Graph Theory 1-HW2

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Exercise. (1)

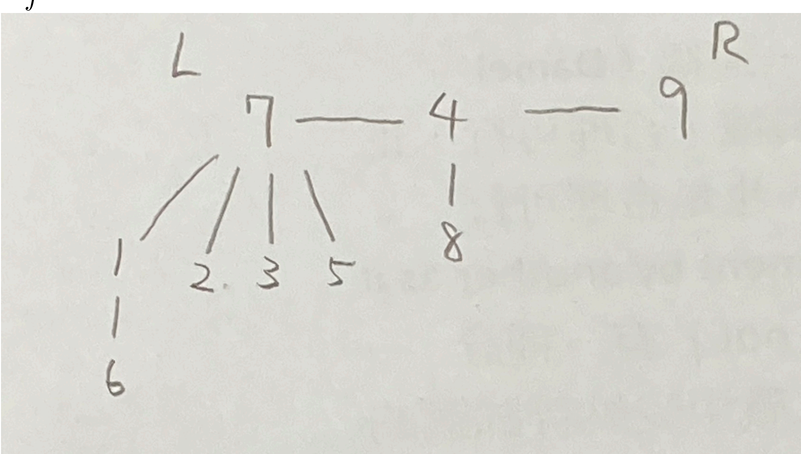
In a tree T, there must exist a vertex v whose degree is $\Delta(T)$. If we remove v, we would get $\Delta(T)$ separate subtrees. Since every tree has at least two leaves, even if one leaf in each subtree is connected to v and thus is not a leaf in the original tree, the other leaf would still contribute to the total number of leaves in T, therefore every tree T has at least $\Delta(T)$ leaves. It is not possible to guarantee $\Delta(T) + 1$ leaves: Consider the case where there is one vertex v with degree $\Delta(T)$ and $\Delta(T)$ other vertices all connected to v. In this case, there are only $\Delta(T)$ leaves.

Exercise. (4)

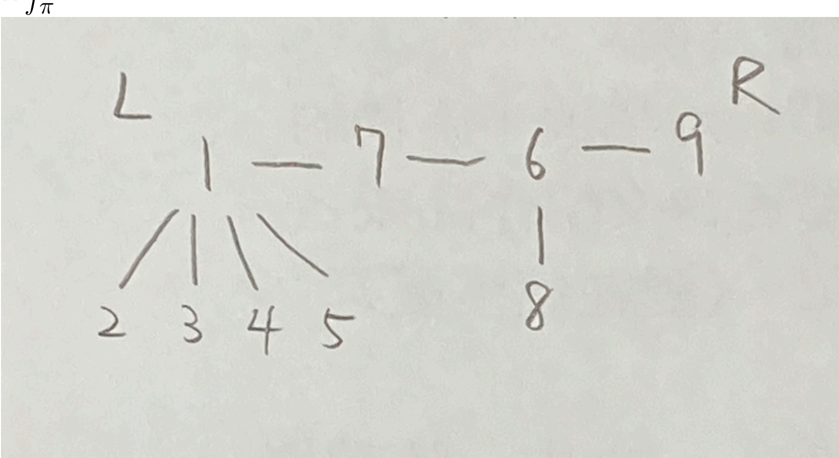
Let $1 \le i \le 9$, $f(i) = \begin{cases} 7 & \text{, if } i = 1 \\ 7 & \text{, if } i = 2 \\ 7 & \text{, if } i = 3 \\ 7 & \text{, if } i = 4 \\ 7 & \text{, if } i = 5 \\ 1 & \text{, if } i = 6 \\ 4 & \text{, if } i = 7 \\ 4 & \text{, if } i = 8 \\ 9 & \text{, if } i = 9 \end{cases}$, $\pi(i) = \begin{cases} 7 & \text{, if } i = 1 \\ 6 & \text{, if } i = 4 \\ 4 & \text{, if } i = 6 \\ 1 & \text{, if } i = 7 \\ i & \text{, otherwise} \end{cases}$

Then using Joyal's bijection, we get T_f and T_{f_π},

T_f:



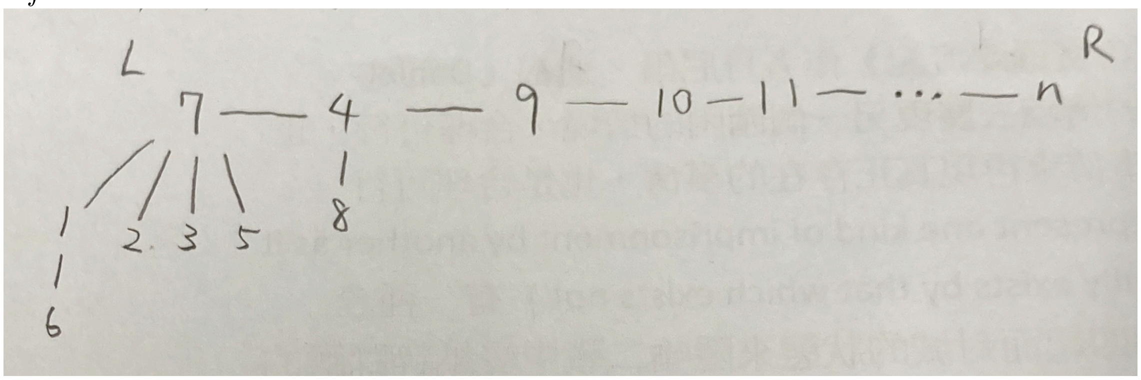
T_{f_π}



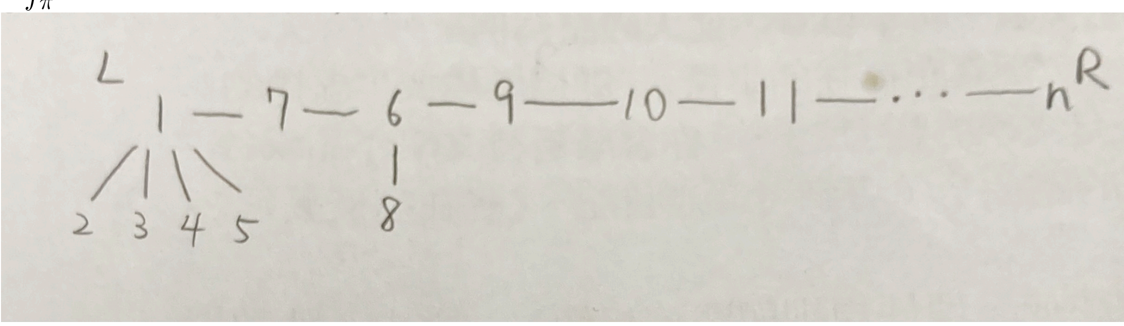
In T_f, there is a degree 5 vertex that shares an edge with a degree 3 vertex, however in T_{f_π}, no such relation can be found (the degree 5 vertex in T_{f_π} is only connected to 4 degree 1 vertices and 1 degree 2 vertex.). Therefore, T_f and T_{f_π} are not isomorphic.

Now we showed the case for $n = 9$, for cases $n > 9$, we can simply let $f(i) = i$, $\pi(i) = i$, $\forall i > 9$, and the 2 graphs are not isomorphic due to the same reasons.

T_f:



T_{f_π}



Exercise. (5)

(a)

For every $d \geq 2$, consider $G_d = K_{d+1}$.

(b)

For every $d \geq 2$, choose any $n \gg d$, then we can construct such a graph H_d: 2 independent K_n subgraphs, which we name M_1 and M_2, with 1 additional degree $2d$ point v which has d edges connected to d points in M_1 and d edges connected to d points in M_2.

In this case, removing v will disconnect the graph $\Rightarrow \kappa(H_d) = 1$; removing d edges that connects v with the same complete subgraphs will disconnect the graph, while removing at most $d-1$ edges in K_n won't disconnect the graph, removing at most $d-1$ edges connecting vertex v and M_1 will leave at least 1 edges still connecting v and M_1, and that $V(M_1)$ will still form a connected subgraph (same for M_2 as well), therefore won't disconnect v and either M_1 or M_2.

$\Rightarrow \kappa'(H_d) = d$; $\delta(H_d) = \min(2d, n-1) > d$;, therefore meeting the requirement.