



- Name: _____
 - Date: _____
 - Section: _____
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ECON 300

Quiz #4

Fall 2023

INSTRUCTIONS:

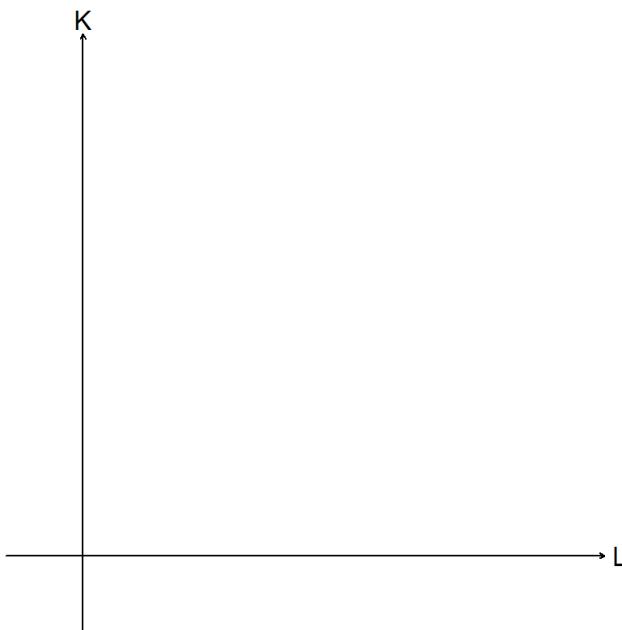
- Please read all questions carefully before you begin answering.
- Answer all questions in the spaces provided on the question sheet. Circle the correct answer for the multiple-choice questions.
- This quiz consists of 7 pages, including this one. There are a total of 5 problems with a total of 20 subquestions.
- This is a closed-book quiz. Please remove all materials from the top of the desk and take any necessary items from your bags before the exam begins.
- The recovery rate for Quiz #4 is 70%.

Problem 1. Production: Part 1

Suppose that a firm has a production process where workers (labor) and machines (capital) are perfectly substitutable. Specifically, each machine (capital) can always perfectly substitute two workers (labor) in producing output in the long run.

1.A. Which type of production function best represent the firm's production technology? (5 points)

1.B. Plot a representative isoquant representing this firm's production technology. (5 points)
The shape and slope of the isoquants are the important elements in this plot.



1.C. Find the Marginal Rate of Technical Substitution between Labor and Capital. (5 points)

- $MRTS_{LK} =$

Problem 2. Production: Part 2

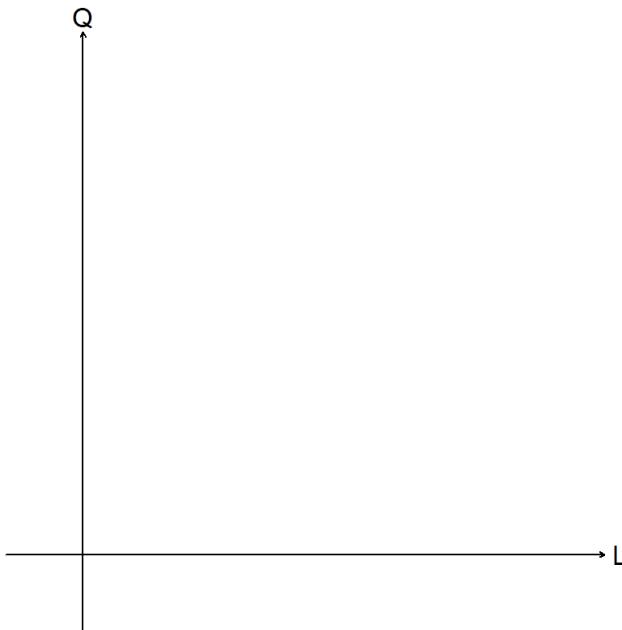
Suppose that a firm uses labor L and capital K inputs to produce output Q . The production technology of the firm can be described using the following production function:

$$F(L, K) = 5LK^2$$

- 2.A. Does this production exhibit increasing, constant, or decreasing returns to scale? (5 points)
How did you draw this conclusion?

- 2.B. Suppose that the firm is currently using $L = 2$ and $K = 3$. (5 points)
If you can add either 1 unit of L or 1 unit of K free of charge, which should you choose and why?

- 2.C. Suppose that the firm's capital input is fixed at $\bar{K} = 3$. (5 points)
Plot the firm's short run production function below (you may approximate the scale).



Problem 3. Cost Functions

Suppose that a firm is interested in calculating its long run cost function. The cost of hiring a worker is $w = 100$, and a machine costs $r = 300$. The firm's production process requires exactly 3 workers to operate each machine, and no more than 3 workers can work on each machine. That is, the production function is given as:

$$F(L, K) = \min\{3K, L\}$$

3.A. Find the firm's optimal input ratio of Labor and Capital. (5 points)

3.B. Find the optimal input of L^* and K^* for an arbitrary production quota Q . (5 points)

- $L^* =$

- $K^* =$

3.C. Find the Total Cost, Average Cost, and Marginal Cost Functions. (5 points)

- $TC(Q) =$

- $AC(Q) =$

- $MC(Q) =$

Problem 4. Cost in the Short Run and Long Run

Suppose that a firm is operating in the short run with its input of capital being fixed at $\bar{K} = 2$. The cost of hiring a worker is $w = 100$, and a machine costs $r = 300$. The firm's production technology is given as:

$$F(L, K) = L \cdot K^3$$

- 4.A. Suppose that the firm's production quota in the short run is $Q = 80$. (5 points)
What is the short run conditional factor demand?

- $L^*(Q, \bar{K}) =$

- 4.B. Calculate the short run total cost for this firm. (5 points)

- 4.C. Suppose that the firm now has enough time to change the input of capital. (5 points)
What is the marginal product of labor and capital?

- $MP_L =$

- $MP_K =$

- 4.D. Assume that $MP_L = K$ and $MP_K = 3L$. (5 points)
Find the expression for the Marginal Rate of Technical Substitution.

- $MRTS_{LK} =$

4.E. Using the $MRTS_{LK}$ from 4.D, find the optimal ratio of Labor and Capital. (5 points)

4.F. Find the optimal units of L and K to produce $Q = 80$. (5 points)

Use the original production function, and optimal ratio from 4.E.

• $L^* =$

• $K^* =$

4.G. Calculate the long run total cost for this firm. (5 points)

4.H. Is the long run cost lower than the short run cost? (5 points)

Explain in your own words why the long run and short run costs are different.

Problem 5. Profit Maximization

Suppose that the output market is perfectly competitive, and each unit of output Q has a market price of $P = 100$. The cost function is given as:

$$TC(Q) = 150 + Q^2$$

5.A. Express the producer's profit as a mathematical equation. (5 points)

- $\Pi(Q) =$

5.B. Find the marginal revenue and marginal cost of production. (5 points)

- $MR(Q) =$

- $MC(Q) =$

5.C. Assume from now on that $MR(Q) = 150$, and $MC(Q) = 5Q$. (5 points)
Find the profit maximizing quantity Q .

- $Q^* =$

- Original Score: _____

- Recovered Score: _____

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