



- Name: _____
 - Date: _____
 - Section: _____
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ECON 300: Intermediate Price Theory

Problem Set #5

INSTRUCTIONS:

- This problem set is not graded.

Problem 1. Cobb-Douglas Production Functions

Suppose that you are producing based on the following technology:

$$F(L, K) = 3L^2K$$

1.A Find the Marginal Product of Labor.

1.B Find the Marginal Product of Capital.

1.C Find the Marginal Rate of Technical Substitution.

1.D Complete the following statement:

If the firm reduces their input of L by 1 unit,
given that the firm keeps output levels constant,
they must increase K by _____ units.

1.E What is the output level when $L = 5$ and $K = 10$?

1.F What is the output level when $L = 10$ and $K = 20$?

1.G Does this production technology display constant returns to scale? Why?

1.H Suppose that the firm's production function is updated to $F(L, K) = 3L^2K^2$. Is this representative of technical progress? Why?

Problem 2. Short Run and Long Run Costs

Suppose that you are aiming to produce 100 units of output using inputs of labor L and capital K . Due to congestion in the supply chain, the amount of capital input you have access to is limited to (and fixed at) 5. The prevailing wage for 1 unit of L is given as $w = 5$, and the rent for 1 unit of capital is given as $r = 10$. Suppose further that you are producing based on the following technology:

$$F(L, K) = 2LK$$

2.A Find the Short Run Conditional Factor Demand.

2.B What is the value of the Short Run Total Cost if your fixed cost is 0?

Starting from question 2.C and beyond, suppose that the supply chain congestion is resolved, and you may freely change the level of capital input in your production. All other variables remain constant. Your target output remains at 100 units, wage is 5, and rent is 10.

2.C Find the Marginal Rate of Technical Substitution between Labor and Capital.

2.D Find the optimal ratio of Labor to Capital.

2.E Express the firm's Isocost as a mathematical equation.

2.F Find the optimal inputs of labor and capital.

Problem 3. Cost Functions

Suppose that you are chosen as the supplier of Monmouth College logo embedded diploma frames. In your woodworking studio, you can hire workers (L) that can work with lumber (K) to produce diploma frames (Q). The market wage is $w = 20$ and the price of each unit of lumber is $r = 10$. Your current production technology can be described as:

$$F(L, K) = K^{\frac{1}{2}}L^{\frac{1}{2}}$$

3.A What is the optimal level of L and K if the target output was $Q = 100$?

3.B What is the optimal level of L and K for an arbitrary level of output Q ?

3.C Find the Total Cost function for producing diploma frames. (*Hint: $TC(Q) = w \cdot L + r \cdot K$*)

3.D Find the Average Total Cost function, $ATC(Q)$.

3.E Find the Marginal Cost function, $MC(Q)$.

3.F Complete the chart below by plotting the three functions $TC(Q)$, $ATC(Q)$, and $MC(Q)$.

