Midterm Exam

Sessions Given

- Monday, October 18, 9:30am - 10:45pm EST

This exam is:

- open notes/books/any printed resources
- open laptop (anything that you have on your own computer, not online storage)

Online resources that you are allowed to access:

- Gradescope
- course website
- language documentation:
    - Java: https://docs.oracle.com/javase/10/docs/api/
    - C++: https://www.cplusplus.com/reference/ and/or https://en.cppreference.com/w/
- anything else you want since we cannot restrict it, but keep in mind that this will take the time away from the time you have to solve the problems

Instructions:

Solve three out of the four problems given on the next pages. You will **not** get extra credit for solving all four problems.

Grading:

Every exam problem is graded out of 10 points. The total exam grade is the weighted sum computed as follows (assume *scoreN* is a score for a particular problem with *score1 >= score2 >= score3*):

*exam = 5 * score1 + 3 * score2 + 2 * score3*

The total score for a problem is determined by the maximum between zero and the sum of scores for individual tests based on their results. The maximum score for each test is determined by *max_score = 10/number_of_tests*.

test outcome	test score
passed test	*max_score*
wrong answer	*- 0.5 max_score*
runtime error	*- 0.5 max_score*
timeout error	*- 0.5 max_score*
presentation error	*0.75 max_score*

Problem 1

Calvin likes to lie in a field and look at the night sky. Since he does not know any real star constellations, he makes them up: if two stars are close to each other, they must belong to the same constellation. He wants to name them all, but fears to run out of names. Can you help him and count how many constellations there are in the sky?

Two stars belong to the same constellation if distance between their projections on a two-dimensional sky plan isn't more than D units.

Input

The first line contains the number of stars `N` (`0 <= N <= 1,000`) and the distance `D` (a real number `0.0 <= D <= 1,000.00`). Next `N` lines have a pair of real coordinates `X Y` (where `-1,000.00 <= X, Y <= 1,000.00`) for each star in the sky. All real numbers in the input will have at most 2 (two) digits after a decimal point.

Output

Output the number `N` that is the number of constellations in Calvin's sky.

Example 1

```
Input:
5 1.5
1.0 0.1
2.0 0.0
5.0 0.2
6.0 0.4
3.0 -0.1

Output:
2
```

Example 2

```
Input:
3 4.0
121.12 254.06
645.04 301.85
912.49 568.96

Output:
3
```

Problem 2

You are an operator of a super computing center and in control of M nodes. One day, a research institute from Light Kingdom submitted N computational tasks. Given the computational power needed for each task, you are to distribute the tasks among the available nodes. Restriction: every node can process up to two tasks. You also want to distribute the workload as evenly as possible, i.e. minimize the following imbalance value

$$Imb = \sum_{i=0}^{M-1} |Avg - Load_i|$$

where *Avg* is average load per node and *Load_i* is the load of the i-th node.

Input

The first line of the input contains two integers M (1 <= M <= 5) and N (1 <= N <= 2M), indicating the number of nodes you control and the number of tasks, respectively.

The second line contains N integers, each of which represents the computational power required for a task. Numbers on these line are between 1 and 1000.

Output

Print one line `IMBALANCE = I` where `I` is the minimum imbalance value rounded to 5 decimal places followed by a new line character.

Example 1

```
Input:
2 3
6 3 8

Output:
IMBALANCE = 1.00000
```

Example 2

```
Input:
3 5
51 19 27 14 33

Output:
IMBALANCE = 6.00000
```

Example 3

```
Input:
5 9
1 2 3 5 7 11 13 17 19

Output:
IMBALANCE = 11.60000
```
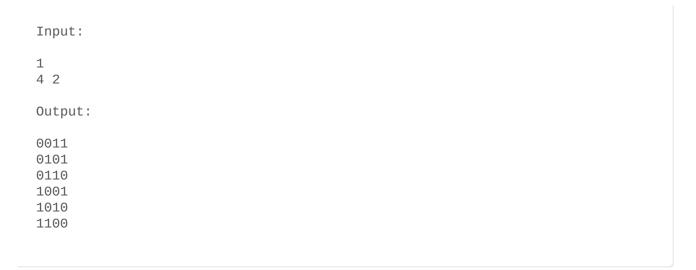
Problem 3

Percy is going to Olympus to return the stolen lightening bolt. To his surprise, each of the steps that lead to Zeus's palace are marked with N bits incrementally starting from 0 to 2^N, $1 \le N \le 16$. Now, to reach the top safely Percy can only use the steps that have exactly H bits set to 1 (he has to step over any other steps), $1 \le H \le N \le 16$. Help Percy find the sequence of all the steps on which he can step or else Poseidon and Zeus will destroy the world of mortals if the lightening bolt is not returned.

Input

The input consists of N and M separated by white space on a single line.

Output

Print the numbers of steps one per line from (numerically) smallest to largest. The output should end with a newline.

```
Input:

1
4 2

Output:

0011
0101
0110
1001
1010
1100
```

Problem 4

We define the operation of splitting a binary number `n` into two numbers `a(n)` and `b(n)` as follows. Let `0 <= i1 < i2 < ... < ik` be the indices of the bits (with the least significant bit having index 0) in `n` **that are 1**. Then the indices of the bits of `a(n)` that are 1 are `i1`, `i3`, `i5`, … and the indices of the bits of `b(n)` that are 1 are `i2`, `i4`, `i6`, …

For example, if `n` is `110110101` in binary then, again in binary, `a = 010010001` and `b= 100100100`.

Input

The input consists of a single integer `n` between `1` and `2^31 - 1` written in standard decimal (base 10) format on a single line.

Output

The output consists of a single **line**, containing the integers `a(n)` and `b(n)` separated by a single space. Both `a(n)` and `b(n)` should be written in decimal format.

Example 1

```
Input:
6
Output:
2 4
```

Example 2

```
Input:
7
Output:
5 2
```

Example 3

```
Input:
32
Output:
32 0
```

Example 4

```
Input:
96
Output:
32 64
```