THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Decision Trees

Lecture 6

STA 371G

Payoff Table

Let's return to the simple example from last time, and see a different way of expressing the problem situation.

	O1	O2	O3
D1	\$10	\$10	\$10
D2	−\$10	\$20	\$30
D3	−\$30	\$30	\$80

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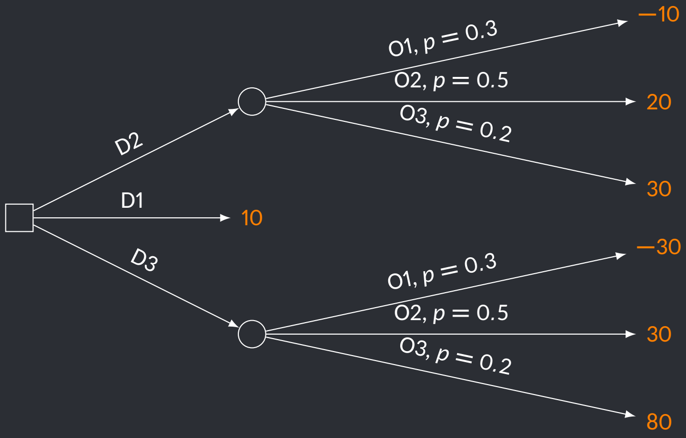
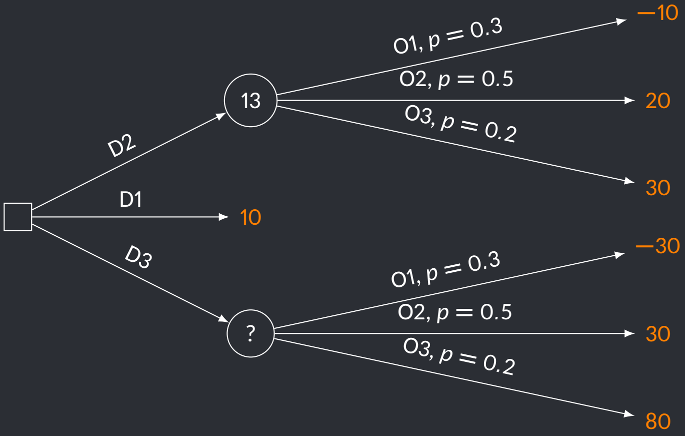
- Monetary values are shown to the right of the leaf (end) nodes.
- Expected monetary values are calculated through a "rolling-back" process.

Example


D2
D1
D3
10
O1, $p = 0.3$
O2, $p = 0.5$
O3, $p = 0.2$
-10
20
30
O1, $p = 0.3$
O2, $p = 0.5$
O3, $p = 0.2$
-30
30
80

Rolling back: Step 1

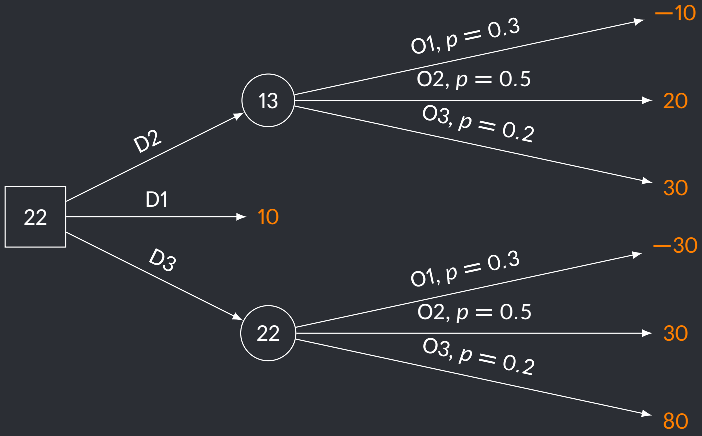
Calculate the expected value at each probability node:



$$E(D1) = .3(-10) + .5(20) + .2(30) = 13$$

Rolling back: Step 2

Calculate the maximum at each decision node:



Take decision D3 since $22 = \max(10, 13, 22)$.

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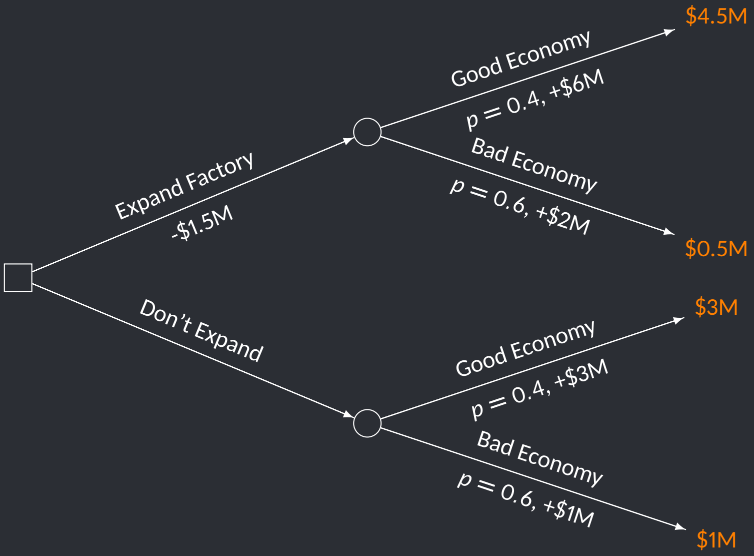
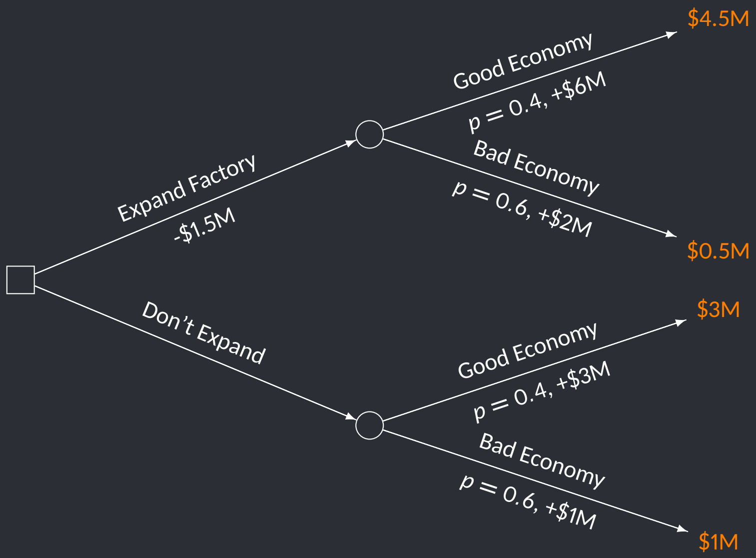
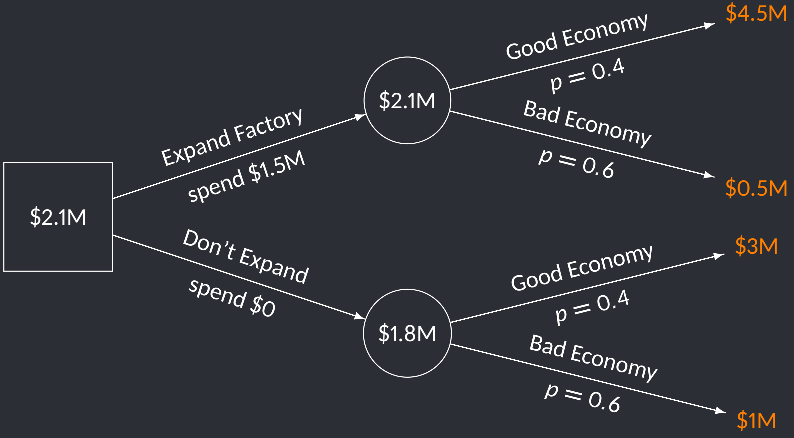
Should they expand the factory or not?


Expand Factory
-$1.5M
Good Economy
p = 0.4, +$6M
$4.5M
Bad Economy
p = 0.6, +$2M
$0.5M
Don't Expand
Good Economy
p = 0.4, +$3M
$3M
Bad Economy
p = 0.6, +$1M
$1M



$E(\text{expand}) = .4(4.5) + .6(0.5) = 2.1$

$E(\text{don't expand}) = .4(3) + .6(1) = 1.8$



$E(\text{expand}) = .4(4.5) + .6(0.5) = 2.1$

$E(\text{don't expand}) = .4(3) + .6(1) = 1.8$

Since \$2.1M > \$1.8M, they should expand!

Sequential decisions

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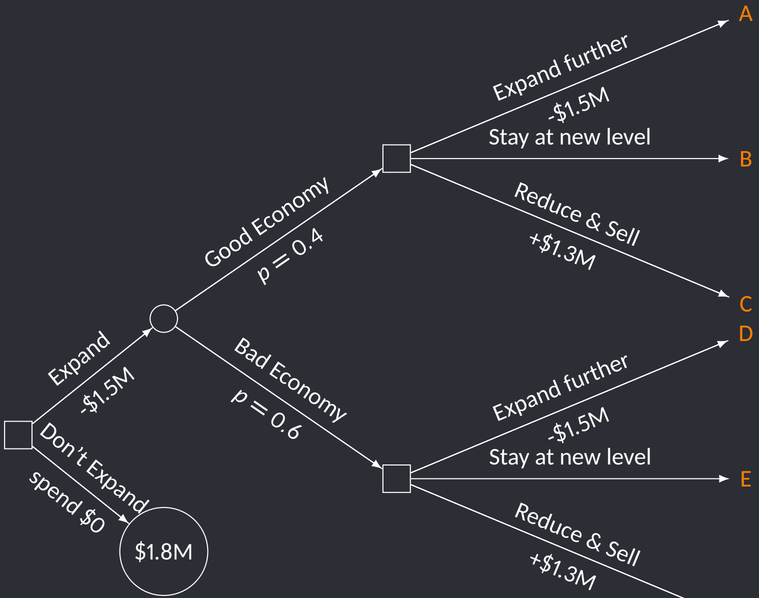
The company later learns that if they expand, they can opt to either:

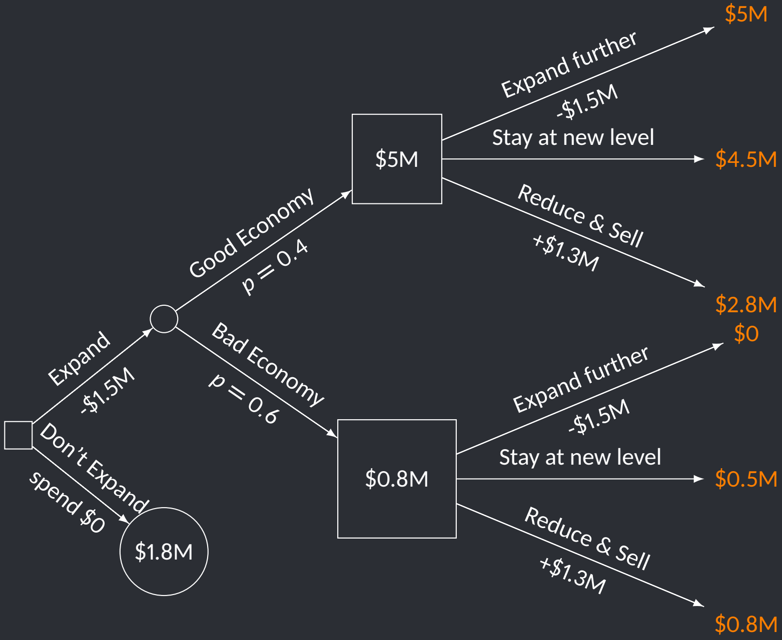
1. expand the factory further, which costs \$1.5M and will yield an extra \$2M in profit if the economy is good, but \$1M if it is bad,
2. abandon the project and sell the equipment they originally bought, for \$1.3M, or
3. do nothing.

How has the decision changed?

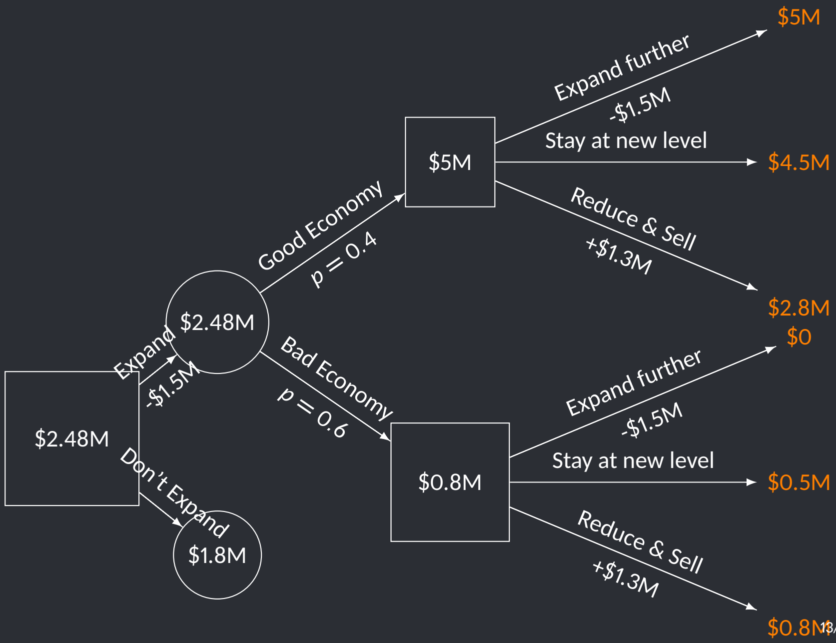


Expand
-$1.5M
Don't Expand
spend $0
$1.8M
Good Economy
p = 0.4
Bad Economy
p = 0.6
Expand further
-$1.5M
Stay at new level
Reduce & Sell
+$1.3M
Expand further
-$1.5M
Stay at new level
Reduce & Sell
+$1.3M
A
B
C
D
E
F

$5M
Expand further
-$1.5M
Stay at new level
$4.5M
Reduce & Sell
+$1.3M
$2.8M
Good Economy
$p = 0.4$
Expand
-$1.5M
Bad Economy
$p = 0.6$
$0
Expand further
-$1.5M
Stay at new level
$0.5M
Reduce & Sell
+$1.3M
$0.8M
Don't Expand
spend $0
$1.8M



$5M
Expand further
-$1.5M
Stay at new level
$5M
$4.5M
Reduce & Sell
+$1.3M
Good Economy
$p = 0.4$
$2.8M
$0
Expand
-$1.5M
Bad Economy
$p = 0.6$
Expand further
-$1.5M
Stay at new level
$0.8M
$0.5M
Don't Expand
spend $0
Reduce & Sell
+$1.3M
$1.8M
$0.8M


$2.48M
Expand
-$1.5M
Don't Expand
$2.48M
$1.8M
Good Economy
$p = 0.4$
Bad Economy
$p = 0.6$
$5M
$0.8M
Expand further
-$1.5M
Stay at new level
Reduce & Sell
+$1.3M
$5M
$4.5M
$2.8M
Expand further
-$1.5M
Stay at new level
Reduce & Sell
+$1.3M
$0
$0.5M
$0.8M

Expected value of the option

The EV of expanding is now

$$(.4(5) + .6(0.8)) = 2.48.$$

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Therefore the value of the option is

$$E(\text{new}) - E(\text{old}) = 2.48 - 2.1 = 0.38,$$

or $380,000.

Does this look familiar?

- This method of valuing a real option is used by corporate strategists, management consultants, and bankers.

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- This method of valuing a real option is used by corporate strategists, management consultants, and bankers.
- A further refinement is to incorporate time into the analysis (e.g. how much is $1M worth today vs in the future).