

# COMP 560 Artificial Intelligence: Probability & Bayesian Networks (Assignment 3)

Ben Hu,<sup>\*</sup> Brian Luong,<sup>†</sup> and Stephen Yan<sup>‡</sup>  
(Dated: October 30, 2015)

We were tasked with solving a simple Bayesian network which required an understanding of independence and causality. By working on a small network it is easier to see how complex a network can get. Thus the value of variable elimination and different sampling methods becomes more clear to see. We also implemented a Naive Bayes Spam Classifier for predicting email into two classes: spam and ham. We used spam/ham email training documents to train our classifier and spam/ham testing documents to try our system. We implemented Bayesian techniques such as *Maximum A Priori (MAP)* estimation and *Maximum Likelihood (ML)* estimation in our model. We performed analysis and testing on our model and observed that spam deprecated as we increased our lexicon threshold. Our results would vary as we changed our Laplace smoothness constant. We further introduced n-gram features to change the way we choose features from emails and thus attempt to improve our classifier accuracy.

## I. WRITTEN QUESTIONS

### A.

$P(J|G) = P(J|G, I)$  is the only equality asserted by the network structure.  $P(B, I, M) = P(B)P(I)P(M)$  is incorrect because I is affected by B and M.  $P(M|G, B, I) = P(M|G, B, I, J)$  is incorrect because knowing J changes the probability of G, changing the probability of M.  $P(J|G) = P(J|G, I)$  is true because G is affected by I, so knowing G already includes the probability of I occurring.

### B.

$P(B, I, \neg M, G, J)$  equals  $P(B) * P(M) * P(I|B, M) * P(G|I, B, M) * P(J|G)$ .  $P(B) = 0.9$ ,  $P(\neg M) = 0.9$ ,  $P(I|B, \neg M) = 0.5$ ,  $P(G|B, \neg M, I) = 0.8$ ,  $P(J|G) = 0.9$ . Thus  $0.9 * 0.9 * 0.5 * 0.8 * 0.9 = 0.2916$ .

### C.

Since we are given B and -M occur, we know the probability of I occurring. From the chart  $P(I) = 0.5$ . Next we would need to calculate the probability of G. Since we know B and -M, we can eliminate both from the probability of G. Thus we can reduce to:

I	P(G)
T	0.8
F	0.0

From this we can eliminate I by combining it with G. This is done by combining probabilities as from the following table:

G	I	P(G, I)
T	T	0.4
T	F	0.0
F	T	0.1
F	F	0.5

Then we can marginalize I, giving us:

G	P(G)
T	0.4
F	0.6

The math so far appears correct as G and -G add up to a probability of 1. Next we will eliminate G and combine it with J to find our desired probability. This elimination will be calculated in the same way as above.

J	G	P(J, G)
T	T	0.36
T	F	0.00
F	T	0.04
F	F	0.60

Then we marginalize J to:

J	P(J)
T	0.36
F	0.64

Thus the probability that this person goes to jail given that they broke the law and the prosecutor is not politically motivated is 0.36. The math checks out as the probability of J and -J is 1.

### D.

Given that there is a guilty verdict as our only evidence, we would use Gibbs sampling. This is done by first doing a forward sample to obtain values for all the variables. Then we would pick a random variable and re-sample based on its markov blanket. Since the Bayesian

<sup>\*</sup> bxhu@live.unc.edu

<sup>†</sup> bluong@email.unc.edu

<sup>‡</sup> sjyan@cs.unc.edu

Network is small, the markov blanket encompasses almost all of the network. There would also be less variables for us to have to sample. Additionally, our evidence is downstream from other variables thus other methods would be less effective.

## II. NAIVE BAYES SPAM CLASSIFICATION

Our task was to implement a Naive Bayes spam classification system for predicting whether an email is spam or ham. In this implementation, we will use Java.

### A. Basics

---

```
public enum Classification {
    HAM,
    SPAM
}
```

---

There are two classes we will classify emails into: SPAM and HAM.

Each email is defined as:

---

```
public class Email {

    private final Classification classification;
    private final String emailFilePath;
    private final String originalEmailString;
    private Classification predictedClassification;
    // should be null for the test documents

    private Map<String, Integer> wordMap;
}
```

---

The field `predictedClassification` will be null for training data and set to either SPAM or HAM for test data based on the result of the classification phase.

`wordMap` is an abstraction of the email into a set of words in the email and the frequency in that email.

---

```
String[] words =
this.originalEmailString.split("[^a-zA-Z]+");
```

---

We extract the words from the plain text of the email from the regular expression above, which indicates that we only take words that consist of characters in the alphabet. We made the decision to disallow strings containing any non-alpha characters because those strings are not defined as "words" within the English dictionary, and therefore may not be useful data.

With these structures set, we can now read in the training emails.

### B. Computing Features

Before training, we must first compute the features that we will use during classification. For this scenario, our features will be a lexicon of words. We will compute a lexicon consisting of the set of unique words occurring more than  $k$  times in the **entire training document collection**. In our experiment, we have varied  $k$  and recorded our observations.

### C. Training and Testing

In machine learning, we must first train our program on training data in order to gain knowledge and then use what was learned to classify new, unseen data.

Our task here is to estimate the likelihood of each word in our feature set occurring for each class (spam and ham). Mathematically, this is written as:

---


$$P(\text{Word} = w \mid \text{class}) = \frac{(\# \text{ of times word } w \text{ occurs in training examples from this class})}{(\text{Total \# of words in training examples from this class})}$$


---

or equivalently in our code:

---

```
Map<String, Parameter> parameters = new
    HashMap<>();
int totalWordsInSpam =
    computeTotalWords(spamEmails);
int totalWordsInHam =
    computeTotalWords(hamEmails);
for (String word : features) {
    double likelihoodSpam =
        computeOccurrences(word, spamEmails) * 1.0
        / totalWordsInSpam;
    double likelihoodHam =
        computeOccurrences(word, hamEmails) * 1.0
        / totalWordsInHam;
    Parameter parameter = new Parameter(word,
        likelihoodSpam, likelihoodHam);
    parameters.put(word, parameter);
}
return parameters;
```

---

In this method, we define parameter objects which wrap a word in the feature set along with its likelihood in occurring within a spam or ham email. The set of these parameter objects is the model that will be used in the testing phase.

In order to smooth the data and reduce the effect of small probabilities, we implemented a technique called *Laplace Smoothing*, which increases the observation count of every value by a constant  $m$ . We experimented with varying the values of  $m$  and have reported our observations.

In our training, we also perform a process called *evaluation* to find the best parameters. Evaluation is the process of splitting the training data into a training and

an evaluation set. The parameters that are learned from the training set will then be applied to the evaluation set. We do this many times while changing the parameters and then choose the parameters that perform the best as our final parameters. Again, the parameters of our model are the likelihoods of a certain word  $w$  in the lexicon appearing in a particular class. These probabilities are smoothed with the Laplace Smoothing Constant. Since we cannot change the contents of the email, the only variables we can change to modify the parameter values are  $k$ , the frequency of a certain word and  $m$ , the Laplace Smoothing Constant.

We experimented with various ratios of training to evaluation emails and we found that they all yield the same results. The results we discuss below are based off of 80% training, 20% evaluation.

In our classification process, we use *maximum a posteriori* (MAP) classification. This is similar to *maximum likelihood* (ML) except we included the *prior* likelihood in our calculation:

$$P(class)P(w_1|class)P(w_2|class)...P(w_{200}|class). \quad (1)$$

```
k:0 laplace: 0
spam correct: 5 ham correct: 20
-----
k:0 laplace: 5
spam correct: 17 ham correct: 17
-----
...

k:5 laplace: 65
spam correct: 15 ham correct: 17
-----
k:5 laplace: 70
spam correct: 15 ham correct: 17
-----
...

k:20 laplace: 40
spam correct: 4 ham correct: 20
-----
k:20 laplace: 45
spam correct: 4 ham correct: 20
-----
...

k:70 laplace: 65
spam correct: 3 ham correct: 20
-----
k:70 laplace: 70
spam correct: 3 ham correct: 20
-----
```

From our observations, we have determined the best parameters with  $k = 0$  and  $w = 5$ . With these parameters, we are able to correctly classify 34/40 of the evaluation emails. An interesting observation is that as  $k$  increased, the correctness of classifying ham emails increased, while the correctness of classifying spam emails

decreased staggeringly. With a low  $k$ , both ham and spam emails were classified correctly on average 85

Having a low  $k$  allows more words to be added to the features (lexicon), and consequently adds more parameters to the model. Having  $k = 0$  means that every word that appears in any email will be added to the feature set. Intuitively, this seems bad because this allows uncommon or seemingly negligible words to affect classification. One hypothesis why  $k = 0$  seems to work particularly well in classifying spam emails is that rare words that would otherwise not be included in the feature set if  $k$  was large are included. By observation, spam emails seem to be composed of many of these rare, uncommon words. Thus, a low  $k$  allows words that are representative of spam emails to be included in the parameters, which in turn increases the correctness of spam classification.

Having a larger  $k$  allows only the likelihoods of very common words to be included in the parameters. Ham emails tend to be composed of more legitimate, common words than spam, so that's perhaps why the correctness of classifying spam converged to 100. This is why the classifier becomes so poor at classifying spam. When classifying a spam email, when a word is encountered that is included in the features set, the likelihood of that word appearing in a ham email is significantly higher than the likelihood of that word appearing in a spam.

A large  $k$  compromises the accuracy of the classifying spam for a marginal increase in accuracy of classifying ham.

Our results on the test data with  $k = 0$  and  $w = 5$ :

The number of HAM emails that were classified correctly: 91

The number of SPAM emails that were classified correctly: 91

The accuracy of classifying Ham is 91.0%

The accuracy of classifying Spam is 91.0%

In a realistic spam filter, we will probably increase  $k$  to prevent ham from being classified as spam. To the user, it is more critical to have nonspam included in the inbox than to have some nonspam in the spam folder.

We built upon our methods to calculate word maps for emails to include n-gram sequencing:

```
private void calculateWordMap(int n) {
    String[] words =
        this.originalEmailString.split("[^a-zA-Z]+");
    wordMap = new HashMap<String, Integer>();
    for(int i = 0; i < words.length - n + 1; i++)
    {
        String phrase = "";
        for(int j = 0; j < n; j++) {
            if(j == n - 1) {
                phrase += words[i + j];
            } else {
                phrase += words[i + j] + " ";
            }
        }
    }
}
```

```

    if(!wordMap.containsKey(phrase)) {
        wordMap.put(phrase, 1);
    } else {
        wordMap.put(phrase, wordMap.get(phrase)
            + 1);
    }
}
}

```

---

A little research suggests that for spam-filtering, n-grams with  $3 < n < 4$  is ideal. However, with most k-Laplace configurations, higher n-grams seem to reduce the accuracy for spam classification and improves the accuracy for ham classification. This is not of much to us since spam classification accuracy fluctuates the most and needs more reinforcement than ham classification accuracy. This is most likely an issue with our spam classification algorithm as well as the nature of the email training set.

### III. FUTURE IMPROVEMENTS

A future improvement in our classifier would involve distinguishing between upper case and lower case words, since we noticed that spam emails generally have more upper case words.

Something that we considered was using ML classification instead of MAP. ML is the same as MAP, without including the *prior* probability. In other words, ML does not take into account any predetermined likelihoods. In our classification task, we determined the *prior* probabilities to be 0.5 because our training data was 50% ham and 50% spam. We experimented with removing this probability and our results were the same. In another scenario where we were given a training set more representative of actual emails (more ham than spams, for example 75% ham and 25% spam), MAP would perform better than ML because the *prior* probabilities in this case are more significant.

### IV. INDIVIDUAL CONTRIBUTIONS

- a. Hu, B.* Bayesian network solutions
- b. Luong, B.* Implementation of Naive Bayes Classifier and analysis of results
- c. Yan, S.* Analysis of ML/MAP estimation, Implementation of extra features (n-gram)

---

## Appendix

---

The value K used in training is 3  
 The Laplace smoothing constant using in training is 25  
 Training is using 1-gram model  
 The number of HAM emails that were classified correctly: 99  
 The number of SPAM emails that were classified correctly: 74  
 The accuracy of classifying Ham is 99.0%  
 The accuracy of classifying Spam is 74.0%  
 An example of HAM email that was classified correctly:  
 An example of HAM email that was classified incorrectly:  
 An example of SPAM email that was classified correctly:  
 Subject: get that new car 8434people nowthe weather or  
 climate in any particular environment can change and affect  
 what people eat and how much of it they are able to eat .  
 An example of SPAM email that was classified incorrectly:  
 Subject: re : feeling tired ?

Altering k-values and Laplace soothing...

```
k:0 laplace: 0
spam correct: 5 ham correct: 20
-----
k:0 laplace: 5
spam correct: 17 ham correct: 17
-----
k:0 laplace: 10
spam correct: 17 ham correct: 16
-----
k:0 laplace: 15
spam correct: 17 ham correct: 16
-----
k:0 laplace: 20
spam correct: 17 ham correct: 16
-----
k:0 laplace: 25
spam correct: 17 ham correct: 16
-----
k:0 laplace: 30
spam correct: 17 ham correct: 16
-----
k:0 laplace: 35
spam correct: 17 ham correct: 16
-----
k:0 laplace: 40
spam correct: 17 ham correct: 16
-----
k:0 laplace: 45
spam correct: 17 ham correct: 16
-----
k:0 laplace: 50
spam correct: 17 ham correct: 16
-----
k:0 laplace: 55
spam correct: 17 ham correct: 16
-----
k:0 laplace: 60
spam correct: 17 ham correct: 16
-----
k:0 laplace: 65
spam correct: 17 ham correct: 16
-----
k:0 laplace: 70
spam correct: 17 ham correct: 16
-----
k:5 laplace: 0
spam correct: 9 ham correct: 18
-----
k:5 laplace: 5
spam correct: 15 ham correct: 19
-----
k:5 laplace: 10
spam correct: 12 ham correct: 19
-----
k:5 laplace: 15
spam correct: 12 ham correct: 19
-----
k:5 laplace: 20
spam correct: 12 ham correct: 19
-----
k:5 laplace: 25
spam correct: 12 ham correct: 19
-----
k:5 laplace: 30
spam correct: 12 ham correct: 18
```

```
-----
k:5 laplace: 35
spam correct: 12 ham correct: 18
-----
k:5 laplace: 40
spam correct: 12 ham correct: 18
-----
k:5 laplace: 45
spam correct: 14 ham correct: 18
-----
k:5 laplace: 50
spam correct: 14 ham correct: 17
-----
k:5 laplace: 55
spam correct: 14 ham correct: 17
-----
k:5 laplace: 60
spam correct: 15 ham correct: 17
-----
k:5 laplace: 65
spam correct: 15 ham correct: 17
-----
k:5 laplace: 70
spam correct: 15 ham correct: 17
-----
k:10 laplace: 0
spam correct: 9 ham correct: 19
-----
k:10 laplace: 5
spam correct: 11 ham correct: 19
-----
k:10 laplace: 10
spam correct: 7 ham correct: 20
-----
k:10 laplace: 15
spam correct: 5 ham correct: 20
-----
k:10 laplace: 20
spam correct: 5 ham correct: 20
-----
k:10 laplace: 25
spam correct: 5 ham correct: 20
-----
k:10 laplace: 30
spam correct: 5 ham correct: 20
-----
k:10 laplace: 35
spam correct: 5 ham correct: 20
-----
k:10 laplace: 40
spam correct: 5 ham correct: 20
-----
k:10 laplace: 45
spam correct: 5 ham correct: 20
-----
k:10 laplace: 50
spam correct: 5 ham correct: 20
-----
k:10 laplace: 55
spam correct: 5 ham correct: 20
-----
k:10 laplace: 60
spam correct: 5 ham correct: 20
-----
k:10 laplace: 65
spam correct: 5 ham correct: 20
-----
k:10 laplace: 70
spam correct: 5 ham correct: 20
-----
k:15 laplace: 0
spam correct: 7 ham correct: 19
-----
k:15 laplace: 5
spam correct: 7 ham correct: 19
-----
k:15 laplace: 10
spam correct: 5 ham correct: 20
-----
k:15 laplace: 15
spam correct: 5 ham correct: 20
-----
k:15 laplace: 20
```

spam correct: 5 ham correct: 20	k:25 laplace: 10
-----	spam correct: 6 ham correct: 19
k:15 laplace: 25	-----
spam correct: 5 ham correct: 20	k:25 laplace: 15
-----	spam correct: 6 ham correct: 20
k:15 laplace: 30	-----
spam correct: 5 ham correct: 20	k:25 laplace: 20
-----	spam correct: 6 ham correct: 20
k:15 laplace: 35	-----
spam correct: 5 ham correct: 20	k:25 laplace: 25
-----	spam correct: 6 ham correct: 20
k:15 laplace: 40	-----
spam correct: 5 ham correct: 20	k:25 laplace: 30
-----	spam correct: 5 ham correct: 20
k:15 laplace: 45	-----
spam correct: 5 ham correct: 20	k:25 laplace: 35
-----	spam correct: 4 ham correct: 20
k:15 laplace: 50	-----
spam correct: 4 ham correct: 20	k:25 laplace: 40
-----	spam correct: 4 ham correct: 20
k:15 laplace: 55	-----
spam correct: 4 ham correct: 20	k:25 laplace: 45
-----	spam correct: 4 ham correct: 20
k:15 laplace: 60	-----
spam correct: 4 ham correct: 20	k:25 laplace: 50
-----	spam correct: 4 ham correct: 20
k:15 laplace: 65	-----
spam correct: 4 ham correct: 20	k:25 laplace: 55
-----	spam correct: 3 ham correct: 20
k:15 laplace: 70	-----
spam correct: 4 ham correct: 20	k:25 laplace: 60
-----	spam correct: 3 ham correct: 20
k:20 laplace: 0	-----
spam correct: 6 ham correct: 19	k:25 laplace: 65
-----	spam correct: 3 ham correct: 20
k:20 laplace: 5	-----
spam correct: 6 ham correct: 19	k:25 laplace: 70
-----	spam correct: 3 ham correct: 20
k:20 laplace: 10	-----
spam correct: 5 ham correct: 20	k:30 laplace: 0
-----	spam correct: 12 ham correct: 19
k:20 laplace: 15	-----
spam correct: 5 ham correct: 20	k:30 laplace: 5
-----	spam correct: 9 ham correct: 19
k:20 laplace: 20	-----
spam correct: 5 ham correct: 20	k:30 laplace: 10
-----	spam correct: 7 ham correct: 19
k:20 laplace: 25	-----
spam correct: 5 ham correct: 20	k:30 laplace: 15
-----	spam correct: 7 ham correct: 20
k:20 laplace: 30	-----
spam correct: 5 ham correct: 20	k:30 laplace: 20
-----	spam correct: 7 ham correct: 20
k:20 laplace: 35	-----
spam correct: 4 ham correct: 20	k:30 laplace: 25
-----	spam correct: 7 ham correct: 20
k:20 laplace: 40	-----
spam correct: 4 ham correct: 20	k:30 laplace: 30
-----	spam correct: 5 ham correct: 20
k:20 laplace: 45	-----
spam correct: 4 ham correct: 20	k:30 laplace: 35
-----	spam correct: 5 ham correct: 20
k:20 laplace: 50	-----
spam correct: 4 ham correct: 20	k:30 laplace: 40
-----	spam correct: 5 ham correct: 20
k:20 laplace: 55	-----
spam correct: 3 ham correct: 20	k:30 laplace: 45
-----	spam correct: 5 ham correct: 20
k:20 laplace: 60	-----
spam correct: 3 ham correct: 20	k:30 laplace: 50
-----	spam correct: 4 ham correct: 20
k:20 laplace: 65	-----
spam correct: 3 ham correct: 20	k:30 laplace: 55
-----	spam correct: 4 ham correct: 20
k:20 laplace: 70	-----
spam correct: 3 ham correct: 20	k:30 laplace: 60
-----	spam correct: 4 ham correct: 20
k:25 laplace: 0	-----
spam correct: 10 ham correct: 19	k:30 laplace: 65
-----	spam correct: 4 ham correct: 20
k:25 laplace: 5	-----
spam correct: 7 ham correct: 19	k:30 laplace: 70
-----	spam correct: 4 ham correct: 20

[illegible]



[illegible]

```
spam correct: 4 ham correct: 20
-----
k:70 laplace: 30
spam correct: 4 ham correct: 20
-----
k:70 laplace: 35
spam correct: 3 ham correct: 20
-----
k:70 laplace: 40
spam correct: 3 ham correct: 20
-----
k:70 laplace: 45
spam correct: 3 ham correct: 20
-----
k:70 laplace: 50
spam correct: 3 ham correct: 20
-----
k:70 laplace: 55
spam correct: 3 ham correct: 20
-----
k:70 laplace: 60
spam correct: 3 ham correct: 20
-----
k:70 laplace: 65
spam correct: 3 ham correct: 20
-----
k:70 laplace: 70
spam correct: 3 ham correct: 20
-----
k:75 laplace: 0
spam correct: 6 ham correct: 18
-----
k:75 laplace: 5
spam correct: 6 ham correct: 18
-----
k:75 laplace: 10
spam correct: 5 ham correct: 19
-----
k:75 laplace: 15
spam correct: 4 ham correct: 20
-----
k:75 laplace: 20
spam correct: 4 ham correct: 20
-----
k:75 laplace: 25
spam correct: 4 ham correct: 20
-----
k:75 laplace: 30
spam correct: 4 ham correct: 20
-----
k:75 laplace: 35
spam correct: 4 ham correct: 20
-----
k:75 laplace: 40
spam correct: 4 ham correct: 20
-----
k:75 laplace: 45
spam correct: 4 ham correct: 20
-----
k:75 laplace: 50
spam correct: 4 ham correct: 20
-----
k:75 laplace: 55
spam correct: 4 ham correct: 20
-----
k:75 laplace: 60
spam correct: 3 ham correct: 20
-----
k:75 laplace: 65
spam correct: 3 ham correct: 20
-----
k:75 laplace: 70
spam correct: 3 ham correct: 20
-----
The value K used in training is 0
The Laplace smoothing constant used in training is 5
The number of HAM emails that were classified correctly: 91
The number of SPAM emails that were classified correctly: 91
The accuracy of classifying Ham is 91.0%
The accuracy of classifying Spam is 91.0%
```

The value K used in training is 3  
 The Laplace smoothing constant using in training is 25  
 Training is using 2-gram model  
 The number of HAM emails that were classified correctly: 100  
 The number of SPAM emails that were classified correctly: 11  
 The accuracy of classifying Ham is 100.0%  
 The accuracy of classifying Spam is 11.0%  
 An example of HAM email that was classified correctly:  
 An example of HAM email that was classified incorrectly:  
 An example of SPAM email that was classified correctly:  
 Subject: 90 % discounts on microsoft , adobe , autodesk ,  
 corel software ! tzvtwhqeldlmicrosoft windows xp professional  
 2002 \$ 50 retail price : \$ 270 . 99 our low price :  
 \$ 50 you save : \$ 220 bromine cheetah adobe photoshop  
 7 . 0 \$ 60 retail price : \$ 609 . 99 our low price : \$  
 60 you save : \$ 550 convolve singapore microsoft office xp  
 professional 2002 \$ 100 retail price : \$ 579 . 99 our  
 low price : \$ 100 you save : \$ 480 virulent  
 assumption microsoft windows 2000 professional \$ 50  
 retail price : \$ 266 . 99 our low price : \$ 50 you save :  
 \$ 216 . 99 shortish ncaa adobe pagemaker 7 . 0 \$ 60 retail  
 price : \$ 404 . 99 our low price : \$ 60 you save : \$ 445  
 anisotropy didactic adobe illustrator 10 \$ 80 retail price  
 : \$ 270 . 99 our low price : \$ 80 you save : \$ 190 stacy  
 convalesce magi 10 th carnage vad arrest centrifuge  
 anchorite posthumous dyingloren armhole fable  
 antithetic affiliate  
 An example of SPAM email that was classified incorrectly:  
 Subject: get that new car 8434people nowthe weather or  
 climate in any particular environment can change and  
 affect what people eat and how much of it they are able  
 to eat .

Altering k-values and Laplace soothing...

```
k:0 laplace: 0
spam correct: 4 ham correct: 20
-----
k:0 laplace: 5
spam correct: 14 ham correct: 19
-----
k:0 laplace: 10
spam correct: 14 ham correct: 19
-----
k:0 laplace: 15
spam correct: 14 ham correct: 19
-----
k:0 laplace: 20
spam correct: 14 ham correct: 19
-----
k:0 laplace: 25
spam correct: 14 ham correct: 19
-----
k:0 laplace: 30
spam correct: 14 ham correct: 19
-----
k:0 laplace: 35
spam correct: 14 ham correct: 19
-----
k:0 laplace: 40
spam correct: 14 ham correct: 19
-----
k:0 laplace: 45
spam correct: 14 ham correct: 19
-----
k:0 laplace: 50
spam correct: 14 ham correct: 19
-----
k:0 laplace: 55
spam correct: 14 ham correct: 19
-----
k:0 laplace: 60
spam correct: 14 ham correct: 19
-----
k:0 laplace: 65
spam correct: 14 ham correct: 19
-----
k:0 laplace: 70
spam correct: 14 ham correct: 19
-----
k:5 laplace: 0
spam correct: 7 ham correct: 19
-----
```

```
k:5 laplace: 5
spam correct: 6 ham correct: 19
-----
k:5 laplace: 10
spam correct: 3 ham correct: 19
-----
k:5 laplace: 15
spam correct: 3 ham correct: 20
-----
k:5 laplace: 20
spam correct: 3 ham correct: 20
-----
k:5 laplace: 25
spam correct: 2 ham correct: 20
-----
k:5 laplace: 30
spam correct: 1 ham correct: 20
-----
k:5 laplace: 35
spam correct: 1 ham correct: 20
-----
k:5 laplace: 40
spam correct: 1 ham correct: 20
-----
k:5 laplace: 45
spam correct: 1 ham correct: 20
-----
k:5 laplace: 50
spam correct: 1 ham correct: 20
-----
k:5 laplace: 55
spam correct: 1 ham correct: 20
-----
k:5 laplace: 60
spam correct: 1 ham correct: 20
-----
k:5 laplace: 65
spam correct: 1 ham correct: 20
-----
k:5 laplace: 70
spam correct: 1 ham correct: 20
-----
k:10 laplace: 0
spam correct: 9 ham correct: 16
-----
k:10 laplace: 5
spam correct: 8 ham correct: 17
-----
k:10 laplace: 10
spam correct: 4 ham correct: 18
-----
k:10 laplace: 15
spam correct: 3 ham correct: 18
-----
k:10 laplace: 20
spam correct: 2 ham correct: 19
-----
k:10 laplace: 25
spam correct: 2 ham correct: 20
-----
k:10 laplace: 30
spam correct: 2 ham correct: 20
-----
k:10 laplace: 35
spam correct: 2 ham correct: 20
-----
k:10 laplace: 40
spam correct: 1 ham correct: 20
-----
k:10 laplace: 45
spam correct: 1 ham correct: 20
-----
k:10 laplace: 50
spam correct: 1 ham correct: 20
-----
k:10 laplace: 55
spam correct: 1 ham correct: 20
-----
k:10 laplace: 60
spam correct: 1 ham correct: 20
-----
k:10 laplace: 65
spam correct: 1 ham correct: 20
-----
```

k:10 laplace: 70 spam correct: 1 ham correct: 20	spam correct: 0 ham correct: 20
k:15 laplace: 0 spam correct: 6 ham correct: 17	k:20 laplace: 60 spam correct: 0 ham correct: 20
k:15 laplace: 5 spam correct: 4 ham correct: 17	k:20 laplace: 65 spam correct: 0 ham correct: 20
k:15 laplace: 10 spam correct: 4 ham correct: 18	k:20 laplace: 70 spam correct: 0 ham correct: 20
k:15 laplace: 15 spam correct: 2 ham correct: 18	k:25 laplace: 0 spam correct: 4 ham correct: 20
k:15 laplace: 20 spam correct: 1 ham correct: 18	k:25 laplace: 5 spam correct: 4 ham correct: 20
k:15 laplace: 25 spam correct: 1 ham correct: 18	k:25 laplace: 10 spam correct: 3 ham correct: 20
k:15 laplace: 30 spam correct: 1 ham correct: 18	k:25 laplace: 15 spam correct: 3 ham correct: 20
k:15 laplace: 35 spam correct: 1 ham correct: 20	k:25 laplace: 20 spam correct: 3 ham correct: 20
k:15 laplace: 40 spam correct: 1 ham correct: 20	k:25 laplace: 25 spam correct: 3 ham correct: 20
k:15 laplace: 45 spam correct: 1 ham correct: 20	k:25 laplace: 30 spam correct: 3 ham correct: 20
k:15 laplace: 50 spam correct: 1 ham correct: 20	k:25 laplace: 35 spam correct: 3 ham correct: 20
k:15 laplace: 55 spam correct: 0 ham correct: 20	k:25 laplace: 40 spam correct: 3 ham correct: 20
k:15 laplace: 60 spam correct: 0 ham correct: 20	k:25 laplace: 45 spam correct: 3 ham correct: 20
k:15 laplace: 65 spam correct: 0 ham correct: 20	k:25 laplace: 50 spam correct: 0 ham correct: 20
k:15 laplace: 70 spam correct: 0 ham correct: 20	k:25 laplace: 55 spam correct: 0 ham correct: 20
k:20 laplace: 0 spam correct: 4 ham correct: 18	k:25 laplace: 60 spam correct: 0 ham correct: 20
k:20 laplace: 5 spam correct: 3 ham correct: 18	k:25 laplace: 65 spam correct: 0 ham correct: 20
k:20 laplace: 10 spam correct: 3 ham correct: 18	k:25 laplace: 70 spam correct: 0 ham correct: 20
k:20 laplace: 15 spam correct: 2 ham correct: 18	k:30 laplace: 0 spam correct: 5 ham correct: 20
k:20 laplace: 20 spam correct: 2 ham correct: 18	k:30 laplace: 5 spam correct: 4 ham correct: 20
k:20 laplace: 25 spam correct: 2 ham correct: 18	k:30 laplace: 10 spam correct: 4 ham correct: 20
k:20 laplace: 30 spam correct: 1 ham correct: 18	k:30 laplace: 15 spam correct: 3 ham correct: 20
k:20 laplace: 35 spam correct: 1 ham correct: 20	k:30 laplace: 20 spam correct: 3 ham correct: 20
k:20 laplace: 40 spam correct: 1 ham correct: 20	k:30 laplace: 25 spam correct: 3 ham correct: 20
k:20 laplace: 45 spam correct: 1 ham correct: 20	k:30 laplace: 30 spam correct: 3 ham correct: 20
k:20 laplace: 50 spam correct: 0 ham correct: 20	k:30 laplace: 35 spam correct: 3 ham correct: 20
k:20 laplace: 55	k:30 laplace: 40 spam correct: 3 ham correct: 20

[illegible]

```

spam correct: 3 ham correct: 20
-----
k:50 laplace: 25
spam correct: 3 ham correct: 20
-----
k:50 laplace: 30
spam correct: 3 ham correct: 20
-----
k:50 laplace: 35
spam correct: 3 ham correct: 20
-----
k:50 laplace: 40
spam correct: 3 ham correct: 20
-----
k:50 laplace: 45
spam correct: 0 ham correct: 20
-----
k:50 laplace: 50
spam correct: 0 ham correct: 20
-----
k:50 laplace: 55
spam correct: 0 ham correct: 20
-----
k:50 laplace: 60
spam correct: 0 ham correct: 20
-----
k:50 laplace: 65
spam correct: 0 ham correct: 20
-----
k:50 laplace: 70
spam correct: 0 ham correct: 20
-----
k:55 laplace: 0
spam correct: 6 ham correct: 20
-----
k:55 laplace: 5
spam correct: 6 ham correct: 20
-----
k:55 laplace: 10
spam correct: 6 ham correct: 20
-----
k:55 laplace: 15
spam correct: 3 ham correct: 20
-----
k:55 laplace: 20
spam correct: 3 ham correct: 20
-----
k:55 laplace: 25
spam correct: 3 ham correct: 20
-----
k:55 laplace: 30
spam correct: 3 ham correct: 20
-----
k:55 laplace: 35
spam correct: 3 ham correct: 20
-----
k:55 laplace: 40
spam correct: 3 ham correct: 20
-----
k:55 laplace: 45
spam correct: 0 ham correct: 20
-----
k:55 laplace: 50
spam correct: 0 ham correct: 20
-----
k:55 laplace: 55
spam correct: 0 ham correct: 20
-----
k:55 laplace: 60
spam correct: 0 ham correct: 20
-----
k:55 laplace: 65
spam correct: 0 ham correct: 20
-----
k:55 laplace: 70
spam correct: 0 ham correct: 20
-----
k:60 laplace: 0
spam correct: 0 ham correct: 20
-----
k:60 laplace: 5
spam correct: 0 ham correct: 20
-----
k:60 laplace: 10
spam correct: 0 ham correct: 20
-----
k:60 laplace: 15
spam correct: 0 ham correct: 20
-----
k:60 laplace: 20
spam correct: 0 ham correct: 20
-----
k:60 laplace: 25
spam correct: 0 ham correct: 20
-----
k:60 laplace: 30
spam correct: 0 ham correct: 20
-----
k:60 laplace: 35
spam correct: 0 ham correct: 20
-----
k:60 laplace: 40
spam correct: 0 ham correct: 20
-----
k:60 laplace: 45
spam correct: 0 ham correct: 20
-----
k:60 laplace: 50
spam correct: 0 ham correct: 20
-----
k:60 laplace: 55
spam correct: 0 ham correct: 20
-----
k:60 laplace: 60
spam correct: 0 ham correct: 20
-----
k:60 laplace: 65
spam correct: 0 ham correct: 20
-----
k:60 laplace: 70
spam correct: 0 ham correct: 20
-----

```

```

-----
k:70 laplace: 0
spam correct: 0 ham correct: 20
-----
k:70 laplace: 5
spam correct: 0 ham correct: 20
-----
k:70 laplace: 10
spam correct: 0 ham correct: 20
-----
k:70 laplace: 15
spam correct: 0 ham correct: 20
-----
k:70 laplace: 20
spam correct: 0 ham correct: 20
-----
k:70 laplace: 25
spam correct: 0 ham correct: 20
-----
k:70 laplace: 30
spam correct: 0 ham correct: 20
-----
k:70 laplace: 35
spam correct: 0 ham correct: 20
-----
k:70 laplace: 40
spam correct: 0 ham correct: 20
-----
k:70 laplace: 45
spam correct: 0 ham correct: 20
-----
k:70 laplace: 50
spam correct: 0 ham correct: 20
-----
k:70 laplace: 55
spam correct: 0 ham correct: 20
-----
k:70 laplace: 60
spam correct: 0 ham correct: 20
-----
k:70 laplace: 65
spam correct: 0 ham correct: 20
-----
k:70 laplace: 70
spam correct: 0 ham correct: 20
-----
k:75 laplace: 0
spam correct: 0 ham correct: 20
-----
k:75 laplace: 5
spam correct: 0 ham correct: 20
-----
k:75 laplace: 10
spam correct: 0 ham correct: 20
-----
k:75 laplace: 15
spam correct: 0 ham correct: 20
-----
k:75 laplace: 20
spam correct: 0 ham correct: 20
-----
k:75 laplace: 25
spam correct: 0 ham correct: 20
-----
k:75 laplace: 30
spam correct: 0 ham correct: 20
-----
k:75 laplace: 35
spam correct: 0 ham correct: 20
-----
k:75 laplace: 40
spam correct: 0 ham correct: 20
-----
k:75 laplace: 45
spam correct: 0 ham correct: 20
-----
k:75 laplace: 50
spam correct: 0 ham correct: 20
-----
k:75 laplace: 55
spam correct: 0 ham correct: 20
-----
k:75 laplace: 60
spam correct: 0 ham correct: 20
-----
k:75 laplace: 65
spam correct: 0 ham correct: 20
-----
k:75 laplace: 70
spam correct: 0 ham correct: 20
-----
The value K used in training is 0
The Laplace smoothing constant used in training is 5
The number of HAM emails that were classified correctly: 98
The number of SPAM emails that were classified correctly: 73
The accuracy of classifying Ham is 98.0%
The accuracy of classifying Spam is 73.0%

```