REVIEW QUESTIONS

Q5.1. What are electromagnetic potentials? How do they arise?

Q5.2. What is the expansion for the gradient of a scalar in Cartesian coordinates? When can a vector be expressed as the gradient of a scalar?

Q5.3. Discuss the physical interpretation for the gradient of a scalar function and the application of the gradient concept for the determination of unit vector normal to a surface.

Q5.4. How is the Laplacian of a scalar defined? What is its expansion in Cartesian coordinates?

Q5.5. Compare and contrast the operations of curl of a vector, divergence of a vector, gradient of a scalar, and Laplacian of a scalar.

Q5.6. How is the Laplacian of a vector defined? What is its expansion in Cartesian coordinates?

Q5.7. Outline the derivation of the differential equations for the electromagnetic potentials.

Q5.8. What is the relationship between the static electric field intensity and the electric scalar potential?

Q5.9. Distinguish between voltage, as applied to time-varying fields, and the potential difference in a static electric field.

Q5.10. Describe the electric potential field of a point charge.

Q5.11. Discuss the determination of the electric field intensity due to a charge distribution by using the potential concept.

Q5.12. Discuss the procedure for the computer plotting of equipotentials due to two (or more) point charges.

Q5.13. Compare the magnetic vector potential field due to a current element to the electric scalar potential due to a point charge.

Q5.14. State Poisson's equation. How is it derived?

Q5.15. Discuss the application of Poisson's equation for the determination of potential due to the space charge layer in a *p-n* junction semiconductor.

Q5.16. State Laplace's equation. In what regions is it valid?

Q5.17. Discuss the application of Laplace's equation for a conducting medium.

Q5.18. Outline the solution of Laplace's equation in one dimension by considering the variation of potential with x only.

Q5.19. Outline the steps in the derivation of the expression for the capacitance of an arrangement of two conductors.

Q5.20. Discuss the relationship between the capacitance, conductance, and inductance per unit length for an infinitely long, parallel conductor arrangement.

Q5.21. Outline the steps in the derivation of the expressions for the capacitance, conductance, and inductance per unit length of an infinitely long parallel cylindrical-wire arrangement.

Q5.22. Distinguish between internal inductance and external inductance. Discuss the concept of flux linkage pertinent to the determination of the internal inductance.

Q5.23. Explain the concept of mutual inductance and discuss an example of its computation.

Q5.24. What is meant by the quasistatic extension of the static field in a physical structure?

Q5.25. Outline the steps involved in the quasistatic extension of the static field in a parallel-plate structure short-circuited at one end.

Q5.26. Discuss the derivation of the condition for the validity of the quasistatic approximation for the parallel-plate structure short-circuited at one end.

Q5.27. Discuss the general condition for the quasistatic approximation of a physical structure.

Q5.28. Discuss the classification of physical structures as electric- and magnetic-field systems.

Q5.29. Discuss the low-frequency behavior of a parallel-plate structure with a lossy medium between the plates.

Q5.30. Discuss the quasistatic behavior of the structure of Fig. 5.20 for $\sigma \approx 0$.

Q5.31. What is a magnetic circuit? Why is the magnetic flux in a magnetic circuit confined almost entirely to the core?

Q5.32. Define the reluctance of a magnetic circuit. What is the analogous electric circuit quantity? Why is the reluctance for a given set of dimensions of a magnetic circuit not a constant?

Q5.33. Discuss the complete analogy between a magnetic circuit and an electric circuit using the example of the toroidal magnetic core versus the toroidal conductor.

Q5.34. How is the fringing of the magnetic flux in an air gap in a magnetic circuit taken into account?

Q5.35. Discuss by means of an example the analysis of a magnetic circuit with three legs and its equivalent-circuit representation.

Q5.36. Discuss by means of an example the phenomenon of electromechanical energy conversion.

Q5.37. Outline the computation of mechanical force of electric origin from considerations of energy balance associated with an electromechanical system.

Q5.38. Discuss by means of an example the computation of energy converted from electrical to mechanical, or vice versa, in an electromechanical system.

PROBLEMS

Section 5.1

P5.1. **Two identities in vector calculus.** Show by expansion in the Cartesian coordinate system that: **(a)** $\nabla \cdot \nabla \times \mathbf{A} = 0$ for any $\mathbf{A}$ and **(b)** $\nabla \times \nabla\Phi = \mathbf{0}$ for any Φ.

P5.2. **Application of identities in vector calculus.** Determine which of the following vectors can be expressed as the curl of another vector and which of them can be expressed as the gradient of a scalar:

(a) $xy\mathbf{a}_x + yz\mathbf{a}_y + zx\mathbf{a}_z$

(b) $(1/r^2)(\cos\phi\,\mathbf{a}_r + \sin\phi\,\mathbf{a}_\phi)$ in cylindrical coordinates

(c) $(1/r)\sin\theta\,\mathbf{a}_\phi$ in spherical coordinates.

P5.3. **Finding a scalar function for which the gradient is a given vector function.** Find the scalar functions whose gradients are given by the following vector functions:

(a) $e^{-y}(\cos x\,\mathbf{a}_x - \sin x\,\mathbf{a}_y)$

(b) $(\cos\phi\,\mathbf{a}_r - \sin\phi\,\mathbf{a}_\phi)$ in cylindrical coordinates

(c) $(1/r^3)(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta)$ in spherical coordinates

P5.4. Application of the gradient concept. By using the gradient concept, show that the unit vector along the line of intersection of two planes

$$a_1x + a_2y + a_3z = c_1$$
$$b_1x + b_2y + b_3z = c_2$$

which are not parallel is given by

$$\pm \frac{(a_2b_3 - a_3b_2)\mathbf{a}_x + (a_3b_1 - a_1b_3)\mathbf{a}_y + (a_1b_2 - a_2b_1)\mathbf{a}_z}{\sqrt{(a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2}}$$

Then find the unit vector along the intersection of the planes $x + y + z = 3$ and $y = x$.

P5.5. Application of the gradient concept. By using the gradient concept, show that the equation of the plane passing through the point (x_0, y_0, z_0) and normal to the vector $(a\mathbf{a}_x + b\mathbf{a}_y + c\mathbf{a}_z)$ is given by

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Then find the equation of the plane tangential to the surface $xyz = 1$ at the point $(\frac{1}{2}, \frac{1}{4}, 8)$.

P5.6. Laplacian of a vector in cylindrical coordinates. Show that the Laplacian of a vector in cylindrical coordinates is given by

$$\nabla^2\mathbf{A} = \left(\nabla^2 A_r - \frac{A_r}{r^2} - \frac{2}{r^2}\frac{\partial A_\phi}{\partial \phi}\right)\mathbf{a}_r + \left(\nabla^2 A_\phi - \frac{A_\phi}{r^2} + \frac{2}{r^2}\frac{\partial A_r}{\partial \phi}\right)\mathbf{a}_\phi + (\nabla^2 A_z)\mathbf{a}_z$$

Section 5.2

P5.7. Equipotential surfaces and direction lines of electric field for a given electric potential. For the static electric potential function $V(x, y) = xy$, discuss the equipotential surfaces and the direction lines of the electric field with the aid of sketches.

P5.8. Electric potential and field for a rectangular quadrupole. An arrangement of point charges known as the rectangular quadrupole consists of the point charges $Q, -Q, Q$, and $-Q$, at the points $(0, 0, 0)$, $(\Delta x, 0, 0)$, $(\Delta x, 0, \Delta z)$, and $(0, 0, \Delta z)$, respectively. Obtain the approximate expression for the electric potential and hence for the electric field intensity due to the rectangular quadrupole at distances r from the origin large compared to Δx and Δz.

P5.9. Electric potential for a finitely long line charge. For a finitely long line charge of uniform density ρ_{L0} C/m situated along the line between $(0, 0, -a)$ and $(0, 0, a)$, obtain the expression for the electric potential at an arbitrary point (r, ϕ, z) in cylindrical coordinates. Further show that the equipotential surfaces are ellipsoids with the ends of the line as their focii.

P5.10. Electric potential for two parallel infinitely long line charges. Show that for two infinitely long line charges parallel to the z-axis, having uniform densities $\rho_{L1} = 2k\pi\varepsilon_0$ C/m and $\rho_{L2} = -2\pi\varepsilon_0$ C/m and passing through $(-1, 0, 0)$ and $(1, 0, 0)$, respectively, the potential is given by $V = \ln (r_2/r_1^k)$, where r_1 and r_2 are distances to the point from the line charges 1 and 2, respectively.

P5.11. Electric potential at the center of a rectangular uniformly distributed surface charge. Consider the surface charge distributed uniformly with density ρ_{S0} C/m^2

on a rectangular-shaped surface of sides a and b. Show that the electric potential at the center of the rectangle is

$$\frac{\rho_{S0}}{2\pi\varepsilon_0}\left(a \ln \frac{\sqrt{a^2 + b^2} + b}{a} + b \ln \frac{\sqrt{a^2 + b^2} + a}{b}\right)$$

Further show that for a square-shaped surface of sides a, the potential at the center is $(\rho_{S0}a/\pi\varepsilon_0) \ln(1 + \sqrt{2})$.

P5.12. Potential difference in the field of an infinitely long strip of surface charge. Consider surface charge of uniform density ρ_{S0} C/m^2 distributed on an infinitely long strip lying between the straight lines $x = -a$, $y = 0$ and $x = a$, $y = 0$. Noting that the electric potential is independent of z, show that the potential difference between two points in the first quadrant of the xy-plane $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ is given by

$$\begin{aligned}\frac{\rho_{S0}}{4\pi\varepsilon_0}\Bigg\{&(a - x_2) \ln[(a - x_2)^2 + y_2^2] + (a + x_2) \ln[(a + x_2)^2 + y_2^2]\\ &- (a - x_1) \ln[(a - x_1)^2 + y_1^2] - (a + x_1) \ln[(a + x_1)^2 + y_1^2]\\ &+ 2y_2\left(\tan^{-1}\frac{a - x_2}{y_2} + \tan^{-1}\frac{a + x_2}{y_2}\right)\\ &- 2y_1\left(\tan^{-1}\frac{a - x_1}{y_1} + \tan^{-1}\frac{a + x_1}{y_1}\right)\Bigg\}\end{aligned}$$

P5.13. Magnetic vector potential and field for a magnetic dipole. Consider a circular current loop of radius a lying in the xy-plane with its center at the origin and with current I flowing in the sense of increasing ϕ, so that the magnetic dipole moment $\mathbf{m}$ is $I\pi a^2\,\mathbf{a}_z$. Show that far from the dipole such that $r \gg a$, the magnetic vector potential is given by

$$\mathbf{A} \approx \frac{\mu\mathbf{m} \times \mathbf{a}_r}{4\pi r^2}$$

and hence the magnetic flux density is given by

$$\mathbf{B} = \frac{\mu m}{4\pi r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta)$$

P5.14. An identity in vector calculus. By expansion in Cartesian coordinates, show that

$$\mathbf{A} \times \nabla\Phi = \Phi\nabla \times \mathbf{A} - \nabla \times (\Phi\mathbf{A})$$

Section 5.3

P5.15. Solution of Poisson's equation for a space-charge distribution in Cartesian coordinates. A space-charge density distribution is given by

$$\rho = \begin{cases} -\rho_0\left(1 + \dfrac{x}{d}\right) & \text{for} \quad -d < x < 0 \\ \rho_0\left(1 - \dfrac{x}{d}\right) & \text{for} \quad 0 < x < d \\ 0 & \text{otherwise} \end{cases}$$

where ρ_0 is a constant. Obtain the solution for the potential V versus x for all x. Assume $V = 0$ for $x = 0$.

P5.16. Solution of Poisson's equation for a space-charge distribution in Cartesian coordinates. A space-charge density distribution is given by

$$\rho = \begin{cases} \rho_0 \sin x & \text{for } -\pi < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

where ρ_0 is a constant. Find and sketch the potential V versus x for all x. Assume $V = 0$ for $x = 0$.

P5.17. Solution of Poisson's equation for a space-charge distribution in spherical coordinates. A space-charge density distribution is given in spherical coordinates by

$$\rho = \begin{cases} \rho_0 & \text{for} \quad a < r < 2a \\ 0 & \text{otherwise} \end{cases}$$

where ρ_0 is a constant. Find and sketch the potential V versus r for all r.

P5.18. Solution of Laplace's equation for a parallel-plate capacitor with two perfect dielectrics. The region between the two plates in Fig. 5.10 is filled with two perfect dielectric media having permittivities ε_1 for $0 < x < t$ (region 1) and ε_2 for $t < x < d$ (region 2). **(a)** Find the solutions for the potentials in the two regions $0 < x < t$ and $t < x < d$. **(b)** Find the capacitance per unit area of the plates.

P5.19. Solution of Laplace's equation for a parallel-plate capacitor with imperfect dielectrics. Assume that the two media in Problem P5.18 are imperfect dielectrics having conductivities σ_1 and σ_2 for $0 < x < t$ and $t < x < d$, respectively. **(a)** What are the boundary conditions to be satisfied at $x = t$? **(b)** Find the solutions for the potentials in the two regions. **(c)** Find the potential at $x = t$.

P5.20. Parallel-plate capacitor with a dielectric of nonuniform permittivity. Assume that the region between the two plates of Fig. 5.10 is filled with a perfect dielectric of nonuniform permittivity

$$\varepsilon = \frac{\varepsilon_0}{1 - (x/2d)}$$

Find the solution for the potential between the plates and obtain the expression for the capacitance per unit area of the plates.

P5.21. Coaxial cylindrical capacitor with a dielectric of nonuniform permittivity. Assume that the region between the coaxial cylindrical conductors of Fig. 5.11(a) is filled with a dielectric of nonuniform permittivity $\varepsilon = \varepsilon_0 b/r$. Obtain the solution for the potential between the conductors and the expression for the capacitance per unit length of the cylinders.

Section 5.4

P5.22. Capacitance per unit length of parallel wire line with large spacing between the wires. For the parallel-wire arrangement of Fig. 5.13(c), show that for $d \gg a$, the capacitance per unit length of the line is $\pi\varepsilon/\ln(2d/a)$. Find the value of d/a for which the exact value of the capacitance per unit length is 1.05 times the value given by the approximate expression for $d \gg a$.

P5.23. Direction lines of electric field for a parallel-wire line. For the line-charge pair of Fig. 5.15, show that the direction lines of the electric field are arcs of circles emanating from the positively charged line and terminating on the negatively charged line.

P5.24. Inductance of a toroid with magnetic core. A filamentary wire carrying current I is closely wound around a toroidal magnetic core of rectangular cross section, as shown in Fig. 5.32. The mean radius of the toroidal core is a and the number of turns per unit length along the mean circumference of the toroid is N. Find the inductance of the toroid.

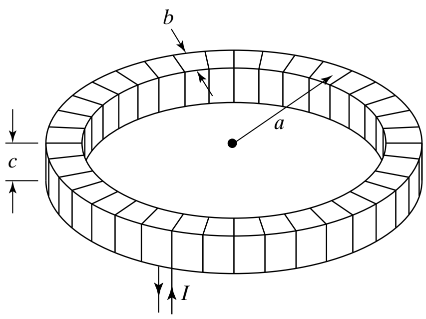

FIGURE 5.32
For Problem P5.24.

P5.25. Inductance per unit length of an infinitely long, uniformly wound solenoid. An infinitely long, uniformly wound solenoid of radius a and having N turns per unit length carries a current I. Find the inductance per unit length of the solenoid. Assume air core ($\mu = \mu_0$).

P5.26. Internal inductance per unit length of a wire with nonuniform current distribution. A current I flows with nonuniform volume density given by

$$\mathbf{J} = J_0\left(\frac{r}{a}\right)^2 \mathbf{a}_z$$

along an infinitely long cylindrical conductor of radius a having the z-axis as its axis. The current returns with uniform surface density in the opposite direction along the surface of an infinitely long, perfectly conducting cylinder of radius $b\ (> a)$ and coaxial with the inner conductor. Find the internal inductance per unit length of the inner conductor.

P5.27. Magnetic energy stored in an infinitely long cylindrical conductor of current. Consider the infinitely long solid cylindrical conductor of Fig. 5.16. Obtain the expression for the energy stored per unit length in the magnetic field internal to the current distribution and show that it is equal to $\frac{1}{2}\mathcal{L}_i I^2$, where I is the total current.

P5.28. Mutual inductance per unit length of two coaxial solenoids. An infinitely long, uniformly wound solenoid of radius a and having N_1 turns per unit length is coaxial with another infinitely long, uniformly wound solenoid of radius $b\ (> a)$ and having N_2 turns per unit length. Find the mutual inductance per unit length of the solenoids. Assume air core ($\mu = \mu_0$).

Section 5.5

P5.29. Input behavior of an inductor at low and high frequencies. For the structure of Fig. 5.19, assume that $l = 10$ cm, $d = 5$ mm, and $w = 5$ cm and free space for the medium between the plates. **(a)** For a current source $I(t) = 1 \cos 10^6 \pi t$ A, find the voltage developed across the source. **(b)** Repeat part (a) for $I(t) = 1 \cos 10^9 \pi t$ A.

P5.30. Frequency behavior of a capacitor beyond the quasistatic approximation. For the structure of Fig. 5.20 with $\sigma = 0$, show that the input behavior for frequencies slightly beyond those for which the quasistatic approximation is valid is equivalent to the series combination of $C(= \varepsilon wl/d)$ and $\frac{1}{3}L$, where $L = \mu\, dl/w$ is the inductance of the structure obtained from static-field considerations with the two plates joined by another conductor at $z = 0$, as in Fig. 5.19.

P5.31. Quasistatic input behavior of a resistor for three different cases. Find the conditions under which the quasistatic input behavior of the structure of Fig. 5.20 is essentially equivalent to that of: **(a)** a single resistor; **(b)** a capacitor $C(= \varepsilon wl/d)$ in parallel with a resistor; and **(c)** a resistor in series with an inductor.

P5.32. Frequency behavior of an inductor with material having nonzero conductivity. For the structure of Fig. 5.19, assume that the medium has nonzero conductivity σ. **(a)** Show that the input behavior correct to the first power in ω is the same as if σ were zero. **(b)** Investigate the input behavior correct to the second power in ω and obtain the equivalent circuit.

P5.33. Frequency behavior of an inductor beyond the quasistatic approximation. For the structure of Fig. 5.19, obtain the equivalent circuit for the input behavior for frequencies for which the fields up to and including the fifth-order terms in ω are significant.

Section 5.6

P5.34. Calculations involving a toroidal magnetic core. A toroidal magnetic core has the dimensions $A = 5\ \text{cm}^2$ and $l = 20$ cm. **(a)** If it is found that for *NI* equal to 200 A-t, a magnetic flux ψ equal to 8×10^{-4} Wb is established in the core, find the permeability μ of the core material. **(b)** If now an air gap of width $l_g = 0.1$ mm is introduced, find the new value of *NI* required to maintain the flux of 8×10^{-4} Wb, neglecting fringing of flux in the air gap.

P5.35. Calculations involving a magnetic circuit with three legs and an air gap. For the magnetic circuit of Fig. 5.27, assume the air gap to be in the center leg. Find the *NI* required to establish a magnetic flux of 9×10^{-4} Wb in the air gap.

P5.36. Calculations involving a magnetic circuit with three legs and two air gaps. For the magnetic circuit of Fig. 5.27, assume that there is an air gap of length 0.2 mm in the left leg in addition to that in the right leg. Find the *NI* required to establish a magnetic flux of 4×10^{-4} Wb in the air gap in the right leg.

P5.37. Magnetic circuit with a center leg and two symmetrical side legs. For the magnetic circuit of Fig. 5.27, assume that there is no air gap. Find the magnetic flux established in the center leg for an applied *NI* equal to 180 A-t.

P5.38. Magnetic circuit with a center leg and two asymmetrical side legs. For the magnetic circuit of Fig. 5.27, assume that there is no air gap and that $A_1 = 5\ \text{cm}^2$, with all other dimensions remaining as specified in Example 5.1. Find the magnetic flux density in the center leg for an applied *NI* equal to 150 A-t.

Section 5.7

P5.39. Finding the mechanical force of electric origin for a parallel-plate capacitor system. In Fig. 5.33, a dielectric slab of permittivity ε sliding between the plates of a parallel-plate capacitor experiences a mechanical force $\mathbf{F}_e$ of electrical origin. Assuming width w for the plates normal to the page and neglecting fringing of fields at the edges of the plates, find the expression for $\mathbf{F}_e$.

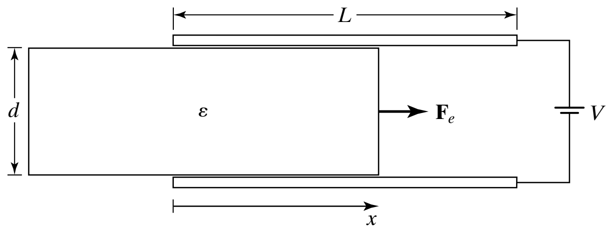

FIGURE 5.33

For Problem P5.39.

P5.40. Finding the mechanical force of electric origin for a cylindrical capacitor system. In Fig. 5.34, a dielectric material of permittivity ε sliding freely in a cylindrical capacitor experiences a mechanical force $\mathbf{F}_e$ of electrical origin in the axial direction. Show that

$$\mathbf{F}_e = \frac{V_0^2 \pi(\varepsilon - \varepsilon_0)}{\ln (b/a)} \mathbf{a}_x$$

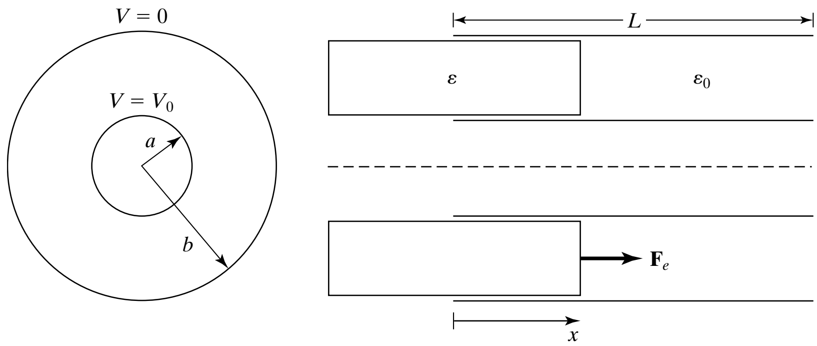

FIGURE 5.34

For Problem P5.40.

P5.41. Energy conversion in a parallel-plate capacitor with a movable plate. Assume that in Example 5.12, the parallel-plate capacitor system of Fig. 5.29 is made to traverse the closed cycle in the V-x plane shown in Fig. 5.35 instead of the closed cycle in the Q-x plane shown in Fig. 5.30. Calculate the energy converted per cycle and determine whether the conversion is from mechanical to electrical or vice versa.

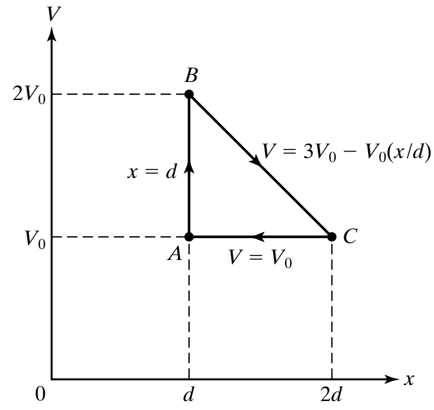

FIGURE 5.35
For Problem P5.41.

P5.42. Energy conversion in a solenoidal coil with sliding magnetic core. Figure 5.36(a) shows a magnetic-field electromechanical device in which the magnetic core is free to slide inside a long air-core solenoidal coil. The solenoid has length l, radius a, and number of turns per meter N, and carries a current I. The magnetic core has length $b < l$, radius a, and permeability $\mu \gg \mu_0$, and extends a distance x into the solenoid. **(a)** Neglect fringing of the field and find the mechanical force $\mathbf{F}_e$ of electric origin on the core. Plot F_{ex} versus x. **(b)** Assume that the device is

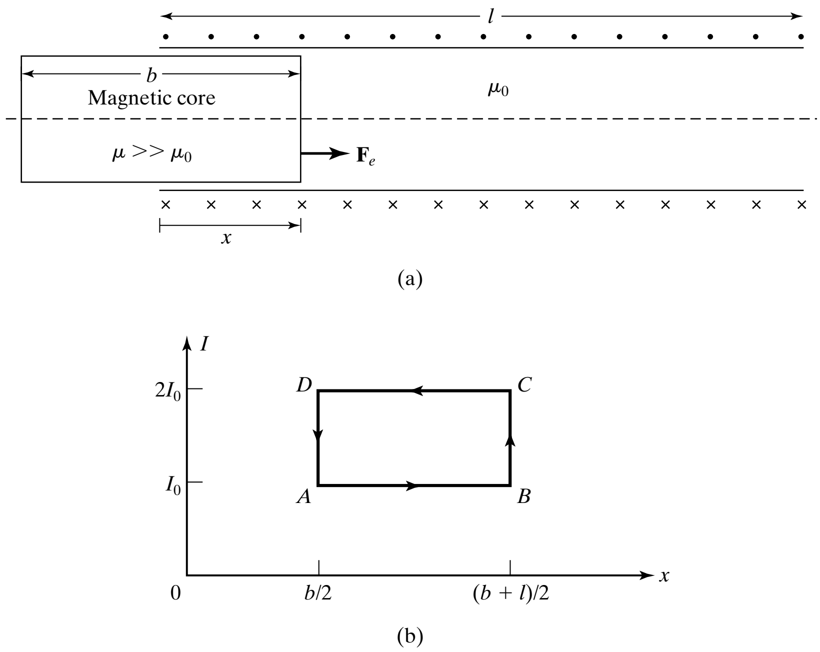

FIGURE 5.36
For Problem P5.42.

made to traverse the closed path in the I-x plane, as shown in Fig. 5.36(b). Find the energy converted per cycle and determine whether it is from mechanical to electrical or vice versa.

REVIEW PROBLEMS

R5.1. Finding the angle between two planes by using the gradient concept. By using the gradient concept, show that the angle α between two planes

$$a_1x + a_2y + a_3z = c_1$$
$$b_1x + b_2y + b_3z = c_2$$

is given by

$$\alpha = \cos^{-1}\frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)}}$$

Then find the angle between the planes $x + y + z = 1$ and $z = 0$.

R5.2. Electric potential due to a circular charged disk of uniform charge density. Consider a circular disk of radius a lying in the xy-plane with its center at the origin and carrying charge of uniform density ρ_{S0} C/m^2. Obtain the expression for the potential V due to the charged disk at a point $(0, 0, z)$ on the z-axis. Verify your answer by considering the limiting cases of $V(z)$ for $|z| \gg a$ and $E_z(z)$ for $|z| \ll a$.

R5.3. Magnetic vector potential and field for an infinitely long straight wire of current. Obtain the magnetic vector potential at an arbitrary point due to an infinitely long straight filamentary wire lying along the z-axis and carrying a current I in the $+z$-direction. Then evaluate $\mathbf{B}$ by performing the curl operation on the magnetic vector potential.

R5.4. Spherical capacitor with a dielectric of nonuniform permittivity. Assume that the region between the concentric spherical conductors of Fig. 5.11(b) is filled with a dielectric of nonuniform permittivity $\varepsilon = \varepsilon_0 b^2/r^2$. Obtain the solution for the potential between the conductors and the expression for the capacitance.

R5.5. Finding the internal inductance per unit length of a cylindrical conductor arrangement. Current I flows with uniform density along an infinitely long, hollow cylindrical conductor of inner radius a and outer radius b and returns with uniform surface density in the opposite direction along the surface of an infinitely long, perfectly conducting cylinder of radius c $(> b)$ and coaxial with the hollow conductor. Find the internal inductance per unit length of the arrangement.

R5.6. Quasistatic input behavior of a short-circuited coaxial cable. An air-dielectric coaxial cable of inner radius $a = 1$ cm, outer radius $b = 2$ cm, and length $l = 1$ m is short-circuited at one end. Obtain the equivalent circuit for the input behavior of the structure for frequencies slightly beyond those for which the quasistatic approximation is valid. Compute the resonant frequency of the equivalent circuit and comment on its value compared to those for which the circuit is valid.

R5.7. Calculations involving a magnetic circuit with three legs. For the magnetic circuit shown in Fig. 5.37, the dimensions of the legs are $A_1 = A_3 = 2$ cm^2, $A_2 = 3$ cm^2, $l_1 = l_3 = 30$ cm, and $l_2 = 10$ cm. The permeability of the core material

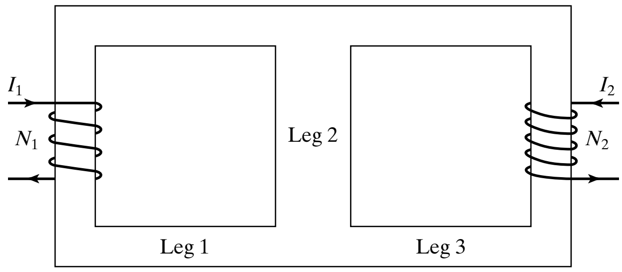

FIGURE 5.37
For Problem R5.7.

can be assumed to be $1000\mu_0$. **(a)** Draw the equivalent electric circuit. **(b)** For $N_1I_1 = 200$ A-t and $N_2I_2 = 100$ A-t, find the magnetic flux in each leg.

R5.8. **For analyzing an electromechanical system set in motion.** In the system shown in Fig. 5.38, the mass M is set in motion in the following manner: (1) the mass is brought to rest at the equilibrium position $x = x_0$ with no charge on the capacitor plates; (2) the mass is constrained to that position and the capacitor plates are charged to $\pm Q$ as shown; and (3) the mass is released, thereby permitting frictionless motion. Obtain the differential equation for the motion of M and find the solution.

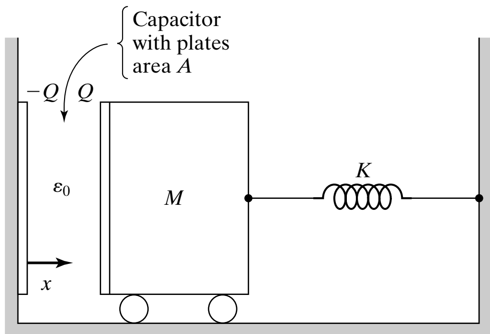

FIGURE 5.38
For Problem R5.8.