THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

# Review of Inference

## Lecture 8

STA 371G

# Updated schedule

- Nothing due this week!
- Quiz 3 next Tuesday
- No Quiz 4
- HW 3 & 4 due due next Monday
- See updated syllabus on Canvas for all details

## About the project

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- You cannot use the same research question as another group
- You'll be assigned to a group, but can choose to work alone (let me know by EOD tomorrow if you want to work alone)
- The project is divided into multiple components; you'll write a group paper in phases over the remainder of the semester

# Project components

- March 11: Initial proposal (5%)
- March 25: Literature review (10%)
- April 1: Final proposal (5%)
- April 15: Exploratory data analysis (10%)
- April 29: Final paper submission (60%)
- May 3, 5, 18: Presentation & peer evaluations (10%)

## Review of Inference

- Today we'll review key terms around statistical inference: hypothesis tests, confidence intervals, and *p*-values
- We'll also touch on how to run simulations in R (which we unfortunately had to cut from the curriculum due to the UT closure)
- The concepts are the most important; the specific details of the applications here are not critical

1. Minecraft speedrunning controversy
2. Sampling distributions and $t$-tests
3. Using resampling to conduct hypothesis tests

## Minecraft video game speedrunning

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- Speedruns are often streamed on YouTube or Twitch and can be big business!
- One Minecraft speedrunner, “Dream,” has 18M subscribers and 1.5B views on YouTube, which could result in millions of dollars in ad revenue
- But Dream was suspected of cheating—how can you prove it?

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- You can obtain an eye of ender by trading for a pearl, plus some “blaze powder”
- Every time you attempt a trade, there is a 4.73% chance you will get a pearl
- As a result, luck plays a big factor in how quickly you can beat the game

## Dream’s luck

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- Assuming no cheating, the number of pearls obtained in 262 trials is Binomial ($n = 262$, $p = 0.0473$)
- We can calculate the chance of having luck this good ($P(\text{number of pearls} \geq 42)$) with R:

```r
1 - pbinom(41, 262, 0.0473)

[1] 5.65e-12
```

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- This indicates that Dream’s claim is implausible (10 runs an hour, 24 hours/day, for 100 years, is just 8.8 million attempts)
- In other words, we can be confident that Dream was cheating

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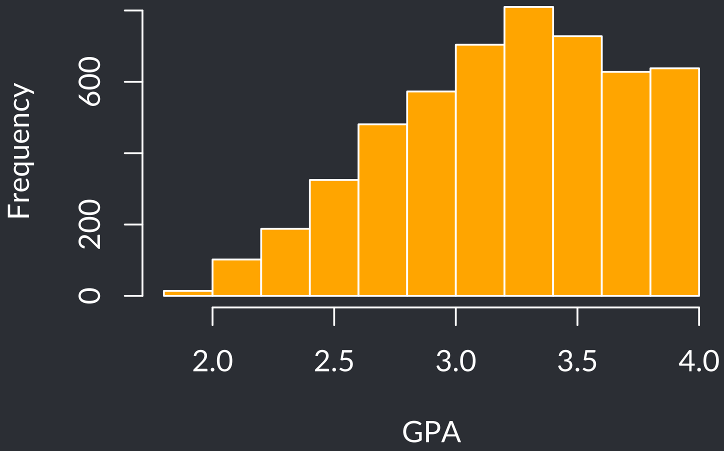
- The $p$-value is the conditional probability of seeing data at least as extreme as what was observed, given that $H_0$ is true: $p = 1$ in 177 billion
- If $p$ is small, we will reject $H_0$ and believe $H_A$. Otherwise, we continue to believe $H_0$: reject $H_0$

1. Minecraft speedrunning controversy

2. Sampling distributions and *t*-tests

3. Using resampling to conduct hypothesis tests

# GPAs of all UT students entering in Fall 2000



# Let's take a sample

Usually, we only have access to a sample of the data. Let's pretend that we only had a sample of $n = 100$ students:

```
sample.gpas <- sample(ut2000$GPA, 100)
mean(sample.gpas)

[1] 3.2
```

Since we have a random sample, it's a good, but not perfect, estimate of the population GPA (3.212).

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Since we have a random sample, it’s a good, but not perfect, estimate of the population GPA (3.212). But normally we don’t have access to the population, so we don’t know how good our estimate is!

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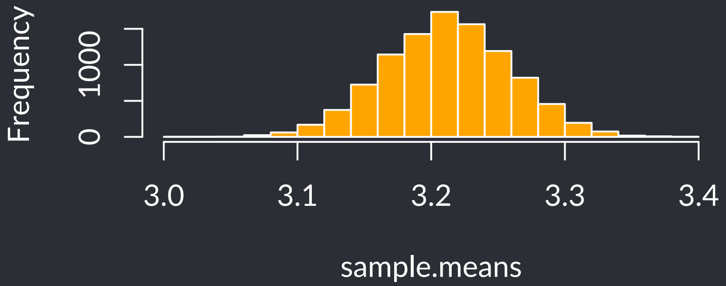
Since we have the population, we can simulate what would happen if we repeatedly took samples over and over from the same population.

```r
sample.means <- replicate(10000, {
  the.sample <- sample(ut2000$GPA, 100)
  return(mean(the.sample))
})
hist(sample.means, main="", col="orange")
```



# Sampling distribution of $\overline{GPA}$

The *sampling distribution* of $\overline{GPA}$ is the distribution of sample means, if we took an infinite number of repeated samples:

$$E(\overline{GPA}) = \mu = 3.212$$
$$\mathrm{SD}(\overline{GPA}) = \frac{\sigma}{\sqrt{n}} = \frac{0.48}{\sqrt{100}} = 0.048$$

The last value quantifies how much the sample mean will vary from sample to sample. But we normally can't compute $\sigma$ since we don't have the whole population, so we estimate it by calculating the SD in the *sample* ($\hat{\sigma}$) and dividing by $\sqrt{n}$; this is the *standard error of the mean*.

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Our sample statistic is $\hat{\mu}$ and our standard error is $\hat{\sigma}/\sqrt{n}$. What is the critical value?

As it turns out, the sampling distribution (of $\hat{\mu}$) is not *quite* Normal. If we standardize the sample means, the distribution of

$$\frac{\hat{\mu} - \mu}{\hat{\sigma}/\sqrt{n}}$$

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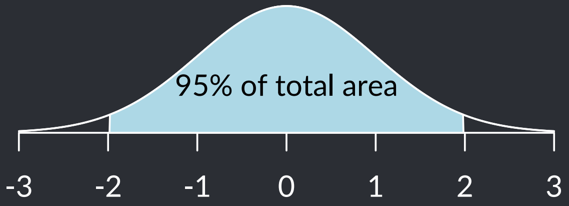
is called a *t*-distribution with $n-1$ degrees of freedom. The critical value for a 95% confidence interval is $t^* = \pm 1.984$, the value that cuts off 95% of the area under the *t*-distribution:



There are two helpful R functions for calculating values around $t$-distributions (like `pnorm` and `qnorm`):

- `pt(x, df)` calculates $P(t < x)$ if we are looking at a distribution with *df* degrees of freedom.

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So the critical value for a 95% confidence interval is `qt(.975, 99)` when $n = 100$.

Using our sample of $n = 100$, we calculate that a 95% confidence interval for the mean population GPA is

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There are two ways to interpret this:

- **Informally,** we are 95% confident that the population mean GPA is between 3.109 and 3.299.
- **Formally,** if we took repeated samples and found the 95% CI within each sample, 95% of the CIs would contain the population mean.

R can do this work for you!

```
t.test(sample.gpas)


One Sample t-test


data:  sample.gpas
t = 67, df = 99, p-value <2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 3.11 3.30
sample estimates:
mean of x
      3.2
```

## Hypothesis tests

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- Let’s pretend we don’t have the population (which is usually the case), and so we can’t know for sure that he is wrong.
- We do have some evidence (our sample) that we can bring to bear on the question.

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- The alternative hypothesis ($H_A$) is what we will believe if it turns out that $H_0$ is false: $\mu \neq 3.15$
- The *p*-value is the conditional probability of seeing data at least as extreme as what was observed, given that $H_0$ is true: if Sooner is correct, how likely is it that in our sample we would see a sample mean (of $\hat{\mu} = 3.204$) that is so far away from his hypothesized value of $\mu = 3.15$?

R can run hypothesis tests for us:

```
t.test(sample.gpas, mu=3.15)


One Sample t-test

data:  sample.gpas
t = 1, df = 99, p-value = 0.3
alternative hypothesis: true mean is not equal to 3.15
95 percent confidence interval:
 3.11 3.30
sample estimates:
mean of x
      3.2
```

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- The $p$-value is the conditional probability of seeing data at least as extreme as what was observed, given that $H_0$ is true: $p = 0.3$
- If $p$ is small, we will reject $H_0$ and believe $H_A$. Otherwise, we continue to believe $H_0$: do not reject $H_0$; Mr Sooner's claim is consistent with our sample data

When doing hypothesis testing, we select an $\alpha$ value in advance, and then reject the null hypothesis if and only if $p < \alpha$.

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$\alpha = .05$ is a good "default" to use unless you have a reason to set it higher or lower.

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## Comparing meeting strategies

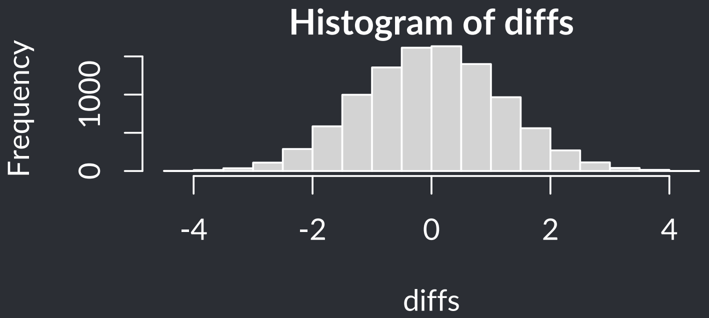
- Renuka holds daily “standup” meetings with her team, under the theory that having the meeting standing up will make it go faster.
- She decides to test this by holding 20 meetings standing up and then 20 meetings sitting down.
- Standing is 3.65 minutes faster, but is that real or just due to random chance?

| Standing | Sitting |
|---|---|
| 27, 29, 30, 30, 31, 31, 31, 32, 33, 33, 33, 34, 34, 35, 35, 35, 36, 37, 37, 40 | 30, 33, 34, 34, 34, 34, 35, 35, 36, 36, 37, 38, 38, 38, 39, 39, 40, 40, 41, 45 |

## Resampling strategy

- Put all 40 meeting times in a blender, and randomly pick out 20 for each group.
- Calculate how often do we get a difference by chance of at least 3.65 minutes?

```r
diffs <- replicate(10000, {
  reordered <- sample(c(standing, sitting), 40)
  group1 <- reordered[1:20]
  group2 <- reordered[21:40]
  return(mean(group1) - mean(group2))
})
hist(diffs)
```



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To calculate it exactly, we count the number of differences that are larger than $\pm 3.65$:

```
sum(diffs <= -3.65 | diffs >= 3.65) / 10000

[1] 0.0016
```