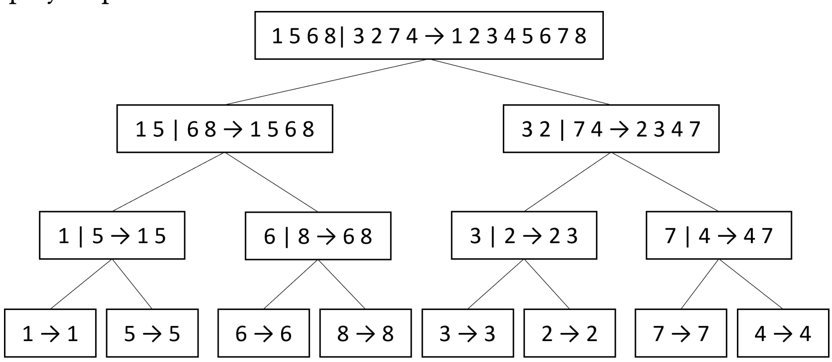
CSCI2100D 2023-24: Solution of Assignment 3[*]

- **Q1. [12 marks]** Let $A[0 \cdots 7] = [1, 5, 6, 8, 3, 2, 7, 4]$.
  - **(i). [6 marks]** Show the process of **mergesort**(A, 0, 7) to sort A in ascending order step by step.



  - **(ii). [6 marks]** Assume that we call **quicksort**(A, 0, 7) to sort A in ascending order, and the pivot position we randomly choose is 1. Show how the partition works and indicate the value of **nextsmallpos** during the invocation of **partition**(A, 0, 7, 1) step by step.

    Since the pivot position is 1, we have **pivotVal** = 5.

Step 1: j = 0, nextSmallpos = 0.	1	4	6	8	3	2	7	5
Step 2: j = 1, nextSmallpos = 1.	1	4	6	8	3	2	7	5
Step 3: j = 2, nextSmallpos = 2.	1	4	6	8	3	2	7	5
Step 4: j = 3, nextSmallpos = 2.	1	4	6	8	3	2	7	5
Step 5: j = 4, nextSmallpos = 3.	1	4	3	8	6	2	7	5
Step 6: j = 5, nextSmallpos = 4.	1	4	3	2	6	8	7	5
Step 7: j = 6, nextSmallpos = 4.	1	4	3	2	6	8	7	5

    Then we swap $A[4]$ and $A[7]$. It yields $A = [1, 4, 3, 2, 5, 8, 7, 6]$.

- **Q2. [12 marks]** Sort array $A[1 \cdots 7] = [9, 1, 10, 3, 2, 8, 4]$ in decreasing order by heap sort. (you may just show the array representation at each step.)
  - **(i). [6 marks]** Show the contents of A in the heap adjust process to make it a min-heap step by step.

Step 1: Adjust node with id 3.	9	1	4	3	2	8	10

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Step 2: Check node with id 2. No violation

Step 3: Adjust node with id 1.

1	9	4	3	2	8	10

Step 4: Adjust node with id 2.

1	2	4	3	9	8	10

– **(ii). [6 marks]** Using the min-heap of Part (i), show the contents of A in the sorting process of swaping elements in the array step by step.

Step 1: Swap node id 7 and node id 1; reduce heap size by 1.

10	2	4	3	9	8	**1**

Step 2: Adjust.

2	3	4	10	9	8	**1**

Step 3: Swap node id 6 and node id 1; reduce heap size by 1.

8	3	4	10	9	**2**	**1**

Step 4: Adjust.

3	8	4	10	9	**2**	**1**

Step 5: Swap node id 5 and node id 1; reduce heap size by 1.

9	8	4	10	**3**	**2**	**1**

Step 6: Adjust.

4	8	9	10	**3**	**2**	**1**

Step 7: Swap node id 4 and node id 1; reduce heap size by 1.

10	8	9	**4**	**3**	**2**	**1**

Step 8: Adjust.

8	10	9	**4**	**3**	**2**	**1**

Step 9: Swap node id 3 and node id 1; reduce heap size by 1.

9	10	**8**	**4**	**3**	**2**	**1**

Step 10: No violation.

Step 11: Swap node id 2 and node id 1; reduce heap size by 1.

10	**9**	**8**	**4**	**3**	**2**	**1**

Step 12: No violation.

Step 13: The heap size now is 1. Sort finishes.

10	**9**	**8**	**4**	**3**	**2**	**1**

- **Q3. [14 marks]** Assume that we have a hash table with size $m = 13$ and the hash function $h(k) = 2k\%13$. We use linear probing to address collisions. Answer the following questions.

  – **(i). [7 marks]** Given an empty hash table, show the hash table when inserting 10, 2, 23, 13, 1, 9, 17 in order step by step. Inserting 10:

							10					

Inserting 2:

				2			10					

Inserting 23:

				2			10	23				

Inserting 13:

13				2			10	23				

Inserting 1:

13		1		2			10	23				

Inserting 9:

13		1		2	9		10	23				

Inserting 17:

13		1		2	9		10	23	17			

  – **(ii). [7 marks]** Given the following hash table, show the records examined when searching for 42.

0				4				8				12
0					22	16	10	30	24	42	25	

When searching for 42, the records 16,10,30,24,42 are examined in order.

- **Q4. [14 marks]** Assume that we have a hash table with size $m = 13$ and we use double hashing to address collisions. The double hashing function is $h(k, i) = (h(k)+i\cdot h'(k))\%m$, where $h(k) = k\%13$ and $h'(k) = 1 + k\%3$. Answer the following questions.
  - **(i). [7 marks]** Given an empty hash table, show the hash table when inserting 14, 2, 18, 36, 31, 23, 42 in order step by step.

Inserting 14: $h(14, 0) = 1$,

	14											

Inserting 3: $h(2, 0) = 2$,

	14	2										

Inserting 18: $h(18, 0) = 5$

	14	2			18							

Inserting 36: $h(36, 0) = 10$,

	14	2			18					36		

Inserting 31: $h(31, 0) = 5$, $h(31, 1) = 7$

	14	2			18		31			36		

Inserting 23: $h(23, 0) = 10$, $h(23, 1) = 0$,

23	14	2			18		31			36		

Inserting 42: $h(42, 0) = 3$

23	14	2	42		18		31			36		

  - **(ii). [7 marks]** Given the following hash table, show the records examined when searching for 44.

0				4				8				12
		15		30	5			21		10	24	38

When searching for 44, since $h(44, 0) = 5$, $h(44, 1) = 8$, and $h(44, 2) = 11$, $h(44, 3) = 1$, the records 5, 21, 24 are examined in order

- **Q5. [38 marks]** Consider the directed graph G_1 as shown in Fig. 1. Answer the following questions.

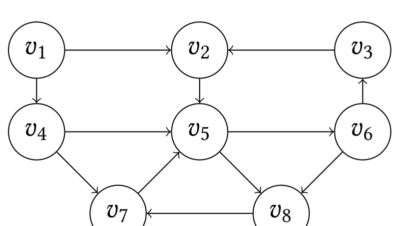

Fig. 1. Directed Graph G_1 for Q5

  - **(i). [2 mark]** Calculate the out-degree and the in-degree of v_5.
    The out-degree of v_5 is 2. The in-degree of v_5 is 3.

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
minlength	0	1	4	1	2	3	2	3
prev	*nil*	v_1	v_6	v_1	v_2	v_5	v_4	v_5

Table 1. **minlength** and **prev** Array of Q5(v)

- **(ii). [2 mark]** Whether the path $v_1 \longrightarrow v_4 \longrightarrow v_5 \longrightarrow v_6 \longrightarrow v_8 \longrightarrow v_7$ is a simple path? Justify your answer.
  Yes, because all the edges exist and all the nodes are distinct.
- **(iii). [8 marks]** For G_1, show both its adjacency list representation and its adjacency matrix representation. (The nodes should be in ascending order of ID.)

$v_1 \rightarrow head \leftrightarrow v_2 \leftrightarrow v_4 \leftrightarrow tail$
$v_2 \rightarrow head \leftrightarrow v_5 \leftrightarrow tail$
$v_3 \rightarrow head \leftrightarrow v_2 \leftrightarrow tail$
$v_4 \rightarrow head \leftrightarrow v_5 \leftrightarrow v_7 \leftrightarrow tail$
$v_5 \rightarrow head \leftrightarrow v_6 \leftrightarrow v_8 \leftrightarrow tail$
$v_6 \rightarrow head \leftrightarrow v_3 \leftrightarrow v_8 \leftrightarrow tail$
$v_7 \rightarrow head \leftrightarrow v_5 \leftrightarrow tail$
$v_8 \rightarrow head \leftrightarrow v_7 \leftrightarrow tail$

Adjacency List for Q5(iii)

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1	0	1	0	1	0	0	0	0
v_2	0	0	0	0	1	0	0	0
v_3	0	1	0	0	0	0	0	0
v_4	0	0	0	0	1	0	1	0
v_5	0	0	0	0	0	1	0	1
v_6	0	0	1	0	0	0	0	1
v_7	0	0	0	0	1	0	0	0
v_8	0	0	0	0	0	0	1	0

Adjacency Matrix for Q5(iii)

Fig. 2. Solution of Q5(iii)

- **(iv). [8 marks]** Traverse G_1 using breadth-first search with v_1 as the source, assuming that the out-neighbors of a node are visited in ascending order of ID. Show the process and the contents of queue Q step by step. You may use 0 to denote the color to be white, 1 to denote the color to be gray, and 2 to denote the color to be black.
  The answer can be found in Figure 3.
- **(v). [8 marks]** According to the results of Part (iv), show the contents of **minlength** array and **prev** array respectively.
  The answer can be found in Table 1.
- **(vi). [4 marks]** Show how to get the minimum length path from the source v_1 to v_7 using the **minlength** array and **prev** array. Justify your answer.
  Get the previous node of v_7 in prev array, which is v_4; get the previous node of v_4, which is v_1, i.e., the source. We get the path: $v_1 \longrightarrow v_4 \longrightarrow v_7$, whose length is 2.
- **(vii). [6 marks]** Draw the BFS tree.
  The answer can be found in Figure 4.

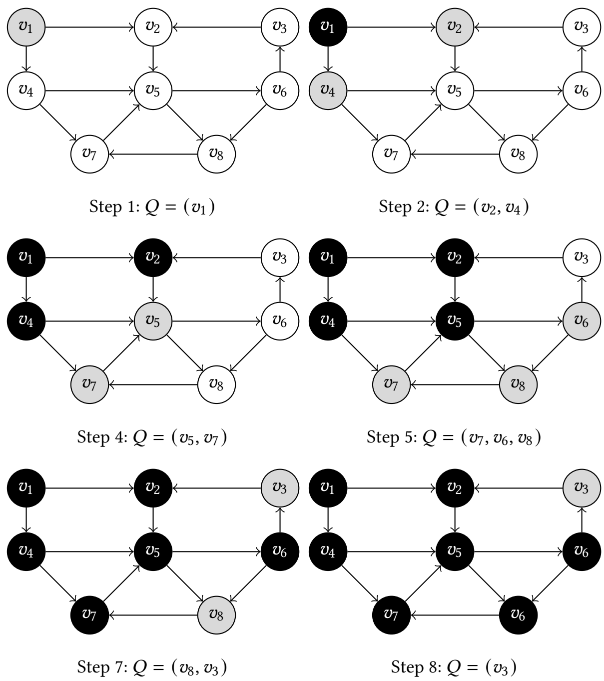

Fig. 3. Solution of Q5(iv)

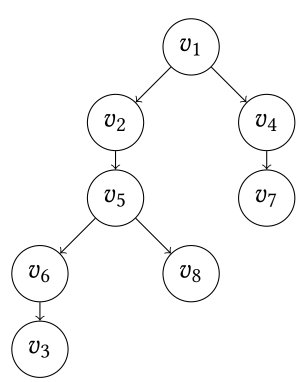

Fig. 4. Solution of Q5(vii)

- **Q6. [10 marks]** Given an undirected graph $G = (V, E)$, A triangle consists of three nodes in G that are pairwise adjacent. More formally, a triangle in G is triplet (u, v, w) where $u, v, w \in V$ and $(u, v), (v, w), (w, u) \in E$. Consider the undirected graph G_2 as shown in Fig. 5. There are two triangles in G_2, (v_0, v_1, v_2) and (v_1, v_2, v_3).

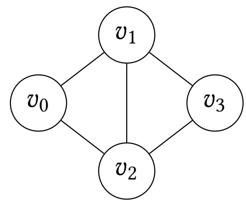

Fig. 5. An Undirected Graph G_2 for Q6

- **(i). [4 marks]** Please design an algorithm TriangleCounting(G) using pseudo-code with the provided Graph ADT, to count the number of triangles in G.

  Graph **ADT**.
  * **`Vertices(G)`:** Lists all vertices u in G.
  * **`Neighbors(G,u)`:** Lists all vertices v such that there is an edge between the vertex u and the vertex v.
  * **`Adjacent(G,u,v)`:** Tests whether there is an edge between the vertex u and the vertex v.

  The algorithm can be designed using iterative approach as follows:

  Algorithm `TriangleCounting`(G)
  $count \leftarrow 0$
  for $u \in Vertices(G)$
      for $v \in Neighbors(G, u)$
          for $w \in Neighbors(G, v)$
              if $Adjacent(G, u, w)$
                  $count \leftarrow count + 1$
  return $count/6$

- **(ii). [2 marks]** Assume that G has n nodes and m edges, and the degree of any node u in G is smaller than d_{max}. What is the time complexity of TriangleCounting(G) expressed in terms of n, m, and d_{max} if the graph ADT is implemented using an adjacency matrix? Justify your answer.
  Since the graph ADT is implemented using an adjacency matrix, the $Vertices(G)$ costs $O(n)$ time, the $Neighbors(G, u)$ costs $O(n)$ time, the $Neighbors(G, v)$ costs $O(n)$ time and the $Adjacent(G, u, v)$ costs $O(1)$ time. Therefore, the time complexity of the algorithm is $O(n \times n \times n \times 1) = O(n^3)$

- **(iii). [2 marks]** Assume that G has n nodes and m edges, and the degree of any node u in G is smaller than d_{max}. What is the time complexity of $TriangleCounting(G)$ expressed in terms of n, m, and d_{max} if the graph ADT is implemented using an adjacency list? Justify your answer.

Since the graph ADT is implemented using an adjacency list, the $Vertices(G)$ and $Neighbors(G, u)$ enumerate each edge exactly once, thus cost $O(m)$ time in total. Besides, the $Neighbors(G, u)$ costs $O(d_{max})$ and the $Adjacent(G, u, v)$ costs $O(d_{max})$ time. Therefore, the time complexity of the algorithm is $O(m \times d_{max} \times d_{max}) = O(md_{max}^2)$

- **(iv). [2 marks]** Assume that the graph ADT is implemented using an adjacency list, and the output of Neighbors(G, u) is guaranteed to be sorted in ascending order by node ID. Please design a more efficient TriangleCounting(G) using pseudo-code.

The algorithm can be designed using iterative approach as follows:

```
Algorithm TriangleCounting(G)
count ← 0
for u ∈ Vertices(G)
   for v ∈ Neighbors(G, u)
      N1 ← Neighbor(G, u)
      N2 ← Neighbor(G, v)
      w1 ← N1.head.next
      w2 ← N2.head.next
      while w1! = N1.tail and w2! = N2.tail
         if w1.element < w2.element
            w1 ← w1.next
         else if w1.element > w2.element
            w2 ← w2.next
         else
            count ← count + 1
            w1 ← w1.next
            w2 ← w2.next
return count/6
```

The time complexity is $O(m \times d_{max}) = O(md_{max})$