Hsieh Model - Version 2

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1 Introduction

The utility of the individual is given by:

$$U(c,s) = c^{\beta}(1-s) \tag{1}$$

Where:

- $c \rightarrow$ consumption
- $s \rightarrow$ time spent at school
- $\beta \rightarrow$ trade between consumption and accumulation of human capital

People working in a region r and occupation i is paid a net wage of $(1-\tau_{ir}^{w})w_{ir}$.

- $w_{ir} \rightarrow$ the net wage per efficiency unit of labor
- $\tau_{ir}^{w} \rightarrow$ is a distortion specific for occupation i and location r

Human capital choices are also distorted due to tax on educational goods.

- $\tau_{ir}^{h} \rightarrow$ tax paid by a person that invest in education

The formation of human capital of a worker in a region r is given by:

$$h_r(e,s) = (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s^{\phi} e^{\eta} \tag{2}$$

- $e \rightarrow$ consumption of educational goods
- $s \rightarrow$ time spent in school
- $H_{tr} \rightarrow$ aggregate human capital of teachers
- $\phi_i \rightarrow$ elasticity of human capital with respect to time in school
- $\eta \rightarrow$ elasticity of education goods in the human capital function
- $\varphi \rightarrow$ elasticity of teacher human capital in the human capital function
- $\alpha \rightarrow$ parameter between zero and one
- $p_{tr} \rightarrow$ is the fraction of people that work in occupation t in region r. This can be view in proposition 1.

Abilities dispersion is modeled as multivariated Fréchet distribution. Let ϵ the abilities dispersion in modeled as a multivariate Fréchet distribution. So, we have:

$$F(\epsilon_1, ..., \epsilon_N) = \exp\left[-\left(\sum_{i=1}^{N} \epsilon_i^{-\theta}\right)\right] \tag{3}$$

- θ skill dispersion

The representative firm has the following production function:

$$Y = \sum_{r=1}^{R}\sum_{i=1}^{N} A_r H_{ir} \tag{4}$$

- $Y \rightarrow$ output
- $A_r \rightarrow$ total factor productivity

1.1 Firm's problem

The firm's problem can be written as:

$$\underset{H_{ir}}{\text{Max}} \quad \sum_{r=1}^{R}\sum_{i=1}^{N} A_r H_{ir} - \sum_{r=1}^{R}\sum_{i=1}^{N} w_{ir} H_{ir} \tag{5}$$

FOCs:

$$\frac{\partial \Pi}{\partial H_{ir}} = \sum_{r=1}^{R}\sum_{i=1}^{N} A_r - \sum_{r=1}^{R}\sum_{i=1}^{N} w_{ir} = 0$$

$$\sum_{r=1}^{R}\sum_{i=1}^{N} A_r = \sum_{r=1}^{R}\sum_{i=1}^{N} w_{ir} \tag{6}$$

If the condition in equation 6 is satisfied, so $H_{ir}^d = x \in \mathbb{R}_+$. If $A_r < w_{ir}$ the profit function will be negative, so $H_{ir}^d = 0$. If $A_r > w_{ir}$, $H_{ir}^d = \infty$, because the profit function is linear in H_{ir}, so, the firms will produce infinitely.

1.2 Worker's problem

Given the occupational choice i for which the individual has an idiosyncratic ability ϵ_i, and taking wage w_{ir} as given, each worker chooses consumption c, e and time spent in school s to solve the following problem:

$$\begin{aligned} &\underset{c,s,e}{\text{Max}} \quad c^{\beta}(1-s_i) \\ st. \quad & c = (1-\tau_{ir}^{w}) h_r(e_{ir}, s_i)\epsilon_i w_{ir} - (1+\tau_{ir}^{h}) e_{ir} \end{aligned} \tag{7}$$

FOCs:

$$U_s = \beta c^{\beta-1} c_s - s_i \beta c^{\beta-1} c_s - c^{\beta} = 0$$

rearranging the terms we have:

$$\beta c^{\beta-1} c_s (1-s_i) - c^{\beta} = 0 \tag{8}$$

replacing $h_r(e_{ir}, s_i)$ in c we have:

$$c = (1-\tau_{ir}^{w})(p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi_i} e_{ir}^{\eta} \epsilon_i w_{ir} - (1+\tau_{ir}^{h}) e_{ir}$$

Derivating c with respect to s_i we obtain:

$$\frac{\partial c}{\partial s_i} = c_s = (1-\tau_{ir}^{w})(p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} \phi_i s_i^{\phi_i - 1} e_{ir}^{\eta} \epsilon_i w_{ir} \tag{9}$$

Plugging equation 9 in 8:

$$\beta c^{\beta-1}[(1-\tau_{ir}^{w})(p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} \phi_i s_i^{\phi_i - 1} e_{ir}^{\eta} \epsilon_i w_{ir}(1-s_i)] - c^{\beta} = 0$$

Rearranging the terms we get c:

$$c = \beta[(1-\tau_{ir}^{w})(p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} \phi_i s_i^{\phi_i - 1} e_{ir}^{\eta} \epsilon_i w_{ir}(1-s_i)] \tag{10}$$

Deriving U with respect e_{ir} we have:

$$U_e = \beta c^{\beta-1} c_e (1-s_i) = 0 \tag{11}$$

Deriving c with respect e_{ir} we have:

$$\frac{\partial c}{\partial e} = c_e = (1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta e_{ir}^{\eta-1} \epsilon_i w_{ir} - (1+\tau_{ir}^h) \tag{12}$$

Plugging equation 12 in 11 we have:

$$\begin{aligned}
\beta c^{\beta-1}[(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta e_{ir}^{\eta-1} \epsilon_i w_{ir} - (1+\tau_{ir}^h)](1-s_i) &= 0 \\
(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta e_{ir}^{\eta-1} \epsilon_i w_{ir} &= (1+\tau_{ir}^h) \\
e_{ir}^{\eta-1} &= \frac{(1+\tau_{ir}^h)}{(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta \epsilon_i w_{ir}} \\
e_{ir} &= \left[\eta \left(\frac{1-\tau_{ir}^w}{1+\tau_{ir}^h} w_{ir}\right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i\right]^\kappa
\end{aligned} \tag{13}$$

Where $\kappa = \dfrac{1}{1-\eta}$. Now, i'll match budget and expression 10.

$$\begin{aligned}
(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} e_{ir}^\eta \epsilon_i w_{ir} - (1+\tau_{ir}^h) e_{ir} &= \beta(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} e_{ir}^\eta \epsilon_i w_{ir}(1-s_i) \\
(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i w_{ir} - (1+\tau_{ir}^h) e_{ir}^{1-\eta} &= \beta(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} \epsilon_i w_{ir}(1-s_i)
\end{aligned}$$

Replacing $e_{ir}^{1-\eta}$ in this last expression we get:

$$\begin{aligned}
(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i w_{ir} - (1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \eta \epsilon_i w_{ir} &= \beta(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} \epsilon_i w_{ir}(1-s_i) \\
(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} \epsilon_i w_{ir}(1-\eta) &= \beta(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \phi_i s_i^{\phi_i-1} \epsilon_i w_{ir}(1-s_i) \\
s_i^{\phi_i}(1-\eta) &= \beta s_i^{\phi_i-1} \phi_i (1-s_i) \\
s_i^* &= \frac{\beta \phi_i}{1-\eta+\beta\phi_i}
\end{aligned}$$

Finally we obtain s_i^*:

$$s_i^* = \left(1 + \frac{1-\eta}{\beta\phi_i}\right)^{-1} \tag{14}$$

Plugging equation 14 in 13 we get:

$$e_{ir} = \left[\eta \left(\frac{1-\tau_{ir}^w}{1+\tau_{ir}^h} w_{ir}\right) (p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi \left(1 + \frac{1-\eta}{\beta\phi_i}\right)^{-\phi_i} \epsilon_i\right]^\kappa \tag{15}$$

1.3 Indirect Utility

Now I will get the indirect utility. Recall that utility is given by:

$$U = c^\beta (1-s)$$

Replace the budget in this expression we have:

$$\begin{aligned}
U &= [(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} e_{ir}^\eta \epsilon_i w_{ir} - (1+\tau_{ir}^h) e_{ir}]^\beta (1-s_i) \\
&= e_{ir}^\beta [(1-\tau_{ir}^w)(p_{tr}^\alpha H_{tr}^{1-\alpha})^\varphi s_i^{\phi_i} e_{ir}^{\eta-1} \epsilon_i w_{ir} - (1+\tau_{ir}^h)]^\beta (1-s_i)
\end{aligned}$$

Replacing $e_{ir}^{\eta-1}$:

$$= e_{ir}^{\beta}\left[\frac{(1-\tau_{ir}^{w})(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_{i}^{\phi_i}(1+\tau_{ir}^{h})\epsilon_i w_{ir}}{(1-\tau_{ir}^{w})(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_{i}^{\phi_i}\eta\epsilon_i w_{ir}} - (1+\tau_{ir}^{h})\right]^{\beta}(1-s_i)$$

$$= e_{ir}^{\beta}\left[\frac{(1+\tau_{ir}^{h})}{\eta} - (1+\tau_{ir}^{h})\right]^{\beta}(1-s_i)$$

$$= \left[e_{ir}(1+\tau_{ir}^{h})\left(\frac{1-\eta}{\eta}\right)\right]^{\beta}(1-s_i)$$

Replacing e_{ir} by equation 15:

$$= \left\{\left[\eta\frac{(1-\tau_{ir}^{w})}{(1+\tau_{ir}^{h})}(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_{i}^{\phi_i}\epsilon_i w_{ir}\right]^{\frac{1}{1-\eta}}(1+\tau_{ir}^{h})\left(\frac{1-\eta}{\eta}\right)\right\}^{\beta}(1-s_i)$$

$$= \left[\eta\frac{(1-\tau_{ir}^{w})}{(1+\tau_{ir}^{h})}(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_{i}^{\phi_i}w_{ir}(1+\tau_{ir}^{h})^{1-\eta}\left(\frac{1-\eta}{\eta}\right)^{1-\eta}\epsilon_i\right]^{\frac{\beta}{1-\eta}}(1-s_i)$$

$$= \left[\eta^{\eta}(1-\eta)^{1-\eta}\left(\frac{(1-\tau_{ir}^{w})}{(1+\tau_{ir}^{h})^{\eta}}w_{ir}\right)(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_{i}^{\phi_i}\epsilon_i\right]^{\frac{\beta}{1-\eta}}(1-s_i)$$

Finally, we get the indirect utility function:

$$D = \left[\bar{\eta}\left(\frac{1-\tau_{ir}^{w}}{(1+\tau_{ir}^{h})^{\eta}}w_{ir}\right)(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_{i}^{\phi_i}(1-s_i)^{\frac{1}{\beta\kappa}}\epsilon_i\right]^{\beta\kappa} \tag{16}$$

where $\bar{\eta} = \eta^{\eta}(1-\eta)^{1-\eta}$ and $\kappa = \frac{1}{1-\eta}$.

1.4 Propositions

Proposition 1 states that the overall occupational share can be obtained by aggregating the individual optimal choice.

Proposition 1. *Aggregating across workers, the solution of individual's occupational choice problem is:*

$$p_{ir} = \frac{\tilde{w}_{ir}^{\theta}}{\sum_{j=1}^{N}\tilde{w}_{jr}^{\theta}} \tag{17}$$

where p_{ir} is the fraction of workers in occupation i in region r, and:

$$\tilde{w}_{ir} = \bar{\eta}\left(\frac{1-\tau_{ir}^{w}}{(1+\tau_{ir}^{h})^{\eta}}w_{ir}\right)(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_{i}^{\phi_i}(1-s_i)^{\frac{1}{\beta\kappa}}$$

Proof. Let:

$$\tilde{w}_{ir} = \bar{\eta}\left(\frac{1-\tau_{ir}^{w}}{(1+\tau_{ir}^{h})^{\eta}}w_{ir}\right)(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_{i}^{\phi_i}(1-s_i)^{\frac{1}{\beta\kappa}}.$$

Then, we can rewrite equation (16) as:

$$D_{ir} = [\tilde{w}_{ir}\epsilon_i]^{\beta\kappa}$$

Therefore, the problem solution of individual i living in region r involves picking the occupation with the highest value of $\tilde{w}_{ir}\epsilon_i$. Without loss of generality, consider the probability of an individual choosing occupation 1:

$$\begin{aligned} p_{ir} &= Pr(\tilde{w}_{1r}\epsilon_1 > \tilde{w}_{ir}\epsilon_i) \quad \forall\, i \neq 1 \\ &= Pr\left(\epsilon_i < \frac{\tilde{w}_{1r}}{\tilde{w}_{ir}}\epsilon_1\right) \quad \forall\, i \neq 1 \\ &= \int F_1(\alpha_1\epsilon, \alpha_2\epsilon, ..., \alpha_N\epsilon)d\epsilon, \end{aligned} \tag{18}$$

where F_1 represents the derivative of equation (3) with respect to its first argument and $\alpha_i = \tilde{w}_{1r}/\tilde{w}_{ir}$ for $i \in \{1, 2, ...N\}$. Taking the derivative of equation (3) with respect to ϵ_1, and evaluating in ϵ:

$$\begin{aligned} F_1 &= \theta\epsilon_1^{-\theta-1}\exp\left(-\epsilon_1\hat{Z}\right) \\ F_1(\epsilon) &= \theta\epsilon^{-\theta-1}\exp\left(-\epsilon\hat{Z}\right) \end{aligned}$$

where $\hat{Z} = \sum_{i=1}^{n}\alpha_i^{-\theta}$. Then, equation (18) can be written as:

$$\begin{aligned} p_{1r} &= \int \frac{\hat{Z}}{\hat{Z}}\theta\epsilon^{-\theta-1}\exp\left(-\epsilon^{-\theta}\hat{Z}\right)d\epsilon \\ &= \frac{1}{\hat{Z}}\int \hat{Z}\theta\epsilon^{-\theta-1}\exp\left(-\epsilon^{-\theta}\hat{Z}\right)d\epsilon \end{aligned}$$

This expression is the derivative of equation (3) with respect to ϵ. Hence, we have:

$$\begin{aligned} p_{1r} &= \frac{1}{\hat{Z}}\int dF(\epsilon) \\ &= \frac{1}{\hat{Z}} \\ &= \frac{\tilde{w}_{1r}^{\theta}}{\sum_{i=1}^{N}\tilde{w}_{ir}^{\theta}} \end{aligned}$$

□

The next proposition defines the workers' human capital in each occupation in a given region.

Proposition 2. *For a given region, the human capital of workers in occupation i is:*

$$H_{ir} = p_{ir}\mathbb{E}[h(e_{ir}, s_i)\epsilon_i|person\ choices\ i], \tag{19}$$

The average quality of workers is:

$$\mathbb{E}[h(e_{ir}, s_i)\epsilon_i|person\ choices\ i] = \bar{\Gamma}\left[\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}}w_{ir}\right)^{\eta}\tilde{h}_{ir}p_{ir}^{-\frac{1}{\theta}}\right]^{\kappa} \tag{20}$$

where $\bar{\Gamma} = \Gamma(1-\kappa/\theta)$ is related to the mean of the Fréchet distribution for abilities, $\tilde{h}_{ir} = [(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_i^{\phi_i}\eta^{\eta}]^{\kappa}$ and $\kappa = 1/(1-\eta)$.

Proof. We have:

$$h(e_{ir}, s_i)\epsilon_i = (p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}\left[\eta\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}}w_{ir}\right)(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_i^{\phi}\epsilon_i\right]^{\eta\kappa}s_i^{\phi_i}\epsilon_i \tag{21}$$

H_{ir} is the total efficiency units of labor supplied to occupation i in region r. Then,

$$\begin{aligned} H_{ir} &= p_{ir}\mathbb{E}\left\{(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}\left[\eta\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}}w_{ir}\right)(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_i^{\phi_i}\epsilon_i\right]^{\eta\kappa}s_i^{\phi_i}\epsilon_i\middle|\text{person choices } i\right\} \\ &= p_{ir}\left\{(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}\left[\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}}w_{ir}\right)\eta(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_i^{\phi_i}\right]^{\eta\kappa}s_i^{\phi_i}\mathbb{E}\left[\epsilon_i^{\kappa}\middle|\text{person choices } i\right]\right\} \\ &= p_{ir}\tilde{h}_{ir}\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}}w_{ir}\right)^{\eta\kappa}\mathbb{E}\left[\epsilon_i^{\kappa}\middle|\text{person choices } i\right] \end{aligned} \tag{22}$$

To calculate this last conditional expectation, we use the Fréchet distribution. For now, we suppress the region index r because this calculation is similar in all regions. Let $y_i = \tilde{w}_i\epsilon_i$. Since we are maximizing y_i, it inherits the extreme value distribution:

$$\begin{aligned} \mathbf{Pr}\left(\underset{i}{\mathrm{Max}}\, y_i < z\right) &= \mathbf{Pr}(\epsilon_i < z/\tilde{w}_i) \quad \forall i \\ &= F(z/\tilde{w}_1, ..., z/\tilde{w}_N) \\ &= \exp\left[-\sum_{i=1}^{N}(z/\tilde{w}_i)^{-\theta}\right] \\ &= \exp\left[-kz^{-\theta}\right] \end{aligned}$$

where $k = \sum_i^N \tilde{w}_i^{\theta}$.

The extreme value also has a Fréchet distribution. After some algebraic manipulations it can be concluded that the distribution of ϵ^*, the workers' ability in their chosen occupation, has a Fréchet distribution:

$$G(x) = \mathbf{Pr}(\epsilon^* < x) = \exp\left[-k^* x^{-\theta}\right] \tag{23}$$

where $k^* = \sum_{i=1}^{N}(\tilde{w}_i/\tilde{w}^*)^{\theta} = 1/p^*$.

Finally, we can calculate the expectation of equation (22). Let i be the occupation the individual chooses, and λ some positive exponent.

$$\begin{aligned} \mathbb{E}(\epsilon_i^{\lambda}) &= \int_0^{\infty}\epsilon_i^{\lambda}dG(\epsilon) \\ &= \int_0^{\infty}\theta\left(\frac{1}{p^*}\right)\epsilon^{(\lambda-\theta-1)}\exp\left[\left(\frac{1}{p^*}\right)\epsilon^{-\theta}\right]d\epsilon \end{aligned}$$

We can set $x = \left(\frac{1}{p^*}\right)\epsilon^{-\theta}$ and rewrite the last expression as:

$$\begin{aligned} \mathbb{E}(\epsilon_i^{\lambda}) &= \left(\frac{1}{p^*}\right)^{\frac{\lambda}{\theta}}\int_0^{\infty}x^{-\frac{\lambda}{\theta}}\exp(-x)dx \\ &= \left(\frac{1}{p^*}\right)^{\frac{\lambda}{\theta}}\Gamma\left(1-\frac{\lambda}{\theta}\right) \end{aligned}$$

Using this result in equation (22) completes the proof. □

Corollary 1. *Let W_{ir} be the gross average wage in occupation i in region r. Then:*

$$W_{ir} = w_{ir}\mathbb{E}[h(e_{ir}, s_i)\epsilon_i] = \bar{\Gamma}\eta\frac{(1-s_i)^{-1/\beta}}{(1-\tau_{ir}^{w})}\left(\sum_{i=1}^{N}\tilde{w}_{ir}^{\theta}\right)^{\frac{\kappa}{\theta}} \tag{24}$$

Definition 1. *A competitive equilibrium in this economy consists of individual choices of $\{c, e, s\}$, an occupational choice by workers, total human capital in each occupation and region H_{ir}, final output Y, and efficiency wages w_{ir} for each occupation and region.*

1. *Given an occupational choice, w_{ir}, and the idiosyncratic ability ϵ, each worker chooses c, e, s to maximize utility in equation (1).*
2. *Given market friction, w_{ir}, H_{it}, and ϵ, a worker chooses the occupation that maximizes D_{ir}.*
3. *A representative firm hires H_{ir} to maximize profits.*
4. *The occupational wage, w_{ir}, clears the labor market in each occupation and region.*
5. *Total output is given by the production function in equation (4).*

1.5 Teacher's Human capital

The fraction of teachers, p_{tr}, can be written as:

$$p_{tr} = \frac{\left[\frac{1-\tau_{tr}^{w}}{(1+\tau_{tr}^{h})^{\eta}} w_{tr} s_t^{\phi_t} (1-s_t)^{\frac{1}{\beta\kappa}}\right]^{\theta}}{\sum_{j=1}^{N}\left[\frac{1-\tau_{jr}^{w}}{(1+\tau_{jr}^{h})^{\eta}} w_{jr} s_j^{\phi_j} (1-s_j)^{\frac{1}{\beta\kappa}}\right]^{\theta}} \tag{25}$$

where $\kappa = 1/(1-\eta)$ and

$$\tilde{h}_{ir} = [(p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi_i} \eta^{\eta}]^{\kappa} \tag{26}$$

So, the teacher's human capital is given by:

$$H_{tr} = p_{tr}^{\frac{\nu}{\pi}} (s_t^{\phi_t} \eta^{\eta})^{\frac{\kappa}{\pi}} \left(\frac{1-\tau_{tr}^{w}}{1+\tau_{tr}^{h}} w_{tr}\right)^{\sigma} \gamma^{\frac{1}{\pi}} \tag{27}$$

where $\kappa = 1/(1-\eta)$, $\pi = 1-(1-\alpha)\varphi\kappa$, $\nu = 1+\alpha\varphi\kappa - \theta/\kappa$, and $\sigma = \eta\kappa/\pi$.

1.6 Problem

So, the problem consist in minimize equation 28, using Nelder-Mead.

$$Dist = \sum_{i=1,r=1}^{N,R}\left(\frac{W_{ir}^{M} - W_{ir}^{T}}{W_{ir}^{T}}\right)^2 + \sum_{i=1,r=1}^{N,R}\left(\frac{p_{ir}^{M} - p_{ir}^{T}}{p_{ir}^{T}}\right)^2 \tag{28}$$

where p_{ir}^{M} and W_{ir}^{M} are given by equations 17 and 24 respectively. On the other hand, p_{ir}^{T} and W_{ir}^{T} are given by PNAD data. The superscript indicate model and target statistics. We assume that $\tau_{1r}^{h} = 0$, $\tau_{1r}^{w} = \tau_1^{w}$, $\forall r$. And $A_R = 1$, i.e, the TPF of the last region is nomalized to 1.