Surreal Quantum Field Theory: A Unified Framework

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Abstract

Surreal Quantum Field Theory (Surreal QFT) is a theoretical framework that leverages surreal numbers embedded into hyperreals to unify quantum mechanics, general relativity, and cosmology within a single four-dimensional formalism. This document presents the complete action, potential, and interaction terms of the theory, along with specified parameters and testable predictions that extend beyond standard quantum field theory.

1 Introduction

Surreal QFT introduces a novel scalar field $\hat{S}$, defined over a surreal-valued spacetime Sur^4, aiming to reconcile quantum field theory with gravitational dynamics and cosmological phenomena. By incorporating surreal numbers—an extension of the reals including infinitesimals and infinities—the theory seeks to provide a comprehensive description of fundamental interactions, inflation, and the cosmological constant.

2 Total Action

The total action S integrates gravitational, Standard Model, and surreal field contributions over a surreal-valued spacetime manifold:

$$S = \int_{\text{Sur}^4} d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R + \mathcal{L}_{\text{SM}} + \frac{1}{2}\epsilon(\partial\hat{S})^2 - V(\hat{S}) + \mathcal{L}_{\text{int}}\right]$$

where:

- $\sqrt{-g}$: Square root of the determinant of the surreal-valued metric tensor $g_{\mu\nu}$.
- d^4x: Four-dimensional volume element in Sur^4.
- $\frac{1}{16\pi G}R$: Einstein-Hilbert term, with R as the Ricci scalar and G as Newton's gravitational constant.
- $\mathcal{L}_{\text{SM}}$: Standard Model Lagrangian density, encompassing gauge fields, fermions, and the Higgs field.
- $\frac{1}{2}\epsilon(\partial\hat{S})^2$: Kinetic term for the surreal field $\hat{S}$, with ϵ as a coupling constant.
- $V(\hat{S})$: Potential energy of the surreal field.
- $\mathcal{L}_{\text{int}}$: Interaction terms coupling $\hat{S}$ to Standard Model fields.

3 Surreal Field Potential

The potential $V(\hat{S})$ governs the dynamics of the surreal field across cosmological and quantum scales:

$$V(\hat{S}) = \epsilon m^4 \left[\frac{\hat{S}^2}{\hat{S}^2 + \sigma^2} - \alpha e^{-(\hat{S}/\mu)^2} \right]$$

3.1 Parameter Specifications

The parameters are defined as follows:

$$\begin{aligned} m &= 10^{15}\,\text{GeV}, \\ \epsilon &= 10^{-4}, \\ \sigma &= 5 \times 10^{14}\,\text{GeV}, \\ \mu &= 10^{13}\,\text{GeV}, \\ \alpha &= 10^{-103}. \end{aligned}$$

These values align the theory with high-energy unification scales and cosmological observations.

3.2 Potential Behavior

- **Large $\hat{S} \gg \sigma, \mu$:**

  $$V(\hat{S}) \approx \epsilon m^4 \left[1 - \alpha e^{-(\hat{S}/\mu)^2} \right] \approx 10^{-4} \times (10^{15})^4 = 10^{56}\,\text{GeV}^4$$

  This plateau supports slow-roll inflation.

- **Small $\hat{S} \to 0$:**

  $$V(0) = -\epsilon m^4 \alpha = -10^{-4} \times 10^{60} \times 10^{-103} = -10^{-47}\,\text{GeV}^4$$

  Adjusted with $\alpha = 10^{-103}$ to match the cosmological constant $\Lambda \approx 10^{-47}\,\text{GeV}^4$.

4 Interaction Terms

The surreal field interacts with Standard Model fields via:

$$\mathcal{L}_{\text{int}} = g_H \hat{S} |D_\mu \phi|^2 + g_W \hat{S} W^a_{\mu\nu} W^{a\mu\nu} + \cdots$$

4.1 Coupling Constants

Example couplings include:

$$g_H = \epsilon \left(\frac{m_h}{m} \right), \quad g_W = g\epsilon \left(\frac{m_W}{m} \right)^{1/2}$$

where:

- $m_h = 125\,\text{GeV}$: Higgs boson mass.
- $m_W = 80\,\text{GeV}$: W boson mass.
- $g \approx 0.65$: Weak coupling constant.

5 Key Predictions

The theory yields experimentally testable predictions:

- **W Boson Mass Shift:** $\Delta m_W \sim 10^{-3}\,\text{GeV}$,
- **Higgs Mass Shift:** $\Delta m_h \sim 10^{-2}\,\text{GeV}$,
- **Gamma-Ray Decay Rate:** $\Gamma_\gamma \sim 10^{-2}\,\text{eV}$ at $10^{15}\,\text{GeV}$,
- **Gravitational Wave Amplitude Boost:** $\delta A/A \sim 10^{-8}$,
- **Photon Phase Shift:** $\delta\phi \sim 10^{-6}$.

6 Conclusion

Surreal QFT integrates a surreal-valued spacetime Sur^4 with a unified action, embedding surreal numbers into hyperreals to describe quantum and gravitational phenomena. The specified potential and interactions provide a framework for inflation, force unification, and the cosmological constant, with predictions testable between 2027 and 2040 using facilities like LHC, FCC-ee, Fermi-LAT, LISA, and quantum optics labs.