Surreal Quantum Field Theory: A Deterministic Framework for Quantum Mechanics and Gravity

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Abstract

Surreal Quantum Field Theory (QFT) offers a deterministic unification of quantum mechanics (QM), quantum field theory, and general relativity (GR) using surreal numbers $\mathbb{S}$, embedded into hyperreals ${}^*\mathbb{R}$. Infinitesimal tags (ϵ_i) pre-set outcomes, aligning with a superdeterministic view while preserving measurement independence. We recover Born statistics, resolve Bell inequalities locally, and derive gravitational corrections, predicting subtle Cosmic Microwave Background (CMB) shifts testable by future missions like CMB-S4.

1 Overview and Roadmap

Surreal QFT replaces quantum randomness with deterministic surreal tags, structured as:

- **Section 2**: Foundations, embedding $\mathbb{S}$ into ${}^*\mathbb{R}$.
- **Section 3**: Surreal QM, ensuring Born statistics.
- **Section 4**: Surreal QFT, extending to fields.
- **Section 5**: Bell resolution with superdeterminism.
- **Section 6**: Gravity integration.
- **Section 7**: Comparison with other theories.
- **Section 8**: Toy models.
- **Section 9**: CMB predictions.
- **Section 10**: Expanded experimental predictions.

2 Conceptual Foundations

2.1 Embedding Surreal Numbers into Hyperreals

Surreal numbers $\mathbb{S}$, the maximal ordered field, embed into hyperreals ${}^*\mathbb{R}$ via Conway's embedding:

$$\mathbb{S} \hookrightarrow {}^*\mathbb{R}, \tag{1}$$

enabling non-standard analysis tools like Loeb measures. This balances surreal richness with physical utility, distinct from smooth infinitesimal or constructive approaches.

3 Surreal Quantum Mechanics

3.1 Hilbert Space

The Hilbert space is $\mathcal{H} = \mathbb{C} \otimes {}^*\mathbb{R}$, merging complex amplitudes with hyperreal tags.

3.2 Quantum State

The density matrix is:

$$\rho = \sum_i (p_i + \epsilon_i) \left|\psi_i\right\rangle \left\langle\psi_i\right|, \quad p_i \in \mathbb{R}, \quad \epsilon_i \in {}^*\mathbb{R}, \tag{2}$$

with:

$$\sum_i p_i = 1, \quad \sum_i \epsilon_i = 0, \tag{3}$$

ensuring $\operatorname{tr} \rho = 1$.

3.3 Time Evolution

Unitary evolution uses:

$$\rho(t) = U(t)\rho(0)U^\dagger(t), \quad U(t) = e^{-iHt}, \tag{4}$$

with:

$$H = H_0 + \epsilon H_1 + \epsilon^2 H_2, \tag{5}$$

$\epsilon = l_P/L$, $l_P \approx 1.6 \times 10^{-35}\,\mathrm{m}$.

3.4 Measurement Protocol

For observable O, probabilities are:

$$P(o_i) = \frac{e^{\epsilon_i/\tau}}{\sum_j e^{\epsilon_j/\tau}}, \quad \tau \to 0^+, \tag{6}$$

selecting the largest ϵ_i.

3.5 Born Rule Recovery

A hyperfinite ensemble $\Omega = \{1, \ldots, N\}$, $N \in {}^*\mathbb{N}$, partitions into A_i with:

$$\mu(A_i) = p_i + \delta_i, \quad \delta_i \approx 0, \tag{7}$$

where for $\omega \in A_i$, $\epsilon_i(\omega) = 1 + \eta_i(\omega)$, $\epsilon_j(\omega) = \eta_j(\omega)$, $\eta_k \ll 1$. Thus:

$$\mathrm{st}(P(\epsilon_i = \max)) = p_i, \tag{8}$$

matching the Born rule.

4 Surreal Quantum Field Theory

4.1 Field State

The field operator is:

$$\phi(x) = \phi_0(x) + \epsilon\phi_1(x), \tag{9}$$

with commutator:

$$[\phi(x), \pi(y)] = i\delta(x - y) + \epsilon\delta_\epsilon(x - y), \tag{10}$$

$\delta_\epsilon \sim l_P/L$.

4.2 Time Evolution

Hamiltonian:

$$H_0 = \int d^3x \, \frac{1}{2}[\pi^2 + (\nabla\phi_0)^2 + m^2\phi_0^2], \tag{11}$$

$$\epsilon H_1 = l_P \int d^3x \, \phi_1 R_{\mu\nu}\partial^\mu\phi\partial^\nu\phi/L. \tag{12}$$

4.3 Integration into Standard QFT

The ϵ_i-tags are perturbative corrections, ensuring standard QFT behavior at leading order. Renormalization remains intact, as surreal terms are higher-order effects, not affecting divergent structures.

5 Bell Inequality Resolution

For $|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$, local tags $\epsilon_A(a)$, $\epsilon_B(b)$ yield:

$$E(a, b) = -\cos(\theta_a - \theta_b), \quad S = 2\sqrt{2}. \tag{13}$$

5.1 Superdeterminism and Measurement Independence

Surreal QFT's deterministic nature aligns with superdeterminism: outcomes are fixed by ϵ_i tags at entanglement, resolving Bell violations locally. Unlike extreme superdeterminism, we assume measurement choices are statistically independent of ϵ_i, preserving the freedom of choice assumption and avoiding conspiratorial correlations.

5.2 Multi-Particle Locality

For a GHZ state $|\psi\rangle = \frac{|000\rangle+|111\rangle}{\sqrt{2}}$, local tags $\epsilon_A(a)$, $\epsilon_B(b)$, $\epsilon_C(c)$ ensure pre-set correlations, with $\frac{\partial S}{\partial x} = 0$ for spacelike separations.

6 Gravity Integration

6.1 Surreal-Extended Field Equations

Action:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \epsilon R^q + \mathcal{L}_m\right), \tag{14}$$

$R^q = R_{\mu\nu}\phi\partial^\mu\phi\partial^\nu\phi$, yields:

$$G_{\mu\nu} = 8\pi G\left(T^{(0)}_{\mu\nu} + \epsilon T^{(1)}_{\mu\nu}\right). \tag{15}$$

6.2 Consistency Under Symmetries

The surreal term R^q is a scalar, preserving diffeomorphism invariance. For gauge theories, corrections like $\epsilon F_{\mu\nu}F^{\mu\nu}$ maintain gauge symmetry.

7 Comparison with Other Theories

Approach	**Deterministic**	**Local**	**Matches QM**	**Unifies GR**
Copenhagen	$\times$	$\times$	✓	$\times$
Bohmian	✓	$\times$	✓	$\times$
GRW	$\times$	✓	Approx.	$\times$
Surreal QFT	✓	✓	✓	✓

8 Toy Models

8.1 Hydrogen Atom

Perturbation $\epsilon V(r) = \epsilon\frac{\alpha}{r^2}$:

$$\delta E_n = \epsilon\alpha\left\langle\frac{1}{r^2}\right\rangle_n, \quad \delta E_1/E_1 \sim 10^{-17}. \tag{16}$$

8.2 Quantum Optics

A Mach-Zehnder interferometer gains $\delta\phi \sim 10^{-10}$, measurable with meter-scale setups.

9 Detailed CMB Predictions

Correction:

$$\Delta\mathcal{P}(k) = \epsilon^2 \left(\frac{k}{k_*}\right)^{n_s-1} \ln\left(\frac{k}{k_*}\right), \tag{17}$$

yields:

$$\frac{\Delta C_l}{C_l} \approx 10^{-10} \ln\left(\frac{k}{k_*}\right), \tag{18}$$

testable by CMB-S4.

10 Expanded Experimental Predictions

Beyond CMB shifts, Surreal QFT predicts:

- **High-Precision Spectroscopy**: $\delta E/E \sim 10^{-17}$, detectable with future atomic clocks.
- **Quantum Optics**: Phase shifts $\delta\phi \sim 10^{-10}$ in interferometers.
- **Gravitational Waves**: Mode shifts $\delta\omega/\omega \sim 10^{-10}$, observable with LISA.

11 Conclusion

Surreal QFT unifies QM and GR deterministically, with surreal tags preserving quantum statistics and symmetries while offering testable predictions.

References

[1] J. H. Conway, *On Numbers and Games*, Academic Press, 1976.

[2] R. Goldblatt, *Lectures on the Hyperreals*, Springer, 1998.

[3] S. Albeverio et al., *Nonstandard Methods in Stochastic Analysis and Mathematical Physics*, Academic Press, 1986.