

Online Appendix to “Misallocation of talent, human capital formation, and development in Brazil”

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In this Online Appendix we provide a complete solution for our General Equilibrium Model. Enjoy yourself !

1 Introduction

The utility of the individual is given by:

$$U(c, s) = c^\beta (1 - s) \quad (1)$$

Where:

- $c \rightarrow$ consumption
- $s \rightarrow$ time spent at school
- $\beta \rightarrow$ trade between consumption and accumulation of human capital

People working in a region r and occupation i is paid a net wage of $(1 - \tau_{ir}^w)w_{ir}$.

- $w_{ir} \rightarrow$ the net wage per efficiency unit of labor
- $\tau_{ir}^w \rightarrow$ is a distortion specific for occupation i and location r

Human capital choices are also distorted due to tax on educational goods.

- $\tau_{ir}^h \rightarrow$ tax paid by a person that invest in education

The formation of human capital of a worker in a region r is given by:

$$h_r(e, s) = (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s^\phi e^\eta \quad (2)$$

- $e \rightarrow$ consumption of educational goods
- $s \rightarrow$ time spent in school
- $H_{tr} \rightarrow$ aggregate human capital of teachers

- $\phi_i \rightarrow$ elasticity of human capital with respect to time in school
- $\eta \rightarrow$ elasticity of education goods in the human capital function
- $\varphi \rightarrow$ elasticity of teacher human capital in the human capital function
- $\alpha \rightarrow$ parameter between zero and one
- $p_{tr} \rightarrow$ is the fraction of people that work in occupation t in region r . This can be view in proposition 1.

Abilities dispersion is modeled as multivariated Fréchet distribution. Let ϵ the abilities dispersion in modeled as a multivariate Fréchet distribution. So, we have:

$$F(\epsilon_1, \dots, \epsilon_N) = \exp \left[- \left(\sum_{i=1}^N \epsilon_i^{-\theta} \right) \right] \quad (3)$$

- θ skill dispersion

The representative firm has the following production function:

$$Y = \sum_{r=1}^R \sum_{i=1}^N A_r H_{ir} \quad (4)$$

- $Y \rightarrow$ output
- $A_r \rightarrow$ total factor productivity

1.1 Firm's problem

The firm's problem can be written as:

$$\text{Max}_{H_{ir}} \quad \sum_{r=1}^R \sum_{i=1}^N A_r H_{ir} - \sum_{r=1}^R \sum_{i=1}^N w_{ir} H_{ir} \quad (5)$$

FOCs:

$$\begin{aligned} \frac{\partial \Pi}{\partial H_{ir}} &= \sum_{r=1}^R \sum_{i=1}^N A_r - \sum_{r=1}^R \sum_{i=1}^N w_{ir} = 0 \\ \sum_{r=1}^R \sum_{i=1}^N A_r &= \sum_{r=1}^R \sum_{i=1}^N w_{ir} \end{aligned} \quad (6)$$

If the condition in equation 6 is satisfied, so $H_{ir}^d = x \in \mathbb{R}_+$. If $A_r < w_{ir}$ the profit function will be negative, so $H_{ir}^d = 0$. If $A_r > w_{ir}$, $H_{ir}^d = \infty$, because the profit function is linear in H_{ir} , so, the firms will produce infinitely.

1.2 Worker's problem

Given the occupational choice i for which the individual has an idiosyncratic ability ϵ_i , and taking wage w_{ir} as given, each worker chooses consumption c , e and time spent in school s to solve the following problem:

$$\begin{aligned} \text{Max}_{c,s,e} \quad & c^\beta(1-s_i) \\ \text{st.} \quad & c = (1-\tau_{ir}^w)h_r(e_{ir}, s_i)\epsilon_i w_{ir} - (1+\tau_{ir}^h)e_{ir} \end{aligned} \quad (7)$$

FOCs:

$$U_s = \beta c^{\beta-1} c_s - s_i \beta c^{\beta-1} c_s - c^\beta = 0$$

rearranging the terms we have:

$$\beta c^{\beta-1} c_s (1-s_i) - c^\beta = 0 \quad (8)$$

replacing $h_r(e_{ir}, s_i)$ in c we have:

$$c = (1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} e_{ir}^\eta \epsilon_i w_{ir} - (1+\tau_{ir}^h)e_{ir}$$

Derivating c with respect to s_i we obtain:

$$\frac{\partial c}{\partial s_i} = c_s = (1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} e_{ir}^\eta \epsilon_i w_{ir} \quad (9)$$

Plugging equation 9 in 8:

$$\beta c^{\beta-1} [(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} e_{ir}^\eta \epsilon_i w_{ir} (1-s_i)] - c^\beta = 0$$

Rearranging the terms we get c :

$$c = \beta [(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} e_{ir}^\eta \epsilon_i w_{ir} (1-s_i)] \quad (10)$$

Deriving U with respect e_{ir} we have:

$$U_e = \beta c^{\beta-1} c_e (1-s_i) = 0 \quad (11)$$

Deriving c with respect e_{ir} we have:

$$\frac{\partial c}{\partial e} = c_e = (1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta e_{ir}^{\eta-1} \epsilon_i w_{ir} - (1+\tau_{ir}^h) \quad (12)$$

Plugging equation 12 in 11 we have:

$$\begin{aligned} \beta c^{\beta-1} [(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta e_{ir}^{\eta-1} \epsilon_i w_{ir} - (1+\tau_{ir}^h)] (1-s_i) &= 0 \\ (1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta e_{ir}^{\eta-1} \epsilon_i w_{ir} &= (1+\tau_{ir}^h) \\ e_{ir}^{\eta-1} &= \frac{(1+\tau_{ir}^h)}{(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta \epsilon_i w_{ir}} \\ e_{ir} &= \left[\eta \left(\frac{1-\tau_{ir}^w}{1+\tau_{ir}^h} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i \right]^\kappa \end{aligned} \quad (13)$$

Where $\kappa = \frac{1}{1-\eta}$. Now, i'll match budget and expression 10.

$$\begin{aligned} (1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} e_{ir}^\eta \epsilon_i w_{ir} - (1+\tau_{ir}^h)e_{ir} &= \beta(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} e_{ir}^\eta \epsilon_i w_{ir} (1-s_i) \\ (1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i w_{ir} - (1+\tau_{ir}^h)e_{ir}^{1-\eta} &= \beta(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} \epsilon_i w_{ir} (1-s_i) \end{aligned}$$

Replacing $e_{ir}^{1-\eta}$ in this last expression we get:

$$\begin{aligned}
(1 - \tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i w_{ir} - (1 - \tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta \epsilon_i w_{ir} &= \beta(1 - \tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} \epsilon_i w_{ir} (1 - s_i) \\
(1 - \tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i w_{ir} (1 - \eta) &= \beta(1 - \tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} \epsilon_i w_{ir} (1 - s_i) \\
s_i^{\phi_i} (1 - \eta) &= \beta s_i^{\phi_i-1} \phi_i (1 - s_i) \\
s_i^* &= \frac{\beta \phi_i}{1 - \eta + \beta \phi_i}
\end{aligned}$$

Finally we obtain s_i^* :

$$s_i^* = \left(1 + \frac{1 - \eta}{\beta \phi_i}\right)^{-1} \quad (14)$$

Plugging equation 14 in 13 we get:

$$e_{ir} = \left[\eta \left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left(1 + \frac{1 - \eta}{\beta \phi_i}\right)^{-\phi_i} \epsilon_i \right]^\kappa \quad (15)$$

1.3 Indirect Utility

Now I will get the indirect utility. Recall that utility is given by:

$$U = c^\beta (1 - s)$$

Replace the budget in this expression we have:

$$\begin{aligned}
U &= [(1 - \tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} e_{ir}^\eta \epsilon_i w_{ir} - (1 + \tau_{ir}^h) e_{ir}]^\beta (1 - s_i) \\
&= e_{ir}^\beta [(1 - \tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} e_{ir}^{\eta-1} \epsilon_i w_{ir} - (1 + \tau_{ir}^h)]^\beta (1 - s_i)
\end{aligned}$$

Replacing $e_{ir}^{\eta-1}$:

$$\begin{aligned}
&= e_{ir}^\beta \left[\frac{(1 - \tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} (1 + \tau_{ir}^h) \epsilon_i w_{ir}}{(1 - \tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta \epsilon_i w_{ir}} - (1 + \tau_{ir}^h) \right]^\beta (1 - s_i) \\
&= e_{ir}^\beta \left[\frac{(1 + \tau_{ir}^h)}{\eta} - (1 + \tau_{ir}^h) \right]^\beta (1 - s_i) \\
&= \left[e_{ir} (1 + \tau_{ir}^h) \left(\frac{1 - \eta}{\eta} \right) \right]^\beta (1 - s_i)
\end{aligned}$$

Replacing e_{ir} by equation 15:

$$\begin{aligned}
&= \left\{ \left[\eta \frac{(1 - \tau_{ir}^w)}{(1 + \tau_{ir}^h)} (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i w_{ir} \right]^{\frac{1}{1-\eta}} (1 + \tau_{ir}^h) \left(\frac{1 - \eta}{\eta} \right) \right\}^\beta (1 - s_i) \\
&= \left[\eta \frac{(1 - \tau_{ir}^w)}{(1 + \tau_{ir}^h)} (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} w_{ir} (1 + \tau_{ir}^h)^{1-\eta} \left(\frac{1 - \eta}{\eta} \right)^{1-\eta} \epsilon_i \right]^{\frac{\beta}{1-\eta}} (1 - s_i) \\
&= \left[\eta^\eta (1 - \eta)^{1-\eta} \left(\frac{(1 - \tau_{ir}^w)}{(1 + \tau_{ir}^h)^\eta} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i \right]^{\frac{\beta}{1-\eta}} (1 - s_i)
\end{aligned}$$

Finally, we get the indirect utility function:

$$D = \left[\bar{\eta} \left(\frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} (1 - s_i)^{\frac{1}{\beta \kappa}} \epsilon_i \right]^{\beta \kappa} \quad (16)$$

where $\bar{\eta} = \eta^\eta (1 - \eta)^{1-\eta}$ and $\kappa = \frac{1}{1-\eta}$.

1.4 Propositions

Proposition 1 states that the overall occupational share can be obtained by aggregating the individual optimal choice.

Proposition 1. *Aggregating across workers, the solution of individual's occupational choice problem is:*

$$p_{ir} = \frac{\tilde{w}_{ir}^\theta}{\sum_{j=1}^N \tilde{w}_{jr}^\theta} \quad (17)$$

where p_{ir} is the fraction of workers in occupation i in region r , and:

$$\tilde{w}_{ir} = \bar{\eta} \left(\frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} (1 - s_i)^{\frac{1}{\beta\kappa}}$$

Proof. Let:

$$\tilde{w}_{ir} = \bar{\eta} \left(\frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} (1 - s_i)^{\frac{1}{\beta\kappa}}.$$

Then, we can rewrite equation (16) as:

$$D_{ir} = [\tilde{w}_{ir}\epsilon_i]^{\beta\kappa}$$

Therefore, the problem solution of individual i living in region r involves picking the occupation with the highest value of $\tilde{w}_{ir}\epsilon_i$. Without loss of generality, consider the probability of an individual choosing occupation 1:

$$\begin{aligned} p_{ir} &= Pr(\tilde{w}_{1r}\epsilon_1 > \tilde{w}_{ir}\epsilon_i) \quad \forall i \neq 1 \\ &= Pr\left(\epsilon_i < \frac{\tilde{w}_{1r}}{\tilde{w}_{ir}}\epsilon_1\right) \quad \forall i \neq 1 \\ &= \int F_1(\alpha_1\epsilon, \alpha_2\epsilon, \dots, \alpha_N\epsilon) d\epsilon, \end{aligned} \quad (18)$$

where F_1 represents the derivative of equation (3) with respect to its first argument and $\alpha_i = \tilde{w}_{1r}/\tilde{w}_{ir}$ for $i \in \{1, 2, \dots, N\}$. Taking the derivative of equation (3) with respect to ϵ_1 , and evaluating in ϵ :

$$\begin{aligned} F_1 &= \theta \epsilon_1^{-\theta-1} \exp(-\epsilon_1 \hat{Z}) \\ F_1(\epsilon) &= \theta \epsilon^{-\theta-1} \exp(-\epsilon \hat{Z}) \end{aligned}$$

where $\hat{Z} = \sum_{i=1}^n \alpha_i^{-\theta}$. Then, equation (18) can be written as:

$$\begin{aligned} p_{1r} &= \int \frac{\hat{Z}}{\hat{Z}} \theta \epsilon^{-\theta-1} \exp(-\epsilon \hat{Z}) d\epsilon \\ &= \frac{1}{\hat{Z}} \int \hat{Z} \theta \epsilon^{-\theta-1} \exp(-\epsilon \hat{Z}) d\epsilon \end{aligned}$$

This expression is the derivative of equation (3) with respect to ϵ . Hence, we have:

$$\begin{aligned} p_{1r} &= \frac{1}{\hat{Z}} \int dF(\epsilon) \\ &= \frac{1}{\hat{Z}} \\ &= \frac{\tilde{w}_{1r}^\theta}{\sum_{i=1}^N \tilde{w}_{ir}^\theta} \end{aligned}$$

□

The next proposition defines the workers' human capital in each occupation in a given region.

Proposition 2. *For a given region, the human capital of workers in occupation i is:*

$$H_{ir} = p_{ir} \mathbb{E}[h(e_{ir}, s_i) \epsilon_i | \text{person choices } i], \quad (19)$$

The average quality of workers is:

$$\mathbb{E}[h(e_{ir}, s_i) \epsilon_i | \text{person choices } i] = \bar{\Gamma} \left[\left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right)^\eta \tilde{h}_{ir} p_{ir}^{-\frac{1}{\theta}} \right]^\kappa \quad (20)$$

where $\bar{\Gamma} = \Gamma(1 - \kappa/\theta)$ is related to the mean of the Fréchet distribution for abilities, $\tilde{h}_{ir} = [(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta]^\kappa$ and $\kappa = 1/(1 - \eta)$.

Proof. We have:

$$h(e_{ir}, s_i) \epsilon_i = (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left[\eta \left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i \right]^{\eta\kappa} s_i^{\phi_i} \epsilon_i \quad (21)$$

H_{ir} is the total efficiency units of labor supplied to occupation i in region r . Then,

$$\begin{aligned} H_{ir} &= p_{ir} \mathbb{E} \left\{ (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left[\eta \left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i \right]^{\eta\kappa} s_i^{\phi_i} \epsilon_i \middle| \text{person choices } i \right\} \\ &= p_{ir} \left\{ (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left[\left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right) \eta (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \right]^{\eta\kappa} s_i^{\phi_i} \mathbb{E} \left[\epsilon_i^\kappa \middle| \text{person choices } i \right] \right\} \\ &= p_{ir} \tilde{h}_{ir} \left(\frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} w_{ir} \right)^{\eta\kappa} \mathbb{E} \left[\epsilon_i^\kappa \middle| \text{person choices } i \right] \end{aligned} \quad (22)$$

To calculate this last conditional expectation, we use the Fréchet distribution. For now, we suppress the region index r because this calculation is similar in all regions. Let $y_i = \tilde{w}_i \epsilon_i$. Since we are maximizing y_i , it inherits the extreme value distribution:

$$\begin{aligned} \mathbf{Pr} \left(\max_i y_i < z \right) &= \mathbf{Pr}(\epsilon_i < z/\tilde{w}_i) \quad \forall i \\ &= F(z/\tilde{w}_1, \dots, z/\tilde{w}_N) \\ &= \exp \left[- \sum_{i=1}^N (z/\tilde{w}_i)^{-\theta} \right] \\ &= \exp \left[-kz^{-\theta} \right] \end{aligned}$$

where $k = \sum_{i=1}^N \tilde{w}_i^{-\theta}$.

The extreme value also has a Fréchet distribution. After some algebraic manipulations it can be concluded that the distribution of ϵ^* , the workers' ability in their chosen occupation, has a Fréchet distribution:

$$G(x) = \mathbf{Pr}(\epsilon^* < x) = \exp \left[-k^* x^{-\theta} \right] \quad (23)$$

where $k^* = \sum_{i=1}^N (\tilde{w}_i/\tilde{w}^*)^\theta = 1/p^*$.

Finally, we can calculate the expectation of equation (22). Let i be the occupation the individual chooses, and λ some positive exponent.

$$\begin{aligned} \mathbb{E}(\epsilon_i^\lambda) &= \int_0^\infty \epsilon_i^\lambda dG(\epsilon) \\ &= \int_0^\infty \theta \left(\frac{1}{p^*} \right) \epsilon^{(\lambda-\theta-1)} \exp \left[\left(\frac{1}{p^*} \right) \epsilon^{-\theta} \right] d\epsilon \end{aligned}$$

We can set $x = \left(\frac{1}{p^*}\right) \epsilon^{-\theta}$ and rewrite the last expression as:

$$\begin{aligned}\mathbb{E}(\epsilon_i^\lambda) &= \left(\frac{1}{p^*}\right)^{\frac{\lambda}{\theta}} \int_0^\infty x^{-\frac{\lambda}{\theta}} \exp(-x) dx \\ &= \left(\frac{1}{p^*}\right)^{\frac{\lambda}{\theta}} \Gamma\left(1 - \frac{\lambda}{\theta}\right)\end{aligned}$$

Using this result in equation (22) completes the proof. \square

Corollary 1. *Let W_{ir} be the gross average wage in occupation i in region r . Then:*

$$W_{ir} = w_{ir} \mathbb{E}[h(e_{ir}, s_i) \epsilon_i] = \bar{\Gamma} \eta \frac{(1 - s_i)^{-1/\beta}}{(1 - \tau_{ir}^w)} \left(\sum_{i=1}^N \tilde{w}_{ir}^\theta \right)^{\frac{\kappa}{\theta}} \quad (24)$$

Definition 1. *A competitive equilibrium in this economy consists of individual choices of $\{c, e, s\}$, an occupational choice by workers, total human capital in each occupation and region H_{ir} , final output Y , and efficiency wages w_{ir} for each occupation and region.*

1. *Given an occupational choice, w_{ir} , and the idiosyncratic ability ϵ , each worker chooses c , e , s to maximize utility in equation (1).*
2. *Given market friction, w_{ir} , H_{it} , and ϵ , a worker chooses the occupation that maximizes D_{ir} .*
3. *A representative firm hires H_{ir} to maximize profits.*
4. *The occupational wage, w_{ir} , clears the labor market in each occupation and region.*
5. *Total output is given by the production function in equation (4).*

1.5 Teacher's Human capital

The fraction of teachers, p_{tr} , can be written as:

$$p_{tr} = \frac{\left[\frac{1 - \tau_{tr}^w}{(1 + \tau_{tr}^h)^\eta} w_{tr} s_t^{\phi_t} (1 - s_t)^{\frac{1}{\beta\kappa}} \right]^\theta}{\sum_{j=1}^N \left[\frac{1 - \tau_{jr}^w}{(1 + \tau_{jr}^h)^\eta} w_{jr} s_j^{\phi_j} (1 - s_j)^{\frac{1}{\beta\kappa}} \right]^\theta} \quad (25)$$

where $\kappa = 1/(1 - \eta)$ and

$$\tilde{h}_{ir} = [(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta^\eta]^\kappa \quad (26)$$

So, the teacher's human capital is given by:

$$H_{tr} = p_{tr}^{\frac{\nu}{\pi}} (s_t^{\phi_t} \eta^\eta)^{\frac{\kappa}{\pi}} \left(\frac{1 - \tau_{tr}^w}{1 + \tau_{tr}^h} w_{tr} \right)^\sigma \gamma^{\frac{1}{\pi}} \quad (27)$$

where $\kappa = 1/(1 - \eta)$, $\pi = 1 - (1 - \alpha)\varphi\kappa$, $\nu = 1 + \alpha\varphi\kappa - \theta/\kappa$, and $\sigma = \eta\kappa/\pi$.

1.6 Problem

So, the problem consist in minimize equation 28, using Nelder-Mead.

$$Dist = \sum_{i=1, r=1}^{N, R} \left(\frac{W_{ir}^M - W_{ir}^T}{W_{ir}^T} \right)^2 + \sum_{i=1, r=1}^{N, R} \left(\frac{p_{ir}^M - p_{ir}^T}{p_{ir}^T} \right)^2 \quad (28)$$

where p_{ir}^M and W_{ir}^M are given by equations 17 and 24 respectively. On the other hand, p_{ir}^T and W_{ir}^T are given by PNAD data. The superscript indicate model and target statistics.