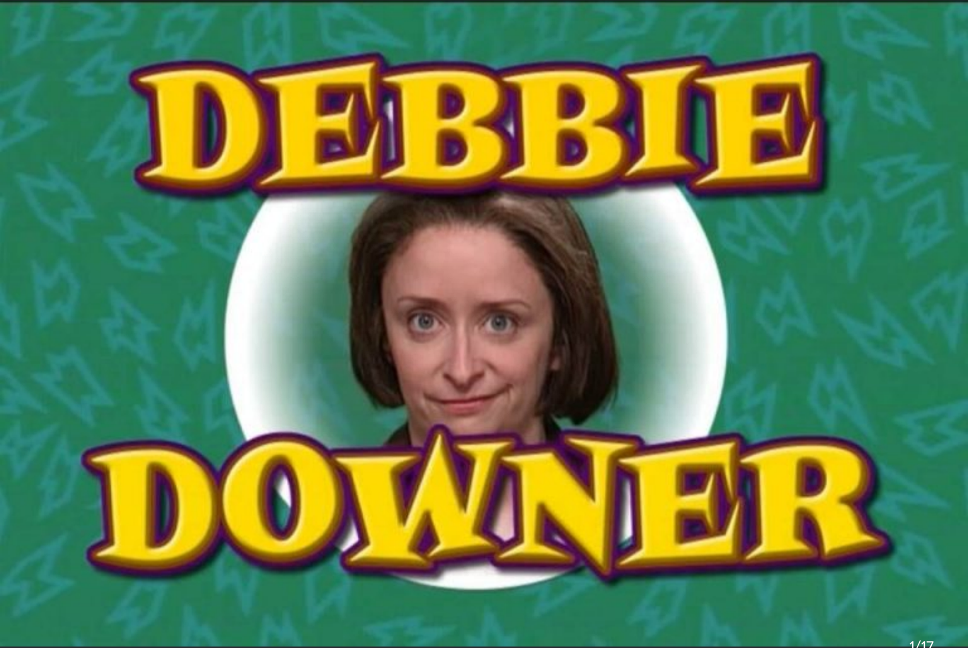
THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

What can go wrong, and how to fix it 2

Lecture 14

STA 371G

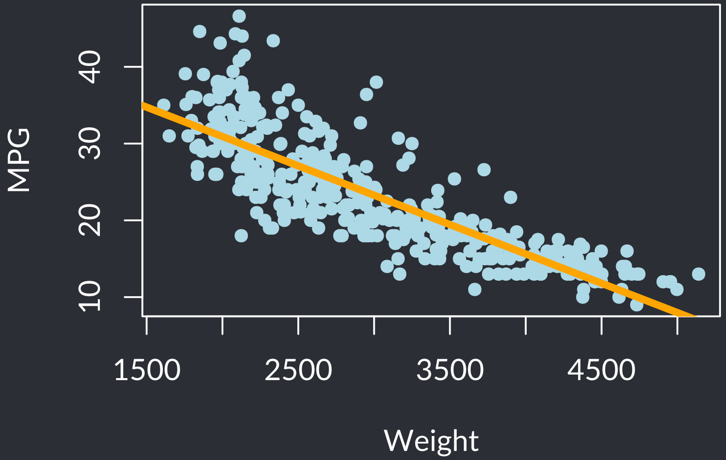

DEBBIE
DOWNER

Mileage efficiency data set

The data set `cars` contains specs for 392 different cars. We'll focus on two variables:

- **MPG** is fuel efficiency, measured in miles per gallon
- **Weight** is the weight of the car, in pounds

What problems do you see here?

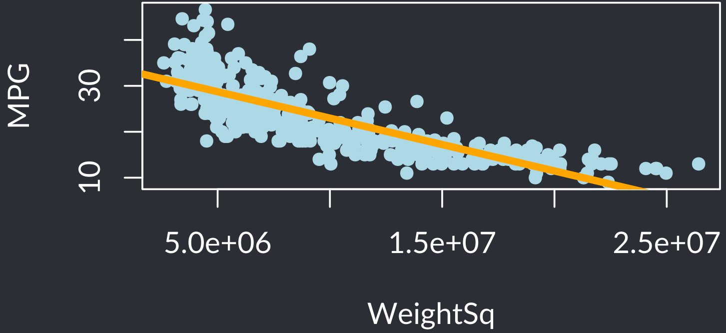


Using transformations to fix problems

- Sometimes, a violation of regression assumptions can be fixed by transforming one or the other of the variables (or both).
- When we transform a variable, we have to also transform our interpretation of the equation.

A bad example

```
cars$WeightSq <- cars$Weight^2
plot(MPG ~ WeightSq, data=cars, pch=16, col="lightblue")
sq.model <- lm(MPG ~ WeightSq, data=cars)
abline(sq.model, col="orange", lwd=4)
```



The log transformation

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LOG!

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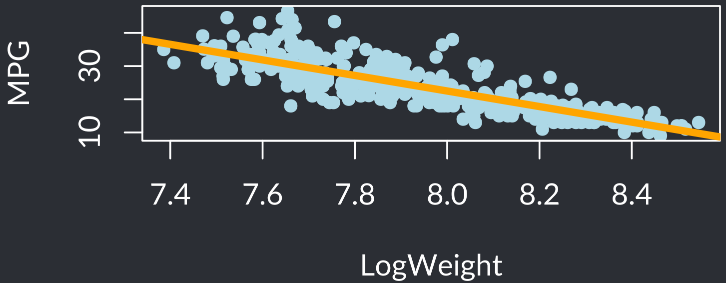
- $\log_b x = y$ when $b^y = x$
- For example, $\log_{10} 1000 = 3$, $\log_{10} 100 = 2$, and $\log_{10} 10 = 1$
- The natural log is $\log_e$, where $e \approx 2.72$ — when we say "log" we will usually mean "natural log" (although for our purposes the base doesn't matter)

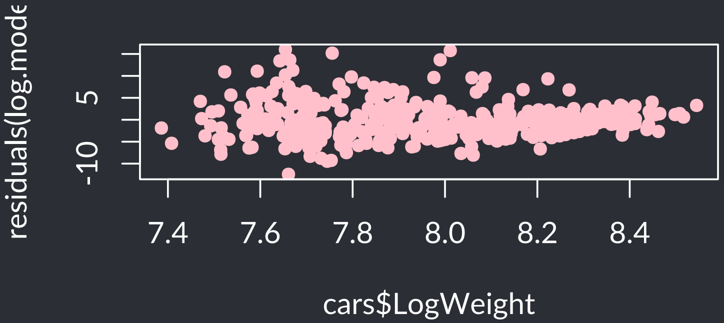
Applying a log transformation

```
cars$LogWeight <- log(cars$Weight)
plot(MPG ~ LogWeight, data=cars, pch=16, col="lightblue")
log.model <- lm(MPG ~ LogWeight, data=cars)
abline(log.model, col="orange", lwd=4)
```



Checking assumptions of our new model

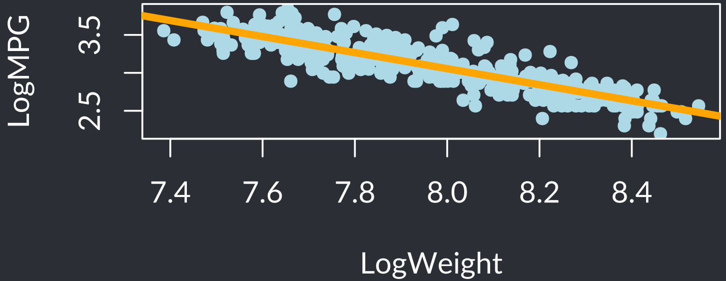
```
plot(cars$LogWeight, residuals(log.model), pch=16, col="pink")
```



Linearity looks good, but homoscedasticity is still not satisfied!

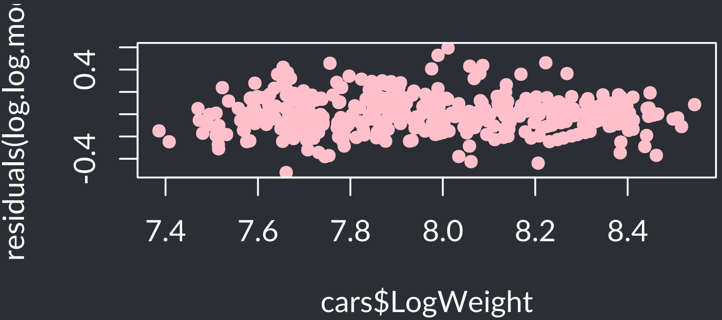
Applying a second log transformation

```
cars$LogMPG <- log(cars$MPG)
plot(LogMPG ~ LogWeight, data=cars, pch=16, col="lightblue")
log.log.model <- lm(LogMPG ~ LogWeight, data=cars)
abline(log.log.model, col="orange", lwd=4)
```



Checking assumptions of the log-log model

```
plot(cars$LogWeight, residuals(log.log.model), pch=16, col="pink"
```

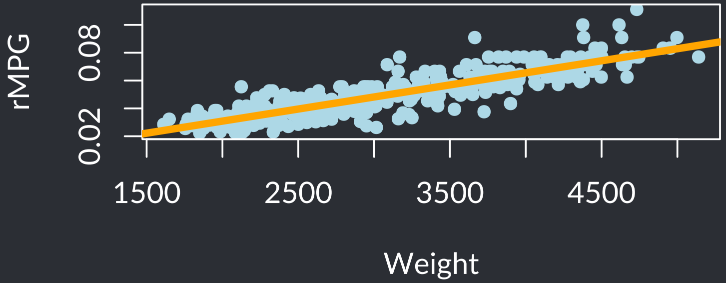


Much better—transforming MPG to log(MPG) gives us both linearity and homoscedasticity!

Another way to transform

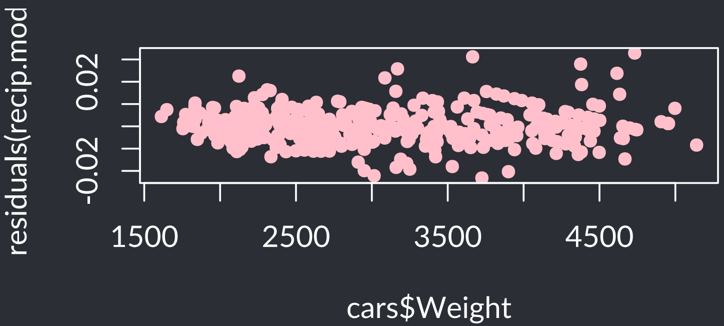
- Instead of transforming using logs, we can also transform using the reciprocal ($x \longrightarrow 1/x$)
- Let's transform Y by making MPG $\longrightarrow$ 1/MPG

```r
cars$rMPG <- 1/(cars$MPG)
plot(rMPG ~ Weight, data=cars, pch=16, col="lightblue")
recip.model <- lm(rMPG ~ Weight, data=cars)
abline(recip.model, col="orange", lwd=4)
```



Checking assumptions of the reciprocal model

```
plot(cars$Weight, residuals(recip.model), pch=16, col="pink")
```



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- There is also a conceptual reason to prefer the reciprocal model: MPG is already a ratio, which tips us off that a reciprocal transformation would be appropriate
- The reciprocal model is also simpler to interpret since only one of the variables is transformed
- In general, log transformations tend to work best when the spacing between values increases as the values increase (e.g., salaries, city population sizes)

Thinking about transformations

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- You might need to transform both X and Y; if so, start by transforming Y to address the heteroscedasticity, and then transform X to address nonlinearity if necessary.
- It's OK to do a little trial and error!