = 3\ \mu$s.

P4.20. Finding the parameters of a perfect dielectric from propagation characteristics. For a uniform plane wave having $\mathbf{E} = E_x(z, t)\mathbf{a}_x$ and $\mathbf{H} = H_y(z, t)\mathbf{a}_y$ and propagating in the $+z$-direction in a perfect dielectric medium, the time variation of E_x in a constant z-plane and the distance variation of H_y for a fixed time are observed to be periodic, as shown in Figs. 4.30(a) and (b), respectively, for two complete cycles. Find the relative permittivity and the relative permeability of the medium.

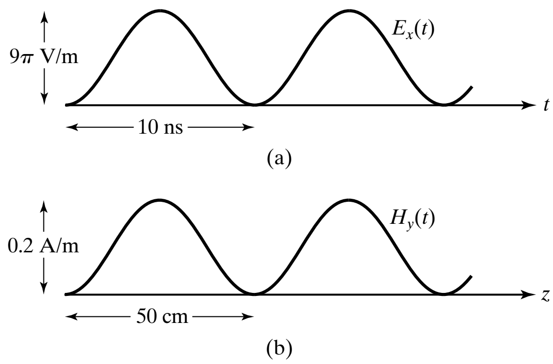

FIGURE 4.30
For Problem P4.20.

P4.21. Computing propagation parameters for a uniform plane wave in ice. For uniform plane wave propagation in ice ($\sigma \approx 10^{-6}$ S/m, $\varepsilon = 3\varepsilon_0$, and $\mu = \mu_0$), compute α, β, v_p, λ, and $\overline{\eta}$ for $f = 1$ MHz. What is the distance in which the fields are attenuated by the factor e^{-1}?

P4.22. Computing propagation parameters for a uniform plane wave in seawater. For uniform plane wave propagation in seawater ($\sigma = 4$ S/m, $\varepsilon = 80\varepsilon_0$, and $\mu = \mu_0$), compute α, δ, β, λ, v_p, and $\overline{\eta}$ for two frequencies: **(a)** $f = 10$ GHz and **(b)** $f = 100$ kHz.

P4.23. Finding the electric field for a nonsinusoidal-wave magnetic field in a material medium. For a uniform plane wave propagating in the $+z$-direction in a material medium, the magnetic field intensity in the $z = 0$ plane is given by

$$[\mathbf{H}]_{z=0} = 0.1 \cos^3 2\pi \times 10^8 t\ \mathbf{a}_y \text{ A/m}$$

Find $\mathbf{E}(z, t)$ for each of the following cases: **(a)** the medium is characterized by $\sigma = 0$, $\varepsilon = 9\varepsilon_0$, and $\mu = \mu_0$; **(b)** the medium is characterized by $\sigma = 10^{-3}$ S/m, $\varepsilon = 9\varepsilon_0$, and $\mu = \mu_0$; and **(c)** the medium is characterized by $\sigma = 10$ S/m, $\varepsilon = 9\varepsilon_0$, and $\mu = \mu_0$.

Section 4.6

P4.24. Verifying consistency of results with boundary conditions. Show that the results obtained for the electric field due to the sheet of charge in Example 1.9 and for the magnetic field due to the sheet of current in Example 1.12 are consistent with the boundary conditions.

P4.25. Applying boundary conditions at interface between dielectric and free space. Medium 1, consisting of the region $r < a$ in spherical coordinates, is a perfect dielectric of permittivity ε_1, whereas medium 2, consisting of the region $r > a$ in spherical coordinates, is free space. The electric field intensities in the two media are given by

$$\mathbf{E}_1 = E_{01}(\cos\theta\,\mathbf{a}_r - \sin\theta\,\mathbf{a}_\theta)$$
$$\mathbf{E}_2 = E_{02}\left[\left(1 + \frac{a^3}{2r^3}\right)\cos\theta\,\mathbf{a}_r - \left(1 - \frac{a^3}{4r^3}\right)\sin\theta\,\mathbf{a}_\theta\right]$$

respectively. Find ε_1.

P4.26. Applying boundary conditions at interface between dielectric and free space. A boundary separates free space from a perfect dielectric medium. At a point on the boundary, the electric field intensity on the free space side is $\mathbf{E}_1 = E_0(4\mathbf{a}_x + 2\mathbf{a}_y + 5\mathbf{a}_z)$, whereas on the dielectric side, it is $\mathbf{E}_2 = 3E_0(\mathbf{a}_x + \mathbf{a}_z)$, where E_0 is a constant. Find the permittivity of the dielectric medium.

P4.27. Applying boundary conditions at interface between magnetic material and free space. Medium 1, consisting of the region $r < a$ in spherical coordinates, is a magnetic material of permeability μ_1, whereas medium 2, consisting of the region $r > a$ in spherical coordinates, is free space. The magnetic flux densities in the two media are given by

$$\mathbf{B}_1 = B_{01}(\cos\theta\,\mathbf{a}_r - \sin\theta\,\mathbf{a}_\theta)$$
$$\mathbf{B}_2 = B_{02}\left[\left(1 + 1.94\frac{a^3}{r^3}\right)\cos\theta\,\mathbf{a}_r - \left(1 - 0.97\frac{a^3}{r^3}\right)\sin\theta\,\mathbf{a}_\theta\right]$$

respectively. Find μ_1.

P4.28. Verification and application of boundary conditions on a perfect conductor surface. In Problem P4.4, show that the applied and secondary fields together satisfy the boundary condition of zero tangential component of electric field on the conductor surface. From the boundary condition for the normal component of **D**, find the charge density on the conductor surface and show that the total induced surface charge per unit width in the z-direction is $-\rho_{L0}$.

P4.29. Applying boundary conditions for a rectangular cavity resonator. The rectangular cavity resonator is a box consisting of the region $0 < x < a$, $0 < y < b$, and $0 < z < d$, and bounded by perfectly conducting walls on all of its six sides. The time-varying electric and magnetic fields inside the resonator are given by

$$\mathbf{E} = E_0 \sin\frac{\pi x}{a}\sin\frac{\pi z}{d}\cos\omega t\,\mathbf{a}_y$$
$$\mathbf{H} = H_{01}\sin\frac{\pi x}{a}\cos\frac{\pi z}{d}\sin\omega t\,\mathbf{a}_x - H_{02}\cos\frac{\pi x}{a}\sin\frac{\pi z}{d}\sin\omega t\,\mathbf{a}_z$$

where E_0, H_{01}, and H_{02} are constants. Find ρ_S and $\mathbf{J}_S$ on all six walls, assuming the medium inside the box to be a perfect dielectric of $\varepsilon = 4\varepsilon_0$.

P4.30. Finding fields for a plane-sheet current source with different media on either side. In Problem P4.13, assume that the region $z > 0$ is free space, whereas the region $z < 0$ is a material medium characterized by $\sigma = 10^{-3}$ S/m, $\varepsilon = 6\varepsilon_0$, and $\mu = \mu_0$. Find **E** and **H** on either side of the current sheet. (*Hint:* Make use of the complex electric and magnetic fields to satisfy the boundary conditions at $z = 0$.)

P4.31. Finding fields for a plane-sheet current source with different dielectrics on either side. An infinite plane sheet lying in the $z = 0$ plane carries a surface current of density

$$\mathbf{J}_s = -0.2 \cos 6\pi \times 10^8 t \, \mathbf{a}_x \text{ A/m}$$

The region $z > 0$ is a perfect dielectric of $\varepsilon = 2.25\varepsilon_0$ and $\mu = \mu_0$, whereas the region $z < 0$ is a perfect dielectric of $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$. Find **E** and **H** on both sides of the sheet.

Section 4.7

P4.32. Normal incidence of a sinusoidal uniform plane wave onto a material medium. Region 1 ($z < 0$) is free space, whereas region 2 ($z > 0$) is a material medium characterized by $\sigma = 10^{-4}$ S/m, $\varepsilon = 5\varepsilon_0$, and $\mu = \mu_0$. For a uniform plane wave having the electric field

$$\mathbf{E}_i = E_0 \cos (3\pi \times 10^5 t - 10^{-3}\pi z) \, \mathbf{a}_x \text{ V/m}$$

incident on the interface $z = 0$ from region 1, obtain the expression for the reflected and transmitted wave electric fields.

P4.33. Normal incidence of a nonsinusoidal uniform plane wave onto a material medium. Repeat Problem P4.32 for the incident wave electric field given by

$$\mathbf{E}_i = E_0 \cos^3 (3\pi \times 10^5 t - 10^{-3}\pi z) \, \mathbf{a}_x \text{ V/m}$$

P4.34. Uniform plane wave reflection and transmission involving three media in cascade. In Fig. 4.31, medium 3 extends to infinity so that no reflected (−) wave exists in that medium. For a uniform plane wave having the electric field

$$\mathbf{E}_i = E_0 \cos (3 \times 10^8 \pi t - \pi z) \, \mathbf{a}_x \text{ V/m}$$

incident from medium 1 onto the interface $z = 0$, obtain the expressions for the phasor electric- and magnetic-field components in all three media.

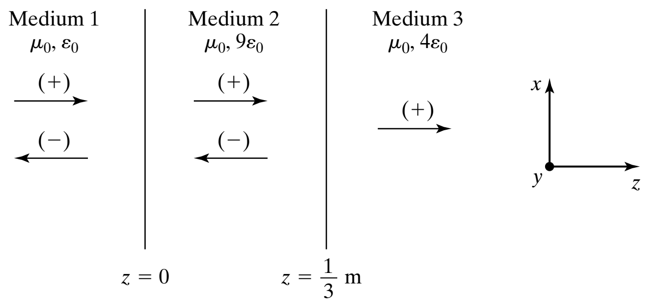

FIGURE 4.31
For Problem P4.34.

P4.35. Plotting field variations for a nonsinusoidal wave incident on a perfect dielectric. A uniform plane wave propagating in the $+z$-direction and having the electric field $\mathbf{E}_i = E_{xi}(t)\mathbf{a}_x$, where $E_{xi}(t)$ in the $z = 0$ plane is as shown in Fig. 4.32, is incident normally from free space ($z < 0$) onto a nonmagnetic ($\mu = \mu_0$), perfect dielectric ($z > 0$) of permittivity $4\varepsilon_0$. Find and sketch the following: **(a)** E_x versus z for $t = 1$ μs and **(b)** H_y versus z for $t = 1$ μs.

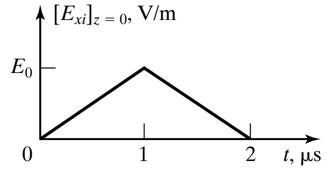

FIGURE 4.32
For Problem P4.35

P4.36. Normal incidence of a uniform plane wave on a perfect conductor surface. The region $z < 0$ is a perfect dielectric, whereas the region $z > 0$ is a perfect conductor, as shown in Fig. 4.33. For a uniform plane wave having the electric and magnetic fields

$$\mathbf{E}_i = E_0 \cos(\omega t - \beta z)\,\mathbf{a}_x$$
$$\mathbf{H}_i = \frac{E_0}{\eta}\cos(\omega t - \beta z)\,\mathbf{a}_y$$

where $\beta = \omega\sqrt{\mu\varepsilon}$ and $\eta = \sqrt{\mu/\varepsilon}$, obtain the expressions for the reflected wave electric and magnetic fields and hence the expressions for the total (incident + reflected) electric and magnetic fields in the dielectric, and the current density on the surface of the perfect conductor.

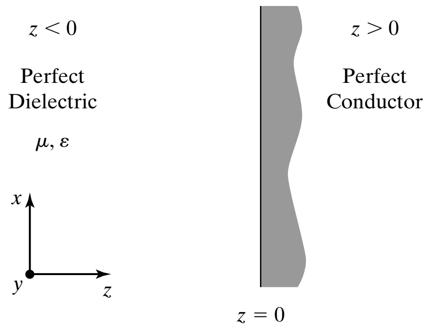

FIGURE 4.33
For Problem P4.36.

REVIEW PROBLEMS

R4.1. Finding surface charge densities for plane conducting slabs between two sheets of charge. Two infinite plane conducting slabs lie between and parallel to two infinite plane sheets of uniform surface charge densities ρ_{SA} and ρ_{SB}, as shown by the cross-sectional view in Fig. 4.34. Find the surface charge densities on all four surfaces of the slabs.

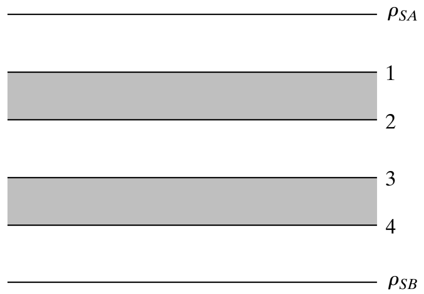

FIGURE 4.34
For Problem R4.1.

R4.2. Characteristic polarizations for an anisotropic dielectric. An anisotropic dielectric material is characterized by the **D** to **E** relationship

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 6.5 & 1.5 & 0 \\ 1.5 & 2.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

Express $\mathbf{E} = E_0(\mathbf{a}_x - \mathbf{a}_y)$ as the linear combination of $\mathbf{E}_1$ and $\mathbf{E}_2$, which correspond to two of the characteristic polarizations of the material.

R4.3. Magnetic dipole moment of a charged rotating disk of nonuniform charge density. Charge Q is distributed with density proportional to r on a circular disk of radius a lying on the xy-plane with its center at the origin and rotating around the z-axis with angular velocity ω in the sense of increasing ϕ. Find the magnetic dipole moment.

R4.4. Finding H and the material parameters of a nonmagnetic medium from E in the medium. The electric field of a uniform plane wave propagating in the $+z$-direction in a nonmagnetic ($\mu = \mu_0$) material medium is given by

$$\mathbf{E} = 8.4e^{-0.0432z} \cos(4\pi \times 10^6 t - 0.1829z)\, \mathbf{a}_x \text{ V/m}$$

Find the magnetic field of the wave. Further, find the values of σ and ε of the medium.

R4.5. Infinite plane current sheet sandwiched between two different perfect dielectric media. An infinite plane current sheet of uniform density $\mathbf{J}_S = -J_S(t)\mathbf{a}_x$ is sandwiched between two perfect dielectric media, as shown in Fig. 4.35(a). If $J_S(t)$ is a triangular pulse of duration 3 μs, the plots of $E_x(t)$ at some value of z equal to z_0 (>0) and $H_y(z)$ for some value of t equal to t_0 (>0) are given by Figs. 4.35(b) and (c), respectively. If $J_S(t) = J_{S0} \cos 6\pi \times 10^8 t$ A/m, instead of being a pulse, find **E** and **H** on both sides of the sheet, and the time-average power radiated by the sheet for unit area of the sheet.

R4.6. Application of boundary conditions on a perfect conductor surface. The region $3x + 4y + 12z < 12$ is occupied by a perfect conductor. If at a point on the perfect conductor surface, the surface charge and current densities at a particular instant of time are ρ_{S0} C/m^2 and $J_{S0}(4\mathbf{a}_x - 3\mathbf{a}_y)$A/m, find **D** and **H** at that point at that instant of time.

R4.7. Application of boundary conditions at interface between dielectric and free space. Medium 1, consisting of the region $r < a$ in spherical coordinates, is a perfect dielectric of permittivity $\varepsilon_1 = 2\varepsilon_0$, whereas medium 2, consisting of the

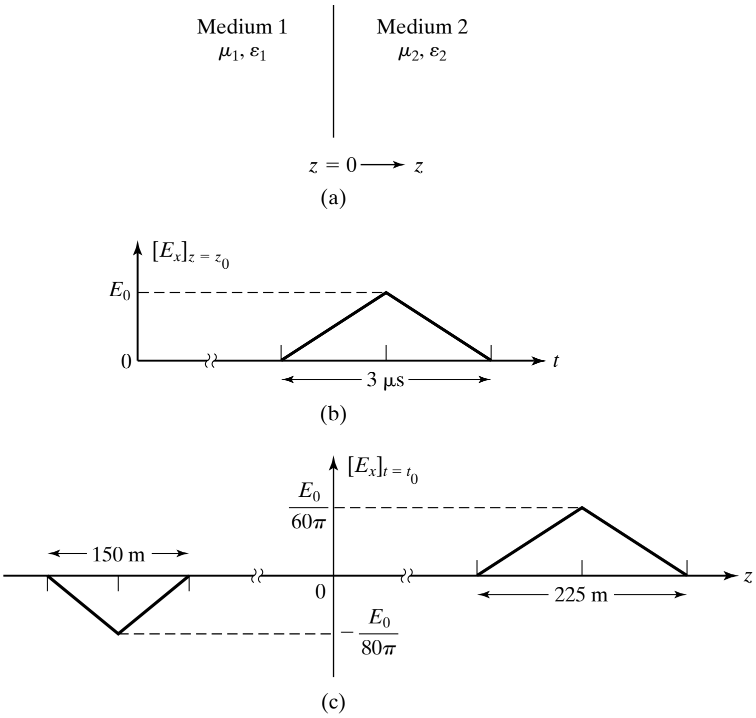

FIGURE 4.35

For Problem R4.5.

region $r > a$, is free space. The electric field intensity in medium 1 is given by $\mathbf{E}_1 = E_0\mathbf{a}_z$. Find the electric field intensity at the points **(a)** $(0, 0, a)$, **(b)** $(0, a, 0)$, and **(c)** $(0, a/\sqrt{2}, a/\sqrt{2})$, in Cartesian coordinates, in medium 2.

R4.8. Normal incidence of a uniform plane wave onto a slab of perfect dielectric. For a sinusoidally time-varying uniform plane wave incident normally from medium 1 on to the interface $z = 0$ in Fig. 4.36, show that there is a minimum value of the frequency for which a wave at that frequency or any integer multiple of that frequency undergoes no reflection at the interface. Further, find the maximum value of the period of a nonsinusoidal periodic wave for which no reflection occurs at the interface. Note that medium 1 and medium 3 are both free space.

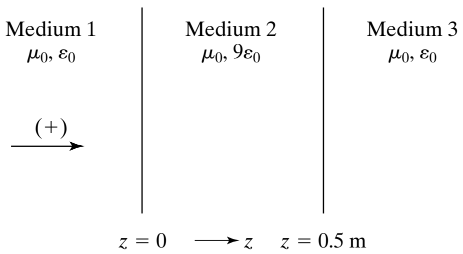

FIGURE 4.36

For Problem R4.8.