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CSC 389

Assignment #7

1. (Reading) Done.
2. 2SAT = {φ:φ is in 2CNF and φ is satisfiable} lies in P

   Similar to what was discussed in class when explaining that $NP \subseteq EXP$ and $P \subseteq NP$ the reason that $NP \subseteq EXP$ is (with an example)

   Example: $L = 3SAT$

   Then $|c| \leq |x|^k + k$ for some k

   We simply try all possible c, and **<u>there are $2^{|x|^k+k}$ many</u>**, and run the verifier on each. If any one of them succeeds, accept, else reject. This is in time $0(2^{n^k})$ for some k. $0(2^{n^k})$ is exponential because we are trying all the possible c for at most 3 different variables/literals within each clause.

   Because $2SAT$ only contains at most 2 variables/literals within each clause then trying all possible c there would only be one choice that is made when deciding for a clause that is either $a \vee b$ or $a \vee \bar{b}$ or the complements of those.

   Ex: A clause of the form $a \rightarrow b$ is $a \vee \bar{b}$ and their complements will always be expressed (since it is in CNF) and in this event if you set $a = T$ then $b = T$ and is the same if $a = F$ then $b = F$.

   So for $2SAT$ we can try all possible c in polynomial time which is in P.

3.
   a) Double-SAT = {ψ: ψ has at least two satisfying truth assignments}
      (i) Double-SAT is in NP the same way as $SAT, 3SAT \in NP$. With Double-SAT we can also have a similar proof. We have a certificate c where we assign two truth assignment to variables and we also have a verifier that checks that the formula is True with both assignments.
      (ii) To prove that Double-SAT is NP-complete we can reduce from SAT.
      We start with a TM M = On input ψ :
         1. Introduce a new variable x.
         2. Let $\bar{\psi}$ be $\psi \vee (x \wedge \bar{x})$ and output $\bar{\psi}$.

      If $\psi \in SAT$ then ψ has at least one satisfying Truth assignment and $\bar{\psi}$ will have at least two satisfying Truth assignments because we can change the variable of x in $(x \wedge \bar{x})$. If $\bar{\psi} \in$ Double-SAT then $\psi \in$ SAT because x is not in ψ.

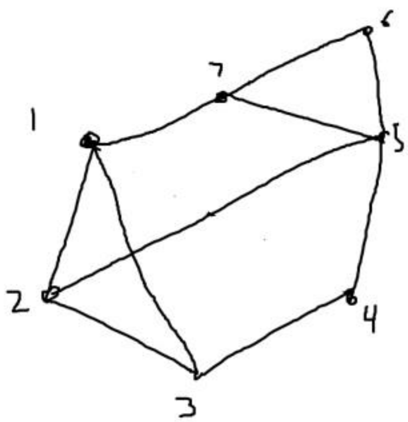
      $\psi \in$ SAT, iff $M(\psi) \in$ Double-SAT

b) WELL_POS = $\{\langle G, k\rangle$: G contains a set of k well-positioned vertices$\}$

(i) WELL_POS is in NP.

Ex:

Graph G:



And the set $A = \{3,7,4\}$ is well positioned because all vertex in the graph are at least distance 1 from the vertex in the set.

We can have a certificate c where we assign k vertices to a set A and then verify whether the rest of the vertices in the graph are at least one edge away from the vertices in our set.

(ii) Construct a graph G and a set A such that ψ is satisfiable iff G has a vertex set A of size k that are well positioned in the graph.

$\psi = (p \vee \bar{q} \vee \bar{r}) \wedge (\bar{p} \vee q \vee s) \wedge (\bar{p} \vee r) \wedge (\bar{r} \vee \bar{s})$

- Create a vertex for each literal in each clause
- Add an edge between two vertices if they come from different clauses and they are not contradictory.
- Let $k =$ # of literal pairs

ψ is satisfiable if and only if G contains a set of size k