THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Inference for Simple Regression 1

Lecture 11

STA 371G

Measuring goodness-of-fit

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- The standard error of the regression s_e can be roughly interpreted as the standard deviation of the residuals.

```r
model <- lm(num.drinks ~ age, data=drinking)
summary(model)


Call:
lm(formula = num.drinks ~ age, data = drinking)

Residuals:
   Min     1Q Median     3Q    Max 
-4.204 -1.853 -0.853  0.810 15.160 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   6.5542     0.2653    24.7   <2e-16 ***
age          -0.1688     0.0159   -10.6   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.96 on 3600 degrees of freedom
  (2902 observations deleted due to missingness)
Multiple R-squared:  0.0304,Adjusted R-squared:  0.0302 
F-statistic:  113 on 1 and 3600 DF,  p-value: <2e-16
```

Interpreting the standard error of the regression

$s_e = 2.96$

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Interpreting the standard error of the regression $s_e = 2.96$

- The residual for the ith case is $Y_i - \hat{Y}_i$ (actual Y value − predicted Y value)
- The residuals are approximately Normally distributed
- The mean of the residuals is 0 (why?)
- Therefore: 95% of the residuals are roughly within $\pm 2s_e$
- In other words, 95% of the time I expect my prediction to be off by at most $2 \cdot 2.96 = 5.93$

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Is this “significant?”

- **Statistical significance:** Can we reject the null hypothesis that the correlation between X and Y in the *population* is zero?
- **Practical significance:** Is the relationship in our sample strong enough to be meaningful?

The overall null hypothesis for a regression model

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- $R^2 = 0$ (in the population)
- $\text{cor}(X, Y) = 0$ (in the population)
- $\beta_1 = 0$
- The model has no predictive power
- Predictions from this model are no better than predicting $\overline{Y}$ for every case

Two ways to test the overall null hypothesis

- The *F*-test (tests $H_0 : R^2 = 0$ in the population vs $H_A : R^2 \neq 0$)
- The *t*-test for the *slope* (β_1) coefficient (tests $H_0 : \beta_1 = 0$ vs $H_A : \beta_1 \neq 0$)
- Note that both tests are two-tailed, since we would care about the null hypothesis being wrong in either direction (i.e. $\beta_1 > 0$ and $\beta_1 < 0$ are both of interest)

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Both of these methods are equivalent; the p-values will be exactly the same!

```
model <- lm(num.drinks ~ age)

Error in eval(predvars, data, env):  object 'num.drinks' not
found

summary(model)


Call:
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Residuals:
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---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3 on 3600 degrees of freedom
  (2902 observations deleted due to missingness)
Multiple R-squared:  0.0304,Adjusted R-squared:  0.0302
F-statistic:  113 on 1 and 3600 DF,  p-value: <2e-16
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- Each additional year you wait to start drinking is associated with consuming 0.17 fewer drinks as an adult.

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- Or: People that start drinking earlier in life consume **significantly more** alcohol when they drink as adults.
- Each additional year you wait to start drinking is associated with consuming 0.17 fewer drinks as an adult.
- Is this relationship **practically significant**?

Practical significance

- To assess **statistical significance**, we look at the *p*-value
- To assess **practical significance**:
  - We only consider it if we already have statistical significance (why?)
  - Look at R^2, the standard error of the regression, and the magnitude of the coefficients
  - It's ultimately a judgement call!

Put a confidence interval on it

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- Our best estimate for the *effect* of a year's postponement of drinking is 0.17 fewer drinks as an adult
- We can use a confidence interval to give a range of plausible values for what this effect size is in the population

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A confidence interval is always of the form

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Recall that the critical value for a 95% confidence interval is the cutoff value that cuts off 95% of the area in the middle of the distribution; the sampling distribution of $\hat{\beta}_1$ is a t-distribution.

```
# n = number of observations (cases)
n <- nobs(model)
# critical value = cutoff values so that 95% of area is cap
tured between them
qt(0.975, n-2)
```

```
[1] 1.960623
```

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model <- lm(num.drinks ~ age)
```

```
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```r
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Put a confidence interval on it

R will also calculate confidence intervals for us:

```
confint(model)
```

```
             2.5 % 97.5 %
(Intercept)    6.0   7.07
age           -0.2  -0.14
```

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R will also calculate confidence intervals for us:

```
confint(model)

             2.5 % 97.5 %
(Intercept)    6.0   7.07
age           -0.2  -0.14
```

In other words, we are 95% confident that the effect of each additional year's delay in starting to drink is between 0.14 and 0.2.

Put a confidence interval on it, part 2

We can also put a confidence interval on a prediction!

Two kinds of intervals:

- Confidence interval for the mean response: Predicting the mean value of Y for a particular X. (Example: Among all people that start drinking at age 21, how many drinks do have on average as adults?)
- Prediction interval: Predicting Y for a single new case. (Example: If Bob started drinking at age 21, how many drinks do we think will have as an adult?)

Confidence intervals for making predictions

- Confidence interval for the mean response (predicting the mean Y for a particular X):

$$\hat{Y} \pm t^* \cdot s_e \sqrt{\frac{1}{n} + \frac{(X - \overline{X})^2}{\sum_{i=1}^{n}(X_i - \overline{X})^2}}$$

- Prediction interval (predicting Y for a single new case):

$$\hat{Y} \pm t^* \cdot s_e \sqrt{1 + \frac{1}{n} + \frac{(X - \overline{X})^2}{\sum_{i=1}^{n}(X_i - \overline{X})^2}}$$

Put a confidence interval on it, part 2

```
predict(model, list(age=21), interval="confidence")

       fit      lwr      upr
1 3.008664 2.83616 3.181167

predict(model, list(age=21), interval="prediction")

       fit       lwr      upr
1 3.008664 -2.802894 8.820221
```

Put a confidence interval on it, part 2

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predict(model, list(age=21), interval="confidence")

       fit     lwr      upr
1 3.008664 2.83616 3.181167

predict(model, list(age=21), interval="prediction")

       fit       lwr      upr
1 3.008664 -2.802894 8.820221
```
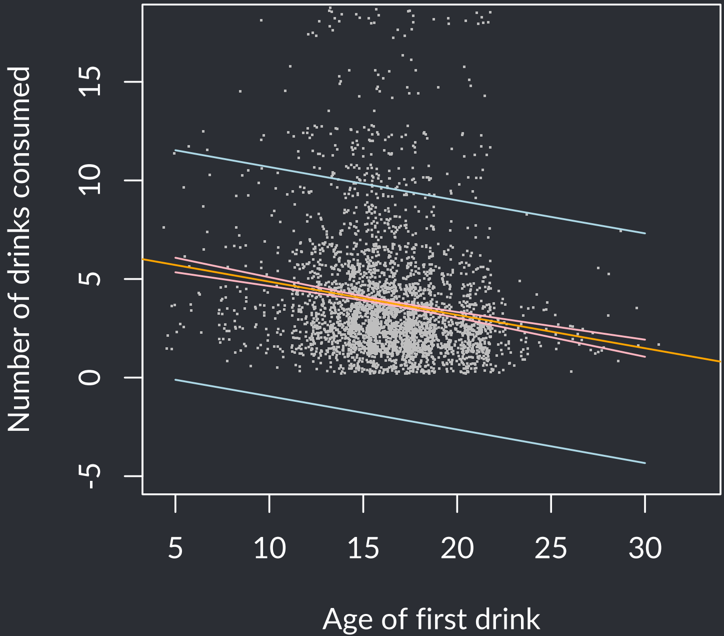

Why is the prediction interval wider?


Number of drinks consumed
-5
0
5
10
15
5
10
15
20
25
30
Age of first drink