CSCI2100D 2023-24: Assignment 3[*]

[#] This assignment is due at 11:59:59pm, 17th April 2024.

- **Q1. [12 marks]** Let $A[0 \cdots 7] = [1, 5, 6, 8, 3, 2, 7, 4]$.
  - **(i). [6 marks]** Show the process of **mergesort**(A, 0, 7) to sort A in ascending order step by step.
  - **(ii). [6 marks]** Assume that we call **quicksort**(A, 0, 7) to sort A in ascending order, and the pivot position we randomly choose is 1. Show how the partition works and indicate the value of **nextsmallpos** during the invocation of **partition**(A, 0, 7, 1) step by step.
- **Q2. [12 marks]** Sort array $A[1 \cdots 7] = [9, 1, 10, 3, 2, 8, 4]$ in decreasing order by heap sort. (you may just show the array representation at each step.)
  - **(i). [6 marks]** Show the contents of A in the heap adjust process to make it a min-heap step by step.
  - **(ii). [6 marks]** Using the min-heap of Part (i), show the contents of A in the sorting process of swaping elements in the array step by step.
- **Q3. [14 marks]** Assume that we have a hash table with size $m = 13$ and the hash function $h(k) = 2k\%13$. We use linear probing to address collisions. Answer the following questions.
  - **(i). [7 marks]** Given an empty hash table, show the hash table when inserting 10, 2, 23, 13, 1, 9, 17 in order step by step.
  - **(ii). [7 marks]** Given the following hash table, show the records examined when searching for 42.

0				4				8				12
0					22	16	10	30	24	42	25	

- **Q4. [14 marks]** Assume that we have a hash table with size $m = 13$ and we use double hashing to address collisions. The double hashing function is $h(k, i) = (h(k) + i \cdot h'(k))\%m$, where $h(k) = k\%13$ and $h'(k) = 1 + k\%3$. Answer the following questions.
  - **(i). [7 marks]** Given an empty hash table, show the hash table when inserting 14, 2, 18, 36, 31, 23, 42 in order step by step.
  - **(ii). [7 marks]** Given the following hash table, show the records examined when searching for 44.

0				4				8				12
		15		30	5			21		10	24	38

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- **Q5. [38 marks]** Consider the directed graph G_1 as shown in Fig. 1. Answer the following questions.

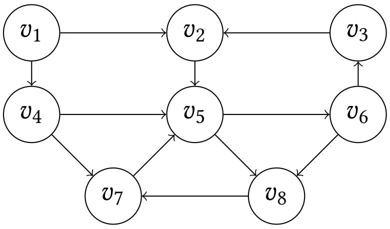

Fig. 1. Directed Graph G_1 for Q5

  - **(i). [2 mark]** Calculate the out-degree and the in-degree of v_5.
  - **(ii). [2 mark]** Whether the path $v_1 \longrightarrow v_4 \longrightarrow v_5 \longrightarrow v_6 \longrightarrow v_8 \longrightarrow v_7$ is a simple path? Justify your answer.
  - **(iii). [8 marks]** For G_1, show both its adjacency list representation and its adjacency matrix representation. (The nodes should be in ascending order of ID.)
  - **(iv). [8 marks]** Traverse G_1 using breadth-first search with v_1 as the source, assuming that the out-neighbors of a node are visited in ascending order of ID. Show the process and the contents of queue Q step by step. You may use 0 to denote the color to be white, 1 to denote the color to be gray, and 2 to denote the color to be black.
  - **(v). [8 marks]** According to the results of Part (iv), show the contents of **minlength** array and **prev** array respectively.
  - **(vi). [4 marks]** Show how to get the minimum length path from the source v_1 to v_7 using the **minlength** array and **prev** array. Justify your answer.
  - **(vii). [6 marks]** Draw the BFS tree of Part (iv).

- **Q6. [10 marks]** Given an undirected graph $G = (V, E)$, A triangle consists of three nodes in G that are pairwise adjacent. More formally, a triangle in G is triplet (u, v, w) where $u, v, w \in V$ and $(u, v), (v, w), (w, u) \in E$. Consider the undirected graph G_2 as shown in Fig. 2. There are two triangles in G_2, (v_0, v_1, v_2) and (v_1, v_2, v_3).

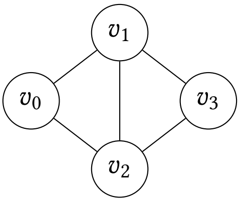

Fig. 2. An Undirected Graph G_2 for Q6

  - **(i). [4 marks]** Please design an algorithm TriangleCounting(G) using pseudo-code with the provided Graph ADT, to count the number of triangles in G.

    Graph **ADT**.
    - `Vertices(G)`: Lists all vertices u in G.
    - `Neighbors(G,u)`: Lists all vertices v such that there is an edge between the vertex u and the vertex v.
    - `Adjacent(G,u,v)`: Tests whether there is an edge between the vertex u and the vertex v.

  - **(ii). [2 marks]** Assume that G has n nodes and m edges, and the degree of any node u in G is smaller than d_{max}. What is the time complexity of TriangleCounting(G) expressed in terms of n, m, and d_{max} if the graph ADT is implemented using an adjacency matrix? Justify your answer.
  - **(iii). [2 marks]** Assume that G has n nodes and m edges, and the degree of any node u in G is smaller than d_{max}. What is the time complexity of TriangleCounting(G) expressed in terms of n, m, and d_{max} if the graph ADT is implemented using an adjacency list? Justify your answer.
  - **(iv). [2 marks]** Assume that the graph ADT is implemented using an adjacency list, and the output of Neighbors(G, u) is guaranteed to be sorted in ascending order by node ID. Please design a more efficient TriangleCounting(G) using pseudo-code.