



# Introduction to Numerical Computing with Numpy

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letter



# Introduction to Numerical Computing with Numpy

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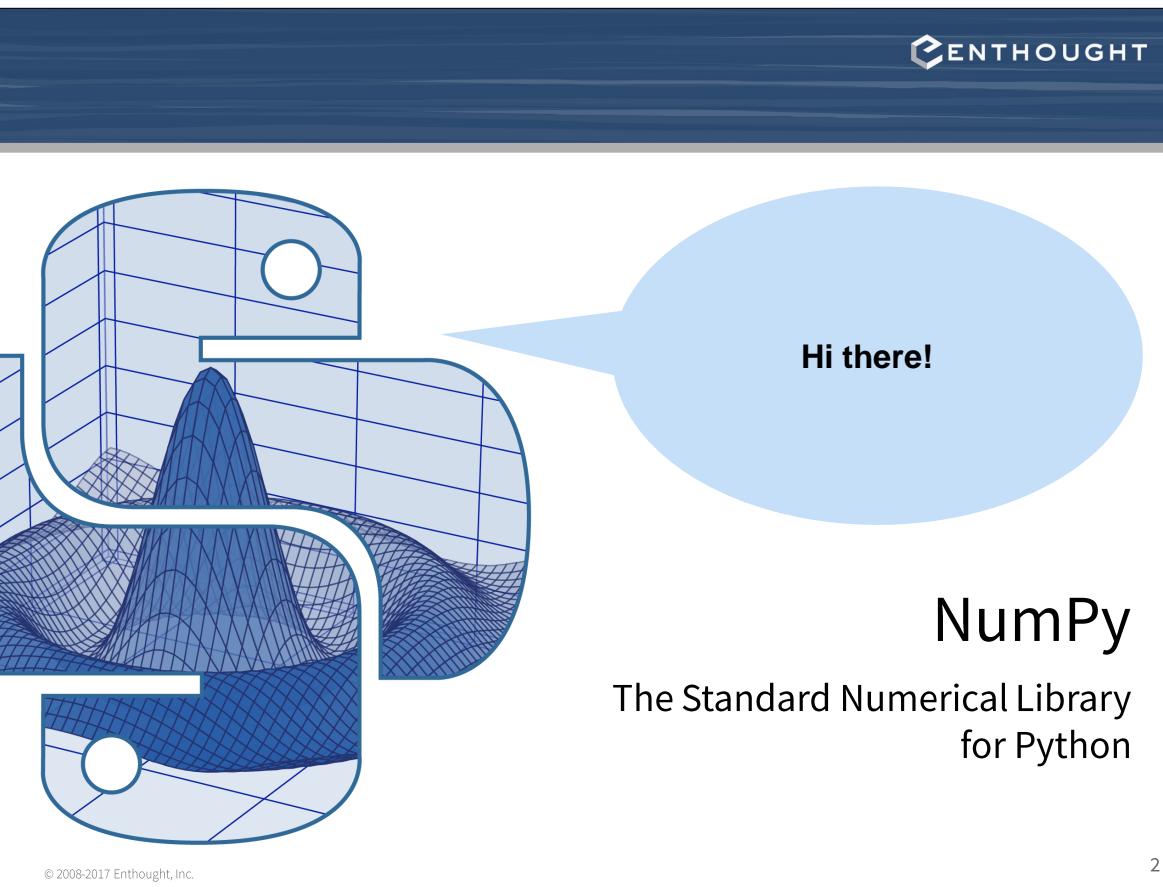
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# Introduction to Numerical Computing with Numpy



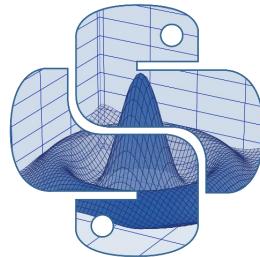
A large blue speech bubble contains the text "Hi there!". To the left of the speech bubble is a stylized Python logo formed by a 3D surface plot. The Python logo is rendered in blue and white, with a grid overlay. The background of the slide is a dark blue gradient at the top, transitioning to white at the bottom.

**NumPy**  
The Standard Numerical Library  
for Python

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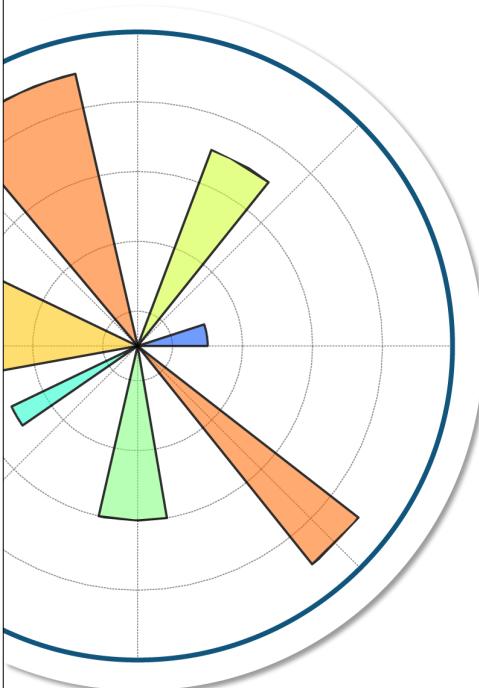
# NumPy Arrays

- Defining Arrays
- Indexing and Slicing
- Creating Arrays
- Array Calculations
- The Array Data Structure
- Advanced NumPy, Overview



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# matplotlib



## Matplotlib

An Interlude

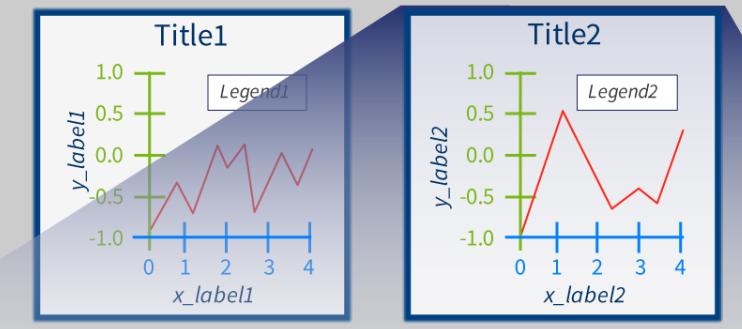
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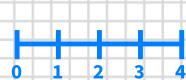
# Matplotlib Object Model

ENTHOUGHT

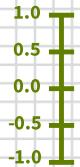
## Figure



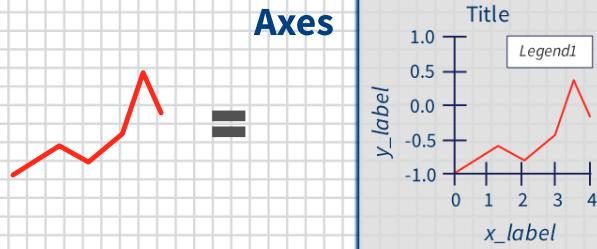
## Axis



## Axis



## Axes



# Matplotlib's "State Machine"

ENTHOUGHT

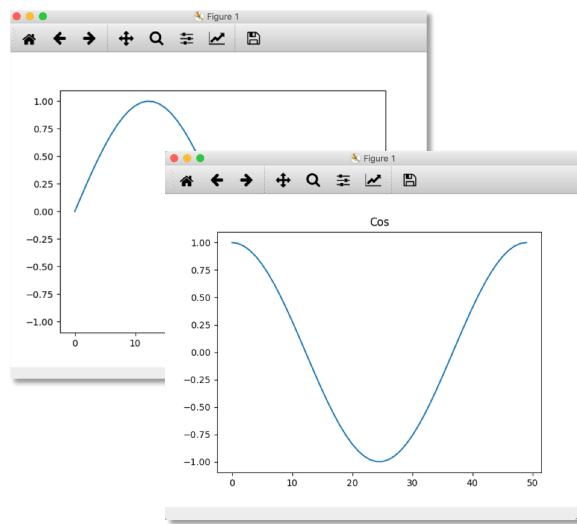
Matplotlib behaves like a state machine.

Any command is applied to the current plotting area:

```
>>> import matplotlib.pyplot as plt
>>> import numpy as np
>>> t = np.linspace(0,2*np.pi,
...                  50)
>>> x = np.sin(t)
>>> y = np.cos(t)

# Now create a figure
>>> plt.figure()
# and plot x inside it
>>> plt.plot(x)

# Now create a new figure
>>> plt.figure()
# and plot y inside it...
>>> plt.plot(y)
# ...and add a title
>>> plt.title("Cos")
```



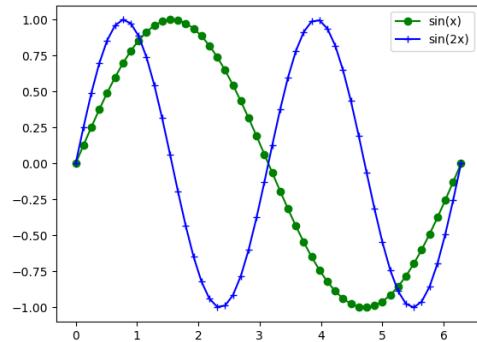
# Line Plots

```
>>> x = np.linspace(0, 2*np.pi, 50)
>>> y1 = np.sin(x)
>>> y2 = np.sin(2*x)

>>> plt.figure() # Create figure
>>> plt.plot(y1)
>>> plt.plot(x, y1)

# red dot-dash circle
>>> plt.plot(x, y1, 'r-o')

# red marker only circle
>>> plt.plot(x, y1, 'ro')
>>> plt.plot(x, y1, 'g-o',
...             x, y2, 'b-+')
>>> plt.legend(['sin(x)',
...             'sin(2x)'])
```



Symbol	Color
b	Blue
g	Green
r	Red
c	Cyan
m	Magenta
y	Yellow
k	Black
w	White

Symbol	Marker
.	Point
o	Circle
< > ^ v	Triangles
8	Octagon
s	Square
*	Star
+	Plus
	...and more

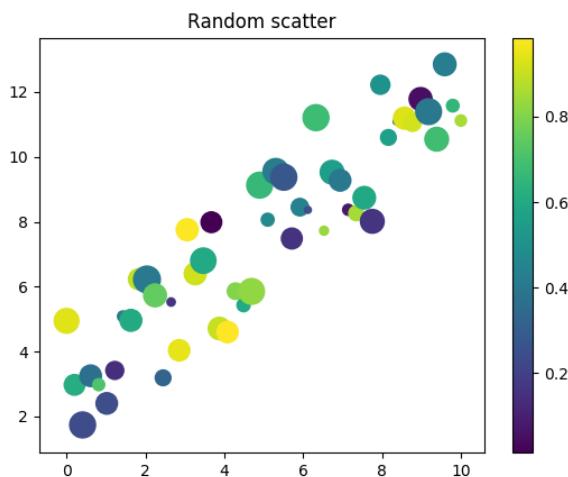
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# Scatter Plots

```
>>> N = 50 # no. of points
>>> x = np.linspace(0, 10, N)
>>> from numpy.random \
...     import rand
>>> e = rand(N)*5.0 # noise
>>> y1 = x + e

>>> areas = rand(N)*300
>>> plt.scatter(x, y1, s=areas)
>>> colors = rand(N)
>>> plt.scatter(x, y1, s=areas,
...               c=colors)
>>> plt.colorbar()
>>> plt.title("Random scatter")
```



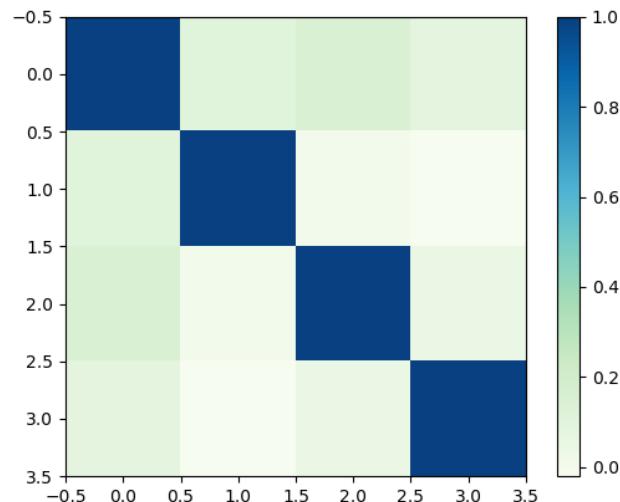
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## Image "Plots"

```
>>> # Create some data
>>> e1 = rand(100)
>>> e2 = rand(100)*2
>>> e3 = rand(100)*10
>>> e4 = rand(100)*100
>>> corrmatrix =
...     np.corrcoef([e1, e2,
...                  e3, e4])
```

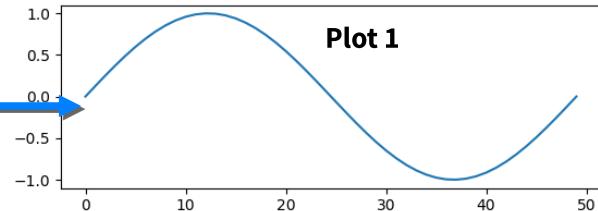
```
>>> # Plot corr matrix as image
>>> plt.imshow(corrmatrix,
...             cmap='GnBu')
>>> plt.colorbar()
```



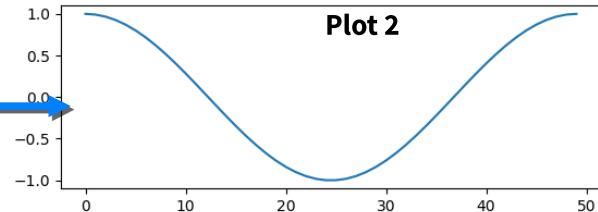
## Multiple Plots Using subplot

```
>>> t = np.linspace(0, 2*np.pi)
>>> x = np.sin(t)
>>> y = np.cos(t)
```

```
# To divide the plotting area
    columns
    |
    >>> plt.subplot(2, 1, 1)
    rows      active plot
    |
    >>> plt.plot(x)
```



```
# Now activate a new plot
# area.
    >>> plt.subplot(2, 1, 2)
    >>> plt.plot(y)
```



# Histogram Plots

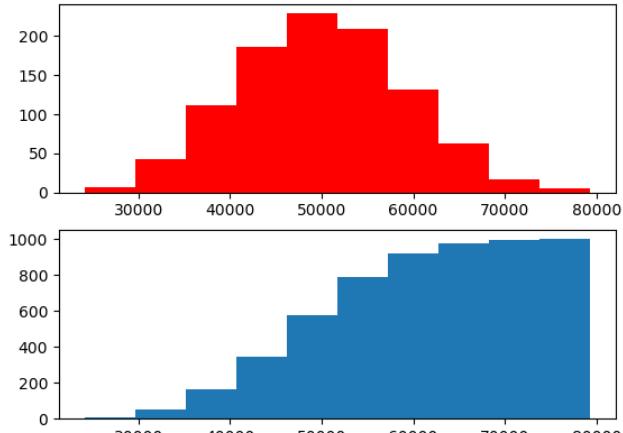
```
>>> # Create array of data
>>> from numpy.random import \
...     randint
>>> data = randint(10000,
...     size=(10,1000))

>>> # Approx norm distribution
>>> x = np.sum(data, axis=0)

>>> # Set up for stacked plots
>>> plt.subplot(2,1,1)
>>> plt.hist(x, color='r')

>>> # Plot cumulative dist
>>> plt.subplot(2,1,2)
>>> plt.hist(x, cumulative=True)

>>> # For multiple histograms:
>>> plt.hist([d1, d2, ...])
```



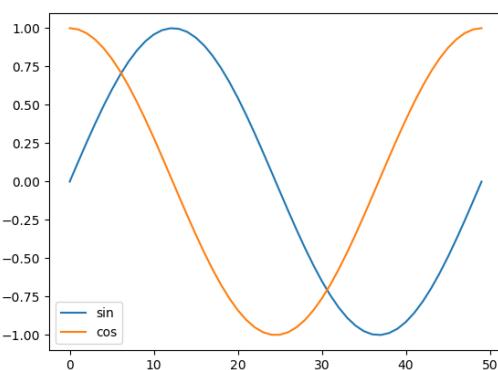
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# Legends, Titles, and Axis Labels

## LEGEND LABELS WITH PLOT

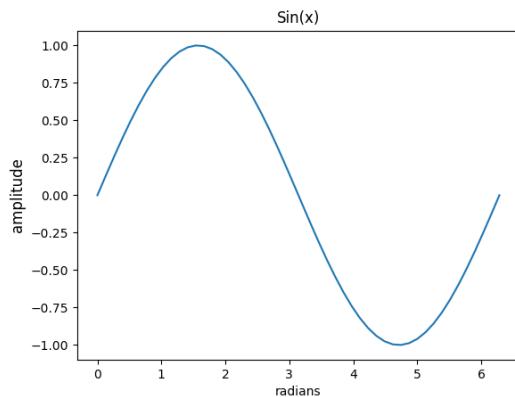
```
# Add labels in plot command.
>>> plt.plot(np.sin(t),
...           label='sin')
>>> plt.plot(np.cos(t),
...           label='cos')
>>> plt.legend()
```



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## TITLES AND AXIS LABELS

```
>>> plt.plot(t, np.sin(t))
>>> plt.xlabel('radians')
# Keywords set text properties.
>>> plt.ylabel('amplitude',
...             fontsize='large')
>>> plt.title('Sin(x)')
```



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# Plotting from Scripts

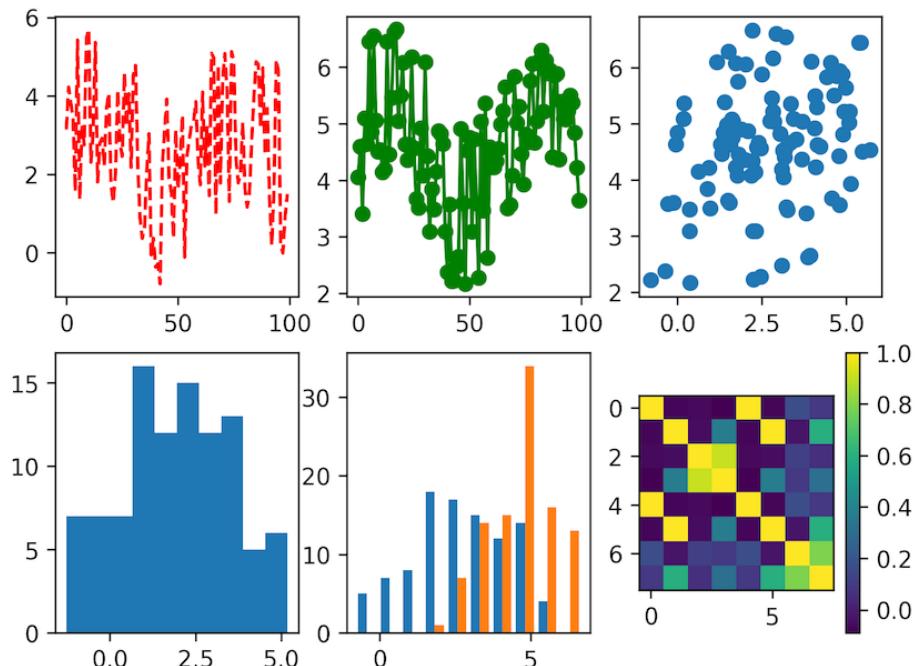
## INTERACTIVE MODE

```
# In IPython, plots show up
# as soon as a plot command
# is issued:
>>> plt.figure()
>>> plt.plot(np.sin(t))
>>> plt.figure()
>>> plt.plot(np.cos(t))
```

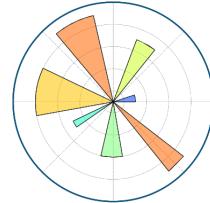
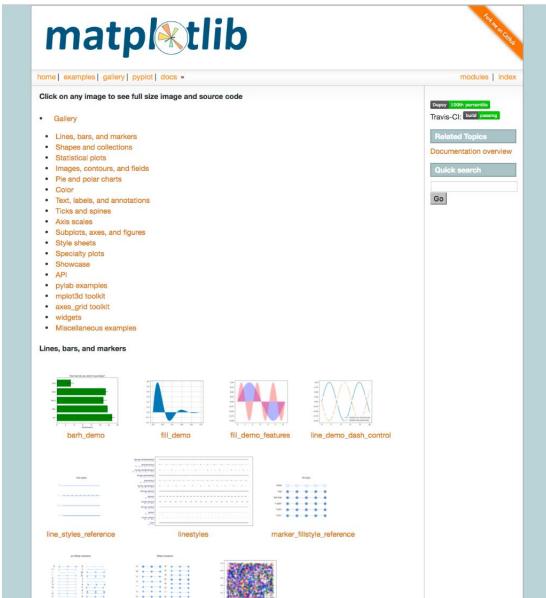
## NON-INTERACTIVE MODE

```
# In a script, you must call
# the show() command to display
# plots. Call it at the end of
# all your plot commands for
# best performance.
import matplotlib.pyplot as plt
import numpy as np
t = np.linspace(0,2*np.pi,
...                           50)
plt.figure()
plt.plot(np.sin(t))
plt.figure()
plt.plot(np.cos(t))
# Plots will not appear until
# this command is run:
plt.show()
```

# MPL Exercise: Desired Output



# Additional Matplotlib Resources



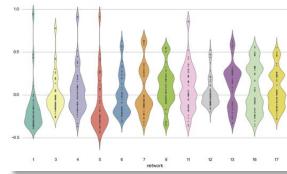
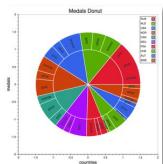
- Simple examples with increasing difficulty  
<http://matplotlib.org/examples/index.html>
- Gallery (huge)  
<http://matplotlib.org/gallery.html>
- See appendix for reference materials
- Usage FAQ  
[http://matplotlib.org/faq/usage\\_faq.html](http://matplotlib.org/faq/usage_faq.html)

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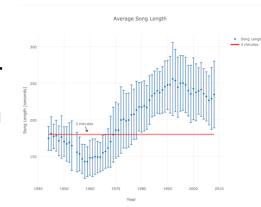
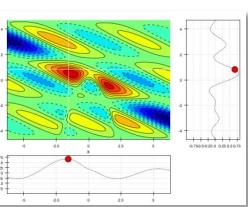
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# Other Visualization Libraries

**Seaborn:** Better looking, high-level plots based on matplotlib



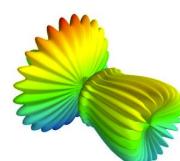
**bokeh:** D3-like visualization in web browsers (HTML file, or inline in IPython notebooks)



**plot.ly:** D3-based visualization with support for multiple languages (R, Python, Matlab, Julia and more.)

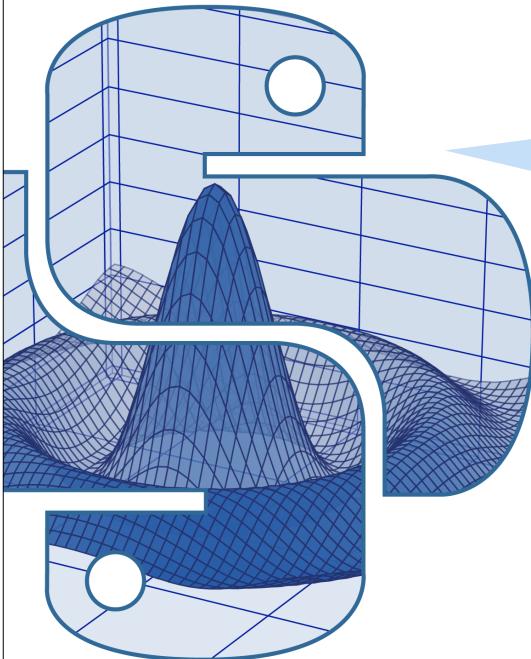
**chaco:** For interactive, custom plots embedded in user applications

**mayavi:** 3D visualization, based on VTK



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a



# NumPy

## Introducing Arrays

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# Introducing NumPy Arrays

## SIMPLE ARRAY CREATION

```
>>> a = np.array([0, 1, 2, 3])
>>> a
array([0, 1, 2, 3])
```

## CHECKING THE TYPE

```
>>> type(a)
numpy.ndarray
```

## NUMERIC "TYPE" OF ELEMENTS

```
>>> a.dtype
dtype('int32')
```

## NUMBER OF DIMENSIONS

```
>>> a.ndim
1
```

## ARRAY SHAPE

```
# Shape returns a tuple
# listing the length of the
# array along each dimension.
>>> a.shape
(4,)
```

## BYTES PER ELEMENT

```
>>> a.itemsize
4
```

## BYTES OF MEMORY USED

```
# Return the number of bytes
# used by the data portion of
# the array.
>>> a.nbytes
16
```

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# Array Operations

## SIMPLE ARRAY MATH

```
>>> a = np.array([1, 2, 3, 4])
>>> b = np.array([2, 3, 4, 5])
>>> a + b
array([3, 5, 7, 9])

>>> a * b
array([ 2,  6, 12, 20])

>>> a ** b
array([ 1,  8, 81, 1024])
```



NumPy defines these constants:  
 $\pi = 3.14159265359$   
 $e = 2.71828182846$

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## MATH FUNCTIONS

```
# create array from 0. to 10.
>>> x = np.arange(11.)

# multiply entire array by
# scalar value
>>> c = (2 * np.pi) / 10.
>>> c
0.62831853071795862
>>> c * x
array([ 0., 0.628, ..., 6.283])

# in-place operations
>>> x *= c
>>> x
array([ 0., 0.628, ..., 6.283])

# apply functions to array
>>> y = np.sin(x)
```

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# Setting Array Elements

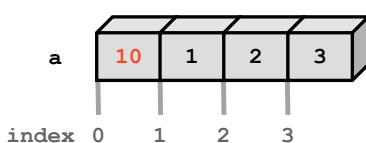
## ARRAY INDEXING

```
>>> a[0]
0

a
index 0 1 2 3
      | | | |
      0 1 2 3
```

```
>>> a[0] = 10
>>> a
array([10, 1, 2, 3])
```



## BEWARE OF TYPE COERCION

```
>>> a.dtype
dtype('int32')

# assigning a float into
# an int32 array truncates
# the decimal part
>>> a[0] = 10.6
>>> a
array([10, 1, 2, 3])

# fill has the same behavior
>>> a.fill(-4.8)
>>> a
array([-4, -4, -4, -4])
```

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# Multi-Dimensional Arrays

## MULTI-DIMENSIONAL ARRAYS

```
>>> a = np.array([[ 0,  1,  2,  3],
...                 [10,11,12,13]])
>>> a
```

```
array([[ 0,  1,  2,  3],
       [10,11,12,13]])
```

2D Array  
a  

$$\begin{matrix} 0 & 1 & 2 & 3 \\ 10 & 11 & 12 & 13 \end{matrix}$$

## SHAPE = (ROWS, COLUMNS)

```
>>> a.shape
(2, 4)
```

dim 1      dim 0  
a  

$$\begin{matrix} 0 & 1 & 2 & 3 \\ 10 & 11 & 12 & 13 \end{matrix}$$

## ELEMENT COUNT

```
>>> a.size
8
```

$2 \times 4 = 8$   
a  

$$\begin{matrix} 0 & 1 & 2 & 3 \\ 10 & 11 & 12 & 13 \end{matrix}$$

## NUMBER OF DIMENSIONS

```
>>> a.ndim
2
```

dim 1      dim 0  
a  

$$\begin{matrix} 0 & 1 & 2 & 3 \\ 10 & 11 & 12 & 13 \end{matrix}$$

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## GET / SET ELEMENTS

```
>>> a[1, 3]
13
```

column  
row

```
>>> a[1, 3] = -1
>>> a
```

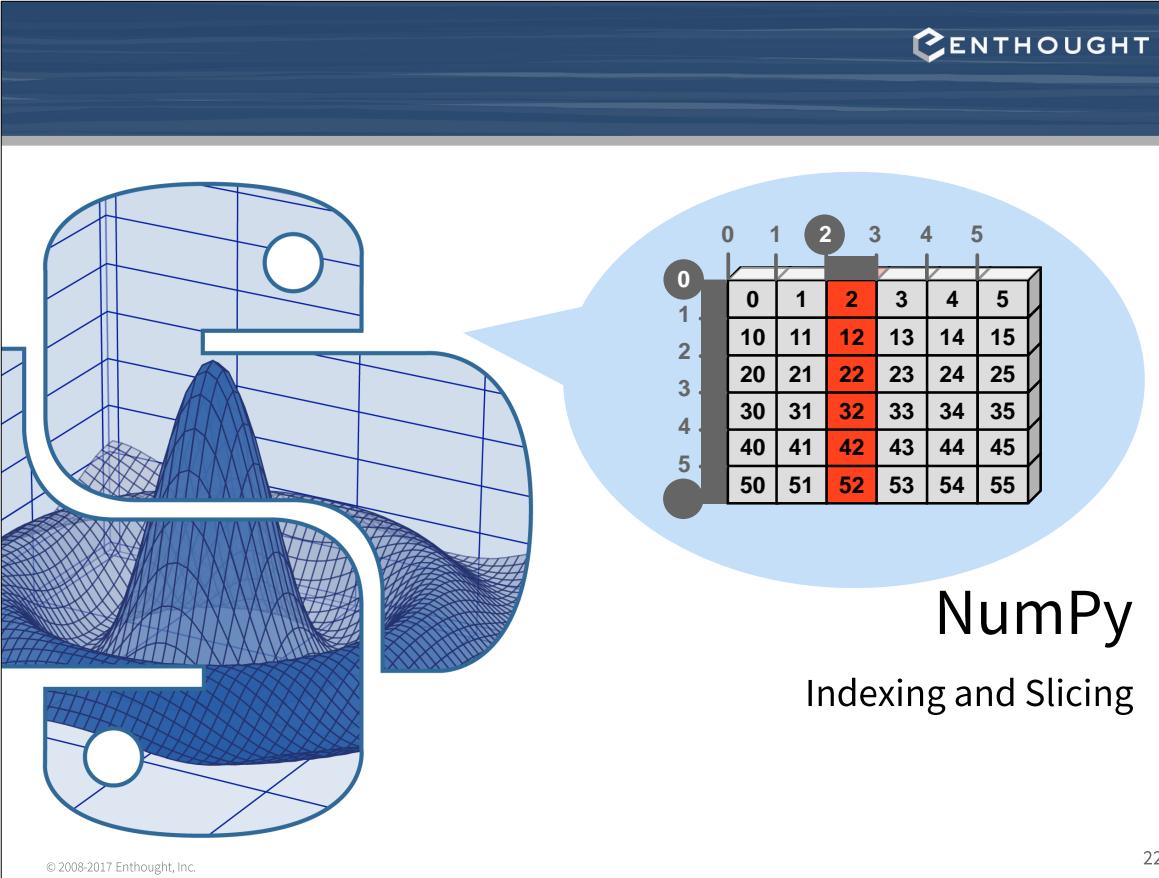
$\begin{matrix} 0 & 1 & 2 & 3 \\ 10 & 11 & 12 & -1 \end{matrix}$

## ADDRESS SECOND (ONETH) ROW USING SINGLE INDEX

```
>>> a[1]
array([10, 11, 12, -1])
```

Python  
a  

$$\begin{matrix} 0 & 1 & 2 & 3 \\ 10 & 11 & 12 & -1 \end{matrix}$$



The image shows a 3D surface plot of a mathematical function, likely  $\sin(x)/x$ , with a white rectangular slice highlighted. An arrow points from this slice to a 6x6 grid of numbers below it, illustrating indexing and slicing in NumPy arrays.

0	1	2	3	4	5
0	1	2	3	4	5
10	11	12	13	14	15
20	21	22	23	24	25
30	31	32	33	34	35
40	41	42	43	44	45
50	51	52	53	54	55

NumPy  
Indexing and Slicing

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# Slicing

**var[lower:upper:step]**

Extracts a portion of a sequence by specifying a lower and upper bound.  
The lower-bound element is included, but the upper-bound element is **not** included.  
Mathematically: [lower, upper). The step value specifies the stride between elements.

## SLICING ARRAYS

```
#           -5 -4 -3 -2 -1
# indices:      0  1  2  3  4
>>> a = np.array([10,11,12,13,14])

# [10, 11, 12, 13, 14]
>>> a[1:3]
array([11, 12])

# negative indices work also
>>> a[1:-2]
array([11, 12])
>>> a[-4:3]
array([11, 12])
```

## OMMITTING INDICES

```
# omitted boundaries are
# assumed to be the beginning
# (or end) of the list

# grab first three elements
>>> a[:3]
array([10, 11, 12])

# grab last two elements
>>> a[-2:]
array([13, 14])

# every other element
>>> a[::2]
array([10, 12, 14])
```

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# Array Slicing

## SLICING WORKS MUCH LIKE STANDARD PYTHON SLICING

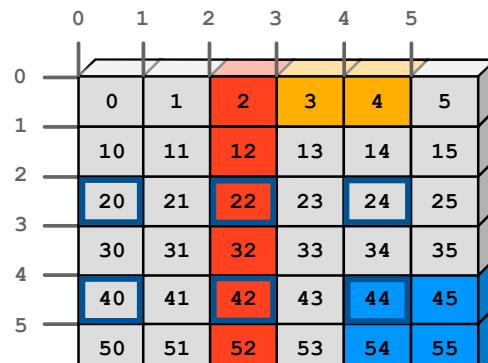
```
>>> a[0, 3:5]
array([3, 4])

>>> a[4:, 4:]
array([[44, 45],
       [54, 55]])

>>> a[:, 2]
array([2, 12, 22, 32, 42, 52])
```

## STRIDED ARE ALSO POSSIBLE

```
>>> a[2::2, ::2]
array([[20, 22, 24],
       [40, 42, 44]])
```



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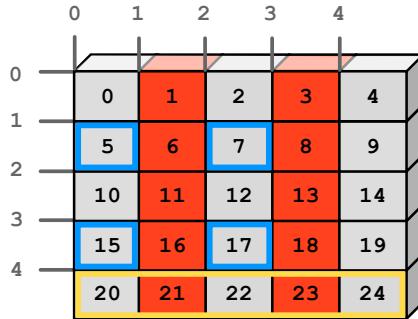
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## Give it a try!

Create the array below with the command

```
a = np.arange(25).reshape(5, 5)
```

and extract the slices as indicated.



## Slices are References

Slices are references to memory in the original array.  
Changing values in a slice also changes the original array.

```
>>> a = np.array([0, 1, 2, 3, 4])
# create a slice containing two elements of a
>>> b = a[2:4]
>>> b
array([2, 3])
>>> b[0] = 10

# changing b changed a!
>>> a
array([ 0,  1, 10,  3,  4])
```

# Fancy Indexing

## INDEXING BY POSITION

```
>>> a = np.arange(0, 80, 10)

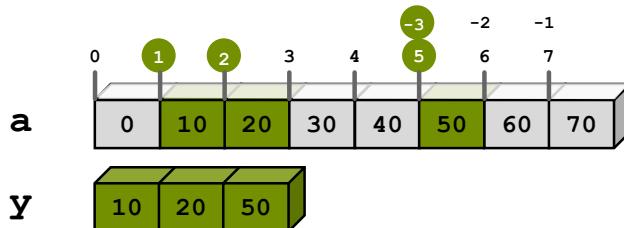
# fancy indexing
>>> indices = [1, 2, -3]
>>> y = a[indices]
>>> print(y)
[10 20 50]
```

## INDEXING WITH BOOLEANS

```
# manual creation of masks
>>> mask = np.array(
...     [0, 1, 1, 0, 0, 0, 1, 0, 0],
...     dtype=bool)

# conditional creation of masks
>>> mask2 = a < 30

# fancy indexing
>>> y = a[mask]
>>> print(y)
[10 20 50]
```



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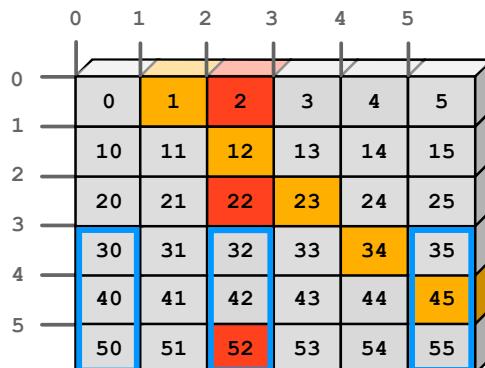
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# Fancy Indexing in 2-D

```
>>> a[[0, 1, 2, 3, 4],
...     [1, 2, 3, 4, 5]]
array([ 1, 12, 23, 34, 45])

>>> a[3:, [0, 2, 5]]
array([[30, 32, 35],
       [40, 42, 45],
       [50, 52, 55]])

>>> mask = np.array(
...     [1, 0, 1, 0, 0, 1],
...     dtype=bool)
>>> a[mask, 2]
array([2, 22, 52])
```



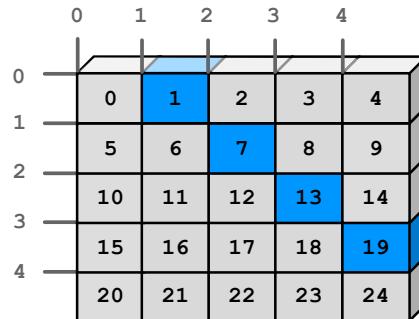
Unlike slicing, fancy indexing creates copies instead of a view into original array.

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## Give it a try!

1. Create the array below with  
 $a = np.arange(25).reshape(5, 5)$   
 and extract the elements indicated in blue.
2. Extract all the numbers divisible by 3 using a boolean mask.

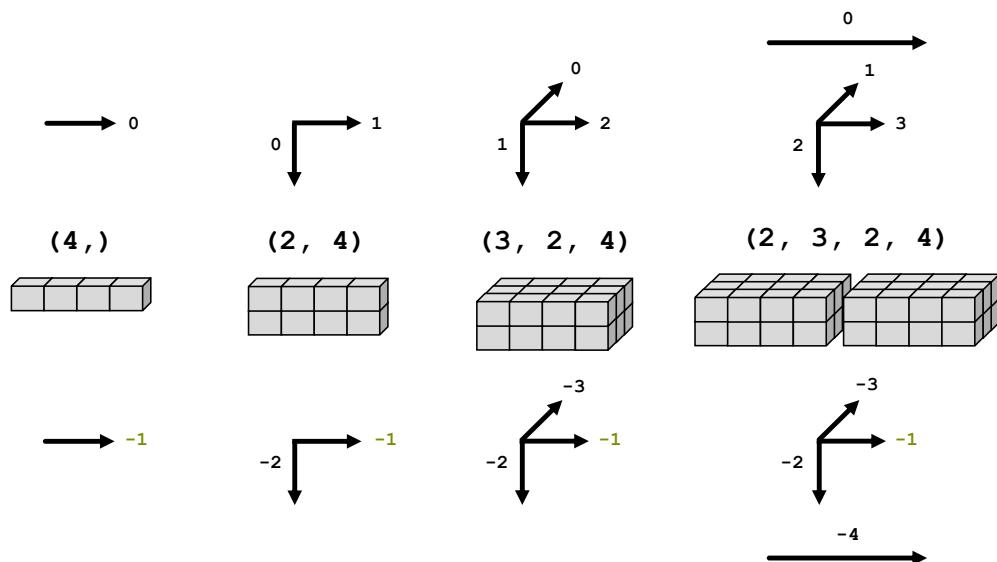


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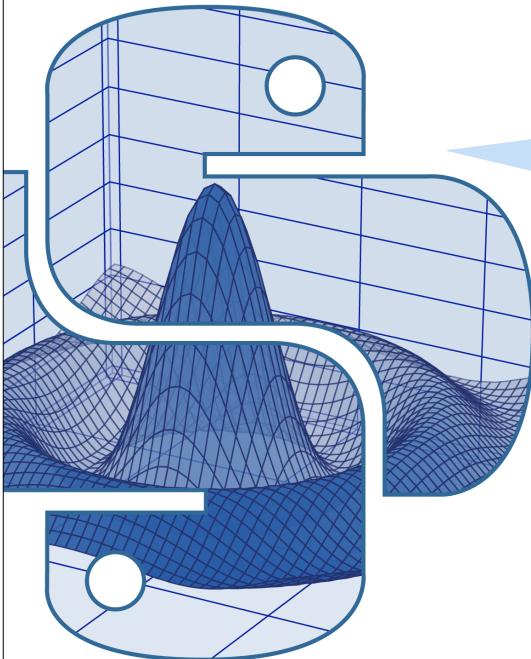
## Multi-Dimensional Arrays

### VISUALIZING MULTI-DIMENSIONAL ARRAYS



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`arange()`  
`linspace()`  
`array()`  
`zeros()`  
`ones()`

## NumPy

### Creating Arrays

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## Array Constructor Examples

### FLOATING POINT ARRAYS

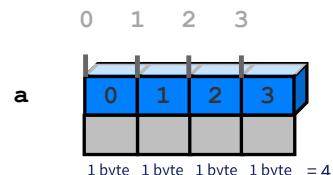
```
# Default to double precision
>>> a = np.array([0,1.0,2,3])
>>> a.dtype
dtype('float64')
>>> a nbytes
32
```

### REDUCING PRECISION

```
>>> a = np.array([0,1.,2,3],
...               dtype='float32')
>>> a.dtype
dtype('float32')
>>> a nbytes
16
```

### UNSIGNED INTEGER BYTE

```
>>> a = np.array([0,1,2,3],
...               dtype='uint8')
>>> a.dtype
dtype('uint8')
>>> a nbytes
4
```



Base 2	Base 10
00000000	-> 0 = 0*2**0 + 0*2**1 + ... + 0*2**7
00000001	-> 1 = 1*2**0 + 0*2**1 + ... + 0*2**7
00000010	-> 2 = 0*2**0 + 1*2**1 + ... + 0*2**7
...	
11111111	-> 255 = 1*2**0 + 1*2**1 + ... + 1*2**7

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# Array Creation Functions

## ARANGE

```
arange([start[, stop[, step]],  
       dtype=None])
```

Nearly identical to Python's `range()`. Creates an array of values in the range [start,stop) with the specified step value. Allows non-integer values for start, stop, and step. Default `dtype` is derived from the start, stop, and step values.

```
>>> np.arange(4)  
array([0, 1, 2, 3])  
>>> np.arange(0, 2*pi, pi/4)  
array([ 0.000,  0.785,  1.571,  
       2.356,  3.142,  3.927,  4.712,  
      5.497])  
  
# Be careful...  
>>> np.arange(1.5, 2.1, 0.3)  
array([ 1.5,  1.8,  2.1])
```

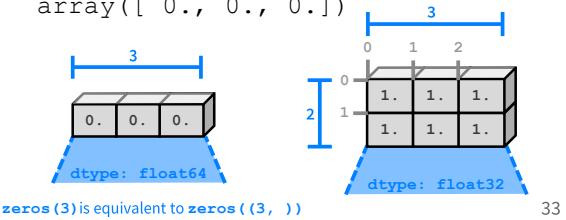
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## ONES, ZEROS

```
ones(shape, dtype='float64')  
zeros(shape, dtype='float64')
```

`shape` is a number or sequence specifying the dimensions of the array. If `dtype` is not specified, it defaults to `float64`.

```
>>> np.ones((2, 3),  
...           dtype='float32')  
array([[ 1.,  1.,  1.],  
      [ 1.,  1.,  1.]],  
      dtype=float32)  
>>> np.zeros(3)  
array([ 0.,  0.,  0.])
```



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# Array Creation Functions (cont'd)

## IDENTITY

```
# Generate an n by n identity  
# array. The default dtype is  
# float64.  
>>> a = np.identity(4)  
>>> a  
array([[ 1.,  0.,  0.,  0.],  
       [ 0.,  1.,  0.,  0.],  
       [ 0.,  0.,  1.,  0.],  
       [ 0.,  0.,  0.,  1.]])  
>>> a.dtype  
dtype('float64')  
>>> np.identity(4, dtype=int)  
array([[ 1,  0,  0,  0],  
       [ 0,  1,  0,  0],  
       [ 0,  0,  1,  0],  
       [ 0,  0,  0,  1]])
```

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## EMPTY AND FILL

```
# empty(shape, dtype=float64,  
#        order='C')  
>>> a = np.empty(2)  
>>> a  
array([1.78021120e-306,  
       6.95357225e-308])  
  
# fill array with 5.0  
>>> a.fill(5.0)  
array([5.,  5.])  
  
# alternative approach  
# (slightly slower)  
>>> a[:] = 4.0  
array([4.,  4.])
```

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# Array Creation Functions (cont'd)

## LINSPACE

```
# Generate N evenly spaced
# elements between (and including)
# start and stop values.
>>> np.linspace(0, 1, 5)
array([0., 0.25, 0.5, 0.75, 1.0])
```

## LOGSPACE

```
# Generate N evenly spaced
# elements on a log scale
# between base**start and
# base**stop (default base=10)
>>> np.logspace(0, 1, 5)
array([1., 1.78, 3.16, 5.62, 10.])
```

## ARRAYS FROM/TO TXT FILES

### Data.txt

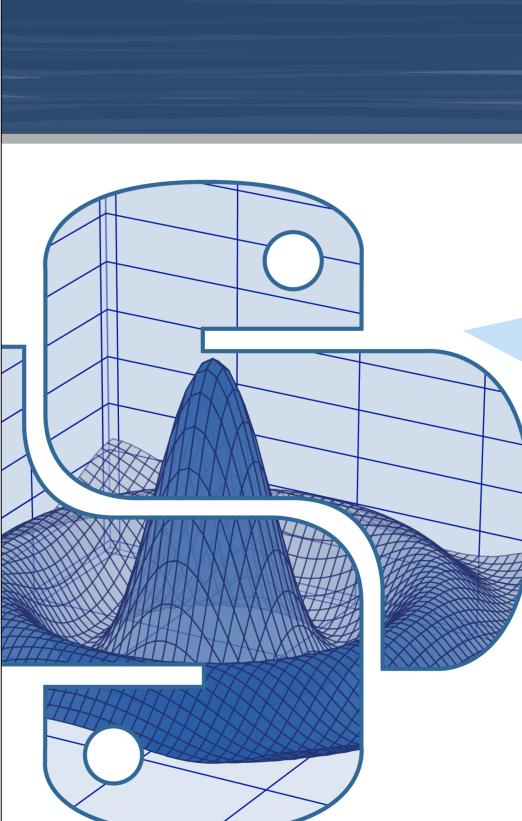
```
-- BEGINNING OF THE FILE
% Day, Month, Year, Skip, Avg Power
01, 01, 2000, x876, 13 % crazy day!
% we don't have Jan 03rd
04, 01, 2000, xfed, 55
```

```
# loadtxt() automatically
# generates an array from the
# txt file
arr = np.loadtxt('Data.txt',
...     skiprows=1,
...     dtype=int, delimiter=",",
...     usecols = (0,1,2,4),
...     comments = "%")

# Save an array into a txt file
np.savetxt('filename', arr)
```

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1	2	3	6
4	5	6	15
5	7	9	

Diagram illustrating a 3x4 NumPy array with indices:

- Row 0: 1, 2, 3, 6
- Row 1: 4, 5, 6, 15
- Row 2: 5, 7, 9, (empty)
- Column 0: 1, 4, 5
- Column 1: 2, 5, 7
- Column 2: 3, 6, 9
- Column 3: 6, 15, (empty)

Arrows indicate indices: Row 0 is labeled 0, Row 1 is labeled 1, Row 2 is labeled 2. Column 0 is labeled -1, Column 1 is labeled 1, Column 2 is labeled 2, Column 3 is labeled 3.

**NumPy**  
Array Calculation Methods

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# Computations with Arrays

- Rule 1:** Operations between multiple array objects are first checked for proper shape match.
- Rule 2:** Mathematical operators (+ - \* / exp, log, ...) apply element by element, on the values.
- Rule 3:** Reduction operations (mean, std, skew, kurt, sum, prod, ...) apply to the whole array, unless an axis is specified.
- Rule 4:** Missing values propagate unless explicitly ignored (nanmean, nansum, ...).

# Array Calculation Methods

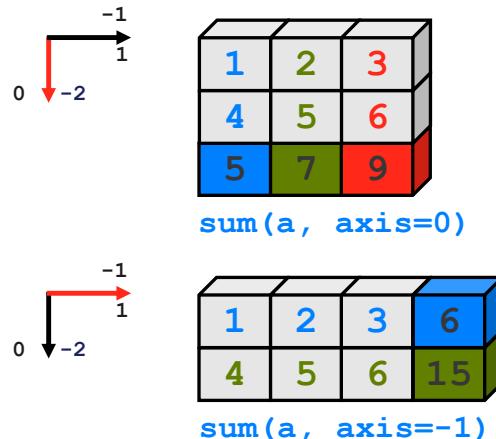
## SUM METHOD

```
# Methods act on data stored
# in the array
>>> a = np.array([[1,2,3],
                  [4,5,6]])

# .sum() defaults to adding up
# all the values in an array.
>>> a.sum()
21

# supply the keyword axis to
# sum along the 0th axis
>>> a.sum(axis=0)
array([5, 7, 9])

# supply the keyword axis to
# sum along the last axis
>>> a.sum(axis=-1)
array([ 6, 15])
```



# Other Operations on Arrays

## SUM FUNCTION

```
# Functions work on data
# passed to it
>>> a = np.array([[1,2,3],
   [4,5,6]])

# sum() defaults to adding
# up all values in an array.
>>> np.sum(a)
21

# supply an axis argument to
# sum along a specific axis
>>> np.sum(a, axis=0)
array([5, 7, 9])
```

## OTHER METHODS AND FUNCTIONS

### Mathematical functions

- sum, prod
- min, max, argmin, argmax
- ptp (max - min)

### Statistics

- mean, std, var

### Truth value testing

- any, all

See the Numpy appendix for more.

# Min/Max

## MIN

```
>>> a = np.array([2.,3.,0.,1.])
>>> a.min(axis=0)
0.0
# Use NumPy's min() instead
# of Python's built-in min()
# for speedy operations on
# multi-dimensional arrays.
>>> np.min(a, axis=0)
0.0
```

## MAX

```
>>> a = np.array([2.,3.,0.,1.])
>>> a.max(axis=0)
3.0

# as a function
>>> np.max(a, axis=0)
3.0
```

## ARGMIN

```
# Find index of minimum value.
>>> a.argmin(axis=0)
2
# as a function
>>> np.argmax(a, axis=0)
2
```

## ARGMAX

```
# Find index of maximum value.
>>> a.argmax(axis=0)
1
# as a function
>>> np.argmax(a, axis=0)
1
```

# Statistics Array Methods

## MEAN

```
>>> a = np.array([[1,2,3],
...                 [4,5,6]])

# mean value of each column
>>> a.mean(axis=0)
array([ 2.5,  3.5,  4.5])
>>> np.mean(a, axis=0)
array([ 2.5,  3.5,  4.5])
```

## STANDARD DEV./VARIANCE

```
# Standard Deviation
>>> a.std(axis=0)
array([ 1.5,  1.5,  1.5])
# For sample, set ddof=1
>>> a.std(axis=0, ddof=1)
array([ 1.22,  1.22,  1.22])

# variance
>>> a.var(axis=0)
array([2.25, 2.25, 2.25])
>>> np.var(a, axis=0)
array([2.25, 2.25, 2.25])
```

## Give it a try!

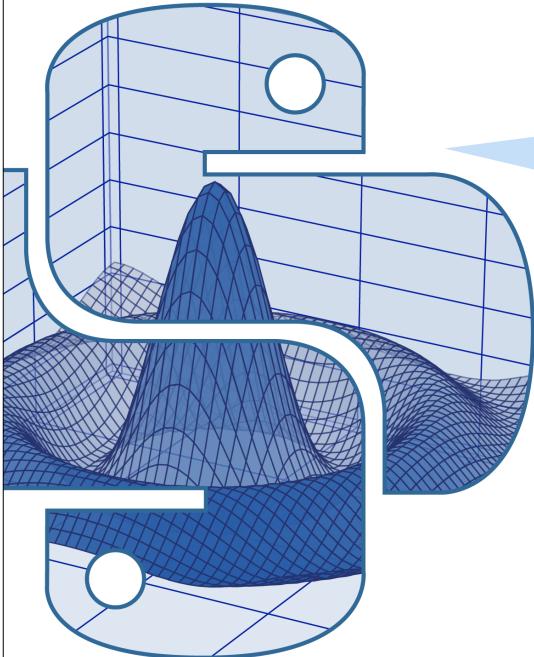
Create the array below with

```
a = np.arange(-15, 15).reshape(5, 6) ** 2
```

and compute:

1. The maximum of each row (one max per row)
2. The mean of each column (one mean per column)
3. The position of the overall minimum (requires 2-3 steps)

	0	1	2	3	4	5
0	225	196	169	144	121	100
1	81	64	49	36	25	16
2	9	4	1	0	1	4
3	9	16	25	36	49	64
4	81	100	121	144	169	196



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$$\begin{matrix} \mathbf{a} & + & \mathbf{b} & = & \mathbf{y} \end{matrix}$$

0	0	0
10	10	10
20	20	20
30	30	30

0	1	2
0	1	2
0	1	2
0	1	2

0	1	2
10	11	12
20	21	22
30	31	32

## NumPy Array Broadcasting

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# Array Broadcasting

NumPy arrays of different dimensionality can be combined in the same expression. Arrays with smaller dimension are **broadcasted** to match the larger arrays, *without copying data*. Broadcasting has **two rules**.

### RULE 1: PREPEND ONES TO SMALLER ARRAY'S SHAPE

```
>>> import numpy as np
>>> a = np.ones((3, 5)) # a.shape == (3, 5)
>>> b = np.ones((5,)) # b.shape == (5,)
>>> b.reshape(1, 5) # result is a (1,5)-shaped array.
>>> b[np.newaxis, :] # equivalent, more concise.
```

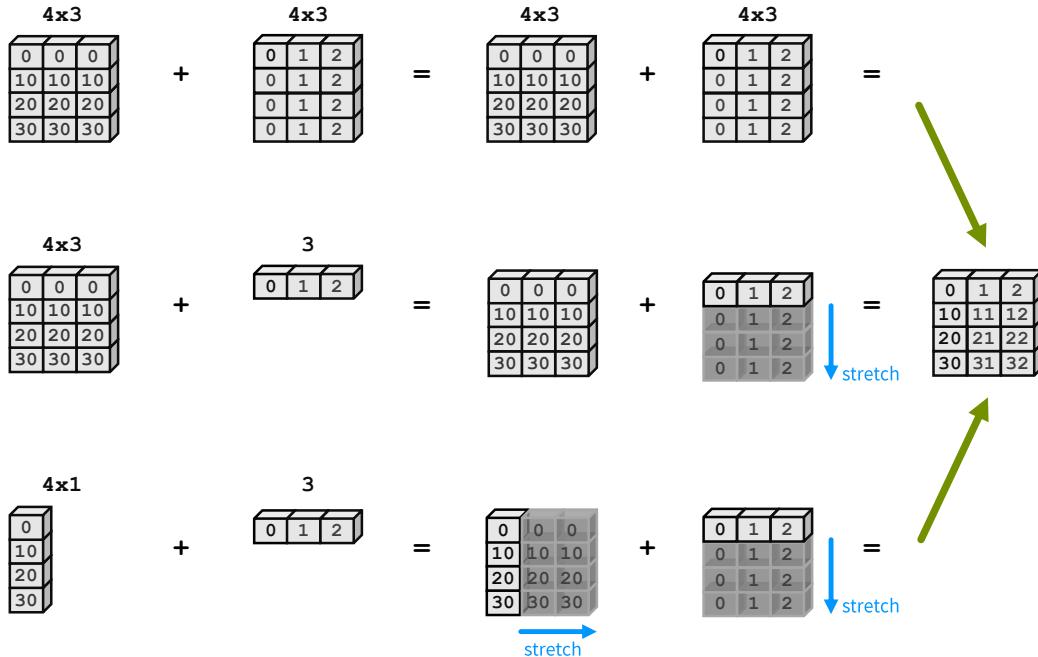
### RULE 2: DIMENSIONS OF SIZE 1 ARE REPEATED WITHOUT COPYING

```
>>> c = a + b # c.shape == (3, 5)
# is logically equivalent to...
>>> tmp_b = b.reshape(1, 5)
>>> tmp_b_repeat = tmp_b.repeat(3, axis=0)
>>> c = a + tmp_b_repeat
# But broadcasting makes no copies of "b"s data!
```

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# Array Broadcasting

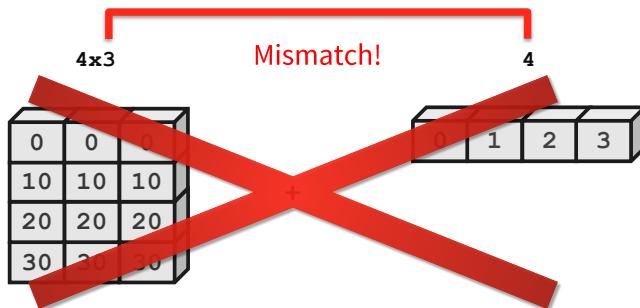


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# Broadcasting Rules

The trailing axes of either arrays must be 1 or both must have the same size for broadcasting to occur. Otherwise, a `"ValueError: shape mismatch: objects cannot be broadcast to a single shape"` exception is thrown.

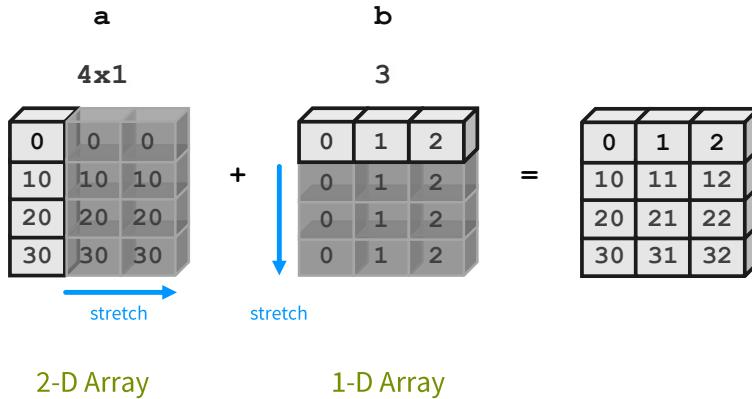


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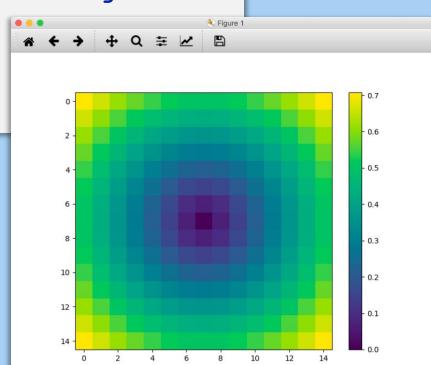
## Broadcasting in Action

```
>>> a = array([0, 10, 20, 30])
>>> b = array([0, 1, 2])
>>> y = a[:, newaxis] + b
```



## Application: Distance from Center

```
>>> import matplotlib.pyplot as plt
>>> a = np.linspace(0, 1, 15) - 0.5
>>> b = a[:, np.newaxis] # b.shape == (15, 1)
>>> dist2 = a**2 + b**2 # broadcasting sum.
>>> dist = np.sqrt(dist2)
>>> plt.imshow(dist)
>>> plt.colorbar()
```



# Broadcasting's Usefulness

Broadcasting can often be used to replace needless data replication inside a NumPy array expression.

`np.meshgrid()` – use `newaxis` appropriately in broadcasting expressions.

`np.repeat()` – broadcasting makes repeating an array along a dimension of size 1 unnecessary.

## MESHGRID: COPIES DATA

```
>>> x, y = \
...     np.meshgrid([1,2],
...                 [3,4,5])
>>> z = x + y
```

## BROADCASTING: NO COPIES

```
>>> x = np.array([1, 2])
>>> y = np.array([3, 4, 5])
>>> z = x[:, np.newaxis] \
...     + y[:, np.newaxis]
```

# Broadcasting Indicies

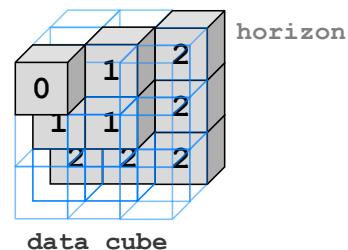
Broadcasting can also be used to slice elements from different “depths” in a 3-D (or any other shape) array. This is a very powerful feature of indexing.

```
>>> yi, xi = np.meshgrid(np.arange(3), np.arange(3),
...                         sparse=True)
>>> zi = np.array([[0, 1, 2],
...                  [1, 1, 2],
...                  [2, 2, 2]])
>>> horizon = data_cube[yi, xi, zi]
```

## Indices

	yi	0   1   2
xi	0	0   1   2
	1	0   1   2
	2	0   1   2

## Selected Data



# Universal Function Methods

The mathematical, comparative, logical, and bitwise operators *op* that take two arguments (binary operators) have special methods that operate on arrays:

```
op.reduce(a, axis=0)
op.accumulate(a, axis=0)
op.outer(a, b)
op.reduceat(a, indices)
```

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## op.reduce()

**op.reduce(a)** applies *op* to all the elements in a 1-D array *a* reducing it to a single value.

For example:

$$\begin{aligned} y &= \text{add.reduce}(a) \\ &= \sum_{n=0}^{N-1} a[n] \\ &= a[0] + a[1] + \dots + a[N-1] \end{aligned}$$

### ADD EXAMPLE

```
>>> a = np.array([1, 2, 3, 4])
>>> np.add.reduce(a)
10
```

### STRING LIST EXAMPLE

```
>>> a = np.array(
    ['ab', 'cd', 'ef'],
    dtype='object')
>>> np.add.reduce(a)
'abcdef'
```

### LOGICAL OP EXAMPLES

```
>>> a = np.array([1, 1, 0, 1])
>>> np.logical_and.reduce(a)
False
>>> np.logical_or.reduce(a)
True
```

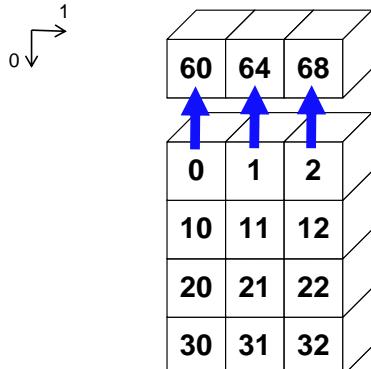
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# op.reduce()

For multidimensional arrays, `op.reduce(a, axis)` applies `op` to the elements of `a` along the specified `axis`. The resulting array has dimensionality one less than `a`. The default value for `axis` is 0.

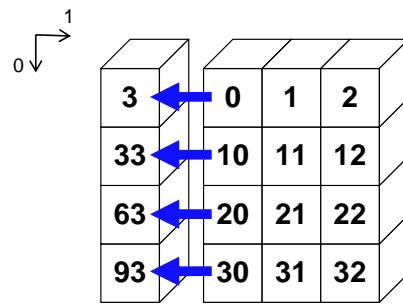
## SUM COLUMNS BY DEFAULT

```
>>> np.add.reduce(a)
array([60, 64, 68])
```



## SUMMING UP EACH ROW

```
>>> np.add.reduce(a, 1)
array([ 3, 33, 63, 93])
```



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# op.accumulate()

`op.accumulate(a)` creates a new array containing the intermediate results of the `reduce` operation at each element in `a`.

For example:

$$y = \text{add.accumulate}(a) \\ = \left[ \sum_{n=0}^0 a[n], \sum_{n=0}^1 a[n], \dots, \sum_{n=0}^{N-1} a[n] \right]$$

## ADD EXAMPLE

```
>>> a = np.array([1,2,3,4])
>>> np.add.accumulate(a)
array([ 1,  3,  6, 10])
```

## STRING LIST EXAMPLE

```
>>> a = np.array(
    ['ab', 'cd', 'ef'],
    dtype='object')
>>> np.add.accumulate(a)
array(['ab', 'abcd', 'abcdef'],
      dtype=object)
```

## LOGICAL OP EXAMPLES

```
>>> a = np.array([True, False, True])
>>> np.logical_and.accumulate(a)
array([True, False, False])
>>> np.logical_or.accumulate(a)
array([True, True, True])
```

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# op.reduceat()

`op.reduceat(a, indices)`

applies `op` to ranges in the 1-D array `a` defined by the values in `indices`. The resulting array has the same length as `indices`.

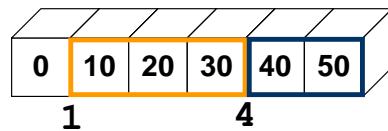
For example:

`y = add.reduceat(a, indices)`

$$y[i] = \sum_{n=indices[i]}^{indices[i+1]} a[n]$$

## EXAMPLE

```
>>> a = np.array([0,10,20,30,40,50])
>>> indices = np.array([1,4])
>>> np.add.reduceat(a, indices)
array([60, 90])
```

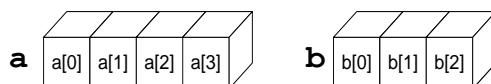


For multidimensional arrays, `reduceat()` is always applied along the *last axis* (sum of rows for 2-D arrays). This is different from the default for `reduce()` and `accumulate()`.

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# op.outer()

`op.outer(a,b)` forms all possible combinations of elements between `a` and `b` using `op`. The shape of the resulting array results from concatenating the shapes of `a` and `b`. (Order matters.)



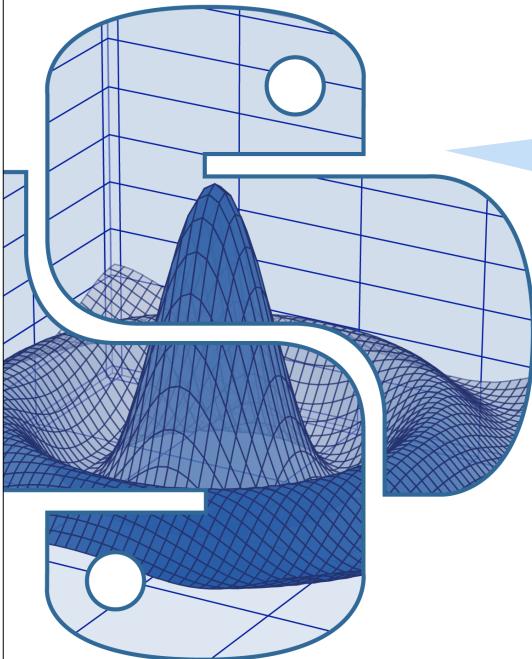
```
>>> np.add.outer(a,b)
```

a[0]+b[0]	a[0]+b[1]	a[0]+b[2]
a[1]+b[0]	a[1]+b[1]	a[1]+b[2]
a[2]+b[0]	a[2]+b[1]	a[2]+b[2]
a[3]+b[0]	a[3]+b[1]	a[3]+b[2]

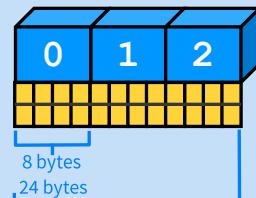
```
>>> np.add.outer(b,a)
```

b[0]+a[0]	b[0]+a[1]	b[0]+a[2]	b[0]+a[3]
b[1]+a[0]	b[1]+a[1]	b[1]+a[2]	b[1]+a[3]
b[2]+a[0]	b[2]+a[1]	b[2]+a[2]	b[2]+a[3]

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Memory Block



# NumPy

## The Array Data Structure

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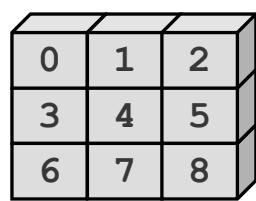
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# Array Data Structure

Memory Block



Python View:

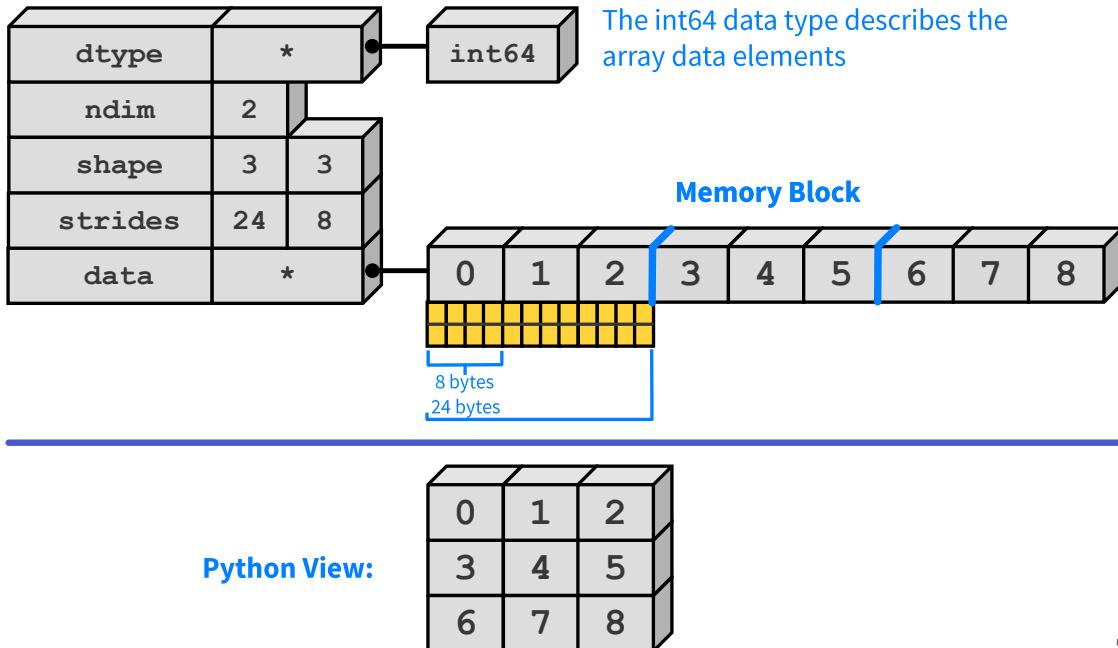


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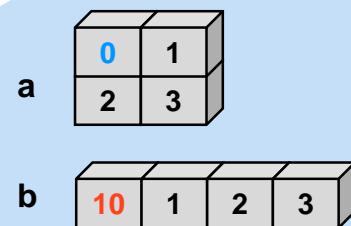
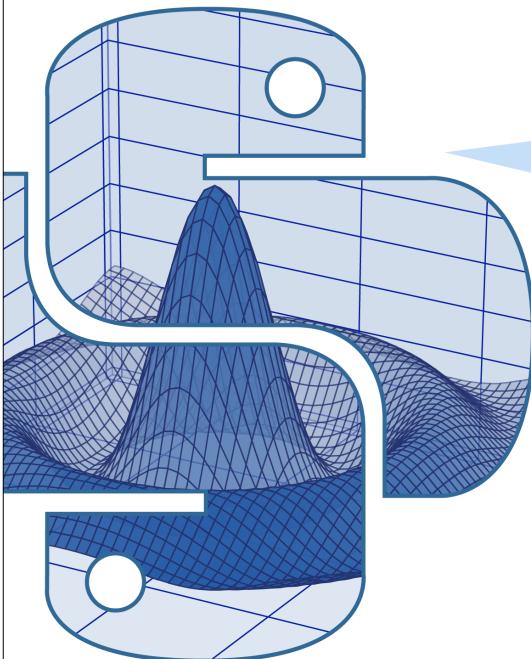
# Array Data Structure

## NDArray Data Structure



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**NumPy**  
Structure Operations

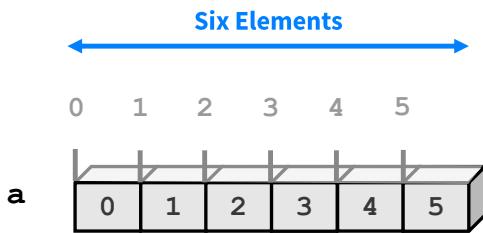
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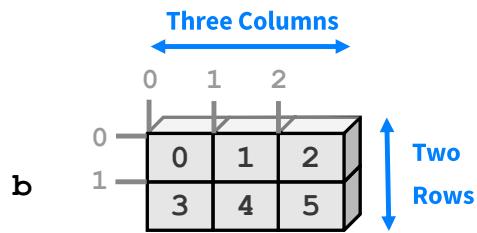
# Operations on the Array Structure

Operations that only affect the array structure, not the data, can be executed without copying memory.

```
>>> a = np.arange(6)
>>> a
```



```
>>> b = a.reshape(2, 3)
>>> b
```



This is not a new array.

The original data does not get reordered.

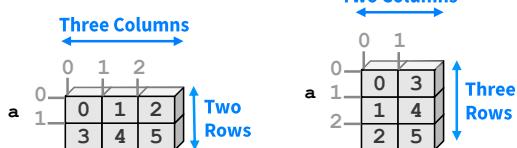
# Transpose

## TRANSPOSE

```
>>> a = np.array([[0,1,2],
...                 [3,4,5]])
>>> a.shape
(2, 3)

# Transpose swaps the order
# of axes.

>>> a.T
array([[0, 3],
       [1, 4],
       [2, 5]])
>>> a.T.shape
(3, 2)
```

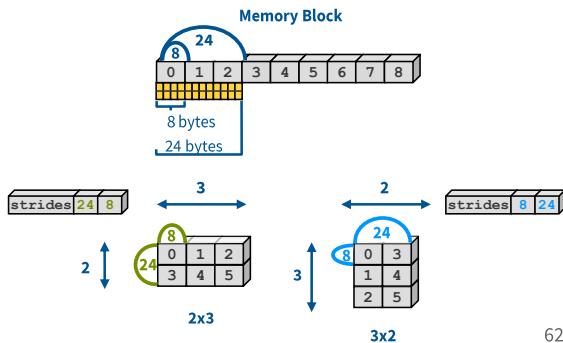


## TRANSPOSE RETURNS VIEWS

```
# Transpose does not move
# values around in memory.
# It only changes the order
# of "strides" in the array

>>> a.strides
(24, 8)

>>> a.T.strides
(8, 24)
```



# Reshaping Arrays

## RESHAPE

```
>>> a = np.array([[0,1,2],
...                 [3,4,5]])

# Return a new array with a
# different shape (a view
# where possible)
>>> a.reshape(3,2)
array([[0, 1],
       [2, 3],
       [4, 5]])

# Reshape cannot change the
# number of elements in an
# array
>>> a.reshape(4,2)
ValueError: total size of new
array must be unchanged
```

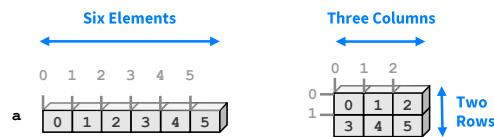
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## SHAPE

```
>>> a = np.arange(6)
>>> a
array([0, 1, 2, 3, 4, 5])
>>> a.shape
(6,)

# Reshape array in-place to
# 2x3

>>> a.shape = (2,3)
>>> a
array([[0, 1, 2],
       [3, 4, 5]])
```



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# Flattening Arrays

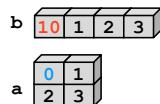
## FLATTEN (SAFE)

`a.flatten()` converts a multi-dimensional array into a 1-D array. The new array is a copy of the original data.

```
# Create a 2D array
>>> a = np.array([[0,1,
...                 [2,3]])

# Flatten out elements to 1D
>>> b = a.flatten()
>>> b
array([0,1,2,3])

# Changing b does not change a
>>> b[0] = 10
>>> b
array([10,1,2,3])
>>> a
array([[0, 1],
       [2, 3]])
```

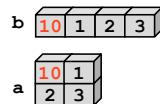


## RAVEL (EFFICIENT)

`a.ravel()` is the same as `a.flatten()`, but returns a reference (or view) of the array if possible (i.e., the memory is contiguous). Otherwise the new array copies the data.

```
# Flatten out elements to 1-D
>>> b = a.ravel()
>>> b
array([0,1,2,3])

# Changing b does change a
>>> b[0] = 10
>>> b
array([10,1,2,3])
>>> a
array([[10, 1],
       [2, 3]])
```



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