

Misallocation of talent, human capital formation, and development in Brazil

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Abstract

In this article, we elaborate a general equilibrium model to study why the relative salary of teachers is higher in regions with low and middle income in Brazil and how it affects the product. In this model, occupational choices, especially for teachers, are affected by distortions in the labor market and in educational goods, which in turn have a direct impact on the formation of human capital. After calibrating the model, using the Generalized Moments Method, for the Brazilian scenario, it was found that there is a misallocation of talent in the Brazilian economy and reallocating more skilled people to the teaching occupation could increase GDP by 68.28 %.

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1 Introduction

Much effort has been made to try to find the causes and barriers to economic growth, and one of the approaches that has stood out is that of misallocation. This approach is used to measure the distortions of the economy, to understand the causes of misallocation of resources and to assess losses in TPF and output. Several papers on misallocation emphasize that, in an economy that presents misallocation of resources, there may be losses in productivity and in the aggregate output. Misallocation of capital, credit, and talent has been pointed out as possible barriers to growth ([Banerjee and Duflo, 2005](#); [Restuccia and Rogerson, 2008](#); [Hsieh and Klenow, 2009](#); [Hsieh et al., 2019](#)).

Misallocation of talent across occupations, and sectors may be a consequence of race and gender discrimination, social norms and culture ([Restuccia and Rogerson, 2017](#)). As can be seen in [Hnatkovska et al. \(2012\)](#) the misallocation of talent in India comes from the caste system. And this system causes a social and economic gap between the privileged and non-privileged castes. On the other hand, to Brazilian data [Café \(2018\)](#) shows that there is overqualification of workers in the public sector in relation to the private sector. This is due to the fact that public tenders cause misallocation of talent, especially those in which the evaluation is not related to the work that will be performed. According to [Café \(2018\)](#), this fact leads to a loss of productivity in the economy.

In an interesting article [Hsieh et al. \(2019\)](#) identifies in US economy that in 1960, 94% of doctors and lawyers were white men and in 2010, the fraction was 62%. So, there are misallocation of talent in both 1960 and 2010. They suppose that distribution of talent for each occupation is identical for whites, blacks, men, and women. And introduce two main elements that will cause individuals to choose occupations where they do not comparative advantage: Discrimination in the labor market and barriers to forming human capital. The main finding is that one fifth of growth in market GDP per person over the last five decades can be traced to declining occupational barriers.

In [Hsieh et al. \(2019\)](#) it is clear the importance of human capital formation for economic growth. Human capital is one of the main elements in the economic growth and development process. An extensive body of literature supports the crucial role of human capital in labor productivity determination by means of an earnings equation in micro and macroeconomic studies as [Patrinos and Psacharopoulos \(2002\)](#), and [Ferreira et al. \(2004\)](#). Human capital is also crucial on economic growth and development by facilitating innovation and diffusion of technology as in [Romer \(1990\)](#), [Mankiw et al. \(1992\)](#), [Borensztein et al. \(1998\)](#) and [Benhabib and Spiegel \(2005\)](#).

The more recent literature has emphasized the relevance of human capital quality in economic growth. [Hanushek and Woessmann \(2012\)](#) argue that the Latin American countries lagged behind over the period 1960-2000 because of their students' poor performance in educational achievement. Latin American countries have been classified as the poorest in score tests across several developed and developing countries, which have been critical to their poor economic performance ([Hanushek and Woessmann, 2012](#)).

Going further, many studies point to the relevance of teachers in the students learning process (Woessmann, 2016; Barros and Delalilbera, 2018; Hanushek et al., 2019). Relying on three different strategies and controlling for several variables that are related to students performance as family background, school inputs, institutional features of the school systems, and cross-country differences in educational inputs, Hanushek et al. (2019) find a robust positive relationship between teacher cognitive skills and student performance measure by the Programme for International Student Assessment (PISA) scores. The cognitive skills of teachers are even more important to students' performance than the cognitive skills of their parents (Hanushek et al., 2019).

Using the student's mathematics test score of the PISA, Woessmann (2016), through an educational production function, points to the relevance of teachers' quality measured by the relative salary and education of teachers on students performance. Woessmann (2016) argues that higher teacher's wages have a positive influence on recruiting higher ability individuals into teaching.

Contrary to what would be expected by the literature, the Brazilian states present an inverse relationship between teachers' relative salary and economic development as can be seen in Barros and Delalilbera (2018). They explain this phenomenon via misallocation of workers where market frictions determine the occupational choice of individuals. They pointed that teachers' human capital is important input for the formation of the human capital of the entire workforce in an economy. And the occupational choice of multi-ability workers is driven by labor market incentives (net wage) and the costs of investment in specific human capital (acquisition of education).

In this context, we follow Barros and Delalilbera (2018) theoretical framework to discuss this inverse relationship between teachers' relative wage and regional economic development across the Brazilian States. However, we modified the Barros and Delalilbera (2018) model to incorporate both the quality and quantity of teachers as inputs for the formation of human capital. The importance of this is clear when we take into account the fact that high distortions in a specific occupation reduce the proportion of workers in that occupation. Therefore, the number of talented people with high human capital choosing this occupation could decrease. And in the specific case of teachers, who are essential inputs for the formation of human capital, this would harm the entire workforce.

Also, we calibrated the model to show how the economy is more sensitive to changes in distortions of the teachers' occupation than in other occupations. Finally, we analyze what would be the best allocation in the economy. Our main result suggest that there are misallocation of talent in Brazilian economy and allocate high skilled people to teacher occupation would increase Brazilian GDP in 68.28%.

To accomplish our goals, this article is divided into four more sections. In section 2 we present the general equilibrium model that we developed based on Barros and Delalilbera (2018) and Hsieh et al. (2019) to study how distortions in the labor and educational markets affect the formation of human capital and consequently the output. In section 3 we explain how

this model was calibrated using data from the Brazilian economy. The results of the calibration and some counterfactual exercises are presented in section 4, and finally, the article is completed in section 5.

2 Model

We build a Roy model based on [Hsieh et al. \(2019\)](#) and [Barros and Delalilbera \(2018\)](#) to study how frictions in labor and educational markets affect the formation of human capital in an economy where teachers have a central role. We consider a country divided into $R \in \mathbb{N}$ independent regions (states). In each region, there is a continuum of workers that choose one of the $N \in \mathbb{N}$ occupations in the economy. A person born in region r can not work in a different region.¹ There is a large number of homogeneous competitive firms, so firms take prices as given. Firms hire workers in all regions and occupations to produce a single good. The production function used by the firms is given by

$$Y = \sum_{r=1}^R \sum_{i=1}^N A_r H_{ir}, \quad (1)$$

where Y is output, A_r is Total Factor Productivity (TFP) of region r and H_{ir} is the aggregate human capital of people working in occupation i at region r . Output can be consumed or used as an educational good, as shall become clear when we present the worker's problem. Thus, the firms problem is choosing labor in terms of efficient units (aggregate human capital) to maximize profit, taking wages (w_{ir}) of each occupation in each region as given.

$$\max_{H_{ir} \geq 0} \left[\sum_{r=1}^R \sum_{i=1}^N A_r H_{ir} - \sum_{r=1}^R \sum_{i=1}^N w_{ir} H_{ir} \right] \quad (2)$$

The solution of the problem described above is simple. The demand for human capital is given by

$$H_{ir}^d = \begin{cases} 0 & \text{if } A_r < w_{ir} \\ x \in \mathbb{R}_+ & \text{if } A_r = w_{ir} \\ \infty & \text{if } A_r > w_{ir} \end{cases} \quad (3)$$

Workers have idiosyncratic abilities for each occupation. In a world with multiple occupations, some workers can have a high talent for many occupations, some for only one, while others may lack the skills for any occupation in the economy. Individuals value consumption and leisure, which we model as the time not spent in school. Each worker is endowed with one

¹According to the Brazilian Census of 2010, almost 80% of the population have never moved from the home state. If we consider people who are in the same state for more than 20 years, this number increases to almost 88%. Therefore, we believe that the assumption that people do not move from the region they are born is not an extreme one.

unit of time to study or to consume as leisure. The utility of a person is given by

$$U(c, s) = c^\beta (1 - s) \quad (4)$$

where c represents consumption, s is time spent at school, and β is a parameter giving the importance of consumption in relation to leisure.

We follow [Hsieh et al. \(2019\)](#) and introduce two frictions (or distortions) in our model. People working in occupation i at region r is paid a net wage of $(1 - \tau_{ir}^w)w_{ir}$ where τ_{ir}^w is a distortion specific for occupation i and location r . One can interpret τ_{ir}^w as an unobserved cost (or benefit) of occupation i at region r . For example, it can represent a social status or a difficulty in finding a job in a given occupation and region.

Educational choices are also distorted due to a ‘tax’ on educational goods, e . For each good invested in education, a person pays τ_{ir}^h as a ‘tax’. We can think of this distortion as representing forces that affect the cost of acquiring human capital in different occupations and regions. For example, τ_{ir}^h might indicate the difficulty of finding training to work in a specific occupation, or it can represent investments to develop math and science skills required by certain occupations.

As in [Barros and Delalilbera \(2018\)](#), we assume that human capital formation of workers depends on teachers’ quality. As a contribution of the present paper, we incorporate the quantity of teachers as another element to workers’ human capital formation. Therefore, the workers’ human capital in each region is given by

$$h_r(e, s) = T_r^\varphi s^{\phi_i} e^\eta \quad (5)$$

where e represents consumption of educational goods, s is time spent in school, η is elasticity of human capital function with respect consumption of educational goods, and $\phi_i > 0$ is the elasticity of human capital concerning time in school. Notice that this parameter varies among occupations and generates differences in schooling. Finally, T_r represents the role of teachers in the workers’ human capital formation. We set $T_r = p_{tr}^\alpha H_{tr}^{1-\alpha}$ where $\alpha \in (0, 1)$, p_{tr} is the fraction of people working as teachers, and H_{tr} is the teachers’ aggregate human capital. We use this functional form to incorporate the quality and quantity of teachers on the human capital formation of workers.²

Following [McFadden \(1974\)](#), [Eaton and Kortum \(2002\)](#), and [Barros and Delalilbera \(2018\)](#), abilities dispersion is modeled as a multivariate Fréchet distribution. Let ϵ_i be the ability of an individual in occupation i , then the distribution of abilities across occupations is:

$$F(\epsilon_1, \dots, \epsilon_N) = \exp \left[- \left(\sum_{i=1}^N \epsilon_i^{-\frac{\tilde{\theta}}{1-\rho}} \right)^{1-\rho} \right], \quad (6)$$

where $\tilde{\theta}$ governs the skill dispersion, and $\rho \in [0, 1]$ gives the correlation of individual’s skills among occupations. A higher $\tilde{\theta}$ implies smaller abilities dispersion. Also, $\rho = 1$ indicates that

²See [Krueger \(2003\)](#) and [Lakdawalla \(2006\)](#) for a discussion on teachers’ quality and quantity.

skills are perfectly correlated, while $\rho = 0$ means that an individual's skills are uncorrelated across occupations. For convenience, let $\theta = \tilde{\theta}/(1 - \rho)$.

The worker's problem can be split into two steps. First, given the occupational choice i , for which the individual has an idiosyncratic ability ϵ_i , and taking wage w_{ir} as given, each worker chooses c , e , and s to solve the following problem:

$$\max_{c,s,e} c^\beta (1 - s) \quad (7)$$

$$\text{s.t. } c = (1 - \tau_{ir}^w)h_r(e, s)\epsilon w_{ir} - (1 + \tau_{ir}^h)e.$$

Solving the problem above, we find the amount of time and goods spent on human capital accumulation:

$$s_i^* = \left(1 + \frac{1 - \eta}{\beta\phi_i}\right)^{-1}, \quad (8)$$

$$e_{ir}^*(\epsilon) = \left[\eta \left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left(1 + \frac{1 - \eta}{\beta\phi_i}\right)^{-\phi_i} \epsilon \right]^{\frac{1}{1-\eta}}. \quad (9)$$

For a given occupation, a higher elasticity of human capital concerning time leads to more time allocated to human capital accumulation. Individuals in high ϕ_i occupations acquire more schooling and have higher wages as compensation. Note that the wage and distortions do not affect schooling because they have the same effect on the return and cost of time.³ However, they can change the returns of investments in human capital goods relative to the costs, with an elasticity that is increasing in η .

Next, we substitute the expressions in equations (8) and (9) and the budget constraint into the utility function to get the following expression for the indirect utility function of occupation i :

$$D_{ir} \equiv D(\tau_{ir}^w, \tau_{ir}^h, H_{tr}, w_{ir}, \epsilon_i) = \left[\psi \left(\frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^\phi (1 - s)^{\frac{1-\eta}{\beta}} \epsilon_i \right]^{\frac{\beta}{1-\eta}}, \quad (10)$$

where $\psi = \eta^\eta (1 - \eta)^{1-\eta}$.

Therefore, the occupational choice problem reduces to picking the occupation that delivers the highest value of D_{ir} . Since talent is drawn from an extreme value distribution, the highest utility can also be characterized by an extreme value distribution (McFadden, 1974).

Proposition 1 states that the overall occupational share can be obtained by aggregating the individual optimal choice.

Proposition 1. *Aggregating across workers, the solution of individual's occupational choice*

³Barros and Delalibera (2018) have evidence that average schooling is similar across Brazilian states giving an occupation. In accord to the authors, the amplitude (max - min) of the mean of years of schooling across states is at most 2.3 years, except for agricultural worker, where this statistic amounts to 3.3

problem is:

$$p_{ir} = \frac{\tilde{w}_{ir}^\theta}{\sum_{j=1}^N \tilde{w}_{jr}^\theta}, \quad (11)$$

where p_{ir} is the fraction of workers in occupation i in region r , and:

$$\tilde{w}_{ir} = \psi \left(\frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} (1 - s_i)^{\frac{1-\eta}{\beta}}$$

Proof. Let:

$$\tilde{w}_{ir} = \psi \left(\frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} (1 - s_i)^{\frac{1-\eta}{\beta}}.$$

Then, we can rewrite equation (10) as:

$$D_{ir} = [\tilde{w}_{ir} \epsilon_i]^{\frac{\beta}{1-\eta}}.$$

Therefore, the problem solution of individual i living in region r involves picking the occupation with the highest value of $\tilde{w}_{ir} \epsilon_i$. Without loss of generality, consider the probability of an individual choosing occupation 1:

$$\begin{aligned} p_{ir} &= Pr(\tilde{w}_{1r} \epsilon_1 > \tilde{w}_{ir} \epsilon_i) \quad \forall i \neq 1 \\ &= Pr\left(\epsilon_i < \frac{\tilde{w}_{1r}}{\tilde{w}_{ir}} \epsilon_1\right) \quad \forall i \neq 1 \\ &= \int F_1(\alpha_1 \epsilon, \alpha_2 \epsilon, \dots, \alpha_N \epsilon) d\epsilon, \end{aligned} \quad (12)$$

where F_1 represents the derivative of equation (6) with respect to its first argument and $\alpha_i = \tilde{w}_{1r}/\tilde{w}_{ir}$ for $i \in \{1, 2, \dots, N\}$. Taking the derivative of equation (6) with respect to ϵ_1 , and evaluating at the appropriate arguments:

$$\begin{aligned} F_1 &= -(1-p) \left(\sum_{i=1}^N \epsilon_i^{-\theta} \right)^{-\rho} \left(\frac{-\theta \epsilon_1^{-\theta-1}}{1-\rho} \right) \exp \left[- \left(\sum_{i=1}^N \epsilon_i^{-\theta} \right)^{1-\rho} \right] \\ &= \theta \epsilon_1^{-(\theta+1)} \left(\sum_{i=1}^N \epsilon_i^{-\theta} \right)^{-\rho} \exp \left[- \left(\sum_{i=1}^N \epsilon_i^{-\theta} \right)^{1-\rho} \right] \\ &= \theta \epsilon_1^{-\theta+1} (\hat{S} \epsilon_1^{-\theta})^{-\rho} \exp[-(\hat{S} \epsilon_1^{-\theta})^{1-\rho}] \\ &= \hat{S}^{-\rho} \theta \epsilon_1^{-\theta(1-\rho)-1} \exp[-(\hat{S} \epsilon_1^{-\theta})^{1-\rho}] \\ &= \hat{S}^{-\rho} \theta \epsilon^{-\theta(1-\rho)-1} \exp[-(\hat{S} \epsilon^{-\theta})^{1-\rho}], \end{aligned}$$

where $\hat{S} = \sum_{i=1}^n \alpha_i^{-\theta}$. Then, equation (12) can be written as:

$$\begin{aligned} p_{1r} &= \int \frac{\hat{S}}{\hat{S}} \hat{S}^{-\rho} \theta \epsilon^{-\theta(1-\rho)-1} \exp[-(\hat{S} \epsilon^{-\theta})^{1-\rho}] d\epsilon \\ &= \frac{1}{\hat{S}} \int \hat{S} \hat{S}^{-\rho} \theta \epsilon^{-\theta(1-\rho)-1} \exp[-(\hat{S} \epsilon^{-\theta})^{1-\rho}] d\epsilon. \end{aligned}$$

This expression is the derivative of equation (6) with respect to ϵ . Hence, we have:

$$p_{1r} = \frac{1}{\hat{S}} \int dF(\epsilon) = \frac{1}{\hat{S}} = \frac{\tilde{w}_{1r}^\theta}{\sum_{i=1}^N \tilde{w}_{ir}^\theta}.$$

□

We can interpret \tilde{w}_{ir} as a net reward of a person from region r and occupation i with mean ability. Therefore, \tilde{w}_{ir} is composed of wage per efficiency unit, schooling, teacher's human capital, and frictions. The next proposition defines the workers' human capital in each occupation in a given region.

Proposition 2. *For a given region, the human capital of workers in occupation i is:*

$$H_{ir} = p_{ir} \mathbb{E}[h(e_{ir}, s_i) \epsilon_i | \text{person choices } i], \quad (13)$$

The average quality of workers is:

$$\mathbb{E}[h(e_{ir}, s_i) \epsilon_i | \text{person choices } i] = \gamma \left[\left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right)^\eta \tilde{h}_{ir} p_{ir}^{-\frac{1}{\theta}} \right]^{\frac{1}{1-\eta}}, \quad (14)$$

where $\gamma = \Gamma(1 - (\theta(1 - \rho)(1 - \eta))^{-1})$ is related to the mean of the Fréchet distribution for abilities and $\tilde{h}_{ir} = [(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta]^\frac{1}{1-\eta}$.

Proof. We have:

$$h(e_{ir}, s_i) \epsilon_i = (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left[\eta \left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left(1 + \frac{1 - \eta}{\beta \phi} \right)^{-\phi} \epsilon_i \right]^{\frac{\eta}{1-\eta}} s_i^{\phi_i} \epsilon_i, \quad (15)$$

where H_{ir} is the total efficiency units of labor supplied to occupation i in region r . Then,

$$\begin{aligned} H_{ir} &= p_{ir} \mathbb{E} \left\{ (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left[\eta \left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i \right]^{\frac{\eta}{1-\eta}} s_i^{\phi_i} \epsilon_i \middle| \text{person choices } i \right\} \\ &= p_{ir} \left\{ (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left[\left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right) \eta (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \right]^{\frac{\eta}{1-\eta}} s_i^{\phi_i} \mathbb{E} \left[\epsilon_i^{\frac{1}{1-\eta}} \middle| \text{person choices } i \right] \right\} \\ &= p_{ir} \tilde{h}_{ir} \left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right)^{\frac{\eta}{1-\eta}} \mathbb{E} \left[\epsilon_i^{\frac{1}{1-\eta}} \middle| \text{person choices } i \right]. \end{aligned} \quad (16)$$

To calculate this last conditional expectation, we use the Fréchet distribution. For now, we suppress the region index r because this calculation is similar in all regions. Let $y_i = \tilde{w}_i \epsilon_i$.

Since we are maximizing y_i , it inherits the extreme value distribution:

$$\begin{aligned}
Pr\left(\text{Max}_i y_i < z\right) &= Pr(\epsilon_i < z/\tilde{w}_i) \quad \forall i \\
&= F(z/\tilde{w}_1, \dots, z/\tilde{w}_N) \\
&= \exp\left[-\left(-\sum_{i=1}^N (z/\tilde{w}_i)^{-\theta}\right)^{1-\rho}\right] \\
&= \exp\left[-\left(\hat{S}z^{-\theta}\right)^{1-\rho}\right].
\end{aligned}$$

The extreme value also has a Fréchet distribution. After some algebraic manipulations it can be concluded that the distribution of ϵ^* , the workers' ability in their chosen occupation, has a Fréchet distribution:

$$G(x) = Pr(\epsilon^* < x) = \exp\left[-\left(\hat{S}^*x^{-\theta}\right)^{1-\rho}\right], \quad (17)$$

where $\hat{S}^* = \sum_{i=1}^N (\tilde{w}_i/\tilde{w}^*)^\theta$.

Finally, we can calculate the expectation of equation (16). Let i be the occupation the individual chooses, and λ some positive exponent.

$$\begin{aligned}
E(\epsilon_i^\lambda) &= \int_0^\infty \epsilon_i^\lambda dG(\epsilon) \\
&= \int_0^\infty \theta(1-\rho)\hat{S}^{*(1-\rho)}\epsilon^{-\theta(1-\rho)-1+\lambda} \exp\left[-\left(\hat{S}^*\epsilon^{-\theta}\right)^{1-\rho}\right] d\epsilon \\
&= \hat{S}^{*\lambda/\theta} \int_0^\infty x^{-\frac{\lambda}{\theta(1-\rho)}} \exp(-x) dx,
\end{aligned} \quad (18)$$

where $x = \left(\hat{S}^*\epsilon^{-\theta}\right)^{1-\rho}$. The last part of equation (18) is a gamma function which amounts to $\Gamma(1 - \lambda(\theta(1-\rho))^{-1})$.

$$\mathbb{E}\left[\epsilon_i^{\frac{1}{1-\eta}} \middle| \text{person choices } i\right] = \left(\frac{1}{p_{ir}}\right)^{\frac{1}{\theta(1-\eta)}} \Gamma\left(1 - \frac{1}{\theta(1-\rho)} \frac{1}{1-\eta}\right). \quad (19)$$

Using this result in equation (16) completes the proof. \square

This result points to a selection effect in the economy. The average quality in equation (14) is inversely related to the share of workers in occupation p_{ir} . If the distortion is high in occupation i and region r , only the most qualified workers are selected for that occupation. For example, in a region where it is easy to become a teacher, their average human capital will be small (intensive margin). On the other hand, holding the average constant, a higher share of workers in an occupation will result in higher aggregate human capital (extensive margin). The net effect depends on the parameters' relative size. If $\theta(1-\eta) > 1$, then the extensive margin dominates. Otherwise, the intensive margin dominates. Next, we solve the model for the average wage in occupation i and region r .

Corollary 1. *Let W_{ir} be the gross average wage in occupation i in region r . Then:*

$$W_{ir} = w_{ir} \mathbb{E}[h(e_{ir}, s_i) \epsilon_i] = \frac{(1 - s_i)^{-1/\beta}}{(1 - \tau_{ir}^w)} \gamma \eta \left(\sum_{i=1}^N \tilde{w}_{ir}^\theta \right)^{\frac{1}{\theta(1-\eta)}} \quad (20)$$

This result is a consequence of Proposition 2. Equation (20) shows that gross average wages in a given region differ among occupations due to schooling and labor market frictions. Occupations with higher workers' schooling or labor market frictions have greater gross average wages. In addition, equation (20) is essential to understand the average wages differences across regions. From equation (3), we conclude that in equilibrium $A_r = w_{ir}$. Then, \tilde{w}_{ir} is a function of A_r , and consequently, W_{ir} is a function of regional TFP. Therefore, frictions, schooling, and TFP are important sources of average wage variation across states. Finally, we use a standard definition of a competitive equilibrium.

Definition 1. *A competitive equilibrium in this economy consists of individual choices of $\{c, e, s\}$, an occupational choice by workers, total human capital in each occupation and region H_{ir} , final output Y , and efficiency wages w_{ir} for each occupation and region.*

- (i) *Given an occupational choice, w_{ir} , and the idiosyncratic ability ϵ , each worker chooses c , e , s to maximize utility in equation (7).*
- (ii) *Given market friction, w_{ir} , H_{it} , and ϵ , a worker chooses the occupation that maximizes D_{ir} .*
- (iii) *A representative firm hires H_{ir} to maximize profits.*
- (iv) *The occupational wage, w_{ir} , clears the labor market in each occupation and region.*
- (v) *Total output is given by the production function in equation (1).*

3 Calibration

Our calibration strategy involves choosing parameters values of our model such that the competitive equilibrium is consistent with the Brazilian states dataset for 2013.⁴ We use four state-level data from National Household Sample Survey (PNAD): years of schooling; work hours; gross earnings; and occupation. After some adjustments,⁵ we have a sample of 101740 individuals distributed among eight big groups of occupation: 1) managers (except public sector); 2) professionals of sciences and arts; 3) middle-level technicians; 4) administrative

⁴We choose this year because it is the last one before a great recession has impacted Brazil.

⁵We drop individuals with no earnings or occupation, and earnings inferior to R\$1.00/h. We select individuals between 25 and 55 years old. If we consider a week with 44 working hours and the minimum wage in Brazil of R\$ 678.00 per month in 2013, this is equivalent to R\$3.59/h. Therefore, we drop individuals that receive considerably less than the minimum wage. Concerning the occupations, we drop individuals with not well-defined occupations and those in the army. There is a code in the Brazilian Occupation Code (CBO) with workers classified in not well-defined occupations.

service; 5) service-sector; 6) sellers and service providers; 7) agriculture; 8) goods and industrial production, services, and repairs-maintenance. We aggregate groups 4, 5, and 6 into the service-sector workers. Finally, we separate those working as teachers. Thus, we have the following categories of occupation:

1. managers (except public sector);
2. professionals of sciences and arts (except teachers);
3. middle-level technicians (except teachers);
4. service-sector;
5. agriculture;
6. goods and industrial production, services and repairs-maintenance;
7. teachers.

The regions are composed of the 26 states⁶ and the Federal District (DF) of Brazil ($R = 27$). Thus, in our calibrated model presents $N = 7$ and $R = 27$.

Table 1: Constant parameters between occupation and region

Parametes	Value	Description	Source
φ	0.25	Elasticity of teacher's human capital in the human capital function	Barros and Delalilbera (2018)
η	0.25	Elasticity of educational goods in the human capital function	Barros and Delalilbera (2018)
θ	3.44	Dispersion of skills	Hsieh et al. (2013)
ρ	0.19	Correlation of an individual's skill	PNAD 2013
α	0.60	Weight of the share of teachers in T_r	Assumption
β	0.69	Consumption preference	Hsieh et al. (2013)

We follow Barros and Delalilbera (2018) and split the parameters into three groups. We calibrate the first group based on Table 1. The ϕ 's compose the second. We use equation (8) and years of schooling to estimate each occupation's ϕ . First, we compute the average years of schooling of each occupation. Then, we calculate the effective time spent in education.⁷ We assume that an individual spends eight hours a day studying during weekdays, giving 2080 hours per year. Thus, an individual spends 24% of his available time studying in a given year. As the schooling period is composed of the first 25 years of the life cycle, we divide the average years of schooling by 25 and multiply it by the time available to education (0.24).

The remaining parameters, τ 's and A 's, are calibrated using the Method of Moments by minimizing the distance between the statistics of our simulated model and the Brazilian states'

⁶Acre (AC), Alagoas (AL), Amapá (AP), Amazonas (AM), Bahia (BA), Ceará (CE), Espírito Santo (ES), Goiás (GO), Maranhão (MA), Mato Grosso (MT), Mato Grosso do Sul (MS), Minas Gerais (MG), Pará (PA), Paraíba (PB), Paraná (PR), Pernambuco (PE), Piauí (PI), Rio de Janeiro (RJ), Rio Grande do Norte (RN), Rio Grande do Sul (RS), Rondônia (RO), Roraima (RR), Santa Catarina (SC), São Paulo (SP), Sergipe (SE), Tocantins (TO).

⁷An agent can consume leisure in the education step.

data⁸. In this group, we have $2NR + R$ parameters. We use two statistics for each occupation and region: the share of workers; and the average gross wage.⁹ In our model those statistics are described in equations 11 and 20. Also, we use the FOC's of firm's problem, where $w_{i,r} = A_r \quad \forall i, r$.

Each occupation's share of workers sum to one in each region, $\sum_{i=1}^N p_{ir} = 1$. This means that we only have $(N - 1)R$ independent statistics in each region. Thus, we assume that $\tau_{1r}^h = 0 \quad \forall r$. Beside, we assume that $\tau_{1r}^w = \tau_1^w \quad \forall r$, i.e., that frictions in occupation 1 are equal across regions. Also, we fix A_R , the last region's TFP.¹⁰ Thus, we have the same number of statistics and parameters to be fitted $(2(N - 1)R + R)$.

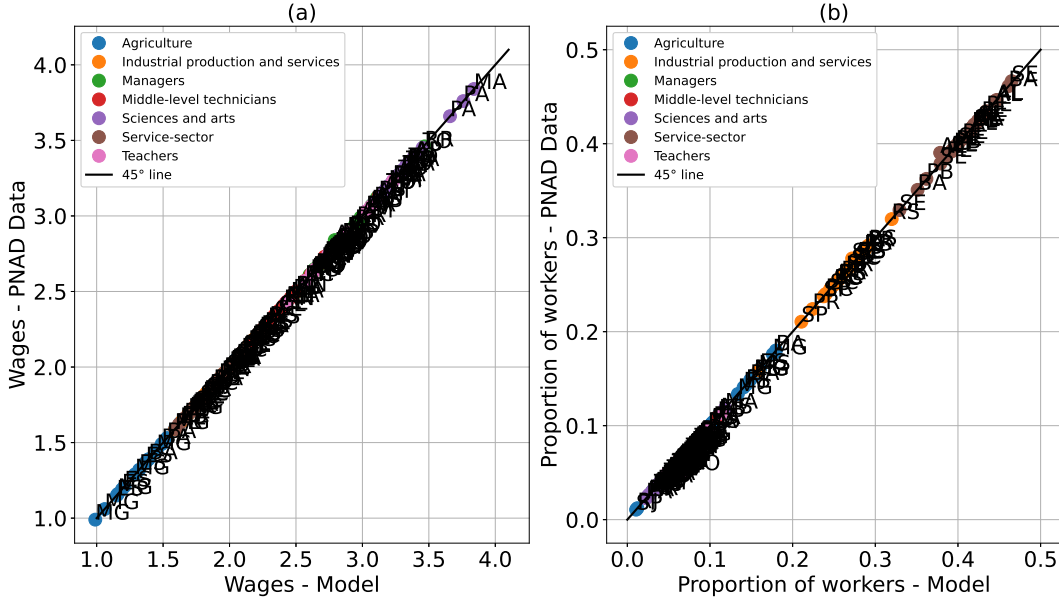
We define the following objective function:

$$\mathcal{M} = \sum_{i=1, r=1}^{N, R} \left(\frac{W_{ir}^M - W_{ir}^D}{W_{ir}^D} \right)^2 + \sum_{i=1, r=1}^{N-1, R} \left(\frac{p_{ir}^M - p_{ir}^D}{p_{ir}^D} \right)^2 \quad (21)$$

where the superscripts M and D indicate model and targets statistics, respectively.¹¹

Our algorithm finds a $\mathcal{M} = 0.0022$, which we consider it a small number, because we have 378 different targets. Figures 1 (a) and (b) presents the adjustment of the average wage of our model to the data and the adjustment of the share of workers of our model to the data, respectively. Note that the model and data have a good adjust, because the points very close to the 45° line in both figures.

Figure 1: Model adjustment to data - wages and share of workers



⁸The calibrated values to τ_{ir}^w , τ_{ir}^h and w_r can be view in Appendix A.

⁹We convert monthly wages into hourly wages considering a 40-hour workweek as a reference. Also, we consider the logarithm of the average gross wage as the target.

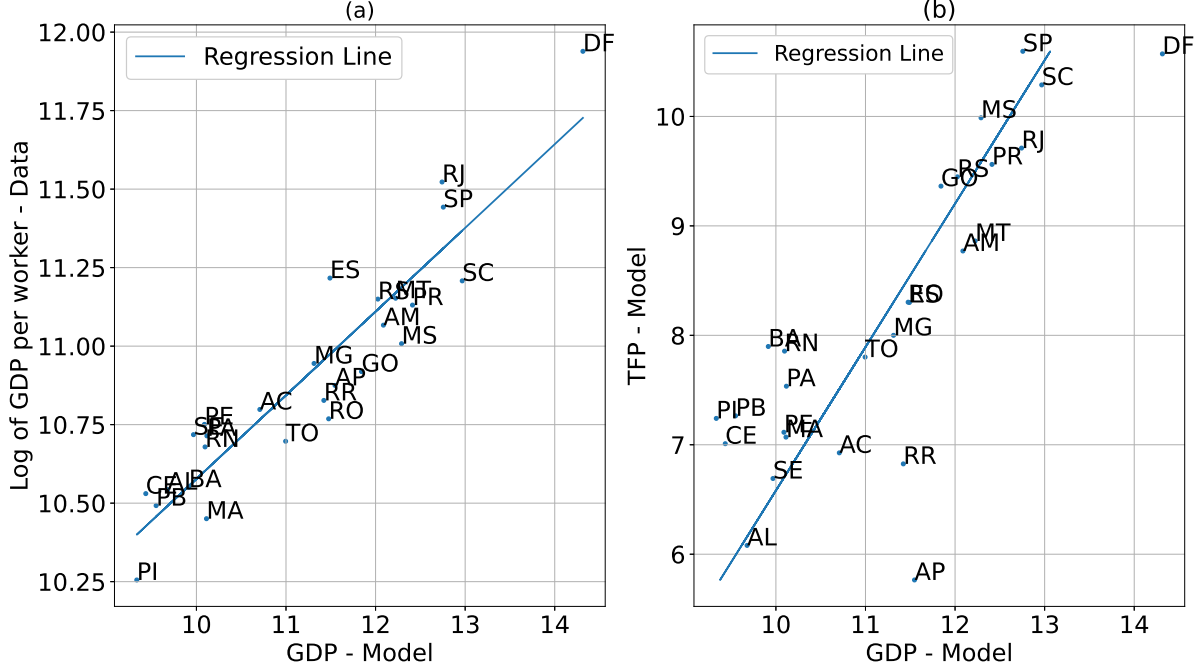
¹⁰Since TFP is equal to a price in our model, we select A_R from a grid. We run the Method of Moments for each values of A_R in a grid between 1 and 10. Then, we select the value to A_R which implies the minimum distance between data and our simulated model. Our algorithm selects $A_R = 8$. This is the TFP of Minas Gerais. We can think that all other TFPs are relative to this value.

¹¹We apply the logarithm in equation (21) to improve the numerical stability of the problem.

4 Results

Our calibrated model has a excellent fit to GDP per worker as can be seen in Figure 2 (a), recall that we do not use GDP as target. It can also be seen in Figure 2 (b) that our model suggests a positive relationship between GDP and TFP. These two features show that the model can be useful to study economic growth.

Figure 2: GDP - Model



The Figure 3, derived from the calibration of the model, gives us an insight into why the relative salary of teachers is higher in Brazilian states with low and medium income compared to states with high income¹². Note that the higher the relative salary of teachers, the greater the proportion of individuals in this occupation. And in low and middle-income states, the relative salary of teachers is higher than in high-income states. Therefore, in these states, being a less attractive teacher than other occupations.

The consequence of this can be seen in the Figure 4, since relative wage affect individual decisions. Since in the low and middle income states the relative wage to be a teacher is greater than in the high income states, the low and middle income states are able to attract more talented people to the teacher occupation compared to the high income states. This fact compromises the formation of human capital and consequently the output of high-income states. It is worth mentioning that, according to Machado and Scorzafave (2016), teachers' wage can also affect their efforts in the classroom and discourage teacher turnover in schools.

Barros and Delalilbera (2018) argue that one of the reasons why the relative salary of teachers is higher in the poorest states is that the occupation of teachers is labor-intensive and

¹²States whose per capita income is greater than R\$ 6400.00 are considered to be high income. Those with income between R\$ 6400.00 and R\$ 4500.00 can be considered as middle income, and those with per capita income below R\$ 4500.00 are those with low income.

is not much affected by technological and structural changes. Therefore, in states where more advanced technologies are present, the relative salary of teachers is lower than in less developed states.

Figure 3: Proportion of workers in teacher occupation and relative wage

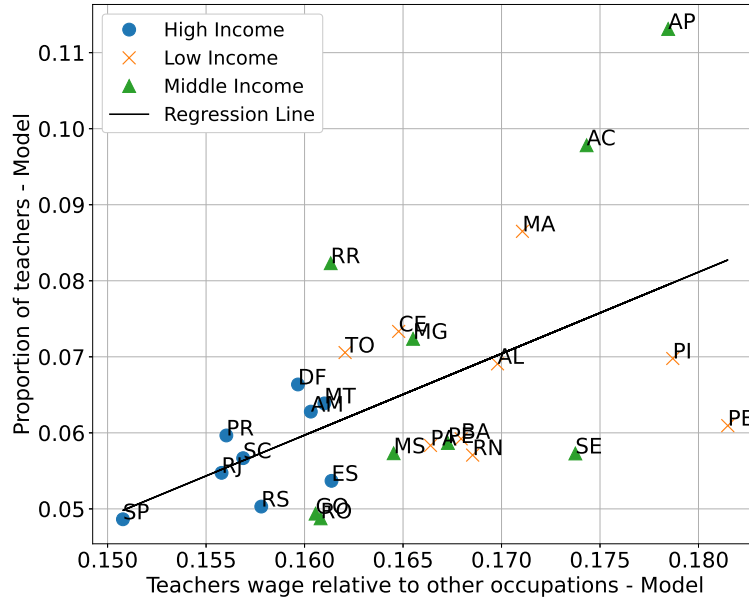
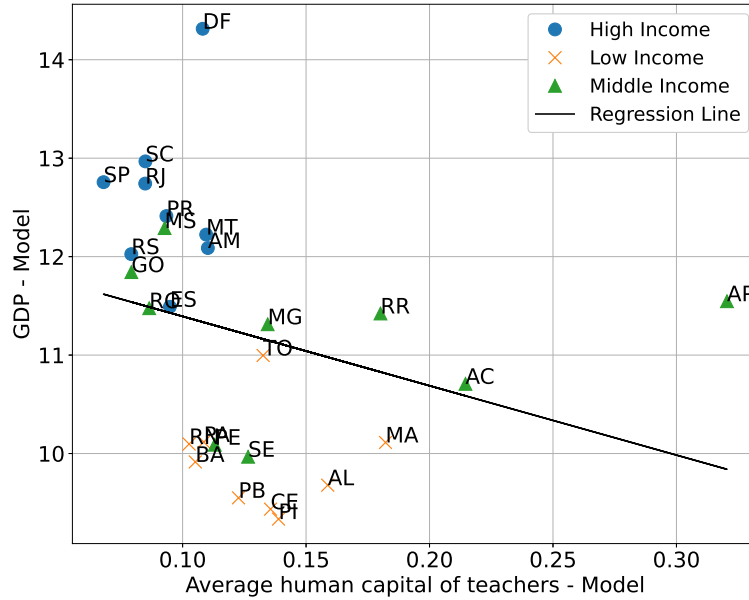


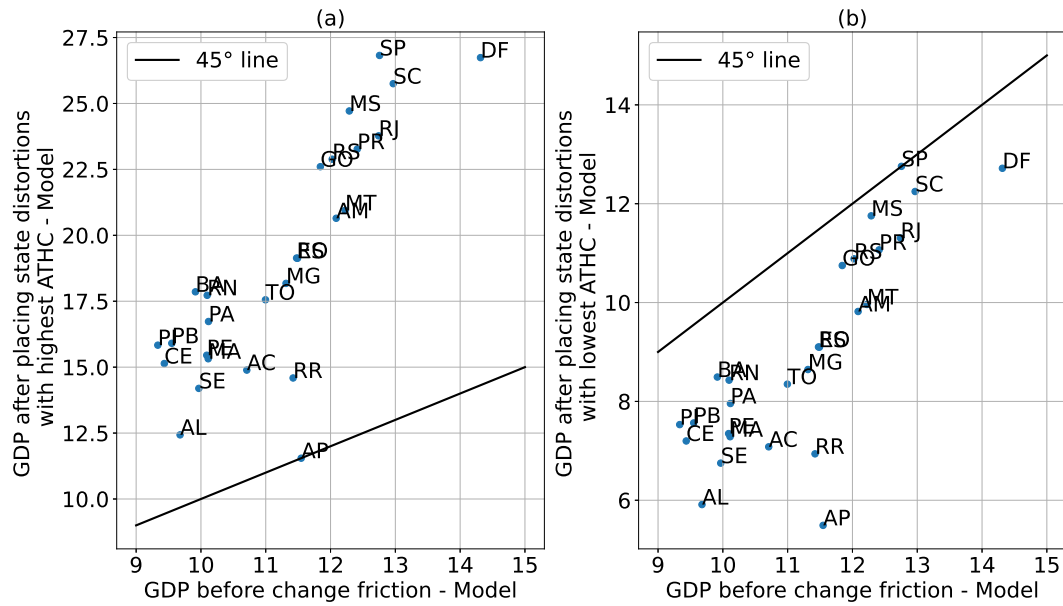
Figure 4: GDP per worker and average human capital



To check how sensitive the economy is to changes in teacher occupation distortions and to analyze what the best allocation in the economy would be, we did some counterfactual exercises. In the first exercise we put the distortions, τ^w and τ^h , of the states with highest and lowest Average Teachers Human Capital (ATHC) in all states to verify the effects on GDP.

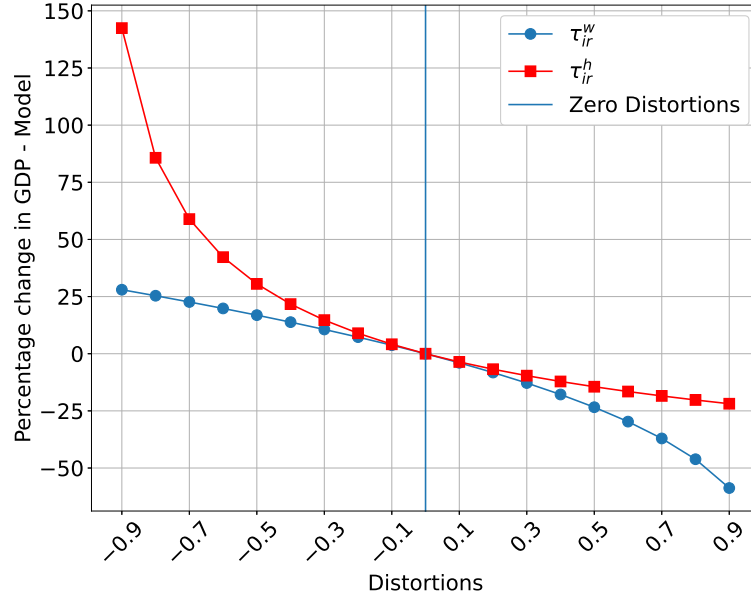
Figure 5 (a) shows that all the states have distortions of Amapá (AP), the state with highest ATHC, as can be seen the GDP of all states increase. In this case, the Brazilian GDP would increase 68.28%. On the other hand, if the states have distortions of São Paulo (SP), the state with lowest ATHC, the GDP of all states decreases (Figure 5 (b)), and the Brazilian GDP would decrease 20.01%. From this, we can infer that there is a problem of misallocation in the economy. When we change distortions, there is a reallocation of talent in occupations. So, if there is externality in choosing to be a teacher a change in distortions makes more talented people choose this occupation, and aggregate GDP increases.

Figure 5: GDP before and after placing state distortions with the highest and lowest ATHC in all states



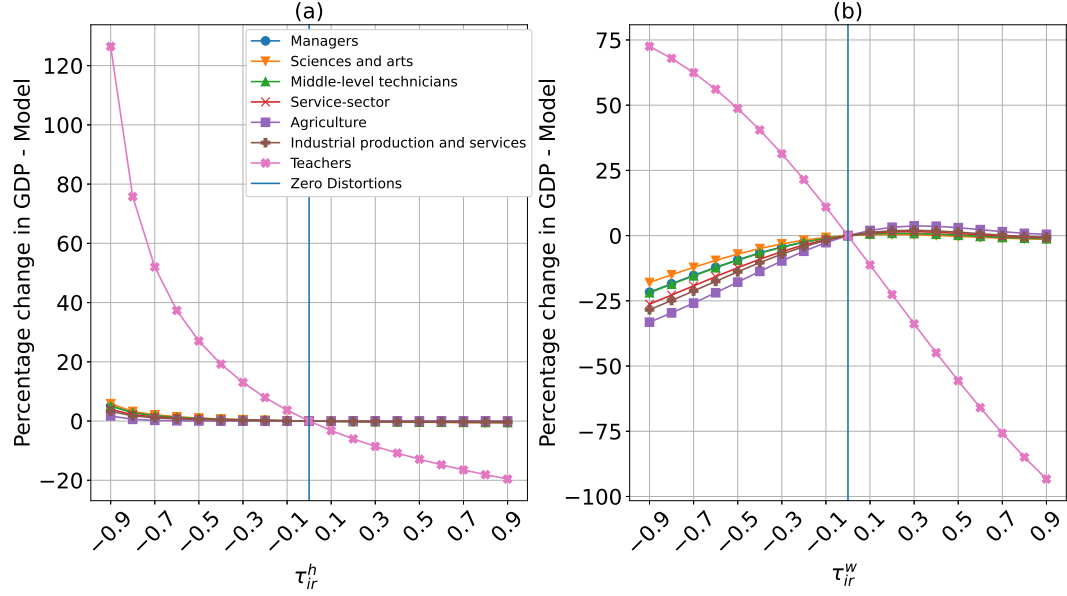
In the next exercise, we look at how increases in distortions in all occupations affect GDP per capita. First, we calculate GDP per capita (reference value) in case the distortions in the labor market and educational goods are zero. Then we keep the distortion in the educational goods market at zero, and we vary the labor market distortion between -0.9 and 0.9 , we calculate GDP, and then we calculate the percentage change in relation to the reference value. So we did the same for the distortion of the educational goods market, *ceteris paribus*. The result can be seen in Figure 6. It is noted that when they become negative, distortions in the educational goods market are more efficient in increasing GDP. On the other hand, labor market distortions have a greater negative effect when they are positive.

Figure 6: Increases in the distortions of all occupations and the percentage effects on GDP - Model



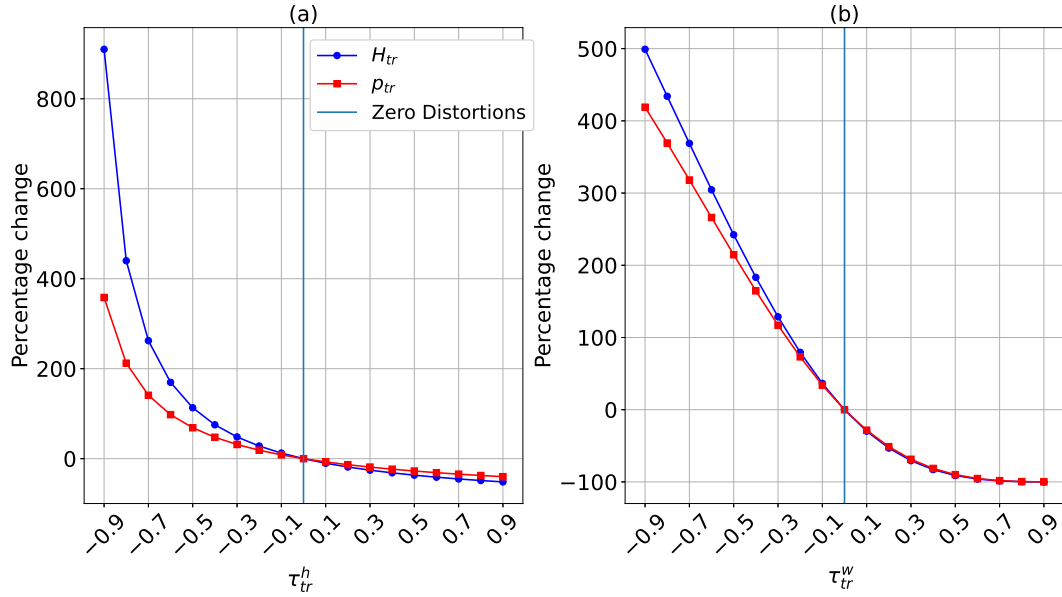
In the next exercise, we did something similar to the previous exercise, but instead of varying the distortions of all occupations simultaneously, we varied the distortions of each occupation keeping the others at zero. The Figures 7(a) and (b) shows that occupation 7, teachers, is more sensitive to increases in τ^w and τ^h than in other occupations. In other words, the greater the distortions of teachers, the lower the GDP. Again, it is noted that, when they become negative, distortions in the educational goods market are more efficient in increasing GDP. On the other hand, labor market distortions have a greater negative effect when they are positive. This leads us to the conclusion that in order to maximize GDP a public policy should make the distortions of teachers occupation reach the minimum limit of -0.99.

Figure 7: Increases in the distortions of each occupation and the percentage effects on GDP



One of the possible causes of this result can be seen in Figure 8. In this exercise, we did something similar to the exercise in Figure 7, but instead of measuring the percentage change in GDP, we verified the percentage change in the human capital of teachers and in the proportion of teachers. It is noted that, increases in distortions cause a decrease in the human capital of teachers and in the proportion of employed teachers.

Figure 8: Increases in distortions and percentual effects on H_{tr} and p_{tr}



Notes: H_{tr} = Human capital of teachers, p_{tr} = proportion of workers in teacher occupation.

5 Conclusion

In this article, we use the [Barros and Delalilbera \(2018\)](#) model to analyze the negative relationship between the relative salary of teachers and GDP per capita in Brazilian states in 2013. However, we modified the model to incorporate not only the quality but also the quantity of teachers as an input of formation of human capital. Our model shows that teachers' human capital is the cornerstone of human capital formation.

Our counterfactual exercises showed that reducing distortions in the labor market and in the educational goods market would increase Brazilian GDP per capita by 68.28 %. In addition, we show that increasing distortions in teacher occupation has a more negative impact on GDP per capita than increases in distortions in other occupations. This is because it reduces the human capital of teachers and the proportion of individuals in this occupation.

In this sense, two public policies could increase the incentives for talented people to be interested in teaching. The first is to increase the relative salary of the profession, this would increase the proportion of individuals in that occupation. The second is to further empower teachers in order to increase both productivity and quality.

In addition, the model used in this article can be used to analyze several macroeconomic problems. For example, the model can be used to analyze how the reallocation of human capital in different regions has impacts on environmental pollution and output. Another possibility is to analyze how distortions in the educational goods market affect human capital and, consequently, sectoral labor productivity.

References

- Banerjee, Abhijit V and Esther Duflo**, “Growth theory through the lens of development economics,” *Handbook of economic growth*, 2005, 1, 473–552.
- Barros, Fernando Jr. and Bruno R. Delalilbera**, “Market frictions, misallocation of talent and development,” *Economics Bulletin*, 2018, 38 (4), 2410–2430.
- Benhabib, Jess and Mark M Spiegel**, “Human capital and technology diffusion,” *Handbook of economic growth*, 2005, 1, 935–966.
- Borensztein, Eduardo, Jose De Gregorio, and Jong-Wha Lee**, “How does foreign direct investment affect economic growth?,” *Journal of international Economics*, 1998, 45 (1), 115–135.
- Café, Renata Motta**, “A Alocação de talentos no setor público brasileiro.” PhD dissertation 2018.
- Eaton, Jonathan and Samuel Kortum**, “Technology, Geography, and Trade,” *Econometrica*, 2002, 70 (5), 1741–1779.

- Ferreira, Pedro Cavalcanti, João Victor Issler, and Samuel de Abreu Pessôa**, “Testing production functions used in empirical growth studies,” *Economics Letters*, 2004, 83 (1), 29–35.
- Hanushek, Eric A and Ludger Woessmann**, “Schooling, educational achievement, and the Latin American growth puzzle,” *Journal of Development Economics*, 2012, 99 (2), 497–512.
- , **Marc Piopiunik, and Simon Wiederhold**, “The value of smarter teachers international evidence on teacher cognitive skills and student performance,” *Journal of Human Resources*, 2019, 54 (4), 857–899.
- Hnatkovska, Viktoria, Amartya Lahiri, and Sourabh Paul**, “Castes and labor mobility,” *American Economic Journal: Applied Economics*, 2012, 4 (2), 274–307.
- Hsieh, Chang-Tai and Peter J Klenow**, “Misallocation and manufacturing TFP in China and India,” *The Quarterly journal of economics*, 2009, 124 (4), 1403–1448.
- , **Erik Hurst, Chad Jones, and Pete Klenow**, “The Allocation of Talent and US Economic Growth,” 2013.
- , —, **Charles I. Jones, and Peter J. Klenow**, “The allocation of talent and US economic growth,” *Econometrica*, 2019, 87 (5), 1439–1474.
- Krueger, Alan B**, “Economic considerations and class size,” *The Economic Journal*, 2003, 113 (485), F34–F63.
- Lakdawalla, Darius**, “The economics of teacher quality,” *The Journal of Law and Economics*, 2006, 49 (1), 285–329.
- Machado, Laura Muller and Luiz Guilherme Dácar da Silva Scorzafave**, “Distribuição de salários de professores e outras ocupações: uma análise para graduados em carreiras tipicamente ligadas à docência,” *Revista Brasileira de Economia*, 2016, 70 (2), 203–220.
- Mankiw, N Gregory, David Romer, and David N Weil**, “A contribution to the empirics of economic growth,” *The quarterly journal of economics*, 1992, 107 (2), 407–437.
- McFadden, Daniel**, “Conditional Logit Analysis of Qualitative Choice Behavior,” in P. Zarembka, ed., *Frontiers of Econometrics*, New York, NY: Academic Press, 1974, pp. 105 – 142.
- Patrinos, Anthony Harry and George Psacharopoulos**, *Returns to investment in education: a further update*, The World Bank, 2002.
- Restuccia, Diego and Richard Rogerson**, “Policy distortions and aggregate productivity with heterogeneous establishments,” *Review of Economic dynamics*, 2008, 11 (4), 707–720.

- **and** — , “The causes and costs of misallocation,” *Journal of Economic Perspectives*, 2017, *31* (3), 151–74.
- Romer, Paul M**, “Endogenous technological change,” *Journal of political Economy*, 1990, *98* (5, Part 2), S71–S102.
- Woessmann, Ludger**, “The importance of school systems: Evidence from international differences in student achievement,” *Journal of Economic Perspectives*, 2016, *30* (3), 3–32.

Appendix A Calibrated distortions

Table A1: Calibrated distortions to the labor market - τ_{ir}^w

States	Managers	Sciences and arts	Middle-level technicians	Service-sector	Agriculture	Industrial production and services	Teachers
AC	0.1546	0.1823	0.0368	-0.2075	-0.4344	-0.2233	0.1360
AL	0.1546	0.3505	-0.0015	-0.1739	-0.3842	-0.1345	0.1663
AM	0.1546	0.2606	0.0552	-0.1867	-0.6765	-0.1188	0.0692
AP	0.1546	0.2608	0.1104	-0.1155	-0.2172	-0.1052	0.2257
BA	0.1546	0.2378	0.0073	-0.3066	-0.7276	-0.1768	0.0843
CE	0.1546	0.2669	-0.0210	-0.2790	-0.8833	-0.2686	0.0533
DF	0.1546	0.2037	0.0634	-0.2580	-0.2843	-0.1978	0.0600
ES	0.1546	0.2518	0.0285	-0.1475	-0.2256	-0.0173	0.1162
GO	0.1546	0.1882	-0.0218	-0.1825	-0.1286	-0.0750	0.0853
MA	0.1546	0.2204	0.0662	-0.3598	-0.9071	-0.2750	0.0871
MG	0.1546	0.2259	0.0600	-0.2003	-0.3025	-0.0681	0.1230
MS	0.1546	0.1980	0.0608	-0.2120	-0.0955	-0.1014	0.1241
MT	0.1546	0.2427	0.0569	-0.1092	0.0020	0.0192	0.1480
PA	0.1546	0.2255	0.0007	-0.2175	-0.4202	-0.1741	0.0965
PB	0.1546	0.2436	0.0744	-0.2971	-0.7162	-0.2942	0.1694
PE	0.1546	0.2682	0.0369	-0.1790	-0.4780	-0.1295	0.1249
PI	0.1546	0.3005	-0.0249	-0.3399	-1.0280	-0.2378	0.1439
PR	0.1546	0.2089	0.0713	-0.1437	-0.2258	-0.0738	0.0688
RJ	0.1546	0.2062	0.0256	-0.1896	-0.3888	-0.1036	0.0356
RN	0.1546	0.2789	0.0556	-0.2240	-0.5724	-0.2340	0.1185
RO	0.1546	0.2610	0.0599	-0.0887	-0.0935	0.0026	0.1413
RR	0.1546	0.3668	0.1307	-0.0901	-0.3412	-0.0967	0.1559
RS	0.1546	0.2275	0.0664	-0.1476	-0.1428	-0.0686	0.0934
SC	0.1546	0.2103	0.0792	-0.0782	-0.0954	-0.0180	0.1042
SE	0.1546	0.3238	0.0963	-0.1803	-0.6612	-0.1576	0.1776
SP	0.1546	0.1552	0.0039	-0.2280	-0.2183	-0.1210	-0.0109
TO	0.1546	0.2338	0.0199	-0.2221	-0.3896	-0.1735	0.0749

Notes: Acre (AC), Alagoas (AL), Amapá (AP), Amazonas (AM), Bahia (BA), Ceará (CE), Distrito Federal (DF), Espírito Santo (ES), Goiás (GO), Maranhão (MA), Mato Grosso (MT), Mato Grosso do Sul (MS), Minas Gerais (MG), Pará (PA), Paraíba (PB), Paraná (PR), Pernambuco (PE), Piauí (PI), Rio de Janeiro (RJ), Rio Grande do Norte (RN), Rio Grande do Sul (RS), Rondônia (RO), Roraima (RR), Santa Catarina (SC), São Paulo (SP), Sergipe (SE), Tocantins (TO).

Table A2: Calibrated distortions to the educational market - τ_{ir}^h

States	Managers	Sciences and arts	Middle-level technicians	Service-sector	Agriculture	Industrial production and services	Teachers
AC	0.0	0.3493	0.4598	-0.5332	4.2901	0.2814	-0.6388
AL	0.0	-0.6807	0.1985	-0.7438	1.5654	-0.5278	-0.7132
AM	0.0	-0.4413	-0.0431	-0.4620	17.7284	-0.2079	-0.0549
AP	0.0	-0.5195	-0.4416	-0.7664	3.0041	-0.5383	-0.8647
BA	0.0	-0.2381	0.3901	-0.3627	10.9199	-0.0448	-0.1800
CE	0.0	-0.3042	0.7835	-0.4958	26.4209	-0.0712	-0.3457
DF	0.0	-0.6530	-0.0003	-0.1479	126.6181	1.8484	0.3834
ES	0.0	-0.4737	0.4676	-0.5922	3.7013	-0.5071	-0.1430
GO	0.0	-0.0057	1.1799	-0.4483	4.2512	-0.3237	0.3014
MA	0.0	-0.3061	0.2255	-0.3251	12.5494	0.1048	-0.5583
MG	0.0	-0.3544	0.4249	-0.3581	5.3596	-0.3042	-0.3154
MS	0.0	0.2166	0.9561	-0.1111	3.8783	0.2081	0.2717
MT	0.0	-0.3632	0.8617	-0.3293	1.1346	-0.2841	-0.0741
PA	0.0	0.0580	0.4374	-0.5490	3.2569	-0.3100	-0.3146
PB	0.0	-0.5335	0.0376	-0.4695	18.7706	0.1349	-0.5076
PE	0.0	-0.5687	0.1549	-0.6745	8.2863	-0.4274	-0.4277
PI	0.0	-0.4422	0.6961	-0.2771	22.6171	0.0577	-0.4907
PR	0.0	-0.2287	0.5173	-0.3218	8.3065	-0.1410	0.3978
RJ	0.0	-0.5760	0.4564	-0.4608	117.4490	-0.0325	0.5320
RN	0.0	-0.3721	0.1025	-0.4955	15.3082	-0.0475	-0.2358
RO	0.0	-0.0941	0.3426	-0.6152	0.5147	-0.5671	-0.1627
RR	0.0	-0.6996	-0.3895	-0.7313	4.6237	-0.3567	-0.6402
RS	0.0	-0.3592	0.3205	-0.4119	4.5424	-0.2264	0.3635
SC	0.0	0.0002	0.6534	-0.1882	7.3352	-0.2143	0.6358
SE	0.0	-0.7390	-0.3087	-0.7265	4.9011	-0.4455	-0.6441
SP	0.0	-0.2983	0.6292	-0.2508	25.5819	0.0002	1.3361
TO	0.0	-0.3374	0.3506	-0.4696	2.7895	-0.0014	-0.3135

Table A3: Calibrated Total Factor Productivity - w

States	TFP
AC	6.9261
AL	6.0804
AM	8.7706
AP	5.7646
BA	7.8980
CE	7.0096
DF	10.5715
ES	8.3002
GO	9.3631
MA	7.0699
MG	8.0000
MS	9.9876
MT	8.8621
PA	7.5350
PB	7.2644
PE	7.1142
PI	7.2410
PR	9.5618
RJ	9.7108
RN	7.8555
RO	8.3028
RR	6.8254
RS	9.4482
SC	10.2880
SE	6.6915
SP	10.5951
TO	7.8012