Monmouth COLLEGE

- Name: ________________
- Date: ________________
- Section: ________________

ECON 300: Intermediate Price Theory

Problem Set #7

INSTRUCTIONS:

- This problem set is not graded.

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Problem 1. Perfect Competition

Suppose that the output market is in perfect competition, and that the demand (Q_x^D) and supply (Q_x^S) functions are given as:

$$\begin{cases} Q_x^D = 500 - P_x \\ Q_x^S = 200 + 2P_x \end{cases}$$

1.A State the five assumptions that define a perfectly competitive market.

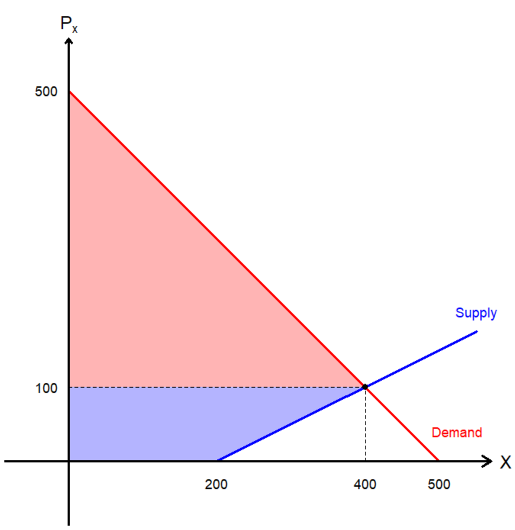
1. Infinitely many buyers and sellers.
2. Homogeneous products.
3. Perfect information.
4. Free entry and exit.
5. No transaction costs.

1.B Find the equilibrium price and quantity.

The market equlibrium price is the price that equalizes market demand and supply:

$$Q_x^D = Q_x^S \quad \Rightarrow \quad 500 - P_x = 200 + 2P_x \quad \Rightarrow \quad 300 = 3P_x \quad \Rightarrow \quad P_x^* = 100$$
$$\Rightarrow \quad Q_x^* = 400$$

1.C Plot the demand and supply curves in the empty chart. You must plot and label all elements clearly:



- The demand curve.
- The supply curve.
- The equilibrium price and quantity.
- ALL intercepts.

1.D What is the value of consumer surplus in this market?

Consumer surplus is the red triangle in the solution for 1.C:

$$CS = 400 \cdot 400 \cdot \frac{1}{2} = 80,000$$

1.E What is the value of producer surplus in this market?

Producer surplus is the blue trapezoid in the solution for 1.C:

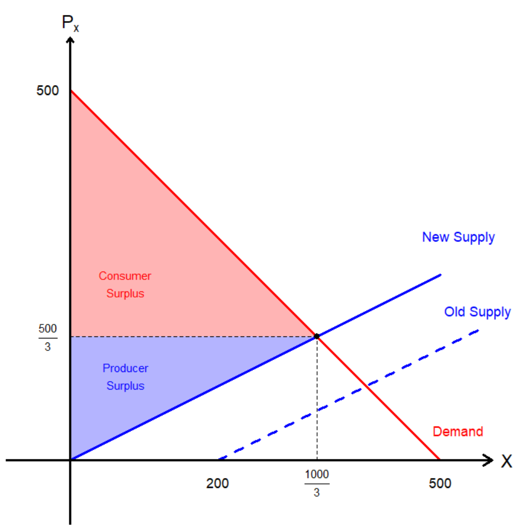
$$PS = (400 + 200) \cdot 100 \cdot \frac{1}{2} = 30,000$$

1.F Find the equilibrium price and quantity when the supply shifts to $Q_x^S = 2P_x$.

The market equlibrium price is the price that equalizes market demand and supply:

$$Q_x^D = Q_x^S \quad \Rightarrow \quad 500 - P_x = 2P_x \quad \Rightarrow \quad 500 = 3P_x \quad \Rightarrow \quad P_x^* = \frac{500}{3}$$
$$\Rightarrow \quad Q_x^* = \frac{1000}{3}$$

1.G Plot the elements listed below in the empty chart. You must plot and label all elements clearly:



- The original demand curve.
- The new supply curve from 1.F.
- The equilibrium price and quantity.
- ALL intercepts.
- Consumer surplus.
- Producer surplus.

Problem 2. Price Controls

Suppose that the output market is in perfect competition with the same parameters as **Problem 1**. The demand (Q_x^D) and supply (Q_x^S) functions are given as:

$$\begin{cases} Q_x^D = 500 - P_x \\ Q_x^S = 200 + 2P_x \end{cases}$$

2.A The government sets a price ceiling of $\bar{P}_x = 80$. Is this price ceiling "binding?"

A price ceiling is *binding* when the ceiling is set below the current market price. We found that the equilibrium price of the market described above is $P_x^* = 100$ in **Problem 1**, so a price ceiling of $\bar{P}_x = 80$ is *binding*.

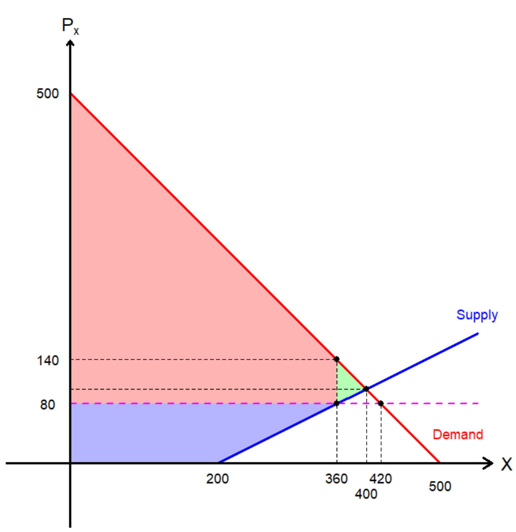
2.B Find the market price and quantity traded in the market following the price control.

The market price will be set at the ceiling, $\bar{P}_x = 80$. At this market price, the quantity demanded and supplied in the market will follow the demand and supply functions:

$$Q_x^D = 500 - \bar{P}_x = 500 - 80 = 420Q_x^S = 200 + 2\bar{P}_x = 200 + 2 \cdot 80 = 360$$

The quantity traded in the market will be the lesser of the two, so it will be $Q_x^T = 360$.

2.C Plot the effect of the price ceiling in the empty chart below. You must plot and label all elements clearly:



- The demand curve.
- The supply curve.
- The market price.
- The quantity traded in the market.
- ALL intercepts.

2.D What is the value of consumer surplus in this market with price controls?

Consumer surplus is the red trapezoid in the solution for 2.C:

$$CS = (420 + 60) \cdot 360 \cdot \frac{1}{2} = 86,400$$

2.E What is the value of producer surplus in this market with price controls?

Producer surplus is the blue trapezoid in the solution for 2.C:

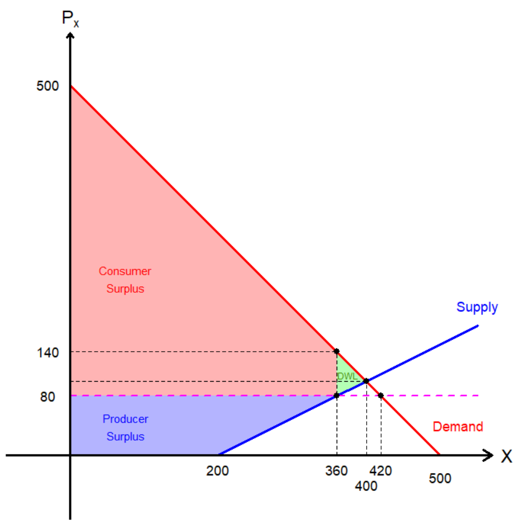
$$CS = (360 + 200) \cdot 80 \cdot \frac{1}{2} = 22,400$$

2.F What is the value of deadweight loss in this market with price controls?

Deadweight loss is the green triangle in the solution for 2.C:

$$DWL = (140 - 80) \cdot (400 - 360) \cdot \frac{1}{2} = 1,200$$

2.G Plot the elements listed below in the empty chart. You must plot and label all elements clearly:

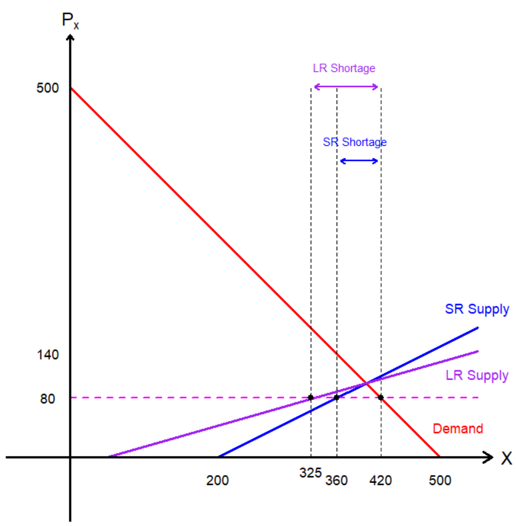


- The demand curve.
- The supply curve.
- The market price.
- The quantity traded in the market.
- ALL intercepts.
- Consumer surplus.
- Producer surplus.
- Deadweight loss.

2.H What happens to the shortage / surplus if this price ceiling is maintained for an excessively long period?

Long run adjustments lead to an increase in the shortage in the market. See the solutions to 2.I for an illustration of this claim.

2.I Plot the short run and long run effects of the price ceiling in the empty chart below. You must plot and label all elements clearly:



- The demand curve.
- The short run supply curve.
- The long run supply curve (approx.).
- Shortage / surplus in the short run.
- Shortage / surplus in the long run.

Problem 3. Taxation

Suppose that the output market is in perfect competition with the same parameters as **Problem 1**. The demand (Q_x^D) and supply (Q_x^S) functions are given as:

$$\begin{cases} Q_x^D = 500 - P_x \\ Q_x^S = 200 + 2P_x \end{cases}$$

3.A What is the equilibrium price and quantity if the government imposes a \$5 per unit tax?

The price that the consumer pays and the price that the producer receives is not the same:

$$\text{Consumer's Price} = \text{Producer's Price} + \text{Tax} \quad \Rightarrow \quad P_x^D = P_x^S + 5$$

Then, set up the condition $Q_x^D = Q_x^S$:

$$500 - P_x^D = 200 + 2P_x^S \;\Rightarrow\; 500 - (P_x^S + 5) = 200 + 2P_x^S \;\Rightarrow\; P_x^S = \frac{295}{3}$$

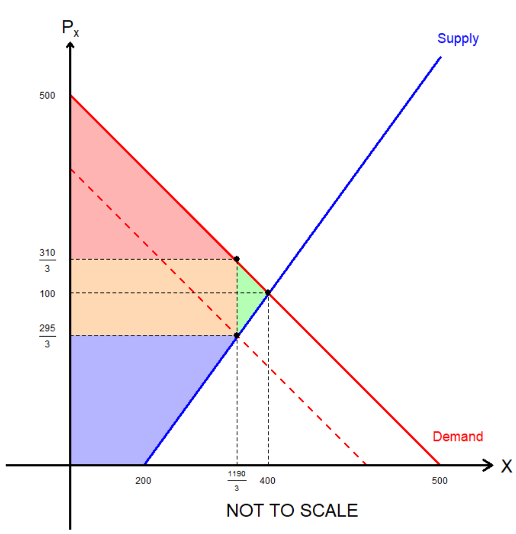
$$\Rightarrow\; P_x^D = \frac{295}{3} + 5 = \frac{310}{3}$$

$$\Rightarrow\; Q_x^* = 500 - P_x^D = \frac{1190}{3}$$

3.B What information do you need to determine whether the consumer or the producers bear a greater burden from taxation?

The own price elasticities of demand and supply. The side that has the lower price elasticity will bear a greater burden.

3.C Plot the effect of taxation in the empty chart below. You must plot and label all elements clearly:



- The demand curve.
- The supply curve.
- The market price.
- The quantity traded in the market.
- ALL intercepts.

3.D What is the value of consumer surplus in this market with taxation?

Consumer surplus is the red triangle in the solution to 3.C:

$$CS = \left(500 - \frac{310}{3}\right) \cdot \frac{1190}{3} \cdot \frac{1}{2} = \frac{708050}{9} \simeq 78,672.22$$

3.E What is the value of producer surplus in this market with taxation?

Producer surplus is the blue trapezoid in the solution to 3.C:

$$PS = \left(200 + \frac{1190}{3}\right) \cdot \frac{295}{3} \cdot \frac{1}{2} = \frac{264025}{9} \simeq 29,336.11$$

3.F What is the value of deadweight loss in this market with taxation?

Deadweight loss is the green triangle in the solution to 3.C:

$$DWL = \left(\frac{310}{3} - \frac{295}{3}\right) \cdot \left(400 - \frac{1190}{3}\right) \cdot \frac{1}{2} = \frac{25}{3} \simeq 8.3$$

3.G What is the value of government revenue in this market with taxation?

Government revenue is the orange rectangle in the solution to 3.C:

$$GR = \left(\frac{310}{3} - \frac{295}{3}\right) \cdot \frac{1190}{3} = \frac{5950}{3} \simeq 1983.33$$

Problem 4. Market Structures: Monopolies

Suppose that you are a profit maximizing monopoly producer of good x. The market demand function is estimated to be $Q_x^D = 500 - P_x$, and your total cost function is given as: $TC(Q_x) = 100 + Q_x^2$.

4.A Find the mathematical expression for your total revenue function.

We first find the inverse demand function:

$$Q_x^D = 500 - P_x \quad \Rightarrow \quad P_x = 500 - Q_x$$

Then the total revenue function can be found as:

$$TR(Q_x) = P_x \cdot Q_x = (500 - Q_x) \cdot Q_x \quad \Rightarrow \quad TR(Q_x) = 500Q_x - Q_x^2$$

4.B Find your marginal revenue function.

Marginal revnue measures "how much does revnue increase when output is increased?":

$$MR(Q_x) = \frac{d}{dQ_x} TR(Q_x) = \frac{d}{dx}\left(500Q_x - Q_x^2\right) \quad \Rightarrow \quad MR(Q_x) = 500 - 2Q_x$$

4.C Find your marginal cost function.

Marginal cost measures "how much does cost increase when output is increased?":

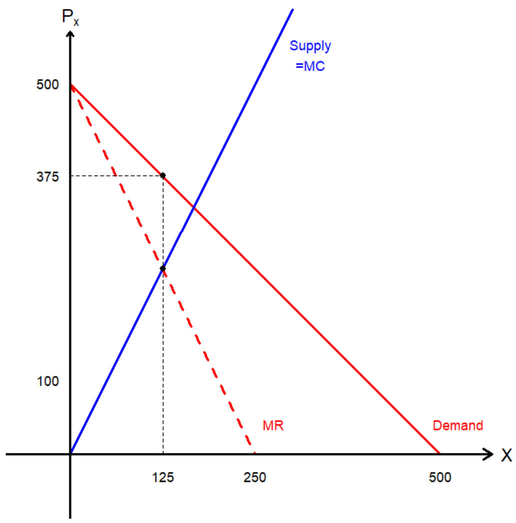
$$MC(Q_x) = \frac{d}{dQ_x} TC(Q_x) = \frac{d}{dx}\left(100 + Q_x^2\right) \quad \Rightarrow \quad MR(Q_x) = 2Q_x$$

4.D What is your profit maximizing quantity and price?

Profit is maximized when $MR = MC$:

$$\begin{aligned} MR(Q_x) = MC(Q_x) \quad \Rightarrow \quad 500 - 2Q_x = 2Q_x \quad &\Rightarrow \quad Q_x^* = 125 \\ &\Rightarrow \quad P_x^* = 375 \quad \because P_x^* = 500 - Q_x^* \end{aligned}$$

4.E Complete the empty chart with the elements listed below. You must plot and label all elements clearly:



- The market demand curve.
- The marginal revenue curve.
- The marginal cost curve.
- The quantity traded in the market.
- ALL intercepts.

4.F What is the source of your firm's market power?

My firm is the only producer of good x.

4.G Suppose that you had the same market conditions given in **Problem 4**, but the output market was perfectly competitive. What would the equilbrium market quantity and price be?

The supply function is the marginal cost function:

$$P_x = 2Q_x$$

Using the supply function above, and the inverse demand function we found in 4.A, we can find:

$$2Q_x = 500 - Q_x \quad \Rightarrow \quad Q_x^* = \frac{500}{3}$$

Then the market price can be found as:

$$P_x^* = 2Q_x \quad \Rightarrow \quad P_x^* = \frac{1000}{3}$$

Problem 5. Market Structures: Cournot Duopolies

Suppose firm A and firm B are competing in the market where the market demand is given as $Q = 500 - P_x$. Competition is over production quantity, and the firms' total cost functions are given as:

$$\begin{cases} TC_A(Q) = 50 + Q \\ TC_B(Q) = 10 + 2Q \end{cases}$$

5.A Find the profit function for firm A.

First we establish that the market supply is the sum of the output of firm A and B. That is:

$$\text{Market Supply} = \text{Firm A's Supply} + \text{Firm B's Supply} \quad \Rightarrow \quad Q = q_A + q_B$$

Then we can find the profit function for firm A as follows:

$$\begin{aligned} \Pi_A = P \cdot q_A - TC_A(q_A) \quad &\Rightarrow \quad \Pi_A = (500 - Q) \cdot q_A - (50 + q_A) \\ &\Rightarrow \quad \Pi_A = (500 - (q_A + q_B)) \cdot q_A - (50 + q_A) \\ &\Rightarrow \quad \Pi_A = 500q_A - q_A^2 - q_A q_B - 50 - q_A \\ &\Rightarrow \quad \Pi_A = 499q_A - q_A^2 - q_A q_B - 50 \end{aligned}$$

5.B Find the profit function for firm B.

Following a similar logic from 5.A:

$$\begin{aligned} \Pi_B = P \cdot q_B - TC_B(q_B) \quad &\Rightarrow \quad \Pi_B = (500 - Q) \cdot q_B - (10 + 2q_B) \\ &\Rightarrow \quad \Pi_B = (500 - (q_A + q_B)) \cdot q_B - (10 + 2q_B) \\ &\Rightarrow \quad \Pi_B = 500q_A - q_A^2 - q_A q_B - 10 - 2q_B \\ &\Rightarrow \quad \Pi_B = 498q_B - q_A^2 - q_A q_B - 10 \end{aligned}$$

5.C Find the best response function of firm A.

The best response function for firm A can be found by solving the profit maximization problem for firm A:

$$\frac{d}{dq_A}\Pi_A = 0 \quad \Rightarrow \quad 499 - 2q_A - q_B = 0$$

Rearranging the terms above, we find:

$$q_A = \frac{499}{2} - \frac{1}{2}q_B$$

5.D Find the best response function of firm B.

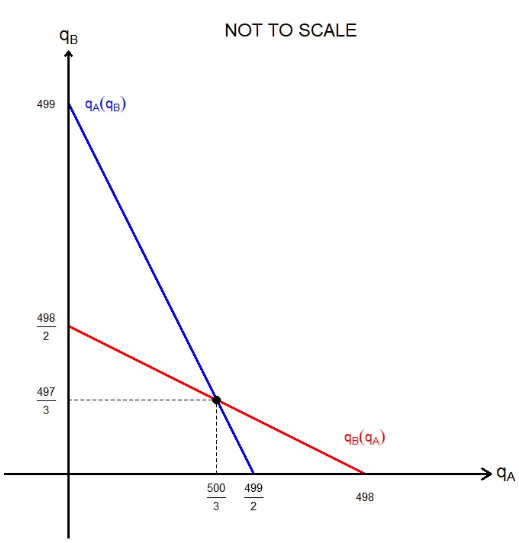
The best response function for firm B can be found by solving the profit maximization problem for firm B:

$$\frac{d}{dq_B}\Pi_B = 0 \quad \Rightarrow \quad 498 - 2q_B - q_A = 0$$

Rearranging the terms above, we find:

$$q_B = \frac{498}{2} - \frac{1}{2}q_A$$

5.E Plot the firms' best response functions in the empty chart below. You must plot and label all elements clearly:



- The best response curve of firm A.
- The best response curve of firm B.
- ALL intercepts.
- The equilibrium quantity.

Problem 6. Static Games of Complete Information

Suppose that there are two players (players 1 and 2) interacting in a static game of complete information. Each player can choose either Up (U) or Down (D).

6.A What does "Static" mean in this context?

Static means that the players in this game will make their moves simultaneously.

6.B What does "Complete Information" mean in this context?

Complete information means that each player has full knowledge on other players' payoffs.

6.C Suppose that the payoff structure is given as follows. Express the game in its normal form representation below, and find all Nash equilibria.

- When both players 1 and 2 play U, their payoffs are 10 each.
- When both players 1 and 2 play D, their payoffs are 5 each.
- When players 1 and 2 play different actions, the player playing U will get a payoff of 2, and the player playing D will get a payoff of 15.

Player 2

Player 1		*U*	*D*
	U	(10,10)	(2,15)
	D	(15,2)	$\boxed{(5,5)}$

6.D Suppose that this game is repeated 5 times. Would the players be able to cooperate for at least one period? Why?

- At the 5th round, each player will have no incentive to stick to choosing U.
- Each player knows this, so cooperation will fall apart at the 5th round of the game.
- Since each player knows that cooperation will cease at the 5th round, they will behave as if the 4th round is the final round.
- At the 4th round, each player will have no incentive to stick to choosing U...
- This repeats, and no cooperation is possible between the players.

6.E Suppose that this game is repeated 500 times. Would the players be able to cooperate for at least one period? Why?

- At the 500th round, each player will have no incentive to stick to choosing U.
- Each player knows this, so cooperation will fall apart at the 500th round of the game.
- Since each player knows that cooperation will cease at the 500th round, they will behave as if the 4th round is the final round.
- At the 499th round, each player will have no incentive to stick to choosing U...
- This repeats, and no cooperation is possible between the players.

6.F Suppose that this game is repeated indefinitely. Would the players be able to cooperate for at least one period? Why?

Since there is no "terminal round," each player will come up with a strategy over time. If the discount factor for both individuals are not excessively high, a trigger strategy will acheive cooperation. Suppose that the individuals depreciate the future by a factor of β, and that the players adopt a trigger strategy. Then, the expected payoff of cooperation will be:

$$\Pi_{coop} = 10 + \beta \cdot 10 + \beta^2 \cdot 10 + \cdots = \frac{10}{1-\beta}$$

Meanwhile, the expected payoff of betrayal will be:

$$\Pi_{betray} = 15 + 0 + 0 + \cdots = 15$$

As long as $\Pi_{coop} > \Pi_{betray}$, cooperation will be sustainable:

$$\Pi_{coop} > \Pi_{betray} \quad \Rightarrow \quad \frac{10}{1-\beta} > 15 \quad \Rightarrow \quad \frac{1}{3} < \beta < 1$$

Problem 7. Static Games of Complete Information

Suppose that two players are playing the following static game of complete information.

		Player 2		
		L	*C*	*R*
Player 1	*U*	(10,2)	(3,8)	(11,10)
	D	(3,5)	(2,12)	(0,12)

7.A Are there any dominant / dominated strategies for player 1?

U is a dominant strategy for player 1.

7.B Are there any dominant / dominated strategies for player 2?

L is a dominated strategy for player 2. Both C and R dominate L.

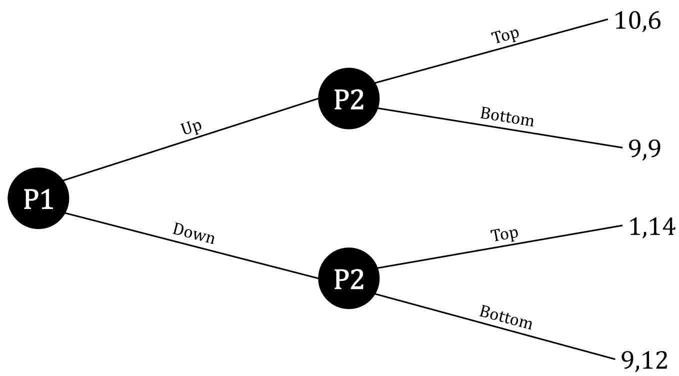
7.C Find all Nash equilibria for this game.

The pure strategy Nash equilibrium is (U, R).

		Player 2		
		L	*C*	*R*
Player 1	*U*	(10,2)	(3,8)	$\boxed{(11,10)}$
	D	(3,5)	(2,12)	(0,12)

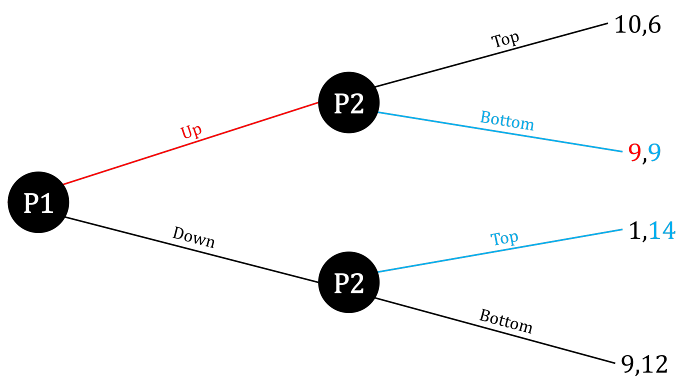
Problem 8. Dynamic Games of Complete Information

Suppose that two players are playing the following dynamic game of complete information.



8.A What are the Nash equilibria of the game described above?

The pure strategy Nash equilibrium is (U, BT).



8.B Represent this game in its normal form below:

Player 2

Player 1

	TT	TB	BT	BB
U	(10,6)	(10,6)	(9,9)	(9,9)
D	(1,14)	(9,12)	(1,14)	(9,12)

8.C Find all Nash equilibria using the normal form found in 8.B.

The pure strategy Nash equilibria are (U, BT) and (U, BB).

		Player 2			
		TT	*TB*	*BT*	*BB*
Player 1	*U*	(10,6)	(10,6)	(9,9)	(9,9)
	D	(1,14)	(9,12)	(1,14)	(9,12)

8.D Does your answer from 8.C match that of 8.A? If not, why?
Hint: Does player 2's strategy BB make sense?

The Nash equlibria we get from the normal form has an additional strategy of (U, BB). However, this strategy is not in player 2's best interest, and should not be considered a valid option in the dynamic game. BB means that "whether player 1 plays U or D, player 2 will play B." But suppose that player 1 plays D. If player 2 plays T at this stage, they will earn 14 as payoff, and if player 2 plays B at this stage, they will earn 12.

Extra: The concept of a "Subgame Perfect Nash Equilibrium (SPNE)" effectively excludes such illogical decisions. However, this will not be covered in detail in this course.