Monmouth COLLEGE

- Name: ______
- Date: ______
- Section: ______

ECON 300: Intermediate Price Theory

Problem Set #5 - Part #2: Suggested Solutions

Fall 2024

Problem 1. The Cost Minimization Problem

Suppose that your firm is producing output Q with using two inputs, labor L and capital K. The wage is given as w and rent is given as r. The firm following technology:

$$F(L,K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$$

1.A. Find the Marginal Product of Labor (MP_L).

- $MP_L = \dfrac{\partial}{\partial L}L^{\frac{1}{2}}K^{\frac{1}{2}} = K^{\frac{1}{2}} \cdot \dfrac{1}{2} \cdot L^{\frac{1}{2}-1} = \boxed{\dfrac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}}$

1.B. Find the Marginal Product of Capital (MP_K).

- $MP_K = \dfrac{\partial}{\partial L}L^{\frac{1}{2}}K^{\frac{1}{2}} = L^{\frac{1}{2}} \cdot \dfrac{1}{2} \cdot K^{\frac{1}{2}-1} = \boxed{\dfrac{1}{2}L^{\frac{1}{2}}K^{-\frac{1}{2}}}$

1.C. Find the Marginal Rate of Technical Substitution $(MRTS_{LK})$

- $MRTS_{LK} = \dfrac{MP_L}{MP_K} = \dfrac{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}}{\frac{1}{2}L^{\frac{1}{2}}K^{-\frac{1}{2}}} = \boxed{\dfrac{K}{L}}$

1.D. Find the firm's optimal ratio of inputs L and K.

The first-order condition for the firm's cost minimization is similar to that of the consumer's utility maximization problem:

$$MRTS_{LK} = \frac{w}{r} \quad \Rightarrow \quad \frac{K}{L} = \frac{w}{r} \quad \Rightarrow \quad w \cdot L = r \cdot K \quad \Rightarrow \quad \boxed{L = \frac{r}{w}K}$$

1.E. Given a production quota of 100, what is the optimal quantities of L and K that minimizes the firm's production costs?

The producer's constraint is the production quota:

$$L^{\frac{1}{2}}K^{\frac{1}{2}} = 100 \quad \Rightarrow \quad \left(\frac{r}{w}K\right)^{\frac{1}{2}}K^{\frac{1}{2}} \quad \Rightarrow \quad \left(\frac{r}{w}\right)^{\frac{1}{2}}K = 100 \quad \Rightarrow \quad \boxed{K = 100 \cdot \left(\frac{r}{w}\right)^{-\frac{1}{2}}}$$

Using the optimal ratio from 1.D, we find that:

$$L = \frac{r}{w}K \quad \Rightarrow \quad \boxed{L = 100 \cdot \left(\frac{r}{w}\right)^{\frac{1}{2}}}$$

Problem 1. The Cost Minimization Problem (continued)

Suppose that your firm is producing output Q with using two inputs, labor L and capital K. The wage is given as w and rent is given as r. The firm following technology:

$$F(L,K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$$

1.F. Given a production quota of $\bar{Q}$, what is the optimal quantities of L and K that minimizes the firm's production costs?

The producer's constraint is the production quota:

$$\bar{Q} = L^{\frac{1}{2}}K^{\frac{1}{2}} \quad \Rightarrow \quad \bar{Q} = \left(\frac{r}{w}K\right)^{\frac{1}{2}} K^{\frac{1}{2}} \quad \Rightarrow \quad \bar{Q} = \left(\frac{r}{w}\right)^{\frac{1}{2}} K \quad \Rightarrow \quad \boxed{K = \bar{Q} \cdot \left(\frac{r}{w}\right)^{-\frac{1}{2}}}$$

Using the optimal ratio from 1.D, we find that:

$$L = \frac{r}{w}K \quad \Rightarrow \quad \boxed{L = \bar{Q} \cdot \left(\frac{r}{w}\right)^{\frac{1}{2}}}$$

1.G. Find the firm's total cost function $TC(Q)$.

The firm's total cost function is:

$$\begin{aligned} TC(\bar{Q}) = w \cdot L + r \cdot K \quad &\Rightarrow \quad TC(\bar{Q}) = w \cdot \left\{\bar{Q} \cdot r^{\frac{1}{2}} \cdot w^{-\frac{1}{2}}\right\} + r \cdot \left\{\bar{Q} \cdot r^{-\frac{1}{2}} \cdot w^{\frac{1}{2}}\right\} \\ &\Rightarrow \quad TC(\bar{Q}) = \bar{Q} \cdot r^{\frac{1}{2}} \cdot w^{\frac{1}{2}} + \bar{Q} \cdot r^{\frac{1}{2}} \cdot w^{\frac{1}{2}} \\ &\Rightarrow \quad \boxed{TC(\bar{Q}) = 2 \cdot \bar{Q} \cdot r^{\frac{1}{2}} \cdot w^{\frac{1}{2}}} \end{aligned}$$

1.H. Find the firm's average total cost function $ATC(Q)$.

Applying the definition of $ATC(Q)$:

$$ATC(Q) = \frac{TC(Q)}{Q} \quad \Rightarrow \quad \boxed{ATC(Q) = 2 \cdot r^{\frac{1}{2}} \cdot w^{\frac{1}{2}}}$$

1.I. Find the firm's marginal cost function $MC(Q)$.

Applying the definition of $MC(Q)$:

$$MC(Q) = \frac{dTC(Q)}{dQ} \quad \Rightarrow \quad MC(Q) = 2 \cdot r^{\frac{1}{2}} \cdot w^{\frac{1}{2}} \cdot 1 \cdot Q^{1-1} \quad \Rightarrow \quad \boxed{MC(Q) = 2 \cdot r^{\frac{1}{2}} \cdot w^{\frac{1}{2}}}$$

Problem 2. The Profit Maximization Problem

Suppose that a firm is operating in a perfectly competitive factor and output market where the unit price of the firm's output is given as $P = 200$. The firm's total cost function is given as:

$$TC(Q) = Q^2 - 10Q + 2500$$

2.A Find the firm's total revenue function $TR(Q)$.

The total revenue is simply the market price multiplied by the units sold:

$$TR(Q) = P \cdot Q \quad \Rightarrow \quad \boxed{TR(Q) = 200Q}$$

2.B Find the firm's marginal revenue function $MR(Q)$.

Applying the definition of $MR(Q)$:

$$MR(Q) = \frac{dTR(Q)}{dQ} \quad \Rightarrow \quad MR(Q) = 200 \cdot 1 \cdot Q^{1-1} \quad \Rightarrow \quad \boxed{MR(Q) = 200}$$

2.C Find the firm's marginal cost function $MC(Q)$.

Applying the definition of $MC(Q)$:

$$\begin{aligned} MC(Q) = \frac{dTC(Q)}{dQ} \quad &\Rightarrow \quad MC(Q) = \frac{d}{dQ}Q^2 - \frac{d}{dQ}10Q + \frac{d}{dQ}2500 \\ &\Rightarrow \quad MC(Q) = 1 \cdot 2 \cdot Q^{2-1} - 10 \cdot 1 \cdot Q^{1-1} + 0 \\ &\Rightarrow \quad \boxed{MC(Q) = 2Q - 10} \end{aligned}$$

2.D If the firm finds that their marginal revenue is greater than their marginal cost of production at the current level of output, should they change their level of output? Why?

Marginal revenue is the extra revenue that the firm will enjoy when they produce (and sell) another unit of output. Marginal cost is the extra cost they incur when they produce another unit of output. Assuming that all produced goods are sold, this means that at the current level of production, the firm is making a profit of each unit produced. Therefore, they should increase their production as it will lead to greater profit.

Problem 2. The Profit Maximization Problem (continued)

Suppose that a firm is operating in a perfectly competitive factor and output market where the unit price of the firm's output is given as $P = 20$. The firm's total cost function is given as:

$$TC(Q) = Q^2 - 10Q + 2500$$

2.E What is the firm's profit maximizing level of output?

Since the profit maximizing condition is $MR(Q) = MC(Q)$:

$$MR(Q) = MC(Q) \quad \Rightarrow \quad 200 = 2Q - 10 \quad \Rightarrow \quad \boxed{Q = 85}$$

2.F What is the firm's production cost associated with the profit maximizing quantity?

Plugging in the profit maximizing quantity into the total cost function $TC(Q)$:

$$TC(85) = 85^2 - 10 \cdot 85 + 2500 \quad \Rightarrow \quad \boxed{TC(85) = 8,875}$$

- Score: ____________________
- Extra Credit: ____________________