THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Probability 2

Lecture 3

STA 371G

Announcements

1. Conditional probabilities

2. Probability trees

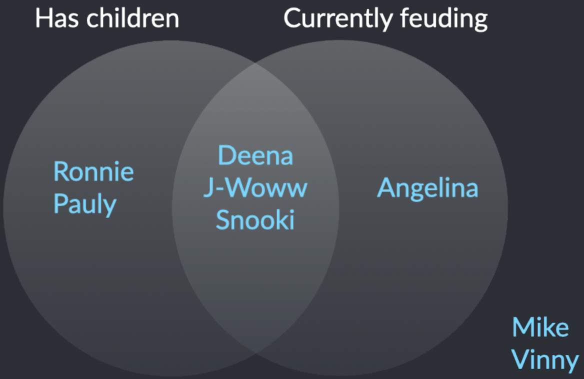
Conditional probability

Definition

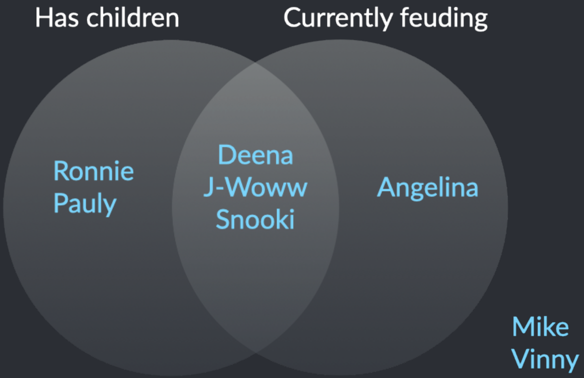
The conditional probability $P(B|A)$ is the probability of B happening if we already know that A has occurred.

Read $P(B|A)$ out loud as “the probability of B given A.”


"A ROLLICKING COMEDY."
-JOHN ANDERSON, VARIETY
WRITTEN, DIRECTED BY AND STARRING LAKE BELL
IN A WORLD...
...where we know A to be true
WINNER
Best Screenplay
SUNDANCE
Film Festival



$$P(C|F) = \frac{\text{Deena, J-Woww, Snooki}}{\text{Deena, J-Woww, Snooki, Angelina}} = \frac{3}{4}$$
$$= \frac{P(C \text{ and } F)}{P(F)} = \frac{3/8}{4/8}$$



$P(C|F) \neq P(F|C)$ — they mean two different things:

- $P(C|F)$ is the proportion of feuding cast members that have children
- $P(F|C)$ is the proportion of cast members with children that are also feuding

Probability rules

1. The chance of an event happening is between 0% and 100%, i.e. $0 \leq P(E) \leq 1$ for any event E.
2. The probabilities for all possible outcomes put together add up to 1.
3. The probability that something doesn't happen is 100% minus the probability that it does happen, i.e. $P(E^c) = 1 - P(E)$.
4. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
5. $P(A \text{ and } B) = P(A)P(B|A)$.

Two multiplication rules are really one

From last time: $P(A \text{ and } B) = P(A)P(B)$, if A and B are independent

From today: $P(A \text{ and } B) = P(A)P(B|A)$

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But these are really the same rule, since $P(B|A) = B$ exactly when A and B are independent!

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But these are really the same rule, since $P(B|A) = B$ exactly when A and B are independent!

This is also how you can check if two events are independent: just check if $P(A)P(B) = P(A \text{ and } B)$.

Reading conditional probabilities off of contingency tables

$P(\text{survived}|\text{female})$ is the proportion of women that survived:

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prop.table(table(titanic$Sex, titanic$Survived), 1)
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          No  Yes
  female 0.25 0.75
  male   0.79 0.21
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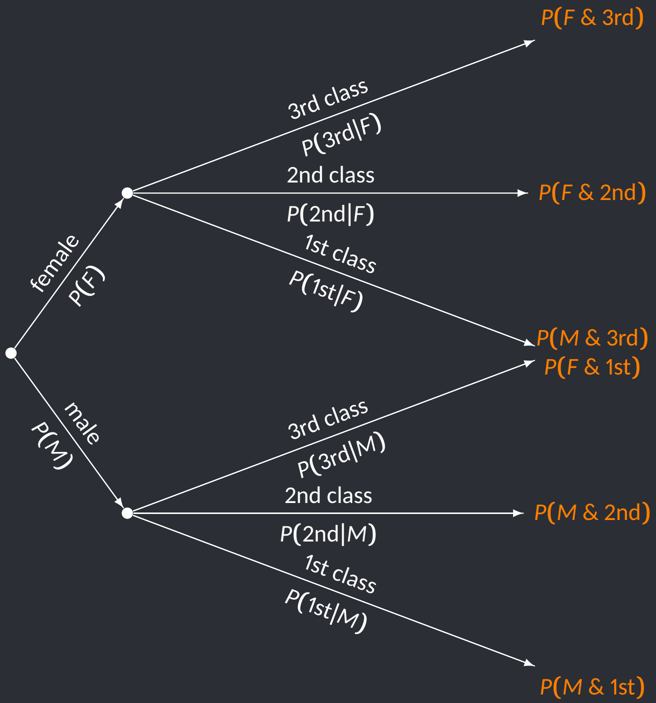
From this we can read off that $P(S|F) = 0.75$.

1. Conditional probabilities

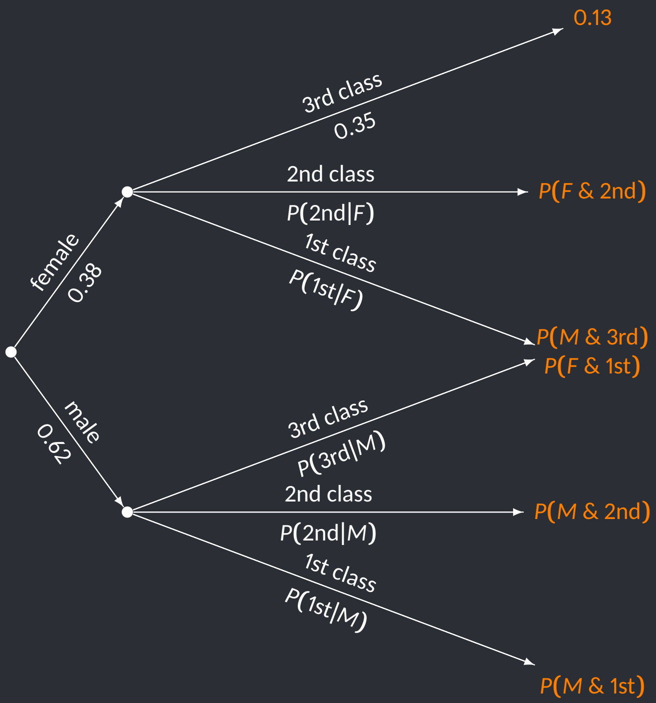
2. Probability trees

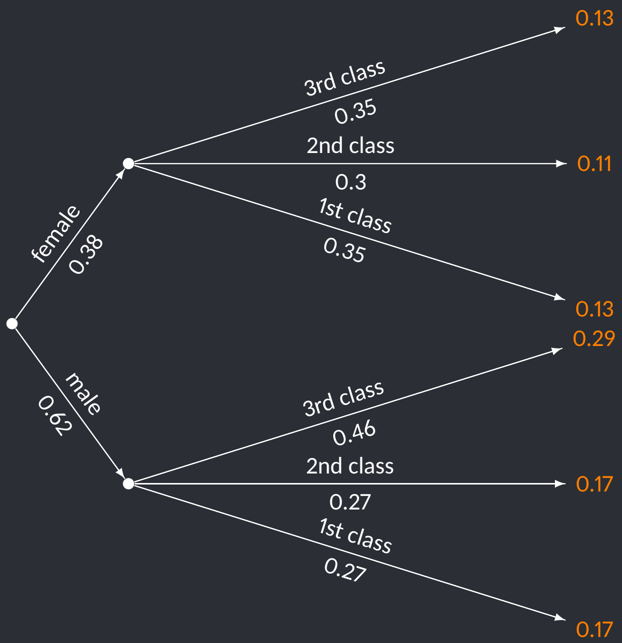
Probability trees

- A way to visualize all joint ($P(A \text{ and } B)$) and conditional ($P(B|A)$) probabilities for a particular situation
- A branch for each possible outcome
- At each level of the tree, assume the things to the left have already happened
- Each branch contains the probability of getting to that branch, conditional on what is to its left
- The leaves of the tree represents probabilities of final outcomes



P(F & 3rd)
3rd class
P(3rd|F)
2nd class
P(F & 2nd)
P(2nd|F)
female
P(F)
1st class
P(1st|F)
P(M & 3rd)
P(F & 1st)
male
P(M)
3rd class
P(3rd|M)
2nd class
P(M & 2nd)
P(2nd|M)
1st class
P(1st|M)
P(M & 1st)


female
0.38
male
0.62
3rd class
0.35
2nd class
P(2nd|F)
1st class
P(1st|F)
3rd class
P(3rd|M)
2nd class
P(2nd|M)
1st class
P(1st|M)
0.13
P(F & 2nd)
P(M & 3rd)
P(F & 1st)
P(M & 2nd)
P(M & 1st)


0.13
3rd class
0.35
2nd class
0.11
0.3
1st class
0.35
female
0.38
0.13
0.29
male
0.62
3rd class
0.46
2nd class
0.17
0.27
1st class
0.27
0.17