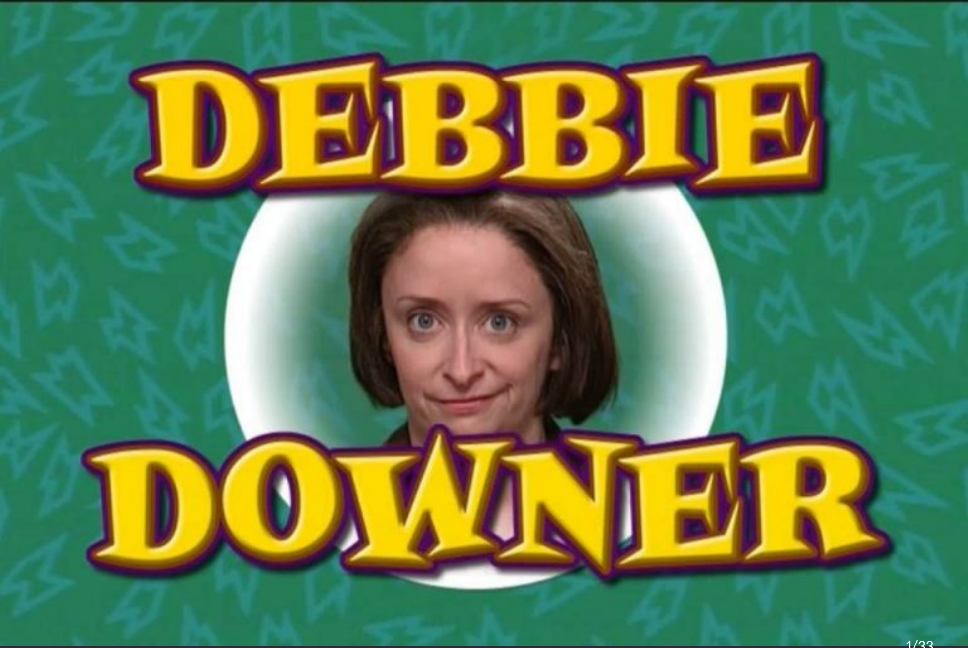
THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

What can go wrong, and how to fix it 1

Lecture 13

STA 371G


DEBBIE
DOWNER

1. Extrapolation

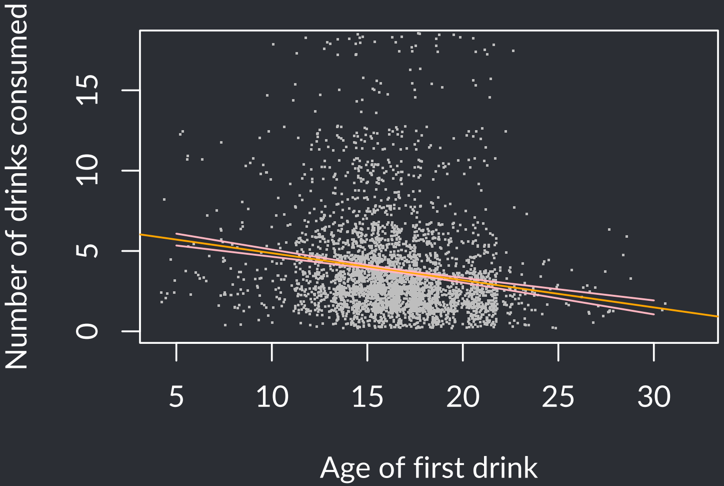
2. Influential observations

3. Autocorrelation

Extrapolation

- Extrapolation means using the regression model to make a prediction outside of the range of the data
- The further we are outside of our “comfort zone” of the data in our sample, the less certain we should be of our prediction

The confidence interval for mean response gets wider as X gets farther from $\overline{X}$:



- There’s nothing wrong with extrapolating a little bit beyond the data

- There’s nothing wrong with extrapolating a little bit beyond the data
- But if we are *very* far outside of the range of our data, prediction intervals and confidence intervals for the mean response will tend to *underestimate* the amount of uncertainty in our estimate

- There’s nothing wrong with extrapolating a little bit beyond the data
- But if we are *very* far outside of the range of our data, prediction intervals and confidence intervals for the mean response will tend to *underestimate* the amount of uncertainty in our estimate
- For example, we wouldn’t want to use this model to estimate the number of drinks consumed for someone that starts drinking at age 40!

1. Extrapolation

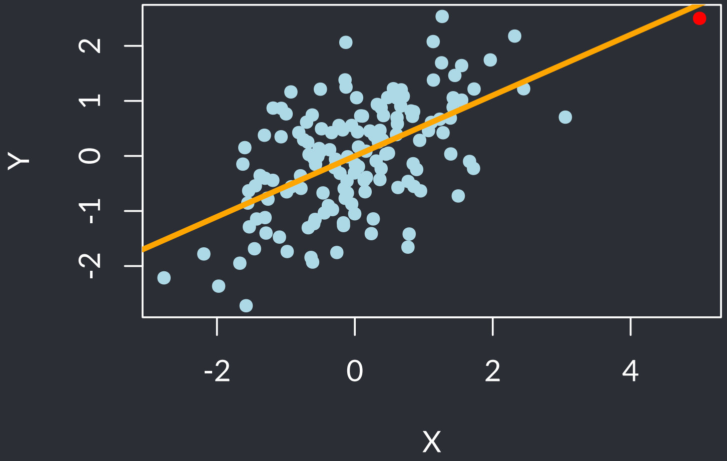
2. Influential observations

3. Autocorrelation

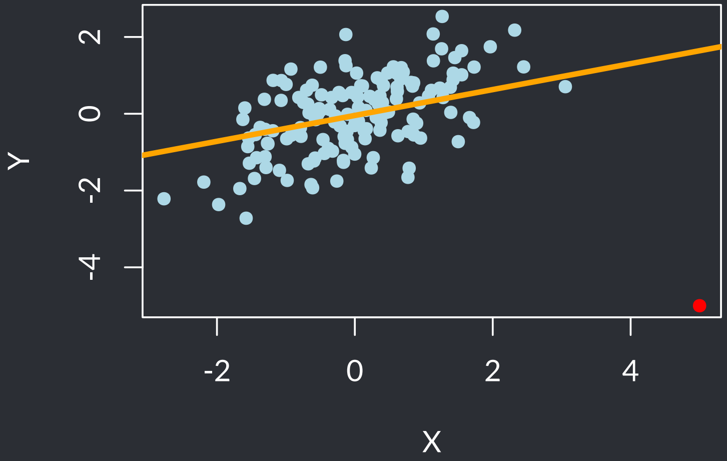
What a single case can do

Even a single case can wreak havoc on the regression line. Let's add one outlier, at $X = 5$, and see what happens with different Y values.

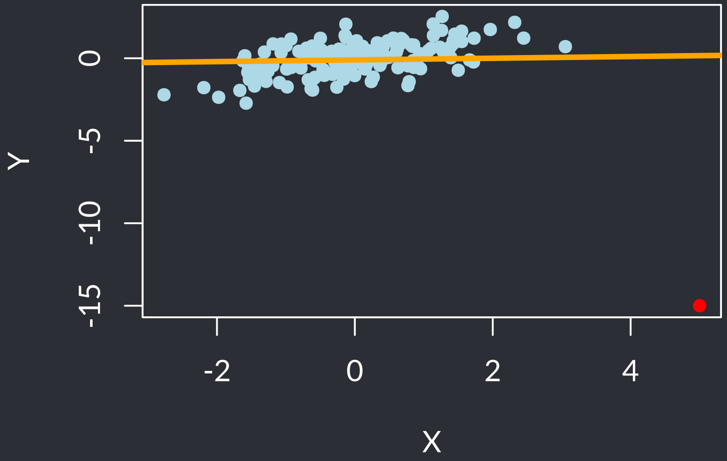
What a single case can do



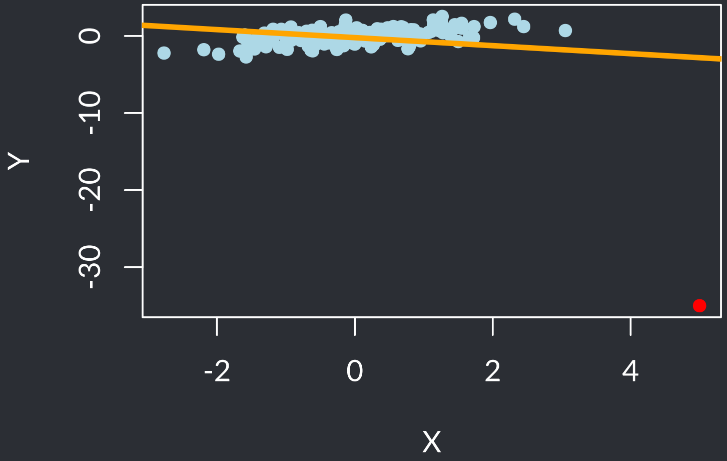
What a single case can do



What a single case can do



What a single case can do



BLACKMAIL

Regression is like blackmail

Blackmail:

- Compromising information gives a blackmailer leverage—the *potential* to have a big impact
- Once the blackmailer uses the information, that gives them influence

Regression is like blackmail

Blackmail:

- Compromising information gives a blackmailer leverage—the *potential* to have a big impact
- Once the blackmailer uses the information, that gives them influence

Regression:

- When a point has a very unusual X value (i.e., far from $\overline{X}$), it has leverage—the *potential* to have a big impact on the regression line
- When that point *also* has a Y value that is out of line with the general trend, it will pull the regression line towards it—giving it influence

How do I know what points are influential?

- Cook's distance can be a useful metric for finding influential points

How do I know what points are influential?

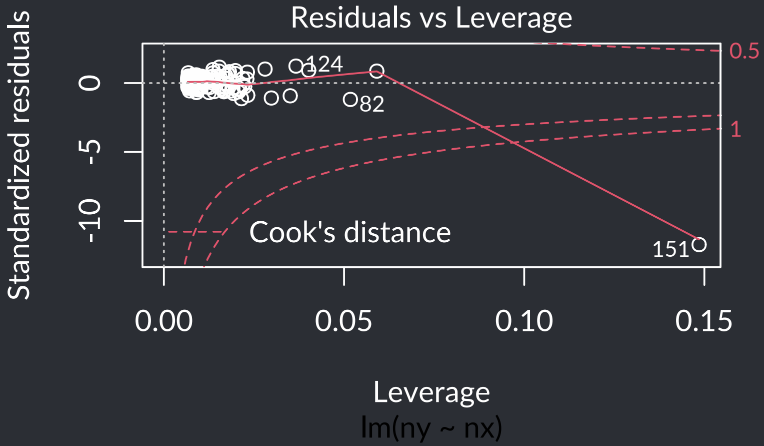
- Cook's distance can be a useful metric for finding influential points
- Larger values of Cook's distance indicate more influential points, but there is no firm cutoff

How do I know what points are influential?

- Cook's distance can be a useful metric for finding influential points
- Larger values of Cook's distance indicate more influential points, but there is no firm cutoff
- Use Cook's distance to help you find points that might be influential, and then run the regression both with and without the point to judge for yourself

Using the Cook's distance plot in R

```r
plot(model, which=5)
```



What can we do about influential observations?

- If removing the influential observation doesn’t make a substantial difference in your analysis, don’t worry about it

What can we do about influential observations?

- If removing the influential observation doesn’t make a substantial difference in your analysis, don’t worry about it
- If it does:

What can we do about influential observations?

- If removing the influential observation doesn't make a substantial difference in your analysis, don't worry about it
- If it does:
  - Consider whether it could be a mistake (happens more than you might think!); if it is, correct the error!

What can we do about influential observations?

- If removing the influential observation doesn’t make a substantial difference in your analysis, don’t worry about it
- If it does:
  - Consider whether it could be a mistake (happens more than you might think!); if it is, correct the error!
  - If not, hold out the influential observation(s) and report on them separately

What can we do about influential observations?

- If removing the influential observation doesn't make a substantial difference in your analysis, don't worry about it
- If it does:
  - Consider whether it could be a mistake (happens more than you might think!); if it is, correct the error!
  - If not, hold out the influential observation(s) and report on them separately
  - Do not just throw out and ignore influential observations!

1. Extrapolation

2. Influential observations

3. Autocorrelation

Residuals

Recall that the residual for the *i*th case in the data is $Y_i - \hat{Y}_i$.

- When the residual is *positive*, the actual Y-value is *higher* than our predicted Y-value.
- When the residual is *negative*, the actual Y-value is *lower* than our predicted Y-value.

Looking at residuals can tell us a lot about how well a model is working, and give us ideas for how to improve it.

What is autocorrelation?

- The first assumption of regression is that the errors are independent.

What is autocorrelation?

- The first assumption of regression is that the errors are independent.
- In many time series data sets, this isn't the case—what happens in one time period is often correlated with those time periods right before or after.

What is autocorrelation?

- The first assumption of regression is that the errors are independent.
- In many time series data sets, this isn’t the case—what happens in one time period is often correlated with those time periods right before or after.
- Autocorrelation is when we can consistently predict the value of a variable at a particular time based on other times.

How can autocorrelation be detected?

- Sometimes, it’s clear that there is likely to be autocorrelation.
- Any time that the value of a time series builds on the previous stage (e.g., daily stock price, annual revenue) autocorrelation is a likely danger.
- But it’s worth checking for autocorrelation in any time series!

What about when it’s not obvious?

The Durbin-Watson test lets us test the null hypothesis that the errors in a regression come from a population where successive errors are uncorrelated.

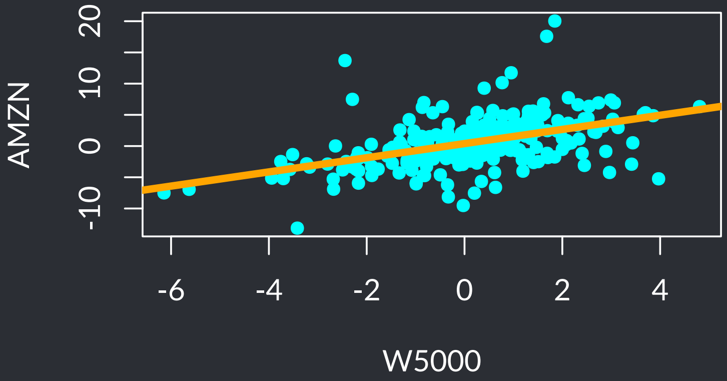
What about when it’s not obvious?

The Durbin-Watson test lets us test the null hypothesis that the errors in a regression come from a population where successive errors are uncorrelated.

If we **reject** the null hypothesis, then there is evidence of autocorrelation, and the independence assumption is violated.

Example

Consider the stock market data, when we regressed a company's weekly return on the weekly return of a market index (Wilshire 5000). Is autocorrelation present?



Example

```
library(lmtest)
model <- lm(AMZN ~ W5000, data=stock.market)
dwtest(model)


Durbin-Watson test

data:  model
DW = 1.9759, p-value = 0.4249
alternative hypothesis: true autocorrelation is greater than 0
```

Here, $p = 0.42 > 0.05$, so we fail to reject the null hypothesis: there is no evidence of (first-order) autocorrelation.

Example

```
library(lmtest)
model <- lm(AMZN ~ W5000, data=stock.market)
dwtest(model)


Durbin-Watson test

data:  model
DW = 1.9759, p-value = 0.4249
alternative hypothesis: true autocorrelation is greater than 0
```

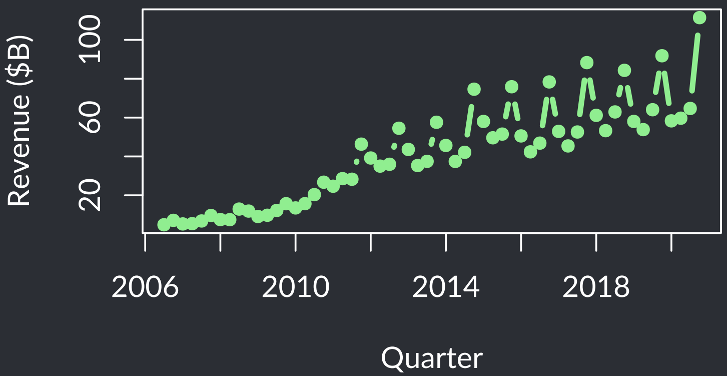
Here, $p = 0.42 > 0.05$, so we fail to reject the null hypothesis: there is no evidence of (first-order) autocorrelation. (Surprised?)

First-order?!

- Durbin-Watson only lets us test for first-order autocorrelation; that is, correlation between the error at time t (today) and the error at time $t - 1$ (yesterday, last month, last year, etc).
- Sometimes the error at time t is correlated not with time $t - 1$ but time $t - 2$ or $t - 3$, etc.

Example

Let’s look at Apple’s quarterly revenue since 2006:



What do you think the peaks correspond to?

The autocorrelation function

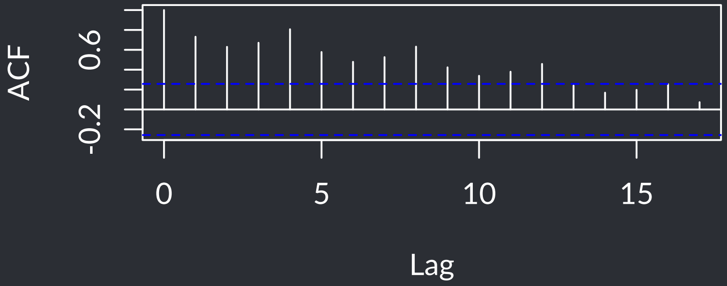
Another approach to suss out autocorrelation is to calculate all possible correlations with the time series and itself, “lagged” back by different time steps:

	Apple revenue ($B)	Lag 1	Lag 2	Lag 3
2006 Q3	4.837	NA	NA	NA
2006 Q4	7.115	4.837	NA	NA
2007 Q1	5.264	7.115	4.837	NA
2007 Q2	5.410	5.264	7.115	4.837
2007 Q3	6.789	5.410	5.264	7.115
2007 Q4	9.608	6.789	5.410	5.264

The autocorrelation function

The autocorrelations are highest for lag 1 and lag 4 (i.e., one quarter and one year ago):

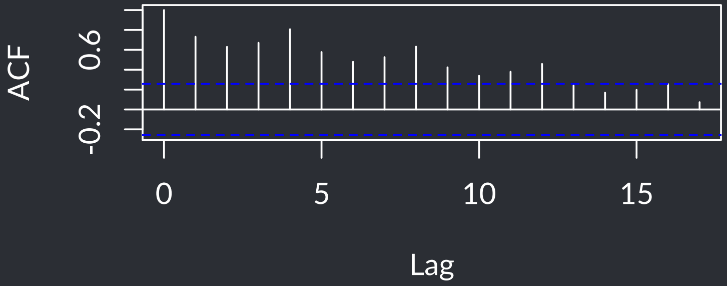
```
acf(apple$Revenue.Billions)
```



The autocorrelation function

The autocorrelations are highest for lag 1 and lag 4 (i.e., one quarter and one year ago):

```
acf(apple$Revenue.Billions)
```



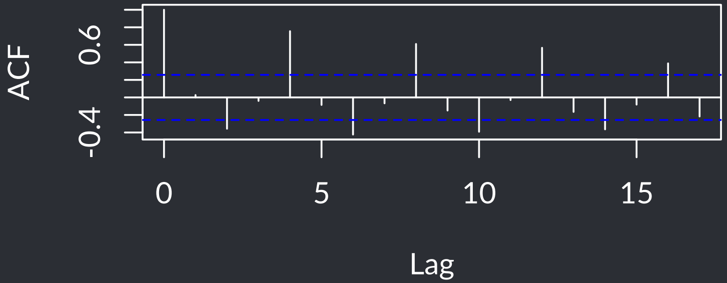
The autocorrelation function, applied to residuals

What we really need to test is the autocorrelation of the *residuals*, not of the revenue itself. Durbin-Watson suggests there is no first-order autocorrelation:

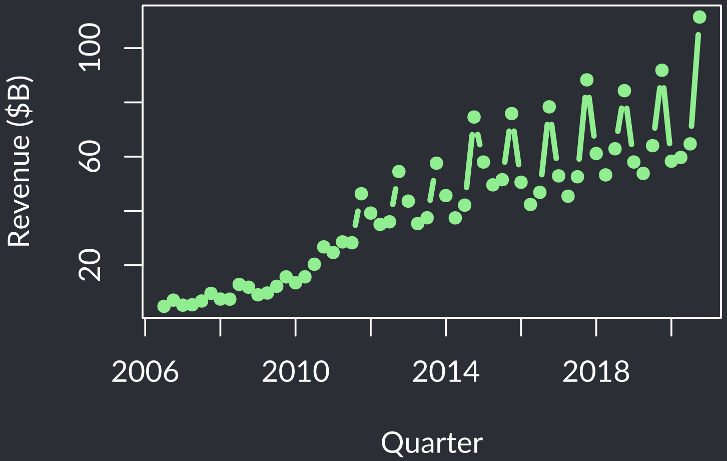
```
model <- lm(Revenue.Billions ~ Time, data=apple)
dwtest(model)


Durbin-Watson test

data:  model
DW = 1.8109, p-value = 0.1938
alternative hypothesis: true autocorrelation is greater than 0
```

But there is clearly second- and fourth-order autocorrelation!

```
model <- lm(Revenue.Billions ~ Time, data=apple)
acf(residuals(model))
```



How do we interpret these–what accounts for this pattern?


Revenue ($B)
20
60
100
2006
2010
2014
2018
Quarter

How to handle autocorrelation

- When time-series data is used, it’s important to check for autocorrelation.

How to handle autocorrelation

- When time-series data is used, it’s important to check for autocorrelation.
- If present, the independence assumption is violated, and we shouldn’t trust the *p*-values and confidence intervals that come out of the regression.

How to handle autocorrelation

- When time-series data is used, it’s important to check for autocorrelation.
- If present, the independence assumption is violated, and we shouldn’t trust the *p*-values and confidence intervals that come out of the regression.
- There may be a way to transform the data to remove the autocorrelation: for example, stock prices have strong autocorrelation, but the percentage changes from day to day (or week to week, etc.) do not.

A final note on regression assumptions

- The purpose of most regression assumptions is ensuring that the *p*-values and confidence intervals are accurate.

A final note on regression assumptions

- The purpose of most regression assumptions is ensuring that the *p*-values and confidence intervals are accurate.
- But nothing prevents you from building a regression even when the assumptions are violated!

A final note on regression assumptions

- The purpose of most regression assumptions is ensuring that the *p*-values and confidence intervals are accurate.
- But nothing prevents you from building a regression even when the assumptions are violated!
- Unless the linearity assumption is violated, the regression equation may still be useful for making predictions.