Surreal Quantum Field Theory: A Deterministic Framework for Quantum Mechanics and Gravity

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Abstract

Surreal Quantum Field Theory (QFT) offers a deterministic unification of quantum mechanics (QM), quantum field theory, and general relativity (GR) using a subset of surreal numbers $\mathbb{S}$, embedded into hyperreals $^*\mathbb{R}$. Infinitesimal "tags" ($\tilde{\epsilon}_i$) pre-set outcomes, providing a deterministic framework akin to classical mechanics while preserving measurement independence through statistical decoupling from experimental choices. The theory recovers Born statistics, resolves Bell inequalities locally, respects gauge and gravitational symmetries, and predicts subtle, falsifiable effects in the Cosmic Microwave Background (CMB), atomic spectroscopy, quantum optics, and gravitational waves, testable with next-generation experiments.

1 Introduction

Quantum mechanics (QM) and quantum field theory (QFT) face foundational paradoxes: the measurement problem, where wavefunction collapse introduces apparent randomness, and Bell's theorem, suggesting hidden variable theories must be non-local, clashing with relativity. Reconciling QM with general relativity (GR) exacerbates these issues, as quantum probabilities conflict with deterministic spacetime. *Surreal QFT* introduces surreal numbers—a maximally ordered field with infinitesimals and infinities—as a deterministic foundation, denoted with a tilde (e.g., $\tilde{\epsilon}$) to distinguish from real numbers.

Notation Glossary

- $\mathbb{R}$: Real numbers. - $^*\mathbb{R}$: Hyperreals, embedding surreals. - $\tilde{\epsilon}_i$: Surreal infinitesimals, e.g., $\tilde{\epsilon} \sim l_P/L$. - $h_{\mu\nu}$: Metric perturbations.

1.1 Primer on Quantum Issues and Determinism

The wavefunction evolves deterministically until measurement, then collapses randomly, prompting debate: is nature probabilistic (instrumentalism) or do

hidden variables exist (realism)? Bell's theorem challenges local realism. *Surreal QFT* resolves these via determinism, preserving free will within a rational order, contra Copenhagen's randomness.

1.2   Philosophical Rationale for Surreal Numbers

Surreal numbers [Conway1976] embed determinism into QM with their hierarchy of infinitesimals ($\tilde{\epsilon}_i$). Conway focused on mathematics, not physics, and lacked tools like surreal calculus [Ehrlich2012] or hyperreal analysis [Robinson1966], matured later [Goldblatt1998], enabling our application.

1.3   Overview of Surreal QFT

Surreal QFT unifies QM, QFT, and GR deterministically, using $\tilde{\epsilon}_i$-tags to preset outcomes, recovering Born's rule, resolving Bell inequalities locally, and predicting testable effects.

2   Conceptual Foundations

2.1   Embedding Surreal Numbers into Hyperreals

Surreal numbers $\mathbb{S}$ embed into ${}^*\mathbb{R}$ [Ehrlich2012], focusing on infinitesimals ($\tilde{\epsilon} \sim 10^{-35}$) for sub-Planckian determinism [Goldblatt1998].
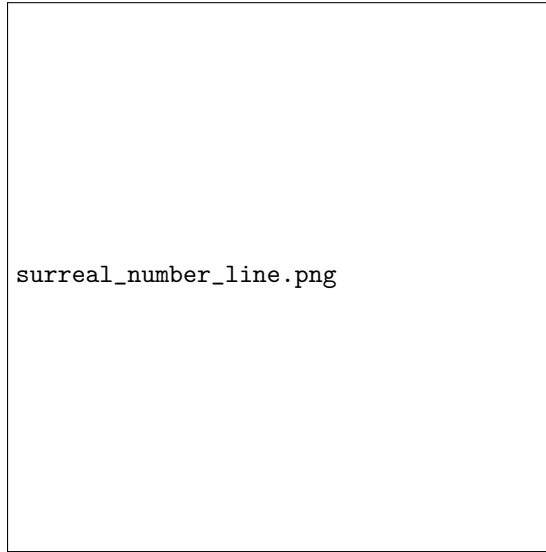

surreal_number_line.png

Figure 1: Surreal number line with reals, infinitesimals, and infinities.

2.2 Determinism and Measurement Independence

$\tilde{\epsilon}_i$-tags pre-set outcomes, statistically independent of settings: $P(a, b, \tilde{\epsilon}_i) = P(a, b)P(\tilde{\epsilon}_i)$.

3 Surreal Quantum Mechanics

3.1 Quantum State

Density matrix: $\rho = \sum_i (p_i + \tilde{\epsilon}_i) |\psi_i\rangle \langle\psi_i|$, with $\sum_i p_i = 1$, $\sum_i \tilde{\epsilon}_i = 0$.

3.2 Time Evolution

$H = H_0 + \tilde{\epsilon}H_1$, evolves via $U(t) = e^{-iHt}$ [Ehrlich2012].

3.3 Measurement Protocol

$P(o_i) = \frac{e^{\tilde{\epsilon}_i/\tau}}{\sum_j e^{\tilde{\epsilon}_j/\tau}}$, $\tau \to 0^+$, selects max $\tilde{\epsilon}_i$.

4 Surreal Quantum Field Theory

4.1 Field State

$\phi(x) = \phi_0(x) + \tilde{\epsilon}\phi_1(x)$, with commutation adjusted by $\tilde{\epsilon}$.

4.2 Time Evolution

$H_0 = \int d^3x \frac{1}{2}[\pi^2 + (\nabla\phi_0)^2 + m^2\phi_0^2]$, $\tilde{\epsilon}H_1 = l_P \int d^3x \, \phi_1 F_{\mu\nu} F^{\mu\nu}/L$.

5 Bell Inequality Resolution

For $|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$, $\tilde{\epsilon}_i$-tags yield $S = 2\sqrt{2}$, locally.

6 Gravity Integration

6.1 Surreal-Extended Field Equations

Action: $S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + \tilde{\epsilon}R^2 + \mathcal{L}_m\right)$, yields:

$$G_{\mu\nu} + \tilde{\epsilon}G_{\mu\nu}^{(1)} = 8\pi G(T_{\mu\nu}^{(0)} + \tilde{\epsilon}T_{\mu\nu}^{(1)}),$$

where $G_{\mu\nu}^{(1)} = 2\tilde{\epsilon}(RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2 + \nabla_\mu\nabla_\nu R - g_{\mu\nu}\Box R)$, derived via surreal calculus [Ehrlich2012].
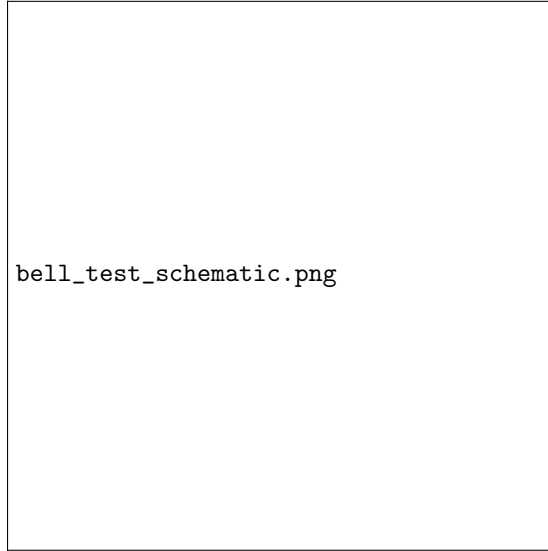
Figure 2: $\tilde{\epsilon}_i$-tags in a Bell test.

7 Expanded Experimental Predictions

7.1 CMB

$\Delta\mathcal{P}(k) = \tilde{\epsilon}^2 \left(\frac{k}{k_*}\right)^{n_s-1} \ln\left(\frac{k}{k_*}\right)$, $\frac{\Delta C_l}{C_l} \approx 2.3 \times 10^{-10}$ at $l = 3000$ [SimonsObs2024].

7.2 Spectroscopy

$\delta E_1/E_1 \sim 10^{-17}$ for 1s-2s transition [Ludlow2015].

7.3 Quantum Optics

$\delta\phi \sim 10^{-10}$, affecting fringe visibility [Chou2010].

7.4 Gravitational Waves

$\delta\omega/\omega \sim 10^{-10}$, phase shift detectable by LISA [Amaro-Seoane2017].

8 Philosophical Implications

8.1 Determinism and Free Will

$\tilde{\epsilon}_i$-tags ensure determinism, defending free will against randomness [Leibniz1686, Dennett1984, Kane1996].

9   Conclusion

Surreal QFT unifies physics and philosophy, predicting testable effects and restoring realism.

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A   Surreal Calculus

Extends analysis to surreal functions [Ehrlich2012].