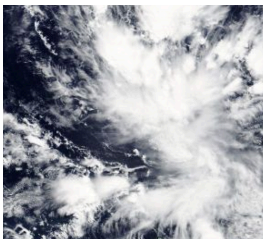
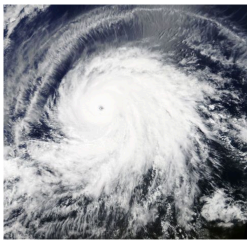
## Chapter 9

# Entity games: coexistence, competition, collaboration

The continuum equations of Part I are a statistical treatment of the interactions among vast ensembles of molecules. That field accounting underpinned the construction of discrete, macroscopic flow *entities*, from cloudy updraft *parcels,* to *thermals* or *bubbles,* to *cells* (Part II), and on into more complex and arbitrary multicellular and even gappy (in terms of cloud) convective *systems* (chapters 7-8).

This chapter explores whether a statistical view of *ensembles of entities* can add new or useful appreciation to the scientific problem of convection -- a *parameterized* view, in the scientific or engineering senses of that term (Arakawa 2004), respectively. The engineering problem is to design better parameterization *schemes*: low-cost convection emulators to run inside scale-truncated (coarse grained) digital atmosphere models for weather and climate. Scientific goals could include formulating a set of field equations for *moist dynamics* that encompasses the expected actions of the ensemble of convective entities.

## 9.1 The dream of a governing statistical principle

Our *hierarchical* diversity of entities, including overlapping classes of entities, invites analogies to disciplines like ecology or economics, as foreshadowed in section 0.5. These are different from the elegant and precise project of statistical mechanics in physics. There, the profound indistinguishability and uncertainty principles of elementary particles align naturally with expressions of our profound lack of knowledge of their detailed states (*maximum entropy* inferences), to beautifully explain long-known empirical laws of thermodynamics (e.g. Jaynes 1957, Mohling 1982).

Some authors with a physical-science bent have tried to reach beyond analogies, seeking to treat planetary-scale flow dynamics as an *extension* of thermodynamic laws. For instance, a *maximized production rate* principle for entropy S can give real teeth to the Second Law's famously enigmatic inequality $dS/dt > 0$. Works reviewed in Paltridge 2001 and Ozawa 2003 offer arguments that nature may be somehow enforcing this optimization in the shaping of fluid motions on macroscopic scales, up to the planetary. But is there a compelling chain-of-causality reason to hope that the forces that act to enforce thermodynamic laws would also be shaping convection? Since gravity remains fundamentally unreconciled with the other forces in physics, and is the main force at play in convection, it seems hard to argue such a case. But we would settle for a useful analogy.

Other meanings of the word "entropy" are also invoked in atmospheric science (reviewed in Liu et al. 2011). Its original sense reflects energy that is *unavailable for macroscopic work* in heat engines (in contrast to *available* or *free* energy[a]). This sense of entropy has clear implications for the efficiency of the gaseous "heat engines" of weather (Emanuel 2005 has a lucid, accessible discussion). Information-theoretic

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[a]Free energy also invites wide-ranging analogizing and extension, for instance in neuroscience (http://en.wikipedia.org/wiki/Free_energy_principle); see Colombo and Wright (2018)'s eloquent restatement and discussion why its "epistemic status is unclear."

entropy, a scalar measure of the breadth of probability distributions best translated as *missing information* (Ben-Naim 2008, exercise 3.8.1), opens a calculus that seems to miraculously transform our profound statement of lack of information into knowledge, as mentioned above (Jaynes 1957). Might information entropy accounting be repurposed usefully in the convecting-flow problem?

## 9.2 Entropy of mini-macro scales, without the microscopic

There are about 500 million square km on earth, and therefore about that many PBL "large eddies" spanning a typical PBL depth of order 1 km. Since the troposphere is roughly 10x deeper, deep convective "cells" are at least 100x fewer -- and fewer than that, since their descent regions are far larger than the updrafts, which tend to have an aspect ratio near 1. Riehl and Malkus (1958) inferred that as few as 1500-5000 "hot towers" of aircraft-observed proportion and properties might do the necessary climatic job of deep convection, before satellites exposed the Earth to our gaze (Fleming 2007). That gaze and other modern tools have transformed thought at least once since then (Zipser 1971).

It seems vain to hope that a maximization principle about *thermodynamic* entropy might be so exacting that it constrain the possible configurations of those mere thousands or millions of convective cells, if their entropy is heedlessly lumped in with the abyss of missing information about the more than $10^{18}$ x $10^{23}$ air molecules in the atmosphere. Even pooling data across time, or under the conceit of an ensemble of possible worlds with different turbulent details, or with numbers boosted by overlapping entities on a hierarchy of scales, the molecular missing information utterly overshadows the number of unconstrained possibilities of convection's configurations on Earth (weather states). Nonetheless, statistical sample numbers in the hundreds or thousands can yield fairly stable mean values. Might it be fruitful to invoke information entropy in a statistical description of only those *mini-macroscales*, as an *analogy* to thermodynamics rather than an *extension*?

To realize such an analogy, we would need to discretize some abstract phase space of state possibilities[b]. Statistical thermodynamics discretizes a 6-dimensional space of molecular positions and momenta, drawing on Heisenberg's uncertainty principle to define the granulation scale of true quantum indeterminacy. Lacking such a fundamental smallest finite scale, we need to decide what sizes of mini-macroscopic (not molecular) entities and activity strengths we wish to treat statistically.

It complicates things that our "entities" are of a wide range of size or even categorical classes. Physics has frameworks for *critical phenomena* that exhibit *phase transitions* in their *order parameter,* in one partially relevant form of multiscale description and dynamics (e.g. Neelin and Peters 19xx). But because *all scales convect* in our atmosphere[c], our entities overlap in a *hierarchy*, different from the picture of merely interacting patches of space, however diverse their size distribution.

## 9.3 Hierarchical ecosystems: concepts and frameworks

Ecology also has to contend with overlapping categories of entities, as individuals really do interact with *populations* of other specis (microbes, sheep, lions), and also with higher-abstraction-level *classes of populations* (food, predators, parasites) whose specific species might be substituted according to local conditions.

In a *Fourier decomposition* of convecting flow, all scales convect, in the sense that each exploits [*Qb*] directly as its total energy source and [*wb*] directly as a kinetic energy source (section 3.6). In this view, the scales are merely *superimposed*: added together as a set of orthogonal basis functions, independent and non-interacting. The *modulation* discussion in section 3 teaches us the fundamental non-uniqueness of this view (section 3.4, exercise 3.8.3). The upward motion in any given

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[b] information theory does not easily extend to continuous probability distributions

[c] Not inevitably: compare the narrow range of sizes in the "mesogranulation" of solar photosphere convection (e.g. Rincon and Reiutord 2018, Kessar et al. 2018).

convective cloud is, by this decomposition logic, *composed of* both high-wavenumber motions expressing runaway parcel buoyancy, plus lower wavenumbers expressing more persistent updrafts of broader scale. Such broad updrafts may play their role through *modulation* of instability in the sense of section 8.1, while their own heat and moisture budgets are satisfied by *rectification* of the net condensation implied by precipitation[d].

In physical space, positive definite precipitation falls both from progressively deepening cumuliform updrafts and from associated stratiform clouds and precipitation (Houze 1997). Together these elements of convective cloud systems function as a gappy or stochastic condensation scheme (in the sense of chapter 8) for larger-scale updrafts. They perform this function just by being entities whose air parcels obey the local laws of motion and thermodynamics, but in the presence of larger-scale or ambient profiles of $b$ and $q$ that subtly shape them. The shaping may happen differently to the mass flux profiles of 'bulk' buoyant vertical drafts vs. to the breadth and slope and longevity of the more laminar saturated flows in stratiform clouds.

A causally chained combination of these elemental entities (shallow convection spawning deep convection spawning stratiform precipitation) acts as a *building block* of larger-scale flows. However, the sub-parts of the building bloc (shallow, deep, stratiform) are systematically extended or elongated (*stretched,* or its opposite stunted) according the phase of larger-scale waves they "compose". The summed effect is a nearly self-similar (building-block-shaped) structure on larger scales (Mapes et al. 2006), remarkably or perhaps unsurprisingly. Some scale dependence of larger-scale waves enters through *aspect ratio*. For instance, with tropospheric depth being the same for all horizontal scales, longer wavelengths imply more inertia and kinetic energy in horizontal flow branches, and more surface drag on those long wind fetches (Kuang 2011, 2012). Also, greater horizontal winds are implied by longer-wavelength patches of divergence (and the vorticity they concentrate,

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[d]with vertical profiles modestly reshaped by eddy fluxes, as in Yanai et al. (1973)

also in a wavelength dependent way). This increasing horizontal wind with wavelength may affect the relative magnitude of moisture variations (governed significantly by horizontal advection or displacements and by windspeed dependent surface fluxes), as compared to density or *b* variations (essentially proportional to vertical displacements). This relative magnitude is an increasing function of wave period (Mapes et al. 2006).

In addition to these overlapping wavelike entities that make Fourier analysis fairly natural, another prominent and entirely different *category* or *manifold* of convective systems must be noted: moist convecting cyclones (noticed in sections 7.6 and 8.2). Bottlenecks to their spontaneous formation make these structures a distinct regime of behaviour, with a portal called *cyclogenesis* (e.g. Nolan 2007). Such rotational wind systems bring in a new fundamental scale: the *deformation radius* (the depth of the flow multiplied by N/vorticity, a ratio which is of order $10^2$ in the ambient subtropics with planetary vorticity *f*). Despite this minimum scale, the diversity of tropical cyclones is breathtaking (Fig. 9.1), and scientifically daunting.

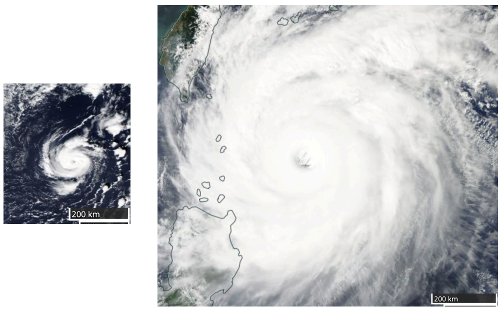

Fig. 9.1. Hurricane Pablo of 2019 (https://go.nasa.gov/2BV4Rfs) and typhoon Jelawat of 2012 (https://go.nasa.gov/31ReAxH). A same-length 200 km scale is in lower right of each image.

Can quantitative frameworks for ecology span these wide ranges of scales and even categories of interacting and overlapping entities? One biophysicist's attempt (Würtz and Annila 2010[e]) envisions a system with an ambient energy level $k_B T$ like thermal energy at temperature $T$. For each species of life (indexed by $j$ or $k$), an intrinsic Gibbs energy (a combination of embodied energy and structural unlikeliness) measures its body structure. Just as in chemical species, the probability of a spontaneous transition from category or species $k$ to species $j$ (perhaps by a $j$ eating $k$ and reproducing) is $e^{-\Delta G_{jk}/k_B T}$. The total probability is that factor multiplied by the number $N_k$ of interchangeable (for this purpose) individuals of species $k$ available. N may be a number *density* per square km, if this is ecology on a landscape. Perhaps several different types of entities (values of $k$) are necessary for a $j$, so the total probability will go as a product of these necessary conditions, with a "degeneracy" $g_{jk}$. that "numbers the $k$-ingredients that remain indistinguishable in the assembled $j$-species."

Ecosystems do not collapse to thermodynamic equilibrium because there is an "external influx" of free energy, which those authors measure in terms of the $k$-to-j transition as $\Delta Q_{jk}$ that "couples to the $jk$-transformation process". Since this influx is "orthogonal...to the system's scalar potential energy differences [$\Delta G_{jk}$] between $j$- and $k$- repository", they use an imaginary notation with $i$ from complex analysis, although it is unclear to this author whether this is a rigorous application of complex numbers. At any rate, within this reasoning framework, they posit a probability $P_j$ as a "measure for the $j$-system"

$$P_j = \left[ \prod_k \left( N_k e^{(-\Delta G_{jk} + i\Delta Q_{jk})/k_B T} \right)^{g_{jk}} / g_{jk}! \right]^{N_j} / N_j!$$

They describe this equation's logic: "The product $N_k$ ... ensures that if any of the $k$-ingredients, e.g., food supplies [individuals in population $k$

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[e] in "maximum energy dispersal" 2nd law arguments, but framing may be generalizable

necessary for individuals of species *j*], is missing, no *j*-species may appear in the ecosystem ($P_j = 0$)." By way of insight, they further offer that "The numerator is raised to the power of $N_j$ because the process may combine the vital $k$- ingredients into any of the indistinguishable $j$-individuals in the population $N_j$. As usual, the division by the factorial $N_j!$ means that the combinatorial configurations of the j-entities in the ecosystem are indistinguishable."

Details aside, one insight of such an equation is that discrete *categories of entities,* each with a *population,* can be accounted with some rigor -- if a single scalar like a Gibbs free energy adjudicates their likelihood, in proportion to a background vigor of random trial-and-error (as measured by temperature and its energy vigor $k_B T$ in statistical physics). As noted above, statistical physics may be best viewed as an *analogy* or source of inspiration than a physical theory to *extend* to complex systems -- if only because our "systems" are convective entities governed by gravity, a force which does not traceably flow from the quantum laws of atomic and molecular physics that govern thermodynamic laws and, arguably, chemistry and biology[f].

Returning to convection, might such a formulation be useful for our "game" of interplay of diverse set of entities, from cells to multicellular but contiguous systems to 'gappy' orchestrated groups of systems? Does this hierarchy of structure play a role in convection doing its one great teleological ‘job', facilitating gravity's relentless project of lowering the center of mass?  Or might convection of such diverse scales and types have *multiple* jobs? After all, living things within ecosystems express more than a single purpose, exploiting the myriad niches created by other living things, even if the entire food web can be described as an "energy dispersal" process (Würtz and Annila 2010) or entropy production system. The Earth's ecologies or *food webs,* in which everything is contingent on everything else, are as distinctively non-unique as Madagascar and Hawaii even under similar gross energy and entropy inputs and outputs. This basic truth further calls into question the dream

[f] The limitations of such "supervenience" arguments bear pondering again (Hoel 2018).

of section 9.1 that a single optimization principle can help us forecast nature's progression or even constrain its statistical character.

## 9.4 Competition and cooperation in predator-prey models

A classic discrete (or *categorical*) mathematical paradigm for entity interactions, borrowed from population ecology, is the Lotka-Volterra equation[g]. It describes the time evolution of the number $n_i$ of individuals in a population of entities of category $i$, within some domain that must be specified, interacting with populations of entities of N other categories (indexed by $j$), here written following Eq. (9) of Nober and Graf (2005):

$$d/dt(n_i) = F_i n_i + \sum_{j=1}^{N} K_{ij}\, n_i n_j \qquad (7.1)$$

If the "food" or forcing coefficient $F_i$ is positive, population $i$ grows exponentially, in the absence of interactions (that is, if $K_{ii} = 0$). If diagonal terms $K_{ii}$ are negative, purely negative interactions occur with other individuals of the same category $i$. Then $(F_i + K_{ii}\, n_i) < 0$ when the population gets too large, and the no-other-interactions solution asymptotes to $n_i = -F_i/K_{ii}$. The fascinations lie with off-diagonal terms in $K_{ij}$. Depending on their signs, the system can express purely cooperative *mutualism* interactions ($K_{ij} > 0$), purely *competitive* negative interactions ($K_{ij} < 0$), or a mixed regime of predator-prey dynamics ($K_{ij}K_{ji} \leq 0$) that acts to redistribute the number of entities among categories.

A similar "master equation" (Eq. 5 of Hagos et al. 2018) allocates "pixels" among entities of types i and j, emphasizing only the off-diagonal terms. With suitable re-weightings, the types could be measured in terms of more continuous budgetable "stuff" like an embodied energy resource or its production rate (biomass or metabolic vigor, in the ecological problem), rather than by a number count. All of this can be normalized per square km of the Earth's surface, in either biological or convective terms, while recognizing that large organisms require a large

[g] https://en.wikipedia.org/wiki/Competitive_Lotka-Volterra_equations

enough domain to exist at all, a problem that vexes scale truncation thinking as discussed below.

All this is only accounting. The question is when and why and how exactly it is *useful* to define entities, and categories of entities, and feedbacks with *self* and with *fellow* category members, and interactions with the truly *other*. In other words, can tallying discrete entities in continuous air truly help us develop a predictive account or at least a deeper appreciation of the natural phenomenon of convection?

The Lotka-Volterra equation discussion of Nober and Graf (2005) continues: “The analogy to convective clouds is straightforward. The reason for convective clouds to form is convective instability (‘food supply’)….each cloud type acts on its environment and tends to reduce instability. Therefore each cloud tends to reduce somehow the ‘food-supply’ for all other cloud types including itself.” Ramirez et al. 1990 clarify this utterly competitive view: “It is observed that the major contribution to the environmental stabilization comes from the drying of the planetary boundary layer induced by subsidence. The thermodynamic effect of nonprecipitating and precipitating convection is to reduce CAPE in the surrounding environment and hence reduce the conditional probability of further convection nearby. A new ... inhibition hypothesis states that, under completely homogeneous external conditions and assuming a spatially random distribution of cloud-triggering mechanisms, the spatial distribution of cumuli in the resulting cloud field must be regular, as opposed to either random or clustered, because cumulus clouds tend to reduce the available energy for convection, thereby inhibiting further convection nearby.”

Nober and Graf's (2005) pure competition view is based on the Arakawa and Schubert (1974, AS74) framework of *cloud work function* (a CAPE to KE conversion categorized by "cloud type" or size) that we encountered briefly in chapter 6. As discussed there, the largest permitted plume or bubble wins the energy game, a propensity the implementers of this framework in parameterization schemes have cleverly battled in numerous ways. The shortcomings of this prominent view flow foreseeably from its ecological postulate of univariate scarcity: The largest permitted plume or bubble tends to dominate, in the absence of dispatcher conditionality or developmental limitations. The exquisitely elaborated but debatable physicality[h] of the early masterpiece (Arakawa and Schubert 1974) has inspired generations. The present book's second artifact of software symbol manipulation-enabled analytic theory, again presented for its texture (a sense of complication level) rather than its face-value meaning requiring symbol definitions, is this culmination in Yano and Plant (2016, their equation 6.9):

$$\begin{aligned} K_{\mathrm{ij}} = {} & \int_{z_B}^{z_{\mathrm{Ti}}} \eta_i \eta_j \left(-\alpha + \epsilon_i \tilde{\mathfrak{a}}_i \frac{\hat{\eta}_i}{\eta_i}\right) \left[\delta_j (s_{\mathrm{vj}}^D - \overline{s}_v) + \frac{\partial \overline{s}_v}{\partial z}\right] \mathrm{dz} + \\ & \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i \hat{\eta}_i \eta_j \left[\tilde{\mathfrak{b}}_i + \tilde{\mathfrak{c}}_i - \frac{\tilde{\mathfrak{d}}_i}{1+\gamma}\left(\frac{\tilde{\eta}_{l_i}}{\hat{\eta}_i}\right)\right] \left[\delta_j (h_j^D - \overline{h}) + \frac{\partial \overline{h}}{\partial z}\right] \mathrm{dz} + \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i \tilde{\mathfrak{d}}_i \tilde{\eta}_{l_i} \eta_j \left[\delta_j (s_j^D - \overline{s}) + \frac{\partial \overline{s}}{\partial z}\right] \mathrm{dz} \\ & - \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i L_v \tilde{\mathfrak{d}}_i \tilde{\eta}_{l_i} \eta_j \left[\delta_j (l_j^D - \overline{l}) + \frac{\partial \overline{l}}{\partial z}\right] \mathrm{dz} \end{aligned} \tag{6.9a}$$

$$\begin{aligned} \left(\frac{\partial A_i}{\partial t}\right)_L = {} & \rho_B \left[\tilde{\mathfrak{a}}_{\mathrm{iB}} \frac{\partial}{\partial t} s_{\mathrm{viB}} + (\tilde{\mathfrak{b}}_{\mathrm{iB}} + \tilde{\mathfrak{c}}_{\mathrm{iB}}) \frac{\partial}{\partial t} h_{\mathrm{iB}} - L_v \tilde{\mathfrak{d}}_{\mathrm{iB}} \frac{\partial}{\partial t} l_{\mathrm{iB}}\right] + \left[\int_{z_B}^{z_{\mathrm{Ti}}} \rho L_v \tilde{\mathfrak{d}}_i \left(\frac{\eta_i}{\tilde{\eta}_{l_i}}\right) \dot{c}_0 \, \mathrm{dz}\right] l_{\mathrm{iB}} + \\ & \int_{z_B}^{z_{\mathrm{Ti}}} \left[\epsilon_i \tilde{\mathfrak{a}}_i \hat{\eta}_i \left(\frac{\partial \tilde{\overline{s}}_{\mathrm{vi}}}{\partial t}\right)_L - \alpha \eta_i \left(\frac{\partial \overline{s}_v}{\partial t}\right)_L\right] \rho \mathrm{dz} + \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i \hat{\eta}_i \left[\tilde{\mathfrak{b}}_i + \tilde{\mathfrak{c}}_i - \frac{\tilde{\mathfrak{d}}_i}{1+\gamma}\left(\frac{\eta_i}{\hat{\eta}_i}\right)\right] \left(\frac{\partial \overline{h}}{\partial t}\right)_L \rho \mathrm{dz} + \\ & \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i \tilde{\mathfrak{d}}_i \eta_i \left(\frac{\partial \overline{s}}{\partial t}\right)_L \rho \mathrm{dz} - \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i L_v \tilde{\mathfrak{d}}_i \eta_i \left(\frac{\partial \overline{l}}{\partial t}\right)_L \rho \mathrm{dz} + \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i L_v \tilde{\mathfrak{d}}_i \eta_i \frac{\partial}{\partial t}\left(\frac{\rho \sigma_i}{\epsilon_i M_i} \tilde{r}_i\right) \rho \mathrm{dz} + \\ & \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i L_v \tilde{\mathfrak{d}}_i \eta_i \int_z^{z_{\mathrm{Ti}}} \dot{c}_0 \, \mathrm{dz}' \, \rho \mathrm{dz} + \dot{z}_{\mathrm{Ti}} G_{\mathrm{Ti}} + \dot{z}_B G_{\mathrm{Bi}} \end{aligned} \tag{6.9b}$$

Is such an artifact the pinnacle of knowledge, or the outer edge of human-accessible meaning? Might more general possibilities have been hidden by the peculiarities of the parameterization *scheme design* problem, forced upon us by coarse-resolution numerical model truncation (and thus cemented in the pioneering classic AS74)?

## 9.5 Can we reject a random patterns null hypothesis?

What would a scientifically minded person, with the continuum equations of Part I and entity notions of Part II to draw on, expect the world to look like, before the first satellite gave us the grand overview

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[h] Presented as a partitioning of *energy* (as clarified in Lord and Arakawa 1980), but the non-orthogonality of "cloud type" (plume radius) categories means that attempting to distribute physical energy over that basis set lacks true rigor.

we now take for granted? Fleming (2007) shows a striking shortcoming of multi-scale imagination in relatively well-informed circles. Another intriguing essay on the satellite-age transition is Zipser (1970)'s "rise of the fourth school of thought." Although it is impossible to un-see the richness of texture, a surprisingly repeatable experience of surprise suggests that our brains have a hard time realistically remembering just how rich[i], in the face of temptingly simple ideas and conceptions.

Suppose we hypothesize that convection has *meaningful patterns*. To test this hypothesis, observational evidence must be shown to stand above some baseline or *null hypothesis* that convection is *meaningless or random*. Defining those null words well is half the battle in the quest for meaning. For instance, images of white clouds over a dark ocean differ obviously from those same dark and light pixels shuffled into maximum-entropy speckle patterns. But who cares?

A more sophisticated entity-based approach is to tally contiguous areas of whiteness or *cloud objects*. We can then ask if their size distribution deviates from some expectation that defines a random null hypothesis, like a pattern of gravel and rocks spilled onto a street from a great height.

One classic study with a well-defined stochastic model for its randomness baseline is Lopez (1977), whose results are adapted in the top panels of Fig. 9.1. The straightness of the plotting symbols (emphasized with line annotations) indicates the *empirical adequacy*[j] of a stochastic model: the multiplication of independent random numbers[k]. However, physical interpretation is nonunique, an art of science. Such multiplication could for instance embody the *independent random dispatch* of bubble-like entities whose superposition causes, in a

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[i] A visit to https://worldview.earthdata.nasa.gov is advisable before thinking too much.

[j] https://plato.stanford.edu/entries/constructive-empiricism/

[k] The *central limit theorem* states that a sum of many independent identically distributed (iid) random variables asymtotes to the normal (Gaussian) distribution. The log of a product of many iid variables becomes a sum, satisfying this theorem.

multiplicative way, the cloud as a whole to reach a greater height. Additional plausible interpretations are discussed in Lopez's paper. Other properties of convective cloud entities (width, duration) also appear to follow such laws adequately in this log-normal line straightness measure.

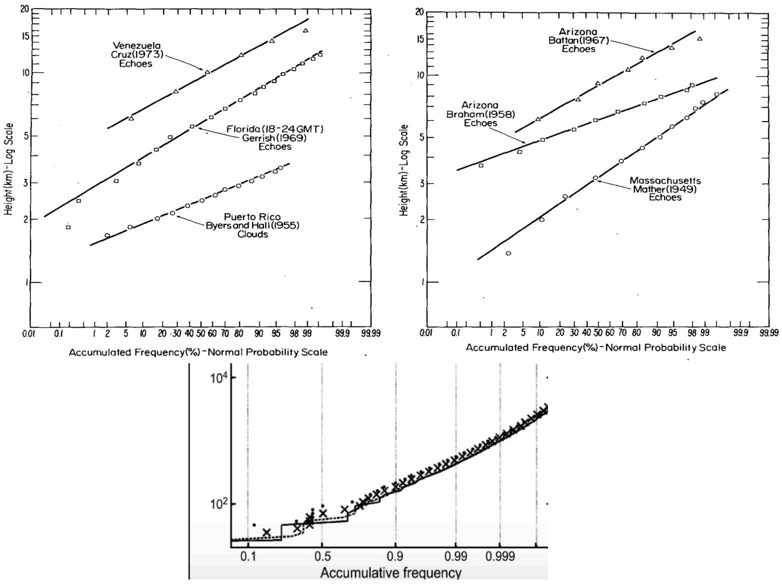



Fig. 9.2. Distributions of cumulus cloud height (top, adapted from Lopez 1977) and MCS anvil area (bottom, adapted from Yuan and Houze 2010), plotted against the cumulative normal function so that straightness of the arrangement of data symbols is indicative of nearness to a lognormal distribution.

In addition to the "inner" interpretive considerations in terms of whether the plotted data is nearly straight on the graph, there is always an "outer" layer of consideration of conditional pre-screening or sampling that must always be considered (section 0.6). How were cloud scenes entering the study screened? Which scenes were excluded, and which others were declared identical enough to be statistically pooled? To what degree do the results reflect patterns in the data, rather than being enforced tacitly by the study's screening assumptions?

Would data generated in a hypothetical *randomworld*, embodying a null hypothesis of maximum entropy or some other expression of lack of meaningful patterns, appear differently from natural data in the plotting space after being conditionally screened and filtered and pooled byababan the same study assumptions? If so, the difference result is precisely as interesting (or not) as the *randomworld* is compelling, in its construction of salient aspects of the data.

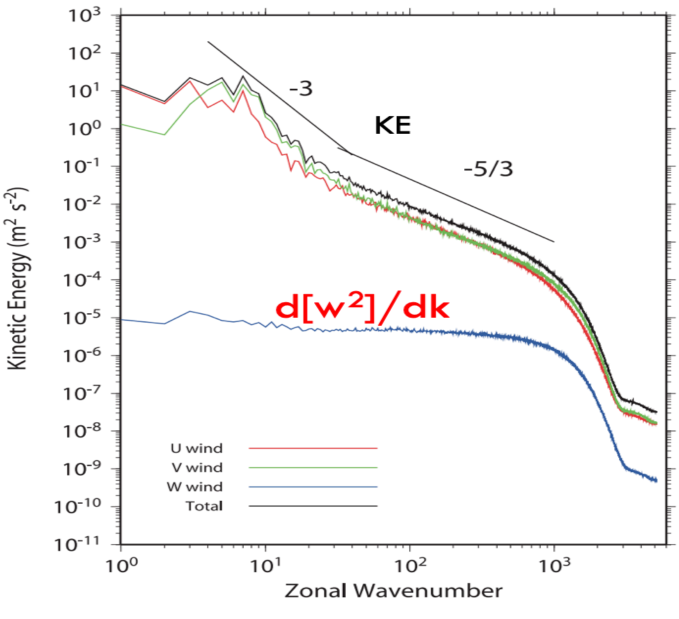
To get beyond the unorganized (by selection) conditions upstream of the top panels of Fig. 9.1, the bottom panel shows the area of MCS high cloud objects observed in all-conditions satellite data over oceanic warm pool areas (Yuan and Houze 2010). This measure of convective entity size has a much wider dynamic range than cumulus cloud height, bounded by the troposphere. While the data curve is not as rigorously straight, again the lesson appears to be that a random multiplicative process, like the combination of *independent* probabilities governing the development of larger entities, is not easy to reject as a null hypothesis for natural convective cloud fields.

Overlap assumptions between vertical layers in cloud-radiation interaction may offer another useful toolkit or at least analogy for defining "organization" in deep convection, even if the dispatcher function is random. Just as the fate of photons traveling vertically depends on the overlap of cloud patches at different levels, so does the fate of ascending parcels in the sense of chapter 4, mixing with air in the layers they traverse or penetrate. Random overlap is known to be drastically excessive for radiation, as even modest fractional cloudiness in each of many layers permits almost no photons to traverse a many-layer atmosphere and reach the surface (for solar radiation) or escape to space (for upwelling infrared). Maximum overlap, on the other extreme, means that the greatest cloud fraction at any single level is the total cloud fraction as seen by radiation. A meaningful concept of "organization" must lie in the space between these extremes, perhaps as a simple linear scale between them (Mapes and Neale 2011), just as the compromise "maximum-random" assumption is better than either extreme in the cloud overlap problem for radiation.

## 9.6 Are entities more real than wavenumbers?

Continuing with the spirit of null hypotheses, undercutting our hopes of special importance and meaning for entities and their agglomerations, we must return to the spectral slope of simple Fourier decompositions (section 0.6). Although somewhat arbitrary[1], Fourier decompositon is powerful, and must be viewed as a baseline for the question: Does discrete entity counting like in Fig. 9.2 pass the test of showing more about nature than about its definitions and assumptions? Does entity counting make more sense than Fourier decomposition?

A useful null hypothesis for non-organized (random) convection in Fourier space is the spectral slope of vertical motion w.




[1] not quite: sinusoids are solutions to linearized versions of our equations (section 2.2.2)

Fig. 9.3. Spectral distribution of velocity variance with respect to spatial wavenumber, adapted from Terasaki et al. (2009).

A complete lack of correlation horizontally among updrafts appears as a *white* (or flat) spectrum of *w* variance with respect to horizontal wavenumber. As Fig. 9.3 shows, this null hypothesis is violated only modestly by the first generation of global nonhydrostatic cloud-resolving models. This finding is yet more evidence that organization of convection is a slight and subtle effect: the answer to section 9.5's titular question again appears to be *barely,* at best.

## 9.7 Lessons from and for large-scale modeling

### 9.7.1 *Popcorn v. typhoons in storm resolving resolving models*

Statistics have their limiations. Power (squared amplitude) spectra in Fourier analysis ignores the phase information that distinguishes a singular, isolated delta function (a highly organized situation) from utterly spatially incoherent speckle patterns. We have no way to measure this difference in phase information in spectral space, except as reconstructions (or the structure of un-decomposed data) in physical space.

One early lesson of global cloud resolving models (Satoh et al. 2019) was that the climatic role of convection could be fulfilled by either random "popcorn" convection or by a population of long-lives super-typhoons, based on whether the PBL scheme had low or high ventillation at its top. This tunable aspect of PBL turbulence was soon tuned toward realism, but its lesson remains: there is a meaningful "game" or competition between tropical cyclones and ordinary convection in otherwise entirely realizable Earth-like atmospheres, as discussed in Zhao et al. (2012). More broadly, the texture differences between a suite of credible global cloud-resolving models are distinctive, yet hard to characterize and hard to distinguish robustly from nature (the "Palmer-Turing test" in the DYAMOND model intercomparison of Stevens et al. 2019). Our measurement tools seem to be inadequate.

### 9.7.2 *Parameterization and its schemes*

A parameterization *scheme* is a low-dimensional surrogate model, much simpler than a real system, which produces almost the same gross or bulk behavior by some carefully defined practical measures. Parameterization schemes for *scale-truncated numerical models* are a particular and peculiar example that has defined the problem space. Whether the scale truncation filter is coarse-grid area averages or horizontal spectral truncation, the missing flow motions that need to be reproduced by a scheme are (i) spectrally *blue* in the horizontal, meaning restricted to sub-grid scales or high wavenumbers; (ii) all-scale in the vertical dimension, a domain which even horizontally coarse models aim to resolve; and (iii) discretized to the model's time step, which is set by the fastest information transmission process between grid cells (usually a wave propagation speed), and therefore strictly local to a model grid column (so that convection must interact with its own "compensating subsidence"). For precipitating convection, there is also (iv) a net (filter-scale) conversion of vapor to heat, which rectifies directly onto large scales, but which depends on the small-scale vertical motions in point (i) so that it enormously amplifies the importance of errors in postulates about those parameterized motions.

The problem of designing such schemes has often overshadowed the underlying scientific problem of devising statistical theories of the whole convecting flow -- even while serving as a strong motivation (Arakawa 2004). The most profound difficulties of scheme design may actually lie in the time domain, followed by the vertical domain: these are the dimensions where we directly care about the scheme's explicit output (the tendencies it produces). Nonetheless, most mechanistic reckonings dwell inordinately on a scheme's inner assumptions and considerations about the *horizontal* scale truncation, assessing those considerations on arguably secondary virtues like the internal consistencies of what are clearly gross assumptions.

This problem is now frustrating its third generation of thinkers, in part because a decent level of success on the largest scales can be accomplished by rather simple ideas. For instance, *convective adjustment* is a concept directly in the time-height domain, local in the horizontal, and one that even mechanistic approaches can be re-interpreted in terms of (Arakawa 2004). The success of the brute-simple Betts-Miller (1984) convective adjustment scheme was humbling for much more painstakingly articulated schemes rooted in fluid-mechanical arguments about Part II's entities. Today such elaborated mass flux schemes (chapter 6) are back to being the leading approach, but only in the unabashedly engineered "bulk" sense of section 6.2. Final closures are usually still teleological claims of adjustment, albeit with a specified time scale whose calibration may be fitted to different grid truncation lengths ("scale-aware" is a lauded contemporary virtue in schemes).

Still there are longtime dilemmas[m] in convections schemes, indicative of an ill-fitting or impoverished framework. For instance, entrainment is simultaneously too great for climate (cloud heights are too low in realistic profiles; and profiles are too unstable in realistic equilibria), yet too small for weather (deep convection occurs too easily in dry conditions). The deeper lesson is that deep convection is too *unconditional* in schemes Opening up the environmental sensitivities of convection to overt barely-physical engineering, such as direct RH dependence (e.g. Hirons et al. 2016), improves performance but severs the literal connection back to convective entity dynamics as a source of essential constraints and claims of rigor. In this engineered approach, a plume becomes merely an adequate updraft-with-mixing framework which enforces the necessary integral constraints of the equation set, with the unfortunate but seemingly numerically inescapable virtue of time independence (a 1-dimensional calculation in z at each time step).

New pinnacles of elegant elaboration (Park 2014, 2019) perform comparably well with much cruder approaches, which is to say not yet breakthrough-level better. Meanwhile, equally elegant attempts to unify all of turbulence (e.g. Bogenschutz et al. 2018) or re-envision the whole dynamical solver to encompass cloudy and clear air volumes (Thuburn et

[m] Mapes and Neale (2011)

al. 2018) have yet to sweep the field. Stochastic replacements for entrainment (Romps 2016) seem promising at this writing, but implementation ends up mattering to model performance as much as the appeal of mathematical formulations. Perhaps any scheme based on a too-rigid, too-specific framing will need external wrappings (stochastic or situational dependences like "effective" entrainment rates) to account for the myriad ways that natural convection escapes the grip of any one formulation as a definably-distinct entity in a well-defined mean environment.

Gross response functions (sensitivities and impacts) may provide the crucial top-level connection between natural convection and parameterization schemes. Diagnostic approaches that point to different values of scheme parameters for different phenomena (like large-scale convection waves or vacillations of different scales or frequencies, e.g. Adames et al. 2018, Wolding et al. 2019) are an indication that our scheme framing may still be collapsed into a too-small subspace of the real physical system. Convective clouds exist and behave without regard to whether they are part of short or long or high or low frequency waves, and indeed their responses are integral to *all* those larger scales simultaneously. Only by insisting that a scheme or surrogate model participate properly in the whole spectrum of overlapping entities and envelopes will we approach skill, and the underlying physical knowledge or appreciation we seek.

Finally, it must be reiterated that the study of severe storms and other particular situations, largely neglected in this statistical view, can also offer stark crises for our too-small conceptualizations that are inadequate to the mystery and power of convection in our atmosphere. Together the study of storms and statistics have many lessons yet to teach us.