STAT3005 Assignment 4
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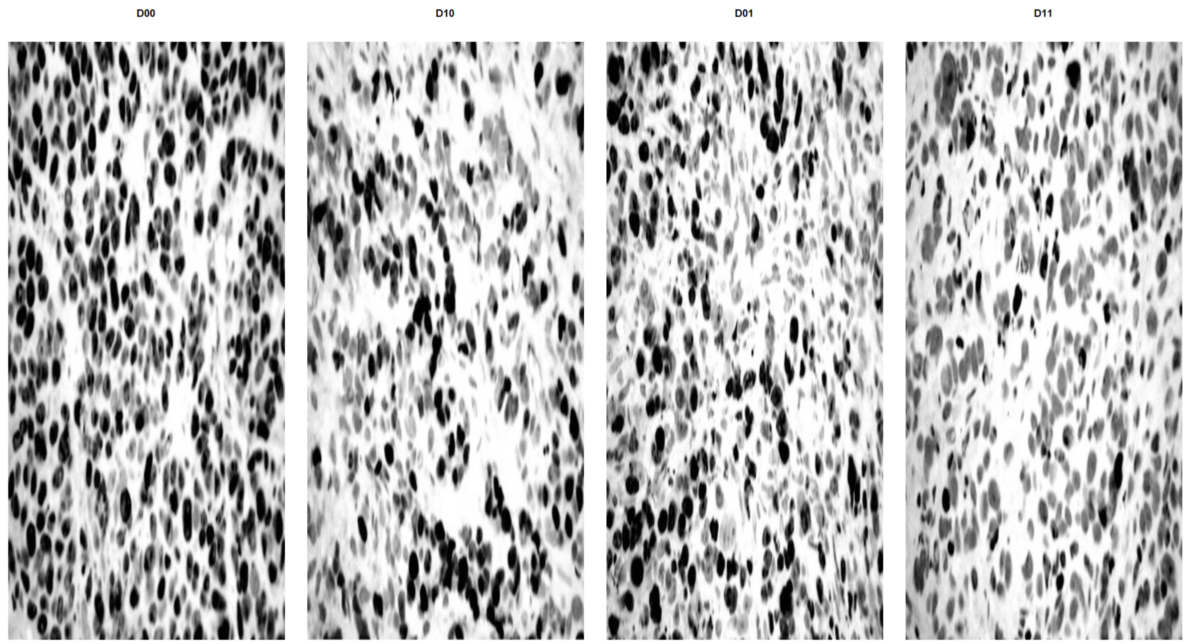
Exercise 4.1.1

The grayscale images can be produced using the following R code:

```r
img = rep(list(NA),4)
img[[1]] = jpeg::readJPEG("D00.jpg")
img[[2]] = jpeg::readJPEG("D10.jpg")
img[[3]] = jpeg::readJPEG("D01.jpg")
img[[4]] = jpeg::readJPEG("D11.jpg")

# Plots of color images
par(mfrow=c(1,4),mar=c(0,0,1,0))
for(i in 1:4){
   plot(0:1,0:1,cex=0,axes=FALSE,main=c("D00","D10","D01","D11")[i])
   rasterImage(img[[i]],0,0,1,1)
}

# Produce grayscale images
get.gray = function(img, w=c(0.22,0.71,0.07)){
   img[,,1]*w[1] + img[,,2]*w[2] + img[,,3]*w[3]
}
img0 = rep(list(NA),4)
for(i in 1:4){
   img0[[i]] = get.gray(img[[i]])
}

# Plot of grayscale images
par(mfrow=c(1,4),mar=c(0,0,1,0))
for(i in 1:4){
   plot(0:1,0:1,cex=0,axes=FALSE,main=c("D00","D10","D01","D11")[i])
   rasterImage(img0[[i]], 0, 0, 1, 1)
}
```

Exercise 4.1.1 (Cont'd)

The following plot will be produced:

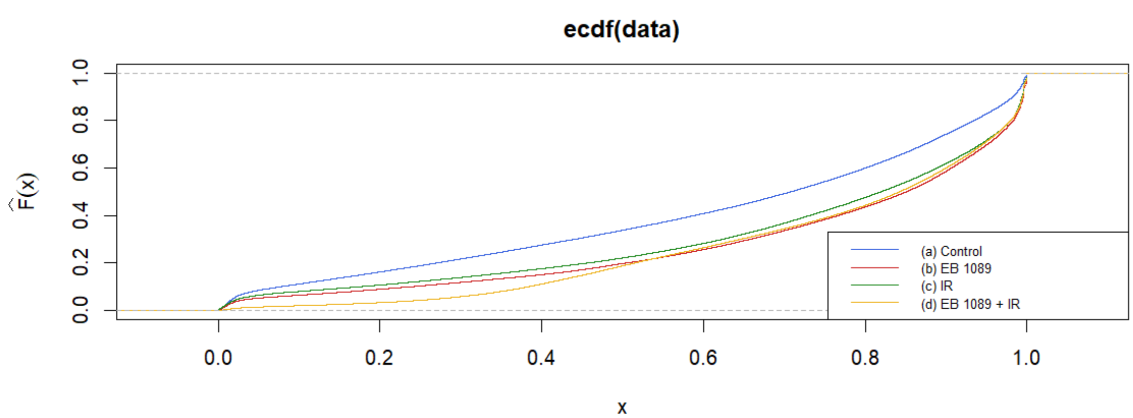


Exercise 4.1.2

The empirical cdfs of the grayscale pixel values can be produced using the following R code:

```
col = c("royalblue","firebrick3","forestgreen","goldenrod2")
par(mfrow=c(1,1),mar=c(4.5,5,3,2))
for(j in 1:4){
   data = c(img0[[j]])
   plot(ecdf(data),
      col=col[j],add=j>1,ylab=expression(widehat(F)(x)),xlab=expression(x))
}
treatment = c("(a) Control","(b) EB 1089","(c) IR","(d) EB 1089 + IR")
legend("bottomright",treatment,col=col,lwd=1,bg="white", cex=0.65)
```

The following plot will be produced:



Exercise 4.1.3

The null hypothesis is

$$H_0 : F_B(\cdot) = F_C(\cdot)$$

and the alternative hypothesis is

$$H_1 : F_B(\cdot) \neq F_C(\cdot)$$

Exercise 4.1.4

The hypotheses can be tested using the following R code:

```
treatment_b = c(img0[[2]])
treatment_c = c(img0[[3]])
ks.test(treatment_b, treatment_c)$p.value # p-value < 2.2e-16
```

As the computed p-value $\approx 0 < \alpha = 0.05$, we reject H_0. So, it is concluded that the cancer treatments (b) EB 1089 and (c) IR have different effects on killing the cancer cells.

Exercise 4.2.1

$$\begin{aligned}
\mathrm{P}(F_A(A) < u) &= \mathrm{P}(A \leq F_A^{-1}(u)) \\
&= F_A(F_A^{-1}(u)) \\
&= u \\
\therefore F_A(A) &= U \sim \mathrm{Unif}(0,1) \\
A &= F_A^{-1}(U)
\end{aligned}$$

Simiarly,

$$\begin{aligned}
\mathrm{P}(F_D(D) < u) &= \mathrm{P}(D \leq F_D^{-1}(v)) \\
&= F_D(F_D^{-1}(v)) \\
&= v \\
\therefore F_D(D) &= V \sim \mathrm{Unif}(0,1) \\
D &= F_D^{-1}(V)
\end{aligned}$$

Exercise 4.2.2

We try to assume the contrary:

$$\begin{aligned}
F_A^{-1}(U) &\geq F_D^{-1}(U) \\
\therefore F_A(F_A^{-1}(U)) &\geq F_A(F_D^{-1}(U)) \text{ as } F_A(\cdot) \text{ is a strictly increasing function} \\
U &\geq F_A(F_D^{-1}(U))
\end{aligned}$$

Besides, the question suppose that

$$\begin{aligned}
F_A(t) &> F_A(t) \\
\therefore F_A(F_D^{-1}(U)) &> F_D(F_D^{-1}(U)) \\
F_A(F_D^{-1}(U)) &> U
\end{aligned}$$

The above inequalities will lead to contradiction:

$$U \geq F_A(F_D^{-1}(U)) > U$$

Thus, the contrary assumed is not true, therefore:

$$F_A^{-1}(U) < F_D^{-1}(U)$$

Exercise 4.2.3

$$\begin{aligned}
F_A^{-1}(U) &< F_D^{-1}(U) \\
\int_0^1 F_A^{-1}(U)\,du &< \int_0^1 F_D^{-1}(U)\,du \\
\mathrm{E}(A) &= \mathrm{E}(D)
\end{aligned}$$

Exercise 4.2.4

The hypotheses can be tested using the following R code:

```
A = c(img0[[1]])        # the pixel value of image corresponding to treatment A
D = c(img0[[4]])        # the pixel value of image corresponding to treatment D
ks.test(A, D, alternative="greater")$p.value  # p-value < 2.2e-16
```

As the computed p-value $\approx 0 < \alpha = 0.05$, we reject H_0. So, it is concluded that treatment D is more effective in killing cancer cells than treatment A.