REVIEW QUESTIONS

Q2.1. How do you find the work done in moving a test charge by an infinitesimal distance in an electric field? What is the amount of work involved in moving the test charge normal to the electric field?

Q2.2. What is the physical interpretation of the line integral of **E** between two points A and B?

Q2.3. How do you find the approximate value of the line integral of a vector field along a given path? How do you find the exact value of the line integral?

Q2.4. Discuss conservative versus nonconservative fields, giving examples.

Q2.5. How do you find the magnetic flux crossing an infinitesimal surface?

Q2.6. What is the magnetic flux crossing an infinitesimal surface oriented parallel to the magnetic flux density vector? For what orientation of the infinitesimal surface relative to the magnetic flux density vector is the magnetic flux crossing the surface a maximum?

Q2.7. How do you find the approximate value of the surface integral of a vector field over a given surface? How do you find the exact value of the surface integral?

Q2.8. Provide physical interpretations for the closed surface integrals of any two vectors of your choice.

Q2.9. State Faraday's law.

Q2.10. What are the different ways in which an emf is induced around a loop?

Q2.11. Discuss the right-hand screw rule convention associated with the application of Faraday's law.

Q2.12. To find the induced emf around a planar loop, is it necessary to consider the magnetic flux crossing the plane surface bounded by the loop? Explain.

Q2.13. What is Lenz's law?

Q2.14. Discuss briefly the motional emf concept.

Q2.15. How would you orient a loop antenna to obtain maximum signal from an incident electromagnetic wave that has its magnetic field directed along the north–south line?

Q2.16. State three applications of Faraday's law.

Q2.17. State Ampère's circuital law.

Q2.18. What is displacement current? Compare and contrast displacement current with current due to flow of charges.

Q2.19. Is it meaningful to consider two different surfaces bounded by a closed path to compute the two different currents on the right side of Ampère's circuital law to find $\oint \mathbf{H} \cdot d\mathbf{l}$ around the closed path?

Q2.20. Discuss the relationship between the displacement current emanating from a closed surface and the current due to flow of charges emanating from the same closed surface.

Q2.21. Give an example involving displacement current.

Q2.22. Discuss briefly the principle of radiation from a wire carrying a time-varying current.

Q2.23. State Gauss' law for the electric field.

Q2.24. How do you evaluate a volume integral?

Q2.25. State Gauss' law for the magnetic field.

Q2.26. What is the physical interpretation of Gauss' law for the magnetic field?

Q2.27. Discuss the dependence of Gauss' law for the magnetic field on Faraday's law.

Q2.28. State the law of conservation of charge.

Q2.29. How is Gauss' law for the electric field dependent on Ampère's circuital law?

Q2.30. Summarize Maxwell's equations in integral form for time-varying fields.

Q2.31. Summarize Maxwell's equations in integral form for static fields.

Q2.32. Are static electric and magnetic fields interdependent? Explain.

Q2.33. Discuss briefly the application of Gauss' law for the electric field to determine the electric field due to charge distributions.

Q2.34. When can you say that the current in a wire enclosed by a closed path is uniquely defined? Give two examples.

Q2.35. Give an example in which the current in a wire enclosed by a closed path is not uniquely defined. Is it correct to apply Ampère's circuital law for the static case in such a situation? Explain.

Q2.36. Discuss briefly the application of Ampère's circuital law to determine the magnetic field due to current distributions.

PROBLEMS

Section 2.1

P2.1. **Evaluation of line integral in Cartesian coordinates.** For the vector field $\mathbf{F} = y\mathbf{a}_x - z\mathbf{a}_y + x\mathbf{a}_z$, find $\int_{(0,0,0)}^{(1,1,1)} \mathbf{F} \cdot d\mathbf{l}$ for each of the following paths from $(0, 0, 0)$ to $(1, 1, 1)$: **(a)** $x = y = z$ and **(b)** $x = y = z^3$.

P2.2. **Evaluation of line integral around a closed path in Cartesian coordinates.** Given $\mathbf{F} = xy\mathbf{a}_x + yz\mathbf{a}_y + zx\mathbf{a}_z$, find $\oint_C \mathbf{F} \cdot d\mathbf{l}$, where C is the closed path comprising the straight lines from $(0, 0, 0)$ to $(1, 1, 1)$, from $(1, 1, 1)$ to $(1, 1, 0)$, and from $(1, 1, 0)$ to $(0, 0, 0)$.

P2.3. **Evaluation of line integral in Cartesian coordinates.** For the vector field $\mathbf{F} = \cos y \; \mathbf{a}_x - x \sin y \; \mathbf{a}_y$, find $\int_{(0,0,0)}^{(1,2\pi,1)} \mathbf{F} \cdot d\mathbf{l}$ in each of the following ways: **(a)** along the straight-line path between the two points; **(b)** along the curved path $x = z = \sin(y/4)$ between the two points; and **(c)** without choosing any particular path. Is the vector field conservative or nonconservative? Explain.

P2.4. **Evaluation of line integral around closed path in cylindrical coordinates.** Given $\mathbf{A} = 2r \sin \phi \; \mathbf{a}_r + r^2\mathbf{a}_\phi + z\mathbf{a}_z$ in cylindrical coordinates, find $\oint_C \mathbf{A} \cdot d\mathbf{l}$, where C is the closed path comprising the straight line from $(0, 0, 0)$ to $(1, 0, 0)$, the circular arc from $(1, 0, 0)$ to $(1, \pi/2, 0)$ through $(1, \pi/4, 0)$, the straight line from $(1, \pi/2, 0)$ to $(1, \pi/2, 1)$, and the straight line from $(1, \pi/2, 1)$ to $(0, 0, 0)$.

P2.5. **Evaluation of line integral in spherical coordinates.** Given $\mathbf{A} = e^{-r}(\cos \theta \; \mathbf{a}_r + \sin \theta \; \mathbf{a}_\theta) + r \sin \theta \; \mathbf{a}_\phi$ in spherical coordinates, find $\int \mathbf{A} \cdot d\mathbf{l}$ for each of the following paths: **(a)** straight-line path from $(0, 0, 0)$ to $(2, 0, 0)$; **(b)** circular arc from $(2, 0, \pi/4)$ to $(2, \pi/2, \pi/4)$ through $(2, \pi/4, \pi/4)$; and **(c)** circular arc from $(2, \pi/6, 0)$ to $(2, \pi/6, \pi/2)$ through $(2, \pi/6, \pi/4)$.

Section 2.2

P2.6. Evaluation of a closed surface integral in Cartesian coordinates. Given $\mathbf{A} = x^2yz\mathbf{a}_x + y^2zx\mathbf{a}_y + z^2xy\mathbf{a}_z$, evaluate $\oint_S \mathbf{A}\cdot d\mathbf{S}$, where S is the surface of the cubical box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.

P2.7. Evaluation of a closed surface integral in Cartesian coordinates. Given $\mathbf{A} = (x^2y + 2)\mathbf{a}_x + 3\mathbf{a}_y - 2xyz\mathbf{a}_z$, evaluate $\oint_S \mathbf{A}\cdot d\mathbf{S}$, where S is the surface of the rectangular box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 2$, $z = 0$, and $z = 3$.

P2.8. Evaluation of a closed surface integral in cylindrical coordinates. Given $\mathbf{A} = r\cos\phi\,\mathbf{a}_r - r\sin\phi\,\mathbf{a}_\phi$ in cylindrical coordinates, evaluate $\oint_S \mathbf{A}\cdot d\mathbf{S}$, where S is the surface of the box bounded by the plane surfaces $\phi = 0$, $\phi = \pi/2$, $z = 0$, $z = 1$, and the cylindrical surface $r = 2$, $0 < \phi < \pi/2$.

P2.9. Evaluation of a closed surface integral in spherical coordinates. Given $\mathbf{A} = r^2\mathbf{a}_r + r\sin\theta\,\mathbf{a}_\theta$ in spherical coordinates, find $\oint_S \mathbf{A}\cdot d\mathbf{S}$, where S is the surface of that part of the spherical volume of radius unity and lying in the first octant.

Section 2.3

P2.10. Induced emf around a closed path in a time-varying magnetic field. Find the induced emf around the rectangular closed path C connecting the points $(0, 0, 0)$, $(a, 0, 0)$, $(a, b, 0)$, $(0, b, 0)$, and $(0, 0, 0)$, in that order, for each of the following magnetic fields:

(a) $\mathbf{B} = \dfrac{B_0a^2}{(x + a)^2}e^{-t}\mathbf{a}_z$

(b) $\mathbf{B} = B_0\sin\dfrac{\pi x}{a}\cos\omega t\,\mathbf{a}_z$

P2.11. Induced emf around a moving loop in a static magnetic field. A magnetic field is given in the xz-plane by $\mathbf{B} = (B_0/x)\mathbf{a}_y$ Wb/m^2, where B_0 is a constant. A rigid rectangular loop is situated in the xz-plane and with its corners at the points (x_0, z_0), $(x_0, z_0 + b)$, $(x_0 + a, z_0 + b)$, and $(x_0 + a, z_0)$. If the loop is moving in that plane with a velocity $\mathbf{v} = v_0\mathbf{a}_x$ m/s, where v_0 is a constant, find by using Faraday's law the induced emf around the loop in the sense defined by connecting the above points in succession. Discuss your result by using the motional emf concept.

P2.12. Induced emf around a closed path in a time-varying magnetic field. A magnetic field is given in the xz-plane by $\mathbf{B} = B_0\cos\pi(x - v_0t)\,\mathbf{a}_y$ Wb/m^2. Consider a rigid square loop situated in the xz-plane with its vertices at $(x, 0, 1)$, $(x, 0, 2)$, $(x + 1, 0, 2)$, and $(x + 1, 0, 1)$. **(a)** Find the expression for the emf induced around the loop in the sense defined by connecting the above points in succession. **(b)** What would be the induced emf if the loop is moving with the velocity $\mathbf{v} = v_0\mathbf{a}_x$ m/s instead of being stationary?

P2.13. Induced emf around a swinging loop in a static magnetic field. A rigid rectangular loop of metallic wire is hung by pivoting one side along the x-axis, as shown in Fig. 2.37. The loop is free to swing about the pivoted side without friction under the influence of gravity and in the presence of a uniform magnetic field $\mathbf{B} = B_0\mathbf{a}_z$ Wb/m^2. If the loop is given a slight angular displacement and released, show that the emf induced around the closed path C of the loop is approximately equal to $B_0ab\omega$, where ω is the angular velocity of swing of the loop

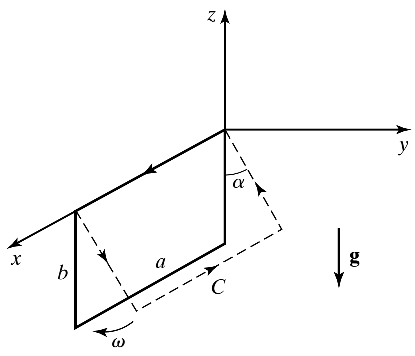

FIGURE 2.37

For Problem P2.13.

toward the vertical. Does the loop swing faster or slower than in the absence of the magnetic field? Explain.

P2.14. A conducting bar rolling down inclined rails in a uniform static magnetic field. A rigid conducting bar of length L, mass M, and electrical resistance R rolls without friction down two parallel conducting rails that are inclined at an angle α with the horizontal, as shown in Fig. 2.38. The rails are of negligible resistance and are joined at the bottom by another conductor, also of negligible resistance, so that the total resistance of the loop formed by the rolling bar and the three other sides is R. The entire arrangement is situated in a region of uniform static magnetic field $\mathbf{B} = B_0\mathbf{a}_z$ Wb/m^2, directed vertically downward. Assume the bar to be rolling down with uniform velocity $\mathbf{v}$ parallel to the rails under the influence of Earth's gravity (acting in the positive z-direction) and the magnetic force due to the current in the loop produced by the induced emf. Show that v is equal to $(MgR/B_0^2L^2)\tan\alpha\sec\alpha$.

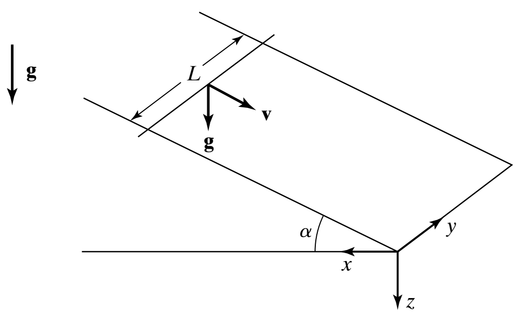

FIGURE 2.38

For Problem P2.14.

P2.15. Induced emf around a revolving loop in a static magnetic field. A rigid rectangular loop of base b and height h situated normal to the xy-plane and with its sides pivoted to the z-axis revolves about the z-axis with angular velocity ω rad/s in the sense of increasing ϕ, as shown in Fig. 2.39. Find the induced emf around the closed path C of the loop for each of the following magnetic fields: **(a)** $\mathbf{B} = B_0\mathbf{a}_y$ Wb/m^2 and **(b)** $\mathbf{B} = B_0(y\mathbf{a}_x - x\mathbf{a}_y)$ Wb/m^2. Assume the loop to be in the xz-plane at $t = 0$.

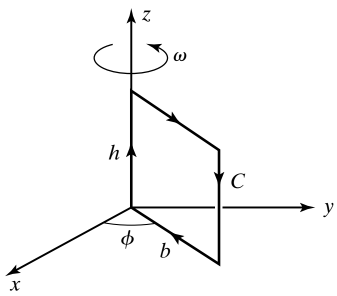

FIGURE 2.39
For Problem P2.15.

P2.16. Induced emf around a loop in a time-varying magnetic field for several cases. A rigid rectangular loop of area A is situated in the xz-plane and symmetrically about the z-axis, as shown in Fig. 2.40, in a region of magnetic field $\mathbf{B} = B_0(\sin \omega t\, \mathbf{a}_x + \cos \omega t\, \mathbf{a}_y)$ Wb/m^2. Find the induced emf around the closed path C of the loop for each of the following cases: **(a)** the loop is stationary; **(b)** the loop revolves around the z-axis in the sense of increasing ϕ with uniform angular velocity of ω rad/s; and **(c)** the loop revolves around the z-axis in the sense of decreasing ϕ with uniform angular velocity of ω rad/s. For parts (b) and (c), assume that the loop is in the xz-plane at $t = 0$.

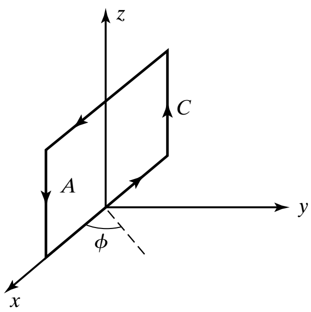

FIGURE 2.40
For Problem P2.16.

Section 2.4

P2.17. Application of Ampere's circuital law in integral form. Given that $\mathbf{H} = \pm H_0(t \mp \sqrt{\mu_0\varepsilon_0}\, z)^2\mathbf{a}_y$ and $\mathbf{D} = \sqrt{\mu_0\varepsilon_0}\, H_0(t \mp \sqrt{\mu_0\varepsilon_0}\, z)^2\mathbf{a}_x$ for $z \gtrless 0$, find the current due to flow of charges enclosed by the rectangular closed path from $(0, 0, 1)$ to $(0, 1, 1)$ to $(0, 1, -1)$ to $(0, 0, -1)$ to $(0, 0, 1)$.

P2.18. Application of Ampere's circuital law in integral form. A current density due to flow of charges is given by $\mathbf{J} = -(x\mathbf{a}_x + y\mathbf{a}_y + z^2\mathbf{a}_z)$ A/m^2. Find the displacement current emanating from each of the following closed surfaces: **(a)** the surface of the cubical box bounded by the planes $x = \pm 2$, $y = \pm 2$, and $z = \pm 2$, and **(b)** the surface of the cylindrical box bounded by the surfaces $r = 1$, $z = 0$, and $z = 2$.

P2.19. Finding rms value of current drawn from voltage source connected to a capacitor. A voltage source connected to a parallel-plate capacitor by means of wires sets up a uniform electric field of $E = 180 \sin 2\pi \times 10^6 t \sin 4\pi \times 10^6 t$ V/m between the plates of the capacitor and normal to the plates. Assume that no field exists outside the region between the plates. If the area of each plate is 0.1 m^2 and the medium between the plates is free space, find the root-mean-square value of the current drawn from the voltage source.

P2.20. Finding rms value of current drawn from voltage source connected to a capacitor. Assume that the time variation of the electric field in Problem P3.19 is as shown in Fig. 2.41. Find and plot versus time the current drawn from the voltage source. What is the root-mean-square value of the current?

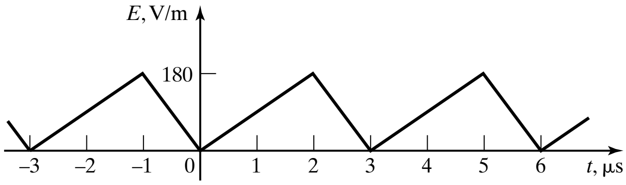

FIGURE 2.41
For Problem P2.20.

Section 2.5

P2.21. Finding displacement flux emanating from a surface enclosing charge. For each of the following charge distributions, find the displacement flux emanating from the surface enclosing the charge: **(a)** $\rho(x, y, z) = \rho_0(3 - x^2 - y^2 - z^2)$ for the cubical box bounded by $x = \pm 1$, $y = \pm 1$, and $z = \pm 1$; and **(b)** $\rho(x, y, z) = \rho_0(xyz)$ for $x > 0$, $y > 0$, $z > 0$, and $x^2 + y^2 + z^2 < 1$.

P2.22. Finding displacement flux emanating from a surface enclosing charge. For each of the following charge distributions, find the displacement flux emanating from the surface enclosing the charge: **(a)** $\rho(r, \phi, z) = \rho_0 e^{-r^2}$ for $r < 1, 0 < z < 1$ in cylindrical coordinates; and **(b)** $\rho(r, \theta, \phi) = (\rho_0/r)\sin^2\theta$ for $r < 1, 0 < \theta < \pi/2$ in spherical coordinates.

P2.23. Application of Gauss' law for the magnetic field in integral form. Using the property that $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$, find the absolute value of the magnetic flux crossing that portion of the surface $y = \sin x$ bounded by $x = 0$, $x = \pi$, $z = 0$, and $z = 1$ for $\mathbf{B} = B_0(y\mathbf{a}_x - x\mathbf{a}_y)$ Wb/m^2.

Section 2.6

P2.24. Application of the law of conservation of charge. Given $\mathbf{J} = (x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z)$ A/m^2, find the time rate of decrease of the charge contained within each of the following volumes: **(a)** volume bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$; **(b)** volume bounded by the cylinders $r = 1$ and $r = 2$ and the planes $z = 0$ and $z = 1$; and **(c)** volume bounded by the spherical surfaces $r = 1$ and $r = 2$ and the conical surface $\theta = \pi/3$.

P2.25. Combined application of several of Maxwell's equations in integral form. Current I flows along a straight wire from a point charge $Q_1(t)$ located at the origin to a point charge $Q_2(t)$ located at (0, 0, 1). Find the line integral of $\mathbf{H}$ along the square closed path having the vertices at (1, 1, 0), (−1, 1, 0), (−1, −1, 0), and (1, −1, 0) and traversed in that order.

P2.26. Combined application of several of Maxwell's equations in integral form. Current I flows along a straight wire from a point charge $Q_1(t)$ at the origin to a point charge $Q_2(t)$ at the point (2, 2, 2). Find the line integral of $\mathbf{H}$ around the triangular closed path having the vertices at (3, 0, 0), (0, 3, 0), and (0, 0, 3) and traversed in that order.

Section 2.7

P2.27. Application of Gauss' law for the electric field in integral form and symmetry. Charge is distributed with density $\rho(x, y, z)$ in a cubical box bounded by the planes $x = \pm 1$ m, $y = \pm 1$ m, and $z = \pm 1$ m. Find the displacement flux emanating from one side of the box for each of the following cases: **(a)** $\rho(x, y, z) = (3 - x^2 - y^2 - z^2)$ C/m^3 and **(b)** $\rho(x, y, z) = \sqrt{|xyz|}$ C/m^3.

P2.28. Electric field due to a cylindrical charge distribution using Gauss' law. Charge is distributed with density $\rho_0 e^{-r^2}$ C/m^3 in the cylindrical region $r < 1$. Find **D** everywhere.

P2.29. Electric field due to a spherical charge distribution using Gauss' law. Charge is distributed with uniform density ρ_0C/m^3 in the region $a < r < 2a$ in spherical coordinates. Find **D** everywhere and plot D_r versus r.

P2.30. Application of Ampere's circuital law in integral form and symmetry. Current flows with density $\mathbf{J}(x, y)$ in an infinitely long thick wire having the z-axis as its axis. The cross section of the wire in the xy-plane is the square bounded by $x = \pm 1$ m and $y = \pm 1$ m. Find the line integral of **H** along one side of the square and traversed in the sense of increasing ϕ for each of the following cases: **(a)** $\mathbf{J}(x, y) = (|x| + |y|)\mathbf{a}_z$ A/m^2 and **(b)** $\mathbf{J}(x, y) = x^2y^2\mathbf{a}_z$ A/m^2.

P2.31. Magnetic field due to a solid wire of current using Ampere's circuital law. Current flows with density $\mathbf{J} = J_0(r/a)\mathbf{a}_z$ A/m^2 along an infinitely long solid cylindrical wire of radius a having the z-axis as its axis. Find **H** everywhere and plot H_ϕ versus r.

P2.32. Magnetic field for a coaxial cable using Ampere's circuital law. A coaxial cable consists of an inner conductor of radius $3a$ and an outer conductor of inner radius $4a$ and outer radius $5a$. Assume the cable to be infinitely long and its axis to be along the z-axis. Current I flows with uniform density in the $+z$-direction in the inner conductor and returns with uniform density in the $-z$-direction in the outer conductor. Find **H** everywhere and plot H_ϕ versus r.

REVIEW PROBLEMS

R2.1. Determination of a specified static vector field to be a conservative field. Show that the vector field given by

$$\mathbf{F} = \cos\theta \sin\phi \, \mathbf{a}_r - \sin\theta \sin\phi \, \mathbf{a}_\theta + \cot\theta \cos\theta \, \mathbf{a}_\phi$$

is a conservative field. Then find the value of $\int \mathbf{F} \cdot d\mathbf{l}$ from the point $(1, \pi/6, \pi/3)$ to the point $(4, \pi/3, \pi/6)$.

R2.2. Induced emf around an expanding loop in a nonuniform static magnetic field. In Fig. 2.42, a rectangular loop of wire with three sides fixed and the fourth side movable is situated in a plane perpendicular to a nonuniform magnetic field $\mathbf{B} = B_0 y \mathbf{a}_z$ Wb/m^2, where B_0 is a constant. The position of the movable side is varied with time in the manner $y = y_0 + a\cos\omega t$, where $a < y_0$. Find the induced emf around the closed path C of the loop. Verify that Lenz's law is satisfied. Show also that the induced emf consists of two frequency components, ω and 2ω.

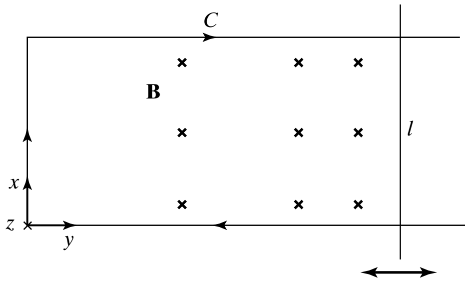

FIGURE 2.42
For Problem R2.2.

R2.3. Finding amplitude of current from sinusoidal voltage source connected to a capacitor. A voltage source is connected by means of wires to a parallel-plate capacitor made up of circular plates of radii a in the $z = 0$ and $z = d$ planes and having their centers on the z-axis. The electric field between the plates is given by

$$\mathbf{E} = E_0 \sin\frac{\pi r}{2a} \cos \omega t \; \mathbf{a}_z \qquad \text{for} \quad r < a$$

Find the amplitude of the current drawn from the voltage source, assuming the region between the plates to be free space and that no field exists outside this region.

R2.4. Combined application of several of Maxwell's equations in integral form. Current I flows along a straight wire from a point charge $Q_1(t)$ located at one of the vertices of a cube to a point charge $Q_2(t)$ at the center of the cube. Find the absolute value of the line integral of $\mathbf{H}$ around the periphery of one of the three sides of the cube not containing the vertex at which Q_1 is located.

R2.5. Electric field due to a spherical charge distribution using Gauss' law. Charge is distributed with density $\rho = \rho_0(r/a)^2$, where ρ_0 is a constant, in the spherical region $r < a$. Find $\mathbf{D}$ everywhere and plot D_r versus r.

R2.6. Magnetic field in the hollow region of wire bounded by two parallel cylindrical surfaces. Current flows axially with uniform density $\mathbf{J}_0$ A/m^2 in the region between two infinitely long parallel, cylindrical surfaces of radii a and b ($<a$), and with their axes separated by the vector distance $\mathbf{c}$, where $|\mathbf{c}| < (a - b)$. Find the magnetic field intensity in the current-free region inside the cylindrical surface of radius b.