Chapter 6

Mass ‘trains’: bulk flux and mixing

6.1 Plumes and entrainment and detrainment

The word *entrainment* (Stommel 1947, 1951) was used in chapter 4 for horizontal thermodynamic mixing: a coefficient in units of % per km, expressing how far the green h_{parcel} curves for ascending parcels in Fig. 4.1 are pulled toward the environmental h curve as they ascend a 1 km distance. This mixing-driven change in a conserved property is just one facet of a *steady-state plume* model, defined by its vertical *mass flux* *M(z)* by

$$dM/dz = M(\epsilon - \delta) \qquad (6.1)$$

where $\epsilon - \delta$ is entrainment minus detrainment (Yanai et al. 1973, de Rooy et al. 2013). Logically, these words imply a “train” with parcels embarking from and disembarking to a larger-scale "environment". Mass continuity requires that M all detrains at the top of the plume, while M at plume base level (usually called *cloud base*) is supplied somehow by horizontal convergence in the subcloud layer.

This crude entity model suffices to serve the mission of studying larger-scale flows in a convecting atmosphere (Part III), providing its M can be translated into a large-scale buoyancy source term Q_b in (2.10) or its symbolically moist version (2.14). If M is ascent at water saturation (moist adiabatic conditions), condensation c is Mdq_{sat}/dz. Other processes like freezing can be included in a proportionate way. The complicated problem of evaporation in precipitation-driven unsaturated downdrafts (section 5.2) can be grossly treated as a lesser, saturated mass flux M_d in an upside-down plume model whose initial mass flux is related back to the precipitation produced by the updraft M. Finally, eddy flux by a plume can be computed from M and the value of other assumed internal scalar properties, assumed to be well mixed within the plume, like h in chapter 4.

This steady plume entity is at the heart of cumulus parameterizations for coarse-grid models, the classic being Arakawa and Schubert (1974) whose Fig. 6 is reproduced below as Fig. 6.1.

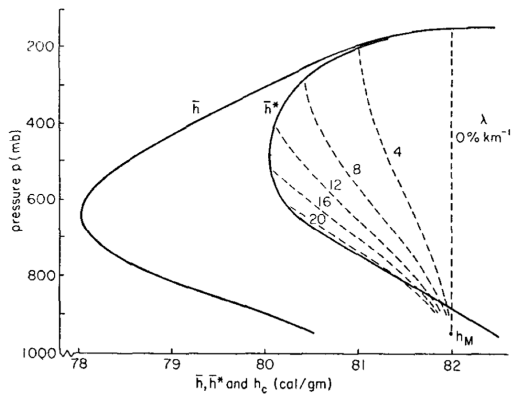

Fig. 6.1. Specific static energy plot like Fig. 4.1 showing how entrainment rate ε (called λ here) maps to the top height of plume buoyancy, in a simple sounding with no intermediate h_{sat} overhangs. Adapted from Fig. 6 of Arakawa and Schubert (1974).

Those authors articulated in symphonic detail the dispatch of plumes spanning a spectrum of different radii (or equivalently, by entrainment rates indexed as $\lambda \sim 0.2/R$). Each steady-state plume was formally envisioned as an average over all stages of the life cycle of an ensemble of same-radius cumulus clouds, mapping ε (λ in Fig. 6.1) directly to cloud top height (where lifted b first or finally vanishes[a]). Detrainment was only at cloud top, while mass convergence below plume base height z_b matched the lower boundary condition M_b there. The potential to

[a] Some elaborations detrain where w vanishes, in a steady w equation with source term b.

kinetic energy conversion term [*w'b'*] depends only on a plume's *M* and *b* profiles, since *w* and *b* are both assumed to be horizontally uniform within the plume. This energy conversion term is used in the scheme's final closure assumption, discussed in more detail in Part III.

Although steady state, Arakawa and Schbuert's (1974) plumes exist only for one time step in an evolving flow. Fortunately for self-consistency, any state-variable changes implied in building and dismantling plume bodies at the beginning and end of the time step are negligible: in substituting *mean* conditions for the *environment,* the *small area approximation* is implied: plume bodies are infinitesimally small[b].

While a plume has realizable mass continuity terms, its thin-layer base convergence and top detrainment ignore the constraints on those inflows and outflows arising from the stratified environment (vertical wavelength dependence) as seen in section 5.1. Nevertheless, the plume entity model remains central for the technical apparatus of so-called *mass flux convection scheme*s, many derived as simplifications of the majestic Arakawa and Schubert 1974 theoretical development. This centrality drives its use as a data analysis framework for simulations (Kuang and Bretherton 2006) and observations (Labbouz et al. 2018).

6.2 "Bulk" plumes as pseudo-ensemble means

The complicated oversimplifications of plume spectrum elaboration can be economized and made more tunable by envisioning a *bulk plume*, with its *M* representing the integrated net vertical flux in an ensemble of individual plumes with different initial radii or other properties. When computational efficiency is prized, the practical simplicity gained is well worth the additional layer of complication in the conceptual model, whose rigor is already quite compromised by tall stacks of idealizations and assumptions. Indeed, doubling down on these compromises begins to

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[b] Finite plume size can be straightforwardly accounted for (Arakawa and Wu 2013) in some ways, but perhaps not rigorously in this aspect (Park 2014).

liberate us into more information-theoretic strategic thinking, like how best to load meaning (distinctions that make a difference) into digital slots or degrees of freedom. Perhaps the ideal conceptual and mathematical framework should have exactly enough inputs and outputs to represent all realizable convective cloud entities or fluxes[c] – provided our appreciation of nature can be wrangled into statements detailed enough to constrain those digital degrees of freedom.

In code terms, the bulk plume approximation is a simplification rather than an elaboration, whose "chief merit is … an equation set with the same structure as that which describes each element" as reviewed by Plant (2010). One main shortcoming may be a required "ansatz" about microphysics, which motivated (Wagner et al. 2011) the predator-prey model of Wagner and Graf (2010) that Part III will revisit. One prominent bulk plume is the equally-weighted integral over plumes in the Community Atmosphere Model scheme of Zhang and McFarlane (1995). While easy to critique in various ways, this scheme remains hard to beat in performance terms, especially after the entire model has been tuned around its behavior for many years.

6.3 Entrainment dilemmas and alternative mixing models

Can a well-mixed plume represent all the scalar quantities in a convective cloud? Must the crucial buoyant air whose rising drives the energetics of convection be a uniform blend of all the properties of all the air that rises? Such a question focuses attention on entrainment (because detrainment does not affect the mixture that comprises the "train"), but that may be misplaced: "despite the focus in the literature on entrainment, it appears that it is rather the detrainment process that determines the vertical structure of the convection in general and the mass flux especially" (De Rooy et al. 2014).

[c] Like a *transilient matrix* (Romps and Kuang 2011).

An early crisis for the entraining plume entity was its inability to reconcile the aircraft-sampled dilution of cloud air with cloud external characteristics like height attained (debated in Warner 1970, Simpson 1971, Warner 1972, Simpson 1972). This cloud-scale dilemma has an echo in global model parameterizations as an unwanted trade-off between unrealistically easy development of deep convection (seen for instance in unrealistic predictions of frequent light rain, and militating for greater entrainment rates) vs. unrealistic dilution of the convection-lofted air entering the upper troposphere (militating for lesser ones). Such false tradeoffs suggest an impoverished framing of the problem, in this case as a single value of a fixed parameter. The process of convective cloud field organization has been suggested as one way to minimally but usefully expand the framework to 2 parameters (Mapes and Neale 2011).

Although instant horizontal mixing of cloud and environment (*entrainment* in Fig. 4.1's sense) is unrealistic, aircraft observations can guide better treatments. Mixtures between two air bodies lie on a straight line in an abstract space of conserved quantities, like points in a q_t vs. h scatter diagram at a given altitude in an aircraft-sampled cloud. This technique has been used to diagnose the multiplicity of mixtures in cumulus clouds (Paluch 1979), including both likely outcomes (random samples by blindly-directed airplanes) and the "lucky parcels" that govern cloud tops, as illustrated lucidly in the extended discussion in chapter 7 of Houze (2014).

These observations were summarized in conceptual models of *stochastic mixing* followed by *buoyancy sorting* to detrain air at its neutral density level (Raymond and Blyth 1986, Kain and Fritsch 1990, Emanuel 1991). Innovation continues in the treatment of mixing, trained on simulations rather than observations, such as Romps (2016) summarizing tracer studies like Romps (2010) and Böing et al. (2019).

6.4 The whole convecting layer as an entity

The end point of our increasingly bulk models of convective entities is whole-layer models, from the humble but profound *mixed-layer* model (all variables well mixed in the vertical) to simple but fully realizable equation-set solutions like Rayleigh-Bénard convection (recently extended to moist air as the "Rainy-Bénard" system in Vallis et al. 2019). A mixed layer embodies the teleological claim that convection is infinitely efficient at neutralizing instability, discussed further in Part III.

6.5 Problems and solutions

6.5.1 *Entraining plume and the concentrations of scalars*

For the entraining plume (no detrainment), integrate (6.1) to show that $M = M_0 exp(\varepsilon z)$.

From the product of (6.1) with a specific (per unit mass) scalar h and its environmental value h_{env}, show that h (green curves in Fig. 4.1, 4.2) can be integrated upward using $h(z + dz) = \epsilon[h_{env}(z) - h(z)]$. Does this relationship depend on whether detrainment is allowed? Explain.