THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Building Models

Lecture 17

STA 371G

My review of the midsemester survey

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- In class: increasing opportunities for practice with R and with concepts in class

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- In class: increasing opportunities for practice with R and with concepts in class
- Getting help outside of class: R info pages, LC solutions, meeting outside of office hours

1. Multicollinearity

2. Selecting the best model

Exploring the data: Multicollinearity

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Correlation between the response and the predictors is good, but correlation between the predictors is not!

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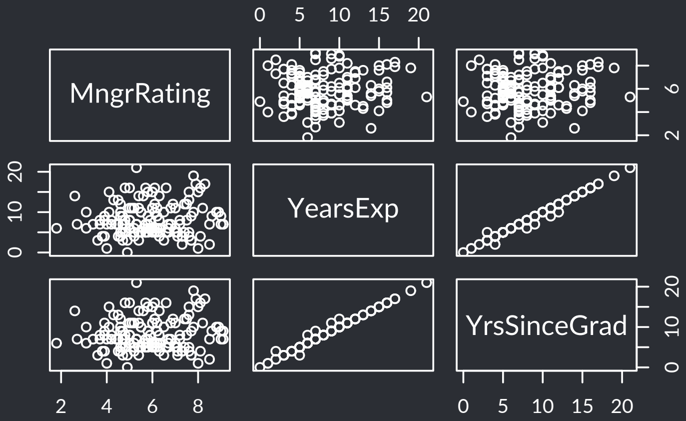
- Any conclusions based on the p-values, coefficients, and confidence intervals of the highly correlated variables will be unreliable.
- These statistics will not be stable: adding new data or predictors to the model could drastically change them.

```r
pairs(~ MngrRating + YearsExp + YrsSinceGrad, data=mclean)
```



```r
model <- lm(Salary ~ MngrRating + YearsExp + YrsSinceGrad,
           data=mclean)
summary(model)
```

```
Call:
lm(formula = Salary ~ MngrRating + YearsExp + YrsSinceGrad, data = mclean)

Residuals:
    Min      1Q  Median      3Q     Max 
-39.181  -4.519   0.630   4.327  25.590 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)   51.5922     2.8185  18.305   <2e-16 ***
MngrRating     4.6929     0.4260  11.016   <2e-16 ***
YearsExp      -1.4579     1.6111  -0.905    0.367    
YrsSinceGrad   0.5036     1.6078   0.313    0.755    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7.799 on 148 degrees of freedom
  (2 observations deleted due to missingness)
Multiple R-squared:  0.5034,Adjusted R-squared:  0.4934 
F-statistic: 50.02 on 3 and 148 DF,  p-value: < 2.2e-16
```

Exploring the data: Multicollinearity

One way to see if two variables are collinear is to check the correlation between the two:

```
cor(mclean$YearsExp, mclean$YrsSinceGrad)

[1] 0.9951195
```

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```
cor(mclean$YearsExp, mclean$YrsSinceGrad)

[1] 0.9951195
```

Any correlation ≥ 0.95 is definitely a problem, but smaller correlations could be problematic too.

Exploring the data: Multicollinearity

A better way to check multicollinearity is using Variance Inflation Factors (VIF).

- The VIF is

$$\mathrm{VIF}(\beta_j) = \frac{1}{1 - R_j^2},$$

  where R_j^2 is the R^2 in a regression predicting X variable j from the other X variables.

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- $\mathrm{VIF}(\beta_j) = 0$ when $R_j^2 = 0$; i.e., the jth predictor variable is completely independent from the others.
- $\mathrm{VIF}(\beta_j)$ increases as R_j^2 does, and is ∞ when there is perfect multicollinearity; i.e., when X_j is perfectly predictable from the other X variables.

Exploring the data: Multicollinearity

```
library(car)
vif(model)

  MngrRating     YearsExp YrsSinceGrad
    1.005517   102.785807   102.831640
```

Predictors with VIF > 5 indicate multicollinearity.

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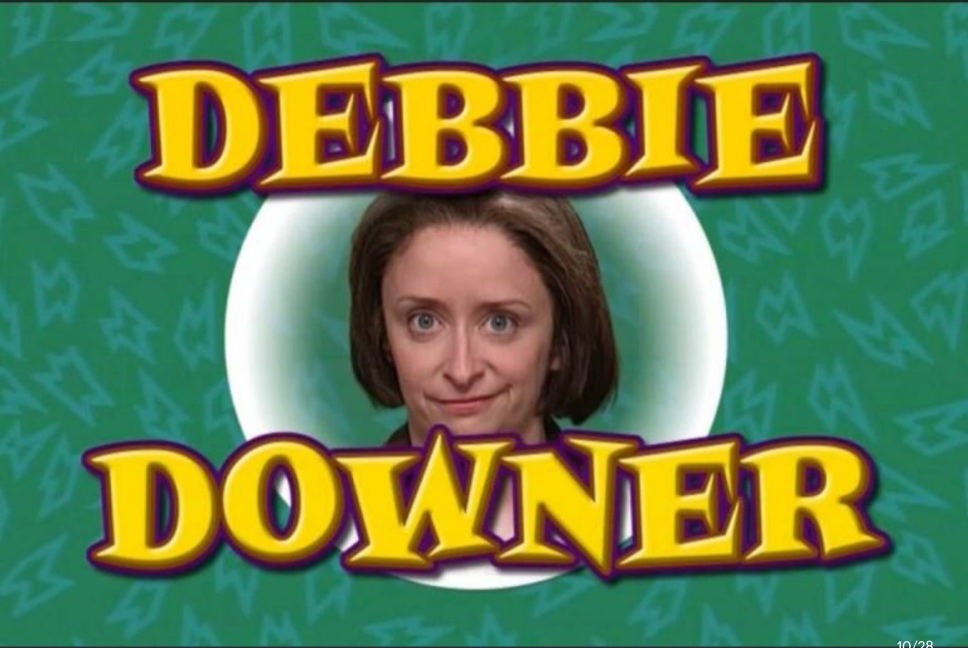
library(car)
vif(model)

  MngrRating     YearsExp YrsSinceGrad
    1.005517   102.785807   102.831640
```

Predictors with VIF > 5 indicate multicollinearity.

Remember: Multicollinearity could exist between more than two predictors.


DEBBIE
DOWNER

Dealing with multicollinearity

There are two general strategies for dealing with multicollinearity:

- Drop a variable with a high VIF factor.
- Combine the variables that correlate into a composite variable.

```
model2 <- lm(Salary ~ MngrRating + YearsExp, data=mclean)
summary(model2)


Call:
lm(formula = Salary ~ MngrRating + YearsExp, data = mclean)

Residuals:
    Min      1Q  Median      3Q     Max 
-39.229  -4.545   0.624   4.303  25.563 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  51.6008     2.8098   18.36  < 2e-16 ***
MngrRating    4.6979     0.4244   11.07  < 2e-16 ***
YearsExp     -0.9557     0.1588   -6.02  1.3e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7.775 on 149 degrees of freedom
  (2 observations deleted due to missingness)
Multiple R-squared:  0.5031,Adjusted R-squared:  0.4964 
F-statistic: 75.43 on 2 and 149 DF,  p-value: < 2.2e-16
```

1. Multicollinearity

2. Selecting the best model

Texas Suffers From A Doctor Shortage

By JONATHAN BAKER • NOV 1, 2017

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When it comes to having a high ratio of doctors to citizens, the State of Texas ranks near the bottom. In fact, as *The Dallas Morning News* reports, 43 states have a higher proportion of primary care physicians to residents than Texas.

And West Texas suffers from a lack of doctors more than other parts of the state. There are 80 counties in Texas with five or fewer practicing doctors - many in West Texas. Thirty-five Texas counties have no doctors at all.

Potential predictor variables

- **LandArea**: Area in square miles
- **PctRural**: Percentage rural land
- **MedianIncome**: Median household income
- **Population**: Population
- **PctUnder18**: Percent children
- **PctOver65**: Percent seniors
- **PctPoverty**: Percent below the poverty line
- **PctUninsured**: Percent without health insurance
- **PctSomeCollege**: Percent with some higher education
- **PctUnemployed**: Percent unemployed

Parsimony

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- We also want a model that is simple, so it's easy to explain to a non-expert.
- The ideal model is parsimonious: a good trade-off between simplicity (as few variables as possible) and a high R^2.

General strategy

1. Use one or more procedures to generate candidate models: possible models that are worth considering.

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2. Select the candidate model with a reasonable tradeoff simplicity and predictive power (high R^2).
3. Check assumptions; apply transformations and other fixes if needed to the final model. If the problems are unfixable, select a different candidate model.

Backward stepwise regression

1. Start with a "full" model containing all of the predictors.
2. Remove the least significant (highest p-value / smallest t-statistic) predictor.
3. Re-run the model with that predictor removed.
4. Repeat steps 2-3 until all predictors are significant.

Forward stepwise regression

1. Start with a “null” model containing none of the predictors.
2. Try adding each predictor, one at a time, and pick the one that ends up being the most significant (lowest p-value / highest t-statistic) predictor.
3. Re-run the model with that predictor added.
4. Repeat steps 2-3 until no more significant predictors can be added.

Other stepwise regression possibilities

- Add (or remove) variables one at a time based on the change in R^2, Adjusted R^2, or another model fit criterion when that variable is added (or removed).
- Run the stepwise regression in both directions, allowing addition or removal of a variable at each step.
- R's `step` function incorporates both of these methods.

The problem with stepwise regression

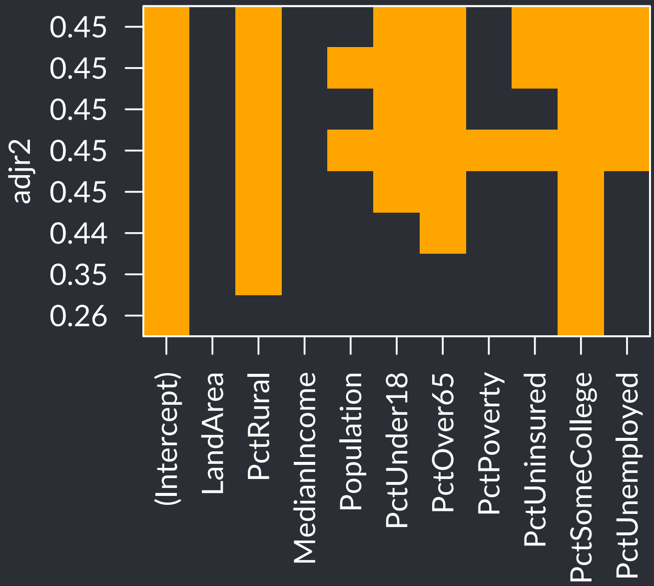
Stepwise regression will not necessarily give you the best model; by only adding or removing one variable at a time, you can get locked into a particular “path” that means you may never consider better models.

Best subsets regression

- Computers are fast! Just let R try out all of the $2^k - 1$ possible models for you.
- R will present you the model with the best Adjusted R^2 for each possible number of predictors.

Best-subsets regression

```r
library(leaps)
plot(regsubsets(PhysiciansPer10000 ~ LandArea + PctRural
                + MedianIncome + Population + PctUnder18
                + PctOver65 + PctPoverty + PctUninsured
                + PctSomeCollege + PctUnemployed,
                data=my.counties), scale="adjr2")
```


adjr2
0.45
0.45
0.45
0.45
0.45
0.44
0.35
0.26
(Intercept)
LandArea
PctRural
MedianIncome
Population
PctUnder18
PctOver65
PctPoverty
PctUninsured
PctSomeCollege
PctUnemployed

- Best-subsets regression presents us with a candidate model for each possible number of predictors.
- The label on the y-axis show the Adjusted R^2 value for the model corresponding to the filled-in squares for that row.

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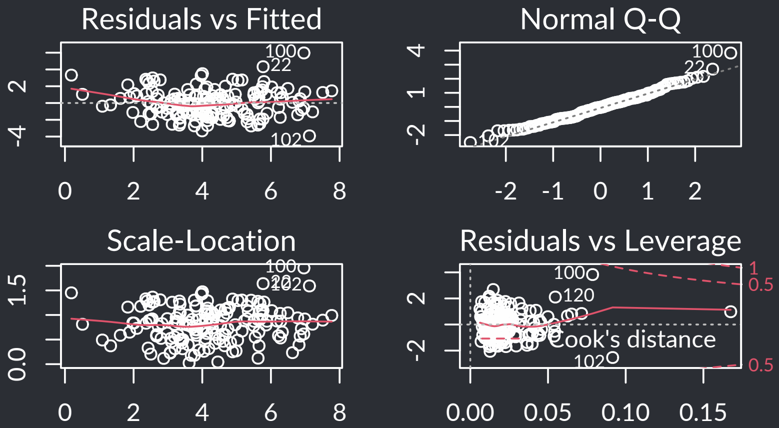
- Look at multiple statistics. They generally say similar things.
- Find the **parsimonious** middle ground between an underspecified model and extraneous variables.
- Fine-tune the model to ensure the model meets assumptions and captures key relationships: you may need to transform predictors and/or add interactions.
- Think about logical reasons why certain predictors might be useful; don't just focus on *p*-values.

Check assumptions of the best model

```
candidate <- lm(PhysiciansPer10000 ~ PctRural + PctOver65
                    + PctSomeCollege, data=my.counties)
plot(candidate)
```



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- Think about the meaning of the variables in your model. Be careful if the model looks too good to be true.
- Do not just use a mechanical process for model selection and call it a day; you need to use your judgement and select a parsimonious model.
- Don’t forget to check the model assumptions for your final model!