Monmouth COLLEGE

- Name: ______________________
- Date: ______________________
- Section: ______________________

ECON 300: Intermediate Price Theory

Problem Set #5 - Part #1: Suggested Solutions

Fall 2024

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Problem 1. Production Technologies

Suppose that your firm is producing output Q with using two inputs, labor L and capital K, using the following technology:

$$F(L, K) = LK^2$$

1.A. In your own words, describe what the Marginal Product of Labor measures.

The marginal product of labor measures "how many units of output Q can be *additionally* produced when one unit of labor L is added to the production process." Mathematically:

$$MP_L \equiv \frac{\partial}{\partial L} F(L, K)$$

1.B. Find the Marginal Product of Labor (MP_L).

- $MP_L = \dfrac{\partial}{\partial L} LK^2 = K^2 \cdot 1 \cdot L^{1-1} = \boxed{K^2}$

1.C. Find the Marginal Product of Capital (MP_K).

- $MP_K = \dfrac{\partial}{\partial L} LK^2 = L \cdot 2 \cdot K^{2-1} = \boxed{2LK}$

1.D. In your own words, describe what the Marginal Rate of Technical Substitution measures.

The marginal rate of technical substitution measures "if the firm reduces their input of L by 1 unit, given that the firm's output level remains constant, they must increase the input of K by $MRTS_{LK}$ units. Mathematically:

$$MRTS_{LK} \equiv \frac{MP_L}{MP_K}$$

1.E. Find the Marginal Rate of Technical Substitution $(MRTS_{LK})$

- $MRTS_{LK} = \dfrac{MP_L}{MP_K} = \dfrac{K^2}{2LK} = \boxed{\dfrac{K}{2L}}$

Problem 1. Production Technologies (continued)

Suppose that your firm is producing output Q with using two inputs, labor L and capital K, using the following technology:

$$F(L, K) = LK^2$$

1.F. How many units of output can you produce if you employ $L = 16$ and $K = 1$?

$$F(16, 1) = 16 \cdot 1 = 16$$

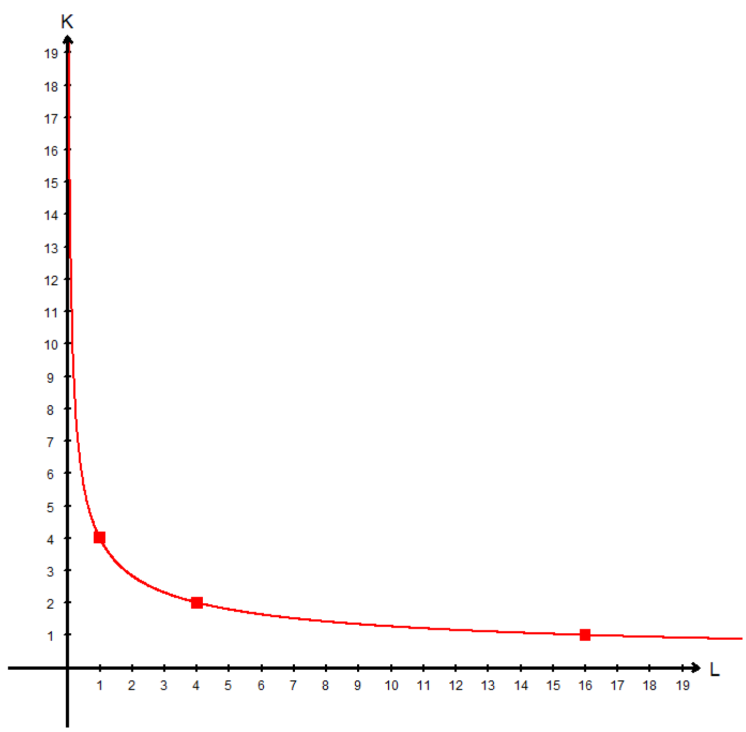
1.G. How many units of output can you produce if you employ $L = 4$ and $K = 2$?

$$F(4, 2) = 4 \cdot 2^2 = 16$$

1.H. How many units of output can you produce if you employ $L = 1$ and $K = 4$?

$$F(1, 4) = 1 \cdot 4^2 = 16$$

1.I. Based on your answers from `1.F` through `1.H`, approximate the isoquant of your firm when your total product is 16.



Problem 2. Returns to Scale

2.A In your own words, define the following terms.

- Constant Returns to Scale (CRS)

  When the producer increases all inputs by a factor of λ, output increases by a factor of λ.

- Increasing Returns to Scale (IRS)

  When the producer increases all inputs by a factor of λ, output increases by a factor of strictly greater than λ.

- Decreasing Returns to Scale (DRS)

  When the producer increases all inputs by a factor of λ, output increases by a factor of strictly less than λ.

2.B Determine if the following production functions display CRS, IRS, DRS, or is inconclusive.

- $F(L, K) = LK^2$

  This production technology can be considered to be increasing returns to scale:

$$F(\lambda L, \lambda K) = \lambda L \cdot (\lambda K)^2 = \lambda^3 LK^2 > \lambda LK^2 = \lambda F(L, K) \text{ when } \lambda > 1.$$

- $F(L, K) = L + K$

  This production technology can be considered to be constant returns to scale:

$$F(\lambda L, \lambda K) = \lambda L + \lambda K = \lambda(L + K) = \lambda F(L, K)$$

- $F(L, K) = L^{\frac{1}{3}} K^{\frac{1}{3}}$

  This production technology can be considered to be constant returns to scale:

$$F(\lambda L, \lambda K) = (\lambda L)^{\frac{1}{3}} \cdot (\lambda K)^{\frac{1}{3}} = \lambda^{\frac{2}{3}} L^{\frac{1}{3}} K^{\frac{1}{3}} < \lambda F(L, K) \text{ when } \lambda > 1.$$

Problem 3. The Short Run, Long Run, and Cost Minimization

Suppose that you are given a production quota of 10000 units of output, where the inputs are labor L and capital K. Due to the short turnaround timeframe, you have no choice b ut to work with the capital stock you are given, which is $\bar{K} = 10$. The wage is given as $w = 10$ and rent for each unit of capital is given as $r = 20$. Your production technology is given as follows:

$$F(L, K) = 4L^2K^2$$

3.A. Find the short run conditional factor demand.

Apply the fixed capital stock in the production function, and figure out how many units of labor must be employed to meet the quota:

$$F(L, 10) = 10000 \quad \Rightarrow \quad 4L^2 \cdot 10^2 = 10000 \quad \Rightarrow \quad L^2 = 25 \quad \Rightarrow \quad \boxed{L = 5}$$

3.B. What is the value of the short run total cost?

The short run total cost (STC) will be the cost of hiring 5 labor and 10 capital:

$$STC(10000) = w \cdot L + r \cdot \bar{K} = 10 \cdot 5 + 20 \cdot 10 = 250$$

Now suppose that you are given a longer turnaround timeframe so that you have some control over the capital stock you would be using in production. All other conditions remain identical to the situation described above.

3.C. Find the Marginal Product of Labor (MP_L).

- $MP_L = \dfrac{\partial}{\partial L} 4L^2K^2 = 4K^2 \cdot 2 \cdot L^{2-1} = \boxed{8LK^2}$

3.D. Find the Marginal Product of Capital (MP_K).

- $MP_K = \dfrac{\partial}{\partial K} 4L^2K^2 = 4L^2 \cdot 2 \cdot K^{2-1} = \boxed{8L^2K}$

3.E. Find the Marginal Rate of Technical Substitution ($MRTS_{LK}$)

- $MRTS_{LK} = \dfrac{MP_L}{MP_K} = \dfrac{8LK^2}{8L^2K} = \boxed{\dfrac{K}{L}}$

Problem 3. The Short Run, Long Run, and Cost Minimization (continued)

Suppose that you are given a production quota of 10000 units of output, where the inputs are labor L and capital K. The wage is given as $w = 10$ and rent for each unit of capital is given as $r = 20$. Your production technology is given as follows:

$$F(L, K) = 4L^2K^2$$

3.F. Find the optimal ratio of Labor and Capital you should use in production.

The condition to find the optimal ratio between L and K is $MRTS_{LK} = \frac{w}{r}$:

$$MRTS_{LK} = \frac{w}{r} \quad \Rightarrow \quad \frac{K}{L} = \frac{10}{20} \quad \Rightarrow \quad \boxed{L = 2K}$$

3.G. Formally express the firm's Isocost.

Assuming the long run total cost can be expressed as C, the isocost can be expressed as follows:

$$\boxed{w \cdot L + r \cdot K = C}$$

3.H. Find the optimal inputs of Labor and Capital you should use in production.

Using the optimal ratio found in `3.F`, and the production function and production quota, we can find the optimal inputs:

$$\begin{aligned} F(L, K) = 10000 \quad &\Rightarrow \quad 4L^2K^2 = 10000 \\ &\Rightarrow \quad 4(2K)^2K^2 = 10000 \\ &\Rightarrow \quad 16K^3 = 10000 \\ &\Rightarrow \quad K^3 = 625 \\ &\Rightarrow \quad \boxed{K = 5\sqrt[3]{5}} \end{aligned}$$

- Score: ______________________
- Extra Credit: ______________________