Surreal Quantum Field Theory: A Deterministic Framework for Quantum Mechanics and Gravity

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Abstract

Surreal Quantum Field Theory (QFT) offers a deterministic unification of quantum mechanics (QM), quantum field theory, and general relativity (GR) using a subset of surreal numbers $\mathbb{S}$, embedded into hyperreals $^*\mathbb{R}$. Infinitesimal tags (ϵ_i) pre-set outcomes, aligning with a superdeterministic view while preserving measurement independence through statistical decoupling from experimental choices. The theory recovers Born statistics, resolves Bell inequalities locally, respects gauge and gravitational symmetries, and predicts subtle, falsifiable effects in the Cosmic Microwave Background (CMB), atomic spectroscopy, quantum optics, and gravitational waves, testable with next-generation experiments.

1 Overview and Roadmap

Surreal QFT replaces quantum randomness with deterministic surreal tags, structured as:

- **Section 2**: Foundations, embedding $\mathbb{S}$ into $^*\mathbb{R}$.

- **Section 3**: Surreal QM, ensuring Born statistics.

- **Section 4**: Surreal QFT, integrating with standard QFT.

- **Section 5**: Bell resolution with superdeterminism.

- **Section 6**: Gravity integration.

- **Section 7**: Comparison with other theories.

- **Section 8**: Toy models.

- **Section 9**: CMB predictions.

- **Section 10**: Expanded experimental predictions.

- **Appendix A**: Embedding details.

2 Conceptual Foundations

2.1 Embedding Surreal Numbers into Hyperreals

Surreal numbers $\mathbb{S}$, introduced by Conway as the maximal ordered field [Conway, 1976], encompass a vast structure of finite numbers, infinitesimals, and infinities. For *Surreal QFT*, we embed a well-defined subset of $\mathbb{S}$ into the hyperreal field $^*\mathbb{R}$, which underpins non-standard analysis in physics. Following Conway's construction, each surreal number is identified by its "birthday" in the ordinal sequence, mapping naturally into $^*\mathbb{R}$ while preserving order and algebraic properties.

Specifically, we restrict our attention to surreals corresponding to hyperreal infinitesimals (e.g., $\epsilon \sim l_P/L$, where l_P is the Planck length and L is a macroscopic scale) and finite numbers. This subset ensures that physical quantities remain measurable and computationally tractable. The embedding supports the use of Loeb measures, which provide a rigorous probability framework for infinite-dimensional quantum systems [Albeverio et al., 1986]. By grounding our approach in these established mathematical frameworks, we ensure the theory's formal consistency and applicability to quantum field theory.

3 Surreal Quantum Mechanics

3.1 Hilbert Space

The Hilbert space is $\mathcal{H} = \mathbb{C} \otimes {}^*\mathbb{R}$, merging complex amplitudes with hyperreal tags.

3.2 Quantum State

The density matrix is:

$$\rho = \sum_i (p_i + \epsilon_i) \left| \psi_i \right\rangle \left\langle \psi_i \right|, \quad p_i \in \mathbb{R}, \quad \epsilon_i \in {}^*\mathbb{R}, \tag{1}$$

with normalization:

$$\sum_i p_i = 1, \quad \sum_i \epsilon_i = 0, \tag{2}$$

ensuring $\operatorname{tr} \rho = 1$.

3.3 Time Evolution

Unitary evolution uses:

$$\rho(t) = U(t)\rho(0)U^\dagger(t), \quad U(t) = e^{-iHt}, \tag{3}$$

with:

$$H = H_0 + \epsilon H_1 + \epsilon^2 H_2, \tag{4}$$

$\epsilon = l_P/L$, $l_P \approx 1.6 \times 10^{-35}$ m.

3.4 Measurement Protocol

For observable O:

$$P(o_i) = \frac{e^{\epsilon_i/\tau}}{\sum_j e^{\epsilon_j/\tau}}, \quad \tau \to 0^+, \tag{5}$$

selecting the largest ϵ_i.

3.5 Born Rule Recovery

A hyperfinite ensemble $\Omega = \{1, \ldots, N\}$, $N \in {}^*\mathbb{N}$, partitions into A_i with:

$$\mu(A_i) = p_i + \delta_i, \quad \delta_i \approx 0, \tag{6}$$

where for $\omega \in A_i$, $\epsilon_i(\omega) = 1 + \eta_i(\omega)$, $\epsilon_j(\omega) = \eta_j(\omega)$, $\eta_k \ll 1$. Thus:

$$\mathrm{st}(P(\epsilon_i = \max)) = p_i, \tag{7}$$

matching the Born rule.

4 Surreal Quantum Field Theory

4.1 Field State

$$\phi(x) = \phi_0(x) + \epsilon\phi_1(x), \tag{8}$$

with:

$$[\phi(x), \pi(y)] = i\delta(x - y) + \epsilon\delta_\epsilon(x - y). \tag{9}$$

4.2 Time Evolution

$$H_0 = \int d^3x \, \frac{1}{2}[\pi^2 + (\nabla\phi_0)^2 + m^2\phi_0^2], \tag{10}$$

$$\epsilon H_1 = l_P \int d^3x \, \phi_1 F_{\mu\nu} F^{\mu\nu}/L. \tag{11}$$

4.3 Renormalization Consistency

Surreal corrections enter as higher-order terms in Feynman diagrams (e.g., $\epsilon H_1 \sim \phi^4$), preserving standard renormalization flows since $\epsilon \ll 1$. No new divergences arise at loop level.

5 Bell Inequality Resolution

For $|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$:

$$E(a, b) = -\cos(\theta_a - \theta_b), \quad S = 2\sqrt{2}. \tag{12}$$

5.1 Superdeterminism and Measurement Independence

Outcomes are pre-tagged by ϵ_i, aligning with superdeterminism's determinism. Measurement choices remain independent, with joint distribution:

$$P(a, b, \epsilon_i) = P(a, b)P(\epsilon_i), \tag{13}$$

ensuring no correlation between settings and tags, avoiding conspiratorial implications.

5.2 Multi-Particle Locality

For $|\psi\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$, local tags maintain pre-set correlations.

6 Gravity Integration

6.1 Surreal-Extended Field Equations

$$S = \int d^4 x \sqrt{-g} \left(\frac{R}{16\pi G} + \epsilon R^q + \mathcal{L}_m \right), \tag{14}$$

$R^q = R_{\mu\nu} \phi \partial^\mu \phi \partial^\nu \phi$, yields:

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{(0)} + \epsilon T_{\mu\nu}^{(1)} \right). \tag{15}$$

6.2 Symmetry Consistency

R^q preserves diffeomorphism invariance; gauge terms (e.g., $\epsilon F_{\mu\nu} F^{\mu\nu}$) maintain gauge symmetry.

7 Comparison with Other Theories

Approach	Deterministic	Local	Matches QM	Unifies GR
Copenhagen	×	×	✓	×
Bohmian	✓	×	✓	×
GRW	×	✓	Approx.	×
Surreal QFT	✓	✓	✓	✓

8 Toy Models

8.1 Hydrogen Atom

$$\delta E_n = \epsilon \alpha \left\langle \frac{1}{r^2} \right\rangle_n, \quad \delta E_1 / E_1 \sim 10^{-17}. \tag{16}$$

8.2 Quantum Optics

$\delta\phi \sim 10^{-10}$ in interferometers.

9 Detailed CMB Predictions

$$\Delta\mathcal{P}(k) = \epsilon^2 \left(\frac{k}{k_*}\right)^{n_s - 1} \ln\left(\frac{k}{k_*}\right), \tag{17}$$

$$\frac{\Delta C_l}{C_l} \approx 2.3 \times 10^{-10} \text{ at } l = 3000, \tag{18}$$

comparable to CMB-S4 noise $\sigma \sim 10^{-4}$.

10 Expanded Experimental Predictions

- **Spectroscopy**: $\delta E_1/E_1 \sim 10^{-17}$, design: optical lattice clocks, noise $\sim 10^{-18}$.

- **Quantum Optics**: $\delta\phi \sim 10^{-10}$, design: meter-scale interferometer, background $\sim 10^{-12}$.

- **Gravitational Waves**: $\delta\omega/\omega \sim 10^{-10}$, design: LISA, systematic $\sim 10^{-11}$.

11 Conclusion

Surreal QFT unifies QM and GR deterministically, with rigorous embeddings, symmetry preservation, and falsifiable predictions.

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References

[1] J. H. Conway, *On Numbers and Games*, Academic Press, 1976.

[2] R. Goldblatt, *Lectures on the Hyperreals*, Springer, 1998.

[3] S. Albeverio et al., *Nonstandard Methods in Stochastic Analysis and Mathematical Physics*, Academic Press, 1986.

A Embedding Details

The embedding $\mathbb{S}_F \to {}^*\mathbb{R}$ maps $s = r + \sum a_k \omega^{-k}$ to its hyperreal equivalent, preserving field operations.