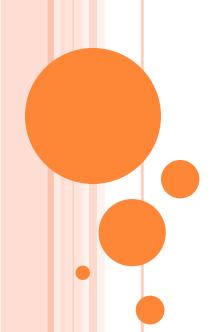


Sharpe Ratio

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Sharpe Ratio (SR)

Definition

• $SR = \frac{\mu - \mu_0}{\sigma}$

- Bank deposit interest rate
- \circ μ : mean yearly return rate, μ_0 : risk-free return rate, σ : standard deviation of yearly return rate
- $\circ \sigma$ is also known as "volatility" or "risk"

Use of Sharpe ratio

- A way to evaluate a long-term portfolio
- The return rate when the volatility is 1%

Assume μ_0 =0

Examples

- SR=0.5
 - Expected return = 0.5% if the volatility is 1%
 - Expected return = 5% if the volatility is 10%

Assume μ_0 =0



Examples of Sharpe Ratio

Comparison of Sharpe ratios of two portfolios



https://rich01.com/what-sharpe-ratio/



Compute SR from Daily Return

- \circ Time resolution of yearly $\mu_{\mathcal{V}}$ and $\sigma_{\mathcal{V}}$ too low \Rightarrow How to compute $\mu_{\mathcal{Y}}$ and $\sigma_{\mathcal{V}}$ based on daily μ_{d} and σ_{d} ?
- $o \ Daily \ return = \frac{Today's \ net \ worth Yesterday's \ net \ worth}{Yesterday's \ net \ worth}$

$$\circ$$
 $Y=X_1+X_2+\cdots+X_{252}$ 252: Average trading days per year

•
$$\mu_y = 252 \, \mu_d$$

•
$$\sigma_v^2 = 252\sigma_d^2$$

$$Y: \mu = \mu_y \text{ and } \sigma = \sigma_y$$

 $X_i: \mu = \mu_d \text{ and } \sigma = \sigma_d$

$$\Rightarrow SR_y = \frac{\mu_y - \mu_{y0}}{\sigma_y} = \frac{252\mu_d - \mu_{y0}}{\sqrt{252}\sigma_d} = \frac{\mu_d - \frac{\mu_{y0}}{252}}{\sigma_d} \times \sqrt{252} = SR_d \times \sqrt{252}$$

Similarly, to compute SR from monthly return:

$$\Rightarrow SR_y = SR_m \times \sqrt{12}$$



Example

- Assume the five days' prices of a stock are [6 8 7 9 8] and the risk-free return is 1%, what is the corresponding Sharpe ratio?
- Solution:
 - return=[2/6, -1/8, 2/7, -1/9];
 - mu=0.0957
 - sigma=0.2477
 - $SR_d = (mu-0.01/252)/sigma=0.3863$
 - $SR_v = sqrt(252) * SR_d = 6.1330$