STAT2005 Programming Languages for Statistics
Exercise for Chapter 5

1. (a) Risky has created a new non-negative integer sequence called Risky Sequence. Risky Sequence $\{R(n)\}$ is defined by

$$R(n) = \begin{cases} 1, & \text{for } n = 0 \text{ or } 1, \\ 4R(n-1) - 3R(n-2), & \text{for odd } n > 1, \\ 3R(n-1) - R(n-2), & \text{for even } n > 1. \end{cases}$$

Write function $R(n)$ to return the nth term in Risky Sequence by recursion.

(b) Padovan Sequence $\{P(n)\}$ is the an integer sequence defined by the recurrence relation

$$P(n) = P(n-2) + P(n-3),$$

with initial values $P(0) = P(1) = P(2) = 1$. Write function $P(n)$ to return the nth term in Padovan Sequence by recursion.

(c) Moser-de Bruijn sequence $\{S(n)\}$ is defined by the recurrence relation

$$S(2n) = 4S(n) \text{ and } S(2n+1) = 4S(n) + 1,$$

with initial values $S(0) = 0 \text{ } and \text{ } S(1) = 1$. Write function $S(n)$ to return the nth term in Moser-de Brujin Sequence by recursion.

2. (a) Company A has n tasks for employees every day. Each task has a corresponding number i, ranging from 1 to n. An employee taking part in task i can earn i dollars. Each employee is initially assigned with a task k, where $1 \leq k \leq n$.

To prevent employees from getting too tired, after taking part in task i, there is a probability of 0.2 and 0.3 to unlock the tasks $2i$ and $2i + 1$ respectively for an employee if the tasks exist. Note that the two events are independent. As the employees are enthusiastic in work, if a task is unlocked, they will take part in it.

Let users input n and k, write function $expected(x)$ to calculate the expected earning for an employee in a day by recursion and calculate the answer of $expected(k)$.

Here are some examples.

If $n = 5$ and $k = 1$, $expected(1) = 1 + 0.2 \times (2 + 0.2 \times 4 + 0.3 \times 5) + 0.3 \times 3 = 2.76$

If $n = 24$ and $k = 6$, $expected(6) = 6 + 0.2 \times (12 + 0.2 \times 24) + 0.3 \times 13 = 13.26$.

If $n = 100$ and $k = 3$, $expected(3) = 17.0811$

(b) Company B also has n tasks for employees every day. Each task has a corresponding number i, ranging from 1 to n. An employee taking part in task i can earn d_i dollars. Every task i has a stamina cost c_i, meaning that taking part in task i costs an employee c_i stamina. $d_1, d_2, \ldots, d_n$ are stored in vector d and $c_1, c_2, \ldots, c_n$ are stored in vector c.

An employee initially has $2n$ stamina. When the stamina becomes 0, the employee cannot work. If the remaining stamina is smaller than c_i, the employee cannot take part in task i. By exhausting every combination of tasks choices using recursion, write

`maxearn(d,c,remainingstamina)` to calculate the maximum possible amount of money an employee can earn.

For example, if $n = 10$, $d = \{1, 3, 7, 6, 8, 9, 2, 4, 4, 8\}$ and $c = \{5, 1, 2, 2, 4, 1, 3, 6, 3, 7\}$, an employee can choose task 2, 3, 4, 5, 6, 9, 10 to earn 45 dollars by using 20 stamina. You may use this example to test your solution.

3. Player 1 and Player 2 are playing a game. In this game, there are n toys on the table, where $n \geq 1$ is any positive integer. This game starts with player 1 and both players take turns to take action. For each action, a player can take 1 to min(5, # of remaining toys) toys away from the table. When a player takes the last toy away from the table, the player wins.

(a) Write R codes to let user input n with the prompt "`Enter the number of toys: `". If the user's input is not a positive integer, print "`Please enter a positive integer!`" and ask him to input again until it is correct.

(b) Write R codes to simulate the whole game.

Program Flow:

1. Input n as described in (a) and begin with player 1.
2. Ask the corresponding player to enter the number of toys to be taken away from the table with prompt "`The current number of toys on the table is i. Player j, please enter the number of toys to be taken away (between 1 to k): `", where i should be the current number of toys on the table, j should be either 1 or 2 depending on which player's turn it is, k is $min(5, i)$. When the player enters an invalid number, the program shall display a warning message: "`Invalid input! `". This phase finishes when the player inputs correctly.
3. Repeat Step 2 until there is no toys left on the table.
4. Display the messages "`Player 1 wins!`" or "`Player 2 wins!`" accordingly.