



- Name: \_\_\_\_\_
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  - Section: \_\_\_\_\_
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## **ECON 300: Intermediate Price Theory**

### **Problem Set #4: Suggested Solutions**

**Fall 2024**

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**Problem 1. Comparative Statics of the UMP: Price**

Suppose that a consumer's utility function  $u(\cdot)$  over two goods  $x$  and  $y$  is given as:

$$u(x, y) = 2xy^4$$

The consumer's budget is \$120, and the unit price of good  $x$  is \$1, and the unit price of good  $y$  is \$4.

- 1.A. Find the marginal utility of good  $x$  and  $y$ .

Take the partial derivative of the utility function with respect to  $x$  and  $y$ .

$$MU_x = \frac{\partial}{\partial x} u(x, y) = \frac{\partial}{\partial x} 2xy^4 = \boxed{2y^4} \quad MU_y = \frac{\partial}{\partial y} u(x, y) = \frac{\partial}{\partial y} 2xy^4 = \boxed{8xy^3}$$

- 1.B. Find the marginal rate of substitution between goods  $x$  and  $y$ .

The marginal rate of substitution is the ratio of the marginal utilities of good  $x$  and  $y$ .

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{2y^4}{8xy^3} = \boxed{\frac{y}{4x}}$$

- 1.C. Find the formal expression for the consumer's budget constraint.

The budget constraint is expressed as follows:

$$\text{Money Spent on Goods} = \text{Total Budget} \Rightarrow \boxed{x + 4y = 120}$$

- 1.D. Find the optimal ratio of goods  $x$  and  $y$  the consumer should purchase to maximize their utility.

The optimal ratio of goods  $x$  and  $y$  can be found using the relative prices of goods (slope of the budget line) and the marginal rate of substitution (slope of the indifference curve).

$$MRS_{xy} = \frac{P_x}{P_y} \Rightarrow \frac{y}{4x} = \frac{1}{4} \Rightarrow 4y = 4x \Rightarrow \boxed{x = y}$$

- 1.E. Find the optimal bundle that the consumer should purchase to maximize their utility.

Use the budget constraint from 1.C., and substitute in the optimal ratio from 1.D. to find the optimal bundle.

$$\underbrace{x + 4y = 120}_{\because x=y \text{ from 1.D}} \Rightarrow \underbrace{x + 4x = 120}_{\because x=y \text{ from 1.D}} \Rightarrow 5x = 120 \Rightarrow \boxed{x^* = 24} \Rightarrow \boxed{y^* = 24}$$

**Problem 1. Comparative Statics of the UMP: Price (continued)**

- 1.F. Suppose that the price of good  $x$  increased from \$1 to \$2. Find the optimal bundle that the consumer should purchase to maximize their utility under this updated price of good  $x$ .

Consumer's  $MRS$  is unchanged, and under the new price of good  $x$ ,  $P_x^1$ , the optimal ratio is:

$$MRS_{xy} = \frac{P_x^1}{P_y} \Rightarrow \frac{y}{4x} = \frac{2}{4} \Rightarrow 4y = 8x \Rightarrow 2x = y$$

Using the updated budget constraint, and the new optimal ratio, we find:

$$2x + 4y = 120 \Rightarrow y + 4y = 120 \Rightarrow 5y = 120 \Rightarrow y^* = 24 \Rightarrow x^* = 12$$

- 1.G. Suppose that the price of good  $x$  increased from \$2 to \$4. Find the optimal bundle that the consumer should purchase to maximize their utility under this updated price of good  $x$ .

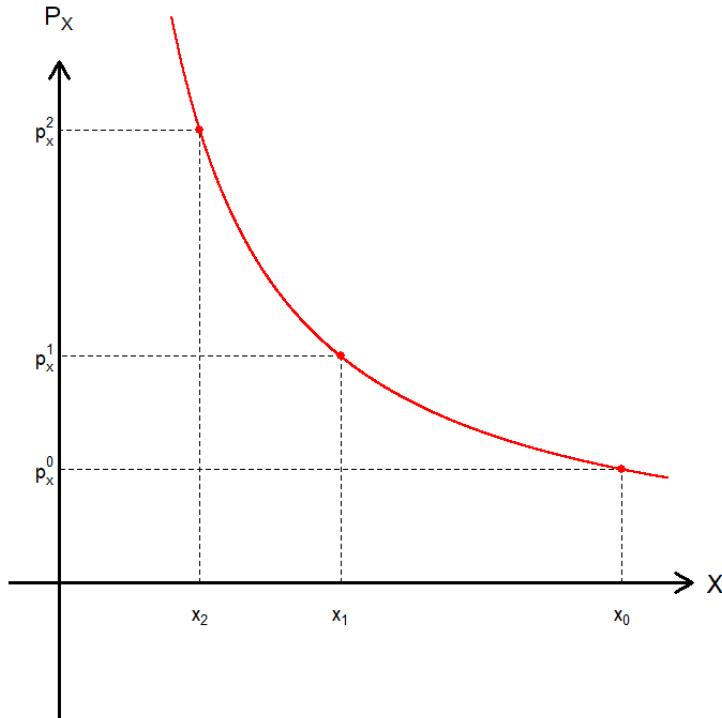
Following the same process as 1.F with new price of  $x$ ,  $P_x^2$ :

$$MRS_{xy} = \frac{P_x^2}{P_y} \Rightarrow \frac{y}{4x} = \frac{4}{4} \Rightarrow 4x = y$$

Using the updated budget constraint, and the new optimal ratio, we find:

$$4x + 4y = 120 \Rightarrow y + 4y = 120 \Rightarrow 5y = 120 \Rightarrow y^* = 24 \Rightarrow x^* = 6$$

- 1.H. Using your answers from 1.E, 1.F, and 1.G, approximate the consumer's demand curve in the empty chart below.



**Problem 2. Comparative Statics of the UMP: Income**

Suppose that a consumer's utility function  $u(\cdot)$  over two goods  $x$  and  $y$  is given as:

$$u(x, y) = 5x^2y$$

The consumer's budget is \$60, and the unit price of good  $x$  is \$1, and the unit price of good  $y$  is \$2.

2.A. Find the marginal utility of good  $x$  and  $y$ .

Take the partial derivative of the utility function with respect to  $x$  and  $y$ .

$$MU_x = \frac{\partial}{\partial x} u(x, y) = \frac{\partial}{\partial x} 5x^2y = \boxed{10xy} \quad MU_y = \frac{\partial}{\partial y} u(x, y) = \frac{\partial}{\partial y} 5x^2y = \boxed{5x^2}$$

2.B. Find the marginal rate of substitution between goods  $x$  and  $y$ .

The marginal rate of substitution is the ratio of the marginal utilities of good  $x$  and  $y$ .

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{10xy}{5x^2} = \boxed{\frac{2y}{x}}$$

2.C. Find the formal expression for the consumer's budget constraint.

The budget constraint is expressed as follows:

$$\text{Money Spent on Goods} = \text{Total Budget} \Rightarrow \boxed{x + 2y = 60}$$

2.D. Find the optimal ratio of goods  $x$  and  $y$  the consumer should purchase to maximize their utility.

The optimal ratio of goods  $x$  and  $y$  can be found using the relative prices of goods (slope of the budget line) and the marginal rate of substitution (slope of the indifference curve).

$$MRS_{xy} = \frac{P_x}{P_y} \Rightarrow \frac{2y}{x} = \frac{1}{2} \Rightarrow \boxed{x = 4y}$$

2.E. Find the optimal bundle that the consumer should purchase to maximize their utility.

Use the budget constraint from 2.C., and substitute in the optimal ratio from 2.D. to find the optimal bundle.

$$\underbrace{x + 2y = 60}_{\because x=4y \text{ from 2.D}} \Rightarrow 4y + 2x = 60 \Rightarrow 6y = 60 \Rightarrow \boxed{y^* = 10} \Rightarrow \boxed{x^* = 40} \underbrace{\qquad\qquad\qquad}_{\because x=4y \text{ from 2.D}}$$

**Problem 2. Comparative Statics of the UMP: Income (continued)**

- 2.F. Suppose that the consumer's income increased from \$60 to \$90. Find the optimal bundle that the consumer should purchase to maximize their utility.

The consumer's  $MRS$  is unchanged, and neither are the prices. Since the optimal ratio depends on these two factors, is also remains unchanged. Plugging the optimal ratio into the updated budget constraint:

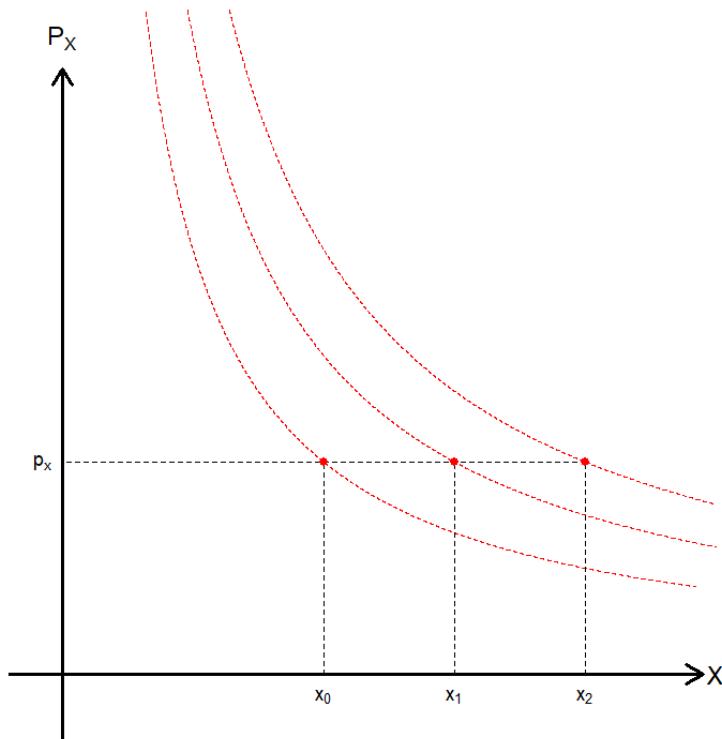
$$x + 2y = 90 \Rightarrow 4y + 2y = 90 \Rightarrow 6y = 90 \Rightarrow y^* = 15 \Rightarrow x^* = 60$$

- 2.G. Suppose that the consumer's income increased from \$90 to \$120. Find the optimal bundle that the consumer should purchase to maximize their utility.

Similar to 2.F, the consumer's  $MRS$  is unchanged, and neither are the prices. Since the optimal ratio depends on these two factors, is also remains unchanged. Plugging the optimal ratio into the updated budget constraint:

$$x + 2y = 120 \Rightarrow 4y + 2y = 120 \Rightarrow 6y = 120 \Rightarrow y^* = 20 \Rightarrow x^* = 80$$

- 2.H. Using your answers from 2.E, 2.F, and 2.G, approximate how the consumer's demand curve reacts to the change in consumers' income in the empty chart below.



**Problem 3. Deriving the Demand Curve**

Suppose that a consumer's utility function  $u(\cdot)$  over two goods  $x$  and  $y$  is given as:

$$u(x, y) = x^2y^4$$

The consumer's budget is  $M$ , and the unit price of good  $x$  is  $P_x$ , and the unit price of good  $y$  is  $P_y$ .

3.A. Find the marginal utility of good  $x$  and  $y$ .

Take the partial derivative of the utility function with respect to  $x$  and  $y$ .

$$MU_x = \frac{\partial}{\partial x} u(x, y) = \frac{\partial}{\partial x} x^2y^4 = \boxed{2xy^4} \quad MU_y = \frac{\partial}{\partial y} u(x, y) = \frac{\partial}{\partial y} x^2y^4 = \boxed{4x^2y^3}$$

3.B. Find the marginal rate of substitution between goods  $x$  and  $y$ .

The marginal rate of substitution is the ratio of the marginal utilities of good  $x$  and  $y$ .

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{2xy^4}{4x^2y^3} = \boxed{\frac{y}{2x}}$$

3.C. Find the formal expression for the consumer's budget constraint.

The budget constraint is expressed as follows:

$$\text{Money Spent on Goods} = \text{Total Budget} \Rightarrow \boxed{P_x \cdot x + P_y \cdot y = M}$$

3.D. Find the optimal ratio of goods  $x$  and  $y$  the consumer should purchase to maximize their utility.

The optimal ratio of goods  $x$  and  $y$  can be found using the relative prices of goods (slope of the budget line) and the marginal rate of substitution (slope of the indifference curve).

$$MRS_{xy} = \frac{P_x}{P_y} \Rightarrow \frac{y}{2x} = \frac{P_x}{P_y} \Rightarrow \boxed{P_y \cdot y = 2 \cdot P_x \cdot x}$$

3.E. Find the expression for the consumer's demand of good  $x$ .

Use the budget constraint from 3.C., and substitute in the optimal ratio from 3.D. to find the optimal bundle.

$$\underbrace{P_x x + P_y y = M}_{\because P_y \cdot y = 2 \cdot P_x \cdot x \text{ from 3.D}} \Rightarrow P_x x + 2P_x x = M \Rightarrow 3P_x x = M \Rightarrow \boxed{x^*(P_x; M) = \frac{1}{3} \cdot \frac{M}{P_x}}$$