# Surreal Quantum Field Theory: A Unified Framework

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#### Abstract

Surreal Quantum Field Theory (Surreal QFT) is a theoretical framework that leverages surreal numbers embedded into hyperreals to unify quantum mechanics, general relativity, and cosmology within a single four-dimensional formalism. This document presents the complete action, potential, and interaction terms of the theory, along with specified parameters and testable predictions that extend beyond standard quantum field theory.

## 1 Introduction

Surreal QFT introduces a novel scalar field  $\hat{S}$ , defined over a surreal-valued spacetime  $\text{Sur}^4$ , aiming to reconcile quantum field theory with gravitational dynamics and cosmological phenomena. By incorporating surreal numbers—an extension of the reals including infinitesimals and infinities—the theory seeks to provide a comprehensive description of fundamental interactions, inflation, and the cosmological constant.

### 2 Total Action

The total action S integrates gravitational, Standard Model, and surreal field contributions over a surreal-valued spacetime manifold:

$$S = \int_{\text{Sur}^4} d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R + \mathcal{L}_{\text{SM}} + \frac{1}{2} \epsilon (\partial \hat{S})^2 - V(\hat{S}) + \mathcal{L}_{\text{int}} \right]$$

where:

- $\sqrt{-g}$ : Square root of the determinant of the surreal-valued metric tensor  $g_{\mu\nu}$ .
- $d^4x$ : Four-dimensional volume element in Sur<sup>4</sup>.
- $\frac{1}{16\pi G}R$ : Einstein-Hilbert term, with R as the Ricci scalar and G as Newton's gravitational constant.
- $\mathcal{L}_{\text{SM}}$ : Standard Model Lagrangian density, encompassing gauge fields, fermions, and the Higgs field.
- $\frac{1}{2}\epsilon(\partial \hat{S})^2$ : Kinetic term for the surreal field  $\hat{S}$ , with  $\epsilon$  as a coupling constant.
- $V(\hat{S})$ : Potential energy of the surreal field.
- $\mathcal{L}_{int}$ : Interaction terms coupling  $\hat{S}$  to Standard Model fields.

### 3 Surreal Field Potential

The potential  $V(\hat{S})$  governs the dynamics of the surreal field across cosmological and quantum scales:

$$V(\hat{S}) = \epsilon m^4 \left[ \frac{\hat{S}^2}{\hat{S}^2 + \sigma^2} - \alpha e^{-(\hat{S}/\mu)^2} \right]$$

### 3.1 Parameter Specifications

The parameters are defined as follows:

$$m = 10^{15} \,\text{GeV},$$
  
 $\epsilon = 10^{-4},$   
 $\sigma = 5 \times 10^{14} \,\text{GeV},$   
 $\mu = 10^{13} \,\text{GeV},$   
 $\alpha = 10^{-103}.$ 

These values align the theory with high-energy unification scales and cosmological observations.

#### 3.2 Potential Behavior

• Large  $\hat{S} \gg \sigma, \mu$ :

$$V(\hat{S}) \approx \epsilon m^4 \left[ 1 - \alpha e^{-(\hat{S}/\mu)^2} \right] \approx 10^{-4} \times (10^{15})^4 = 10^{56} \,\text{GeV}^4$$

This plateau supports slow-roll inflation.

• Small  $\hat{S} \to 0$ :

$$V(0) = -\epsilon m^4 \alpha = -10^{-4} \times 10^{60} \times 10^{-103} = -10^{-47} \,\text{GeV}^4$$

Adjusted with  $\alpha = 10^{-103}$  to match the cosmological constant  $\Lambda \approx 10^{-47} \, \text{GeV}^4$ .

# 4 Interaction Terms

The surreal field interacts with Standard Model fields via:

$$\mathcal{L}_{\text{int}} = g_H \hat{S} |D_\mu \phi|^2 + g_W \hat{S} W^a_{\mu\nu} W^{a\mu\nu} + \cdots$$

# 4.1 Coupling Constants

Example couplings include:

$$g_H = \epsilon \left(\frac{m_h}{m}\right), \quad g_W = g\epsilon \left(\frac{m_W}{m}\right)^{1/2}$$

where:

- $m_h = 125 \,\text{GeV}$ : Higgs boson mass.
- $m_W = 80 \,\text{GeV}$ : W boson mass.
- $q \approx 0.65$ : Weak coupling constant.

# 5 Key Predictions

The theory yields experimentally testable predictions:

• W Boson Mass Shift:  $\Delta m_W \sim 10^{-3} \, \mathrm{GeV}$ ,

• Higgs Mass Shift:  $\Delta m_h \sim 10^{-2} \, \mathrm{GeV}$ ,

• Gamma-Ray Decay Rate:  $\Gamma_{\gamma} \sim 10^{-2} \, \mathrm{eV}$  at  $10^{15} \, \mathrm{GeV}$ ,

• Gravitational Wave Amplitude Boost:  $\delta A/A \sim 10^{-8}$ ,

• Photon Phase Shift:  $\delta \phi \sim 10^{-6}$ .

# 6 Conclusion

Surreal QFT integrates a surreal-valued spacetime Sur<sup>4</sup> with a unified action, embedding surreal numbers into hyperreals to describe quantum and gravitational phenomena. The specified potential and interactions provide a framework for inflation, force unification, and the cosmological constant, with predictions testable between 2027 and 2040 using facilities like LHC, FCC-ee, Fermi-LAT, LISA, and quantum optics labs.