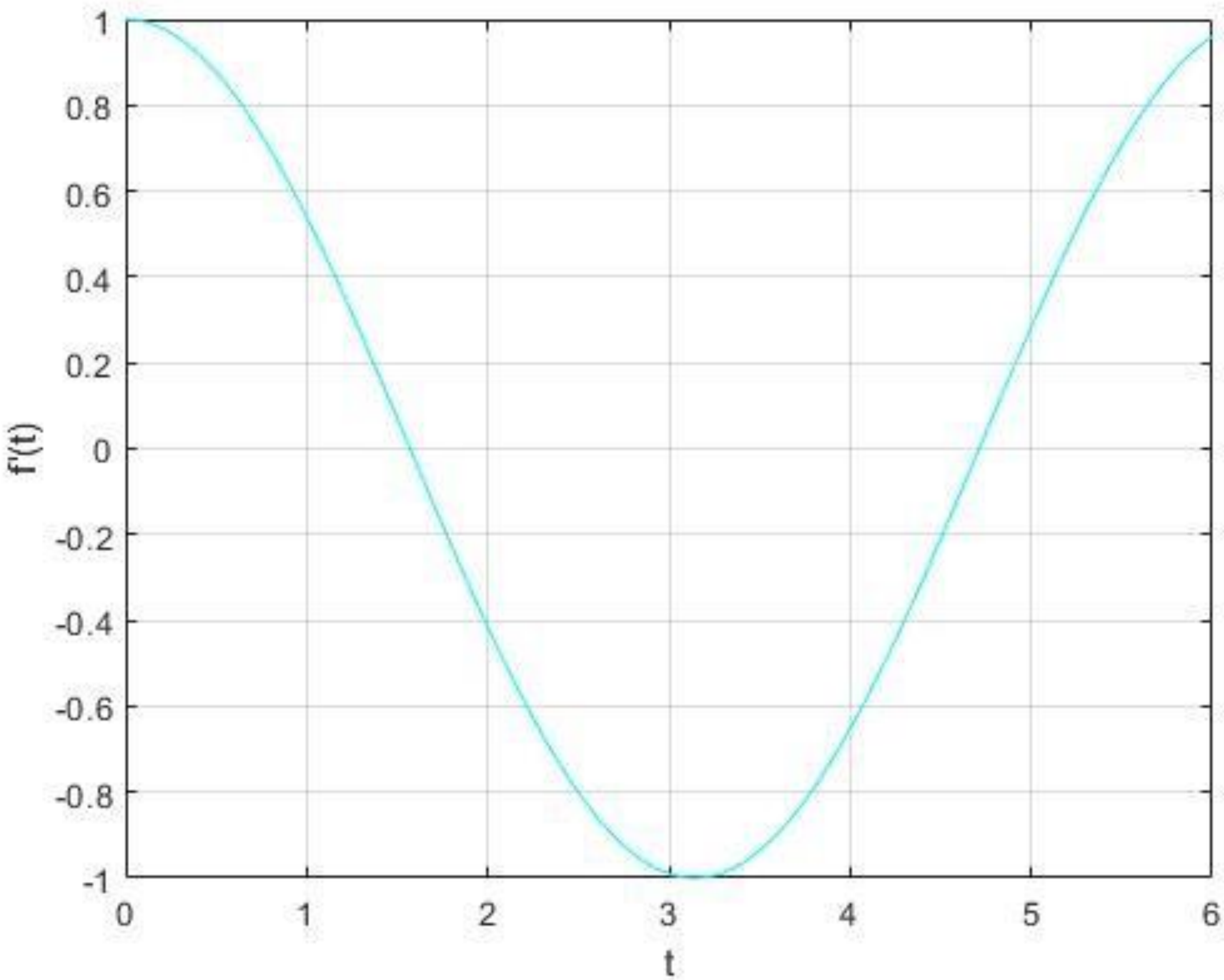
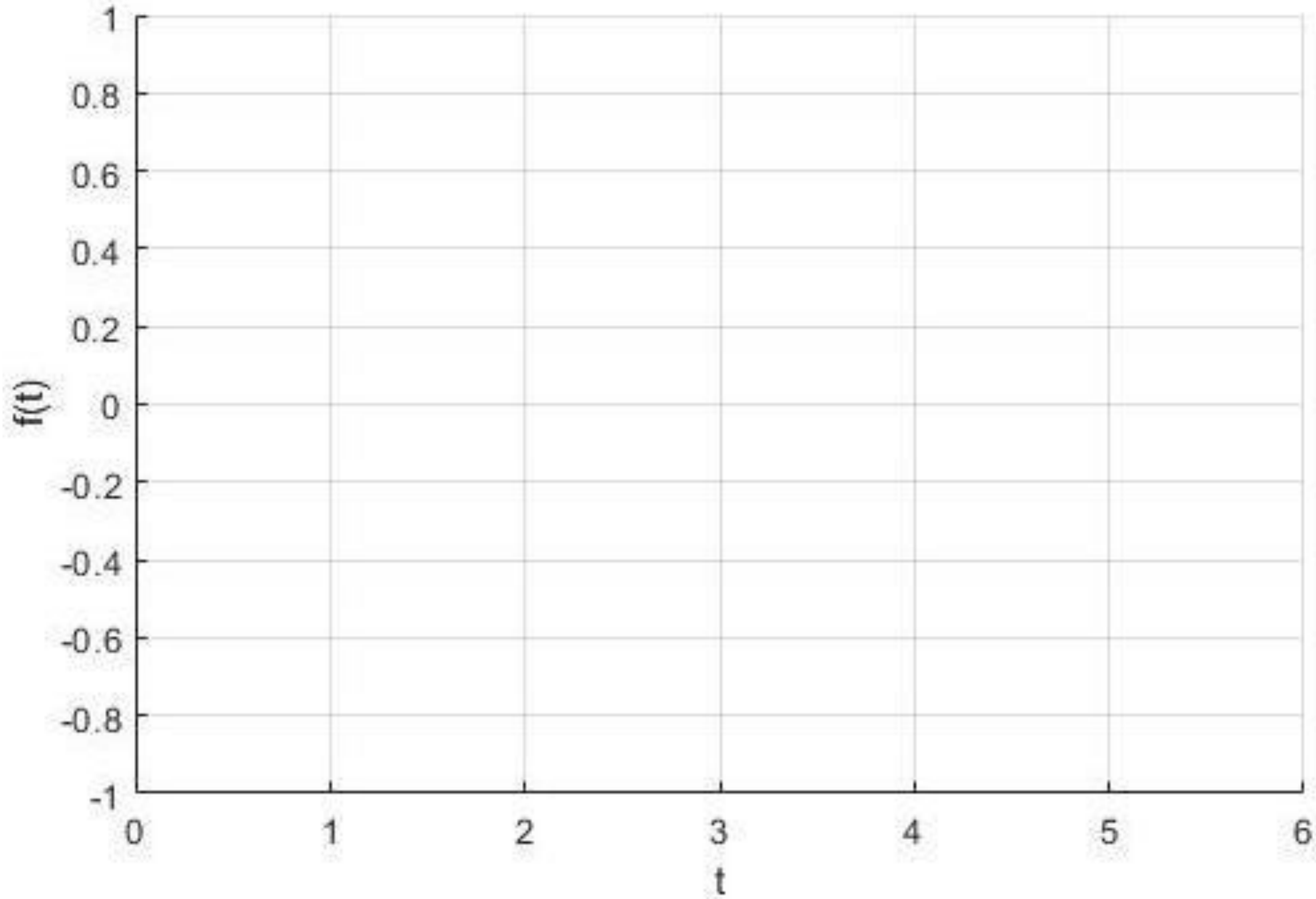
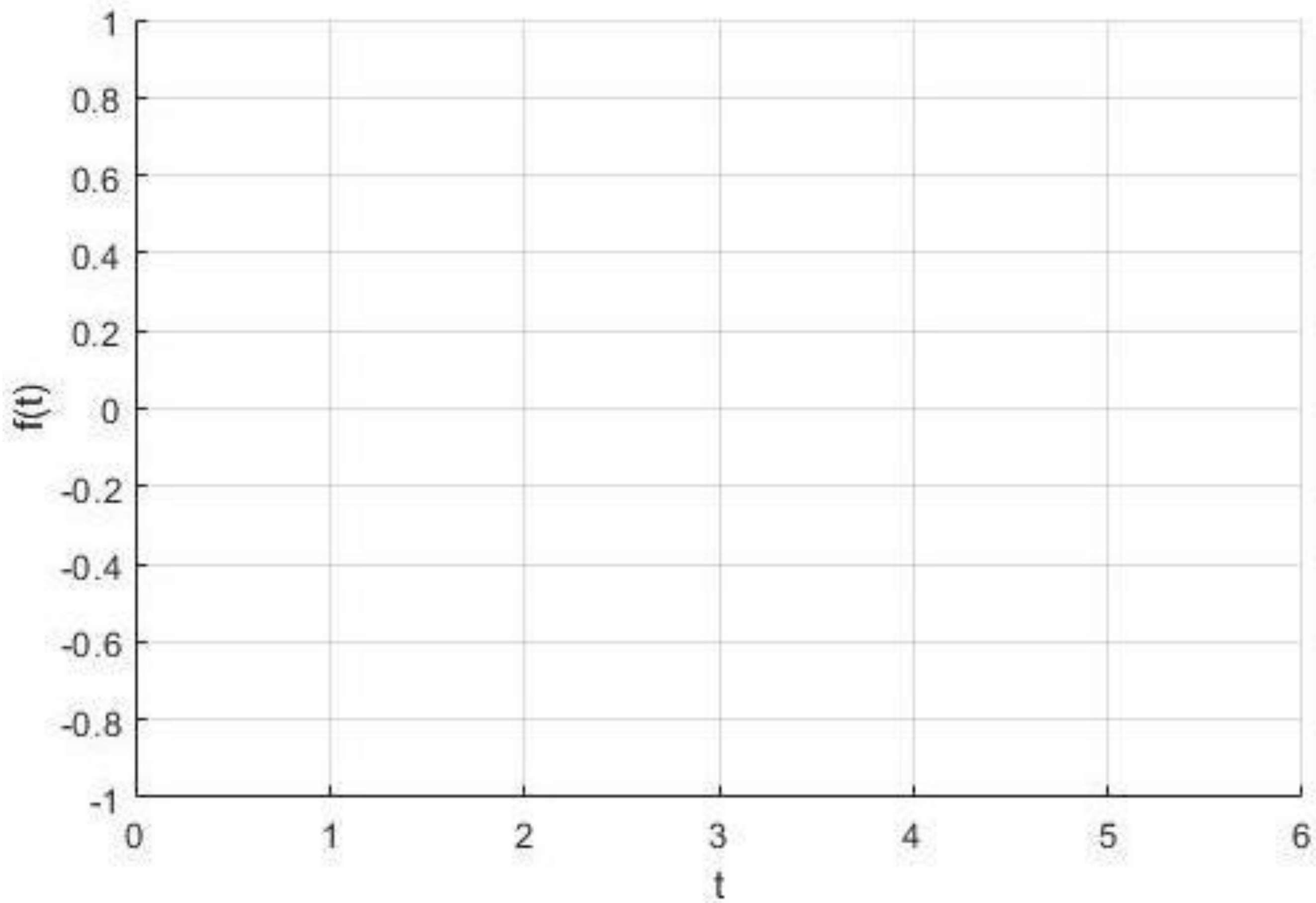
1. **Visual Numerical Methods** Consider the graph of $f^0(t)$ below.



(a) Use Euler's method to draw an approximation of the graph of $f(t)$. Use $h = 1$, $n = 6$, and $f(0) = 0$. Estimate values from the graph of $f^0(t)$. Show your work.



(b) Use Improved Euler's method to draw an approximation of the graph of $f(t)$. Use $h = 1$, $n = 6$, and $f(0) = 0$. Estimate values from the graph $f^0(t)$. Show your work.



2. (5 points) **Numerical Approximation Sometimes Fails**

   When we use numerical approximation techniques we are mainly concerned with how accurate a given numerical algorithm is, i.e. how small the error is for a given step size h. In some cases though, a numerical method might result in a solution that is completely wrong. To see this, consider the IVP:

   $$y^0 = -8y, \qquad y(0) = 1,$$

   where y is a function of time, t, with domain $0 \le t \le 2$.

   Use the Euler's method to create a plot. Also provide slope field and create another plot. Every plot should have the exact solution curve: y(t) = exp(-8*t) plotted in blue. In Python it will be y(t) = np.exp(-8*t).

   (a) Explain why Euler's method fails so badly when h = 0.5.

Method	t_0	y_0	h	n	color

euler	0.0	1.0	0.5	4	red

(b) Explain why Euler's method fails when $h = 0.25$, but suddenly starts working fairly well when $h = 0.1$.

Method	t_0	y_0	h	n	color
euler	0.0	1.0	0.25	8	orange
euler	0.0	1.0	0.1	20	green

Print your plots and staple them to this lab.

3. If we drop a package from a helicopter with a parachute attached, the wind resistance provided by the parachute is not really linearly dependent on the velocity. Assume that for a certain type of parachute the velocity initial value problem is modeled by:

$$v^0(t) = 9 - 0.2v - 0.27v^{1.5}\ v(0)$$

$$= 1$$

(a) What is the terminal velocity in this model?

(b) This is a differential equation which does NOT have an elementary solution. Use Euler and improved Euler with time steps 0.5 and 0.1. Plot your results and comment about apparent accuracy of the three methods with these different choices of time step size.

(c) Based on your work above, approximately what percent of the terminal velocity is attained at $t = 1$ and $t = 4$ seconds?