Monmouth COLLEGE

- Name: ____________________
- Date: ____________________
- Section: ____________________

ECON 300: Intermediate Price Theory

Problem Set #6

INSTRUCTIONS:

- This problem set is not graded.

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Problem 1. Budget Constraints

Suppose that you as the consumer are participating in a market for good x and good y. Each unit of good x is \$5, and each unit of good y is \$4. Your income to spend on purchasing good x and y is \$200.

1.A Find the mathematical expression for your budget constraint.

1.B If you spent all of your money on good y, how many units can you purchase?

1.C Calculate the slope of the budget constraint.

1.D Interpret the meaning of the slope of the budget constraint by completing the following statement:

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If I am already spending all of my money,
should I purchase one extra unit of good x,
I must sell off ______ units of good y.
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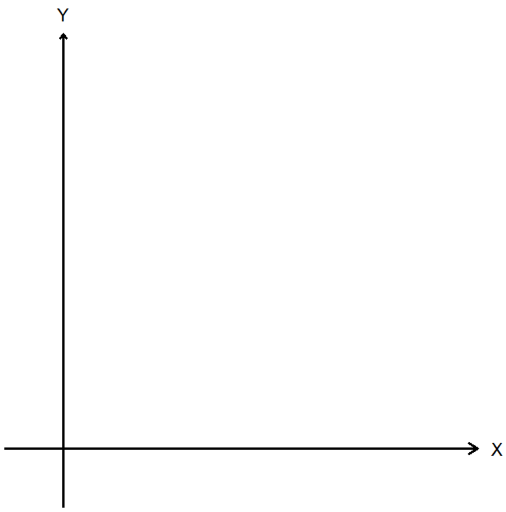
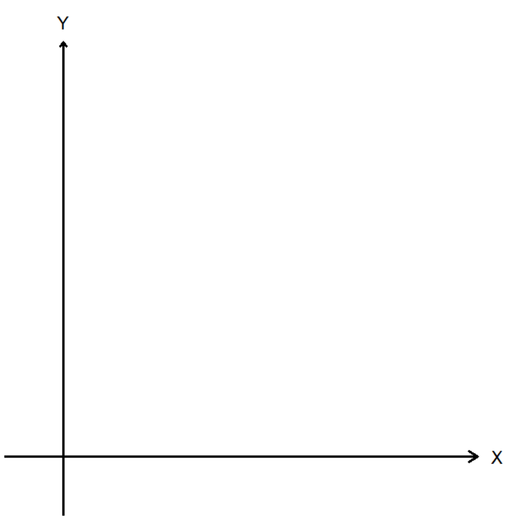
1.E Plot your budget constraint below:



- The budget line.
- The x intercept value.
- The y intercept value.

1.F Find the mathematical expression for your budget constraint when the price of good x falls to 4.

1.G Plot the updated budget constraint from 1.F below, and describe how it is different from your answer in 1.E.



Problem 2. Preferences and Utility

2.A Fill out the following table using the definition of utility and marginal utility.

Quantity	Utility	Marginal Utility
1	100	-
2	190	
3	270	80
4		70
5		10

2.B Describe the concept of the Law of Diminishing Marginal Utility.

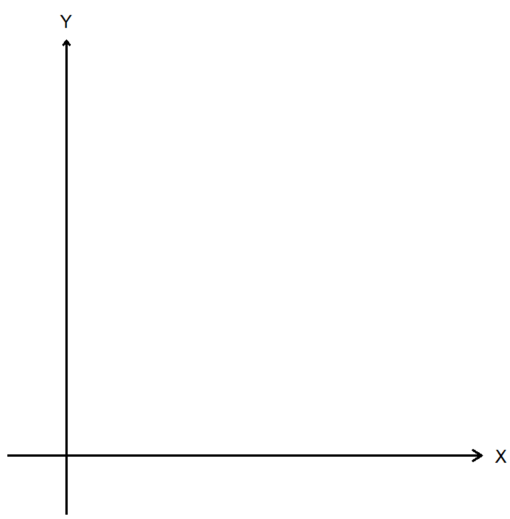
2.C Describe what it means for utility to be an ordinal concept.

2.D Suppose you have two utility functions $u_1(x, y) = 2xy$ and $u_2(x, y) = 4xy$. Can you these two utility functions represent the same underlying preference relation?

2.E Identify and briefly explain the two axioms that are required for a preference relationship to be rational?

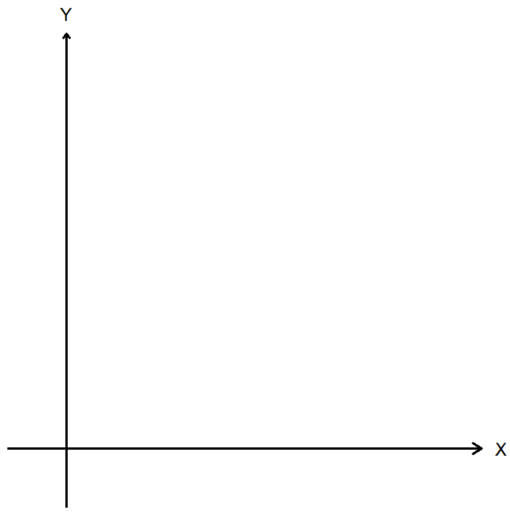
2.F Suppose that our preferences follow the rule that "when $A \succsim B$, and $B \succsim C$, then $A \succsim C$." Which axiom does this refer to?

2.G Suppose that consumers believe that goods x and y are completely interchangeable at a 3:1 ratio. That is, every 3 units of good x is perfectly substituted by 1 unit of good y. Provide a rough plot of the indifference curves.



- Two levels of indifference curves.
- Label the indifference curve representing the lower utility as IC_{low}.
- Label the indifference curve representing the greater utility as IC_{high}.
- Intercept values / ratio indicators.

2.H Suppose that consumers believe that goods x and y must be used strictly following a 3:1 ratio. That is, every 3 units of good x must be accompanied by 1 unit of good y. Provide a rough plot of the indifference curves.



- Two levels of indifference curves.
- Label the indifference curve representing the lower utility as IC_{low}.
- Label the indifference curve representing the greater utility as IC_{high}.
- Intercept values / ratio indicators.

2.I What is the definition of the marginal rate of substitution? How would you find the marginal rate of substitution on a graph?

2.J Suppose that the MRS_{xy} is measured to be 2. Complete the following statement:

While keeping the consumer's utility constant,
when the consumer gets one extra unit of good x,
they will be willing to give up ______ units of good y.

Problem 3. Utility Maximization: Leontief

Suppose you are participating in a market with goods x and y. The market price of good x and y are P_x and P_y, respectively and your income is M. Assume that your utility function is given as:

$$u(x, y) = min\{2x, 3y\}$$

3.A What happens to utility when $2x > 3y$?

3.B What happens to utility when $2x < 3y$?

3.C Is there an optimal ratio of goods x and y?
If so, what is the optimal ratio? If not, what is the rule to follow?

3.D What is the demand function for good x?

Problem 4. Utility Maximization: Linear

Suppose you are participating in a market with goods x and y. The market price of good x and y are $P_x = 10$ and $P_y = 5$, respectively and your income is $M = 100$. Assume that your utility function is given as:

$$u(x, y) = 5x + 2y$$

4.A Calculate the marginal utility of x and y.

4.B Is there an optimal ratio of goods x and y?
If so, what is the optimal ratio? If not, what is the rule to follow?

4.C What is the amount of good x that the consumer should purchase?

4.D What happens to the optimal amount of good x when P_y falls to 3?

Problem 5. Utility Maximization: Cobb-Douglas

Suppose you are participating in a market with goods x and y. The market price of good x and y are P_x and P_y, respectively and your income is M. Assume that your utility function is given as:

$$u(x, y) = 3xy^2$$

5.A Calculate the marginal utility of x and y.

5.B Assuming that $MU_x = y$ and $MU_y = 2x$, find the marginal rate of substitution.

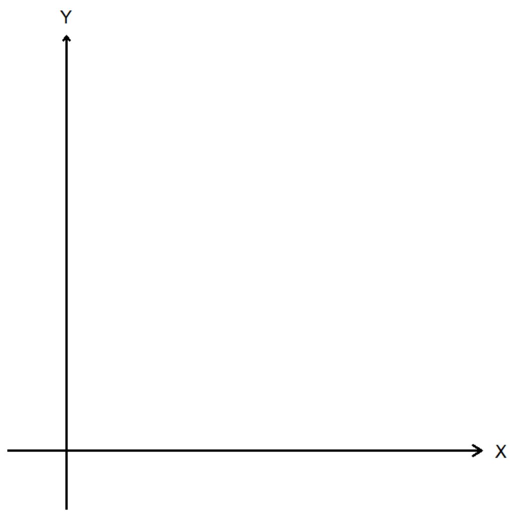
5.C Is there an optimal ratio of goods x and y?
If so, what is the optimal ratio? If not, what is the rule to follow?

5.D What is the demand function for good x?

Problem 6. Comparative Statics of the UMP

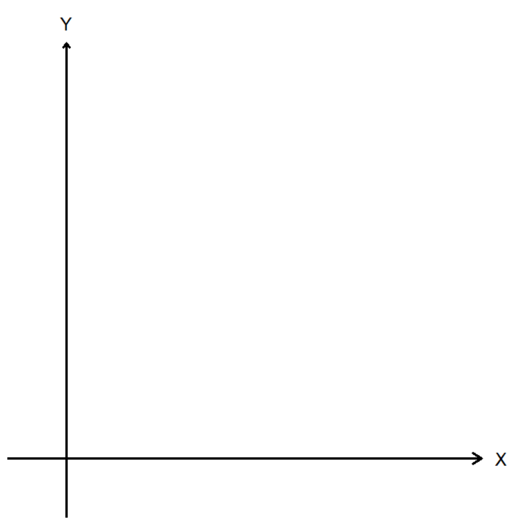
Suppose that the consumer's utility function is Cobb-Douglas, and that the price of good x is P_x, the price of good y is P_y, and income is M.

6.A Depict the consumer's utility maximization problem in the empty chart below. You must plot and label the following elements:



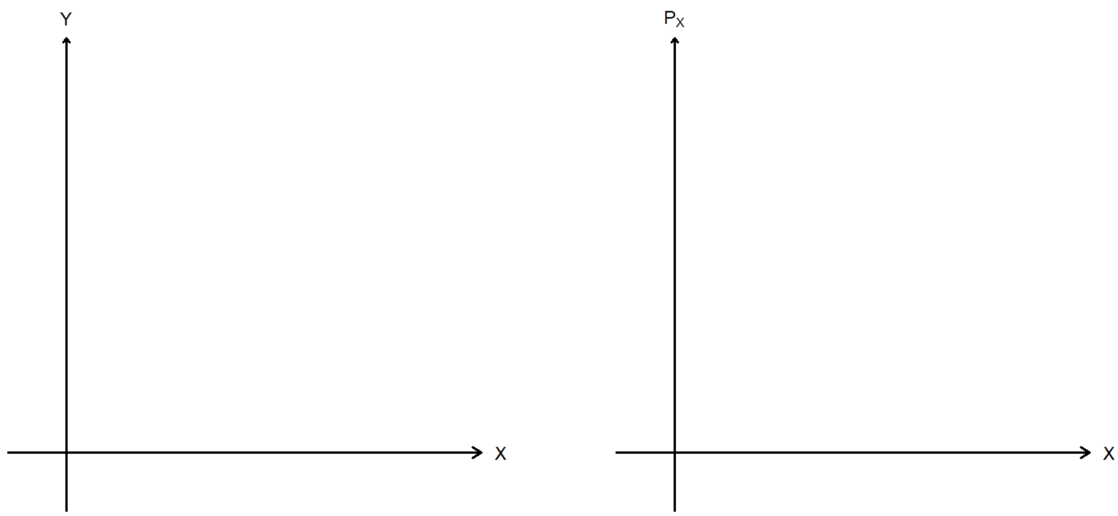
- The budget constraint (approximate).
- The indifference curve (approximate).
- The optimal bundle. The bundle's coordinates need not be exact.
- All intercept values.

6.B Suppose good x is a normal good, and that that the consumer's income increased to M'. Depict what would happen to the visualization of the consumer's utility maximization problem in the empty chart below:

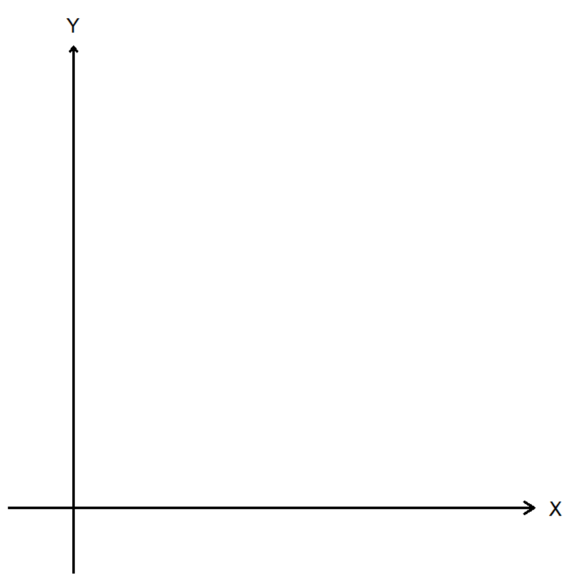


- The original and new budget constraint (approximate).
- The original and new indifference curve (approximate).
- The original and new optimal bundle. The bundle's coordinates need not be exact.
- All intercept values.

6.C Suppose that good x is an ordinary good, and that the price of good x increased. In the diagram to the left, plot the individual consumer's utility maximization problem following the rules of 6.B. To the right, plot the individual consumer's demand curve based on your answers on the diagram to the left. The diagrams need not be to scale.



6.D Suppose good x is a ordinary good, and that that the price of good x increased. Plot the diagram that demonstrates the Hicksian income and substitution effects in the empty chart below:



- The original and new budget constraint.
- The "Hicksian" budget constraint.
- The original and new indifference curve (approximate).
- The original and new optimal bundle. The bundle's coordinates need not be exact.
- The "Hicksian" optimal bundle.
- The substitution and income effects in arrows.

Problem 7. Production Functions: Linear

Suppose you are a producer with production technology that allows you to perfectly substitute between labor and capital at a 1:2 ratio. Specifically, your production function takes the form:

$$F(L, K) = 2L + K$$

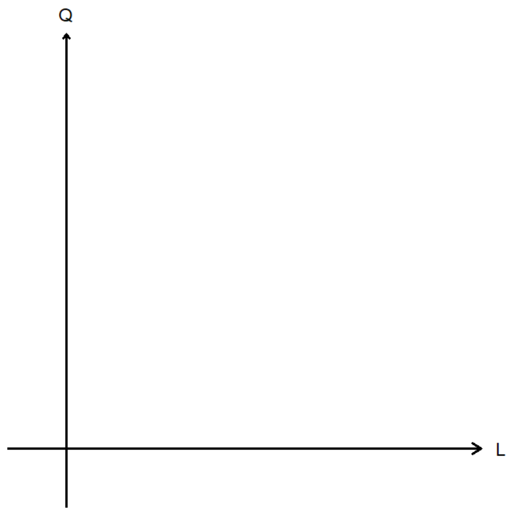
7.A How many units of capital is required to replace one unit of labor?

7.B Calculate the marginal product of labor and capital.

7.C Calculate the marginal rate of technical substitution. How does this relate to your answer in 7.A?

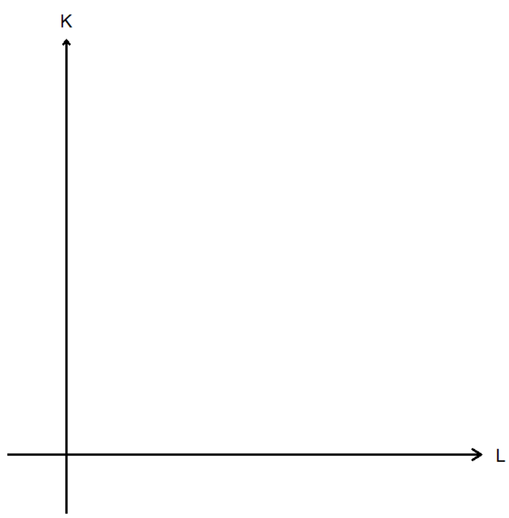
7.D Does this production technology exhibit increasing, decreasing, or constant returns to scale?

7.E Suppose that in the short run, capital is fixed at 5. Plot the short run production function in the empty chart below:



- The short run production function.
- The intercepts.

7.F Suppose now in the long run, capital is flexible. Plot the isoquant of the production technology in the empty chart below:



- Two levels of isoquants.
- Label the isoquant representing the lower level of production as IQ_{low}.
- Label the isoquant representing the greater level of production as IQ_{high}.
- Intercept values / ratio indicators.

Problem 8. Production Functions: Leontief

Suppose you are a producer with production technology that allows requires you to use labor and capital following a strict 3:2 mix. Specifically, for each 3 units of labor and 2 units of capital, you get 6 units of output. That is, your production function takes the form:

$$F(L, K) = min\{2L, 3K\}$$

8.A How many units of capital is required to replace one unit of labor?

8.B Does this production technology exhibit increasing, decreasing, or constant returns to scale?

8.C Suppose that your production technology shifts to $F_1(L, K) = min\{4L, 6K\}$. Does this change represent technical progress?

Problem 9. Cost Functions

Suppose you are a producer of some good, and the only two inputs you use are labor and capital. Labor is paid wages (w), and capital is paid rent (r).

9.A What is the mathematical expression of your total cost function $TC(Q)$?

9.B How would you calculate your average cost function $ATC(Q)$?

9.C How would you calculate your marginal cost function $MC(Q)$?

9.D In your own words, explain why the marginal cost curve will pass through the lowest point of the average cost curve.

Problem 10. Cost Minimization

Suppose you are a producer of some good, and the only two inputs you use are labor and capital. Wage is $w = 100$ and rent is $r = 50$. Your production technology is given as:

$$F(L, K) = L^2 K$$

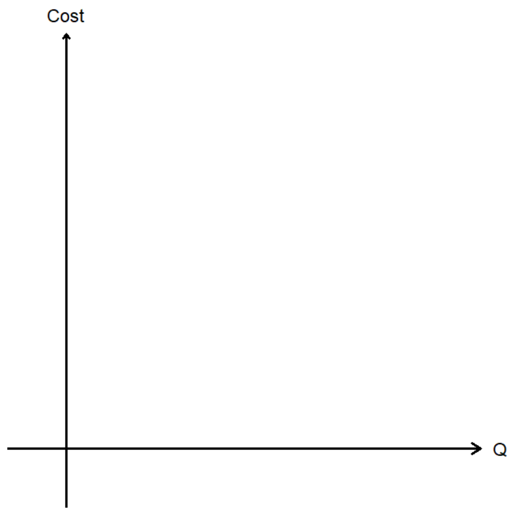
10.A Find the marginal product of labor and capital.

10.B Assuming $MP_L = 2K$ and $MP_K = L$, find the marginal rate of technical substitution.

10.C Is there an optimal ratio of inputs L and K?
If so, what is the optimal ratio? If not, what is the rule to follow?

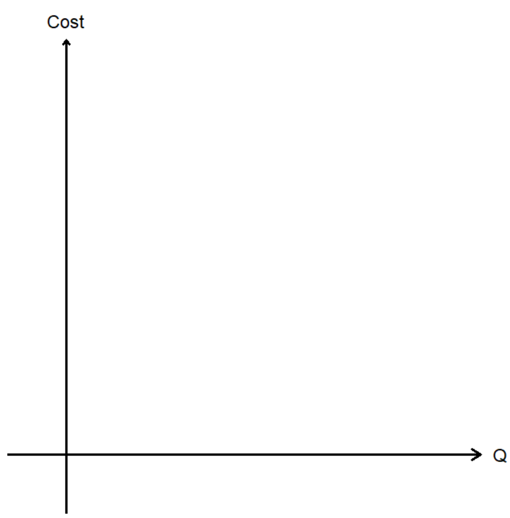
10.D If your target output is $Q = 1,000$, what is your total cost of production?

10.E Plot the producer's cost minimization problem below:



- The isocost.
- The isoquant (approximate).
- The optimal input bundle.

10.F For this question only, assume that rent increased. Plot the producer's cost minimization problem below:

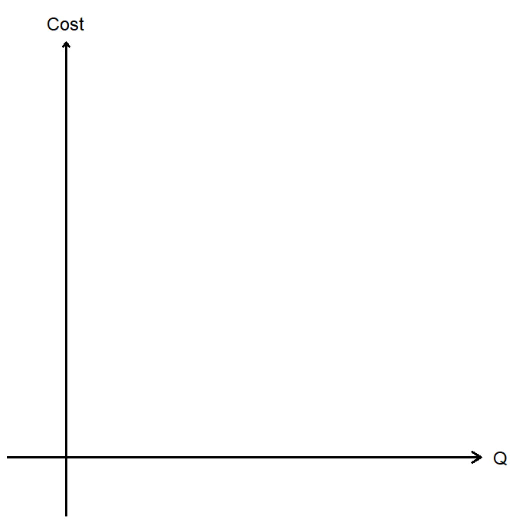


- The old and new isocosts.
- The isoquant (approximate).
- The old and new optimal input bundle.

10.G If your target output is some arbitrary Q, what is your total cost of production?

10.H Find the average total cost function and marginal cost function.

10.I Plot the total, average, and marginal cost curves below:



- The total cost curve.
- The average total cost curve.
- The marginal cost curve.

Problem 11. Cost Minimization

Suppose you are a producer of some good, and the only two inputs you use are labor and capital. Wage is $w = 50$ and rent is $r = 100$. Your production technology is given as:

$$F(L, K) = min\{2L, K\}$$

11.A Identify the type of production function we are given.

11.B Is there an optimal ratio of inputs L and K?
If so, what is the optimal ratio? If not, what is the rule to follow?

11.C If your target output is some arbitrary Q, what is your total cost function?

11.D Find the average total cost function and marginal cost functions.

Problem 12. Cost Minimization

Suppose you are a producer of some good, and the only two inputs you use are labor and capital. Wage is $w = 50$ and rent is $r = 100$. Your production technology is given as:

$$F(L, K) = L + 3K$$

12.A Identify the type of production function we are given.

12.B Find the marginal product of labor and capital.

12.C Is there an optimal ratio of inputs L and K?
If so, what is the optimal ratio? If not, what is the rule to follow?

12.D If your target output is some arbitrary Q, what is your total cost function?

Problem 13. Profit Maximization

Suppose you are a producer of some good, and the only two inputs you use are labor and capital. The output market is perfectly competitive and the market price of your output is $P_x = 100$. Furthermore, your total cost function was calculated as:

$$TC(Q) = 1000 + 30Q + 2Q^2$$

13.A In your own words, explain why $MR(Q) = MC(Q)$ would be the optimization condition for profit maximizing producers. Think about what happens when $MR(Q) > MC(Q)$, and what happens when $MR(Q) < MC(Q)$.

13.B What is the expression for total revenue $TR(Q)$?

13.C What is the expression for marginal revenue $MR(Q)$?

13.D What is the expression for marginal cost $MC(Q)$?

13.E What is the optimal amount to produce if you are a profit maximizer?

13.F Suppose that the market price of your output increased to $P'_x = 150$. What would be your updated optimal amount to produce as a profit maximizer?