

# Problems with $p$ -values

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Lecture 21

STA 371G

## A quick refresher on *p*-values

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- For example, in marketing, we might A/B test two potential web online store designs.
- We randomly show design A (the existing design) or B (the new design) to customers, and then track how much money they spend.
- A t-test would be used to test the null hypothesis that the average amount spent with design A is the same as the amount spent with design B.
- If we find  $p < .05$  (and that the new design results in significantly higher revenue than the old design) then we redo our store with the new design.

The hypotheses we are testing:

$$H_0 : \mu_A = \mu_B \quad \text{vs} \quad H_A : \mu_A < \mu_B$$

And:

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So a small  $p$ -value means that null hypothesis  $H_0$  is not a tenable assumption—we'll reject it and assume instead that the alternative hypothesis  $H_A$  is true.

Remember: a  $p$ -value is a **conditional probability** that represents how likely it is that we would see a difference as extreme as the results we saw in the sample, under the assumption that  $H_0$  is true.

As a result, it is sensitive to:

- **Sample size:** Seeing the same difference in a larger sample will tend to result in a smaller  $p$ -value.
- **Sampling variation:** More variation in the population will tend to result in a larger  $p$ -value.

1. The consequence of running too many hypothesis tests
2. Changing your analysis strategy based on a hypothesis test
3. Stopping an experiment based on a hypothesis test
4. Survivorship bias
5. Practical versus statistical significance

# ENTRANCE To MYSTERY SPOT

OPEN

PLEASE SMOKE

50¢

SANTA

CHEESEBURGER

## How reliable is $R^2$ ?

- The mystery data set contains 20 predictor variables X1-X20.

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.0048932	0.0315499	0.155	0.87678
X1	-0.0242680	0.0311808	-0.778	0.43658
X2	0.0094610	0.0325268	0.291	0.77121
X3	-0.0164687	0.0316452	-0.520	0.60289
X4	0.0332419	0.0315227	1.055	0.29190
X5	-0.0003434	0.0320555	-0.011	0.99146
X6	-0.0235806	0.0311449	-0.757	0.44916
X7	0.0239592	0.0314237	0.762	0.44597
X8	-0.0153691	0.0308775	-0.498	0.61878
X9	0.0188475	0.0301545	0.625	0.53210
X10	0.0816945	0.0310096	2.634	0.00856 **
X11	0.0300185	0.0314272	0.955	0.33972
X12	-0.0189656	0.0307947	-0.616	0.53812
X13	-0.0657305	0.0311022	-2.113	0.03482 *
X14	0.0076949	0.0318495	0.242	0.80914
X15	0.0227637	0.0316354	0.720	0.47197
X16	0.0689893	0.0332165	2.077	0.03807 *
X17	-0.0239150	0.0312032	-0.766	0.44361
X18	-0.0002470	0.0316023	-0.008	0.99376
X19	-0.0502899	0.0305908	-1.644	0.10051
X20	-0.0628928	0.0325095	-1.935	0.05333 .

```
parsimonious.model <- lm(Y ~ X10 + X13 + X16, data=mystery)
summary(parsimonious.model)
```

Call:

```
lm(formula = Y ~ X10 + X13 + X16, data = mystery)
```

Residuals:

Min	1Q	Median	3Q	Max
-2.5839	-0.6636	-0.0255	0.6312	3.5081

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.006384	0.031188	0.205	0.8379
X10	0.074640	0.030694	2.432	0.0152 *
X13	-0.065601	0.030809	-2.129	0.0335 *
X16	0.071064	0.032880	2.161	0.0309 *
---				
Signif. codes:	0 ****	0.001 ***	0.01 **	0.05 *
	.	.	.	1

Residual standard error: 0.9857 on 996 degrees of freedom

Multiple R-squared: 0.01434, Adjusted R-squared: 0.01137

F-statistic: 4.829 on 3 and 996 DF, p-value: 0.00242

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- If we have 20 predictor variables, we are running 20 hypothesis tests, and we'd expect to see one of them be significant, just by chance.
- That means that when you are building a model from a large data set with many variables, you should expect to see some significant coefficients no matter what.

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- Maybe it would be significant for customers in Oklahoma?
  - Yes— $p = 0.03!$
- So it must be true that there is a significant gender difference in propensity to buy among customers in Oklahoma...
- ...or, you just got a false positive by running a lot of tests!

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- In some sense, this is a natural part of the data analysis process—you don't necessarily know what effects are out there unless you look for them.
- But running too many tests can result in false positives!



# Hack Your Way To Scientific Glory



You're a social scientist with a hunch: **The U.S. economy is affected by whether Republicans or Democrats are in office.** Try to show that a connection exists, using real data going back to 1948. For your results to be publishable in an academic journal, you'll need to prove that they are "statistically significant" by achieving a low enough p-value.

## 1 CHOOSE A POLITICAL PARTY

Republicans

Democrats

## 2 DEFINE TERMS

Which politicians do you want to include?

- Presidents
- Governors
- Senators
- Representatives

How do you want to measure economic performance?

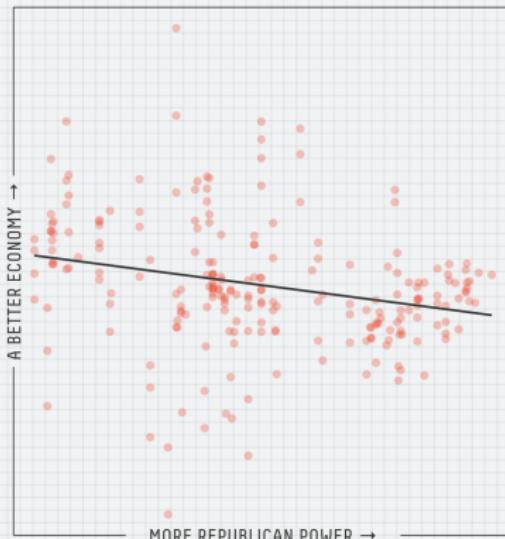
- Employment
- Inflation
- GDP
- Stock prices

Other options

- Factor in power  
Weigh more powerful positions more heavily
- Exclude recessions  
Don't include economic recessions

## 3 IS THERE A RELATIONSHIP?

Given how you've defined your terms, does the economy do better, worse or about the same when more Republicans are in power? Each dot below represents one month of data.



## 4 IS YOUR RESULT SIGNIFICANT?

If there were no connection between the economy and politics, what is the probability that you'd get results at least as strong as yours? That probability is your p-value, and by convention, you need a **p-value of 0.05 or less** to get published.



## Result: Publishable

You achieved a p-value of **less than 0.01** and showed that **Republicans have a negative effect on the economy**. Get ready to be published!

If you're interested in reading real (and more rigorous) studies on the connection between politics and the economy, see the work of Larry Bartels and Alan Blinder and Mark Watson.

Data from The @unitedstates Project, National Governors Association, Bureau of Labor Statistics, Federal Reserve Bank of St. Louis and Yahoo Finance.

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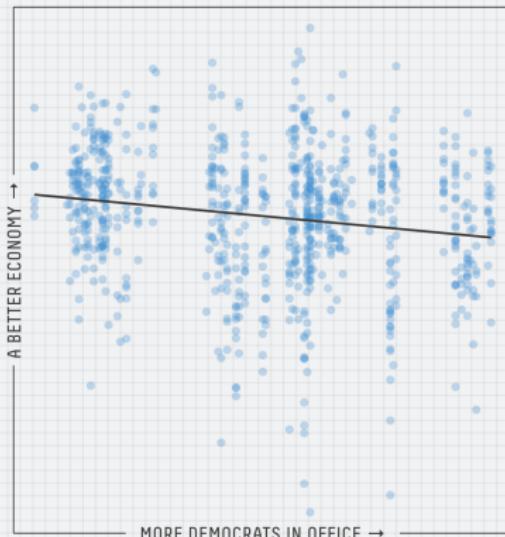
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# Feeling the Future: Experimental Evidence for Anomalous Retroactive Influences on Cognition and Affect

Daryl J. Bem  
Cornell University

The term *psi* denotes anomalous processes of information or energy transfer that are currently unexplained in terms of known physical or biological mechanisms. Two variants of psi are *precognition* (conscious cognitive awareness) and *premonition* (affective apprehension) of a future event that could not otherwise be anticipated through any known inferential process. Precognition and premonition are themselves special cases of a more general phenomenon: the anomalous retroactive influence of some future event on an individual's current responses, whether those responses are conscious or nonconscious, cognitive or affective. This article reports 9 experiments, involving more than 1,000 participants, that test for retroactive influence by "time-reversing" well-established psychological effects so that the individual's responses are obtained before the putatively causal stimulus events occur. Data are presented for 4 time-reversed effects: precognitive approach to erotic stimuli and precognitive avoidance of negative stimuli; retroactive priming; retroactive habituation; and retroactive facilitation of recall. The mean effect size ( $d$ ) in psi performance across all 9 experiments was 0.22, and all but one of the experiments yielded statistically significant results. The individual-difference variable of stimulus seeking, a component of extraversion, was significantly correlated with psi performance in 5 of the experiments, with participants who scored above the midpoint on a scale of stimulus seeking achieving a mean effect size of 0.43. Skepticism about psi, issues of replication, and theories of psi are also discussed.

**Keywords:** psi, parapsychology, ESP, precognition, retrocausation

## What did the Bem study do?

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- A typical example: participants see two images and pick one; the computer then randomly picks one of the images as the “correct” image, and the participant sees a “positive” image as a reward (or a “negative” image as punishment) if they happened to select the right one in advance.
- Participants picked the right image 51.7% of the time ( $t = 2.39$ ,  $p = .009$ )

## How did he prove that ESP exists?!

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- Bem reports on 9 experiments that came out with  $p < .05$ , but he doesn't report on all of the failed attempts! This is called the **file-drawer problem**.
- Many researchers later tried to replicate Bem's results, and failed.

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## Running A/B testing until we get significance

- Consider a similar situation: we run an A/B test where a customer is presented with two different offers.

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## Running A/B testing until we get significance

- Consider a similar situation: we run an A/B test where a customer is presented with two different offers.
- After each customer, we test the null hypothesis that the proportion of customers that prefer offer A is the same as the proportion that prefer offer B.
- We try this with up to 10,000 customers, but stop if we get a  $p$ -value less than .05.

Let's simulate a situation where the customers in the population are just selecting randomly between offers A and B.

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In theory, the proportion of customers choosing offer A should stay near 0.5 and we should get through all 10,000 customers with  $p > .05$  each time.

The pink line shows the  $p$ -value after each customer, and the blue line shows the proportion of customers that chose offer A.



- Running the hypothesis test after each customer provides too many opportunities for false positives.
- Better: decide in advance how long you'll run the test.

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# This is how many fund managers actually beat index funds

Published: May 13, 2017 10:46 a.m. ET



Aa

Roughly 1 in 20 actively managed domestic funds beat index funds



## MOST POPULAR



Baby boomers commit the '7 deadly sins' of retirement planning



Move over, Ferrari and Lamborghini — this Italian electric car's 1,900 horsepower makes it faster than a Formula 1 racer



My dying father gave his girlfriend his debit card to buy food — and she

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- Some managed funds do outperform the S&P 500—but the high performers are **different** every year.
- Why are there no funds that consistently outperform the market?

## Survivorship bias

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- The worst performers are the ones most likely to be shut down every year.
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- This phenomenon is called **survivorship bias**: the survivors are not necessarily representative of the whole.
- Survivorship bias is why we should be careful about trying to emulate the characteristics of the most successful people or companies!

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- Design B results in a mean of \$41 spent, with an SD of \$20.
- A one-sample t-test of the null hypothesis  $\mu_A = \mu_B$  against  $\mu_A < \mu_B$  gives a  $p$ -value of 0.0002.
- So we should redesign the store with design B, right?

But:

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- What if there is another design that would cost just as much to implement but would deliver a much higher ROI?

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- A small  $p$ -value means that we can be confident that designs A and B do not result in exactly the same revenue per user—is the difference **statistically significant**...
- ...but that’s not answer to the question we really have: is the difference large enough for us to care about—is the difference **practically significant**?

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- Design B might be statistically significantly better than design A, but the online store redesign might cost millions to execute.
- One fund might outperform the market in a statistically significant way, but the difference might be partially (or completely!) offset by the fund’s fees.