Chapter 6

Mass 'trains': bulk flux and mixing

6.1 Plumes and entrainment and detrainment

The word *entrainment* (Stommel 1947, 1951) was used in chapter 4 for horizontal mixing of thermodynamic quantities: a coefficient with units of % per km, expressing how far the green h_{parcel} curves for ascending parcels in Fig. 4.1 are pulled toward the environmental *h* curve as they ascend a 1 km distance. This mixing-driven change in a conserved property is just one facet of a *steady-state plume* model, defined by its vertical *mass flux M(z)* by

$$dM/dz = M(\epsilon - \delta) \tag{6.1}$$

where $\epsilon - \delta$ is entrainment minus detrainment (Yanai et al. 1973, de Rooy et al. 2013). Logically, these words imply a "train" with parcels embarking from and disembarking to a larger-scale "environment". Mass continuity requires that M must all detrain at the top of the plume, while M at plume base level (usually called *cloud base*, although it need not be at the LCL of surface air) is supplied somehow by horizontal convergence in the *sub-cloud layer*.

This crude entity model suffices as a framework for larger-scale flows in a convecting atmosphere, providing its *M* can be translated into a large-scale buoyancy source term Q_b in (2.10), or in its symbolically moist version (2.14) with a corresponding *h*-conserving term in a specific humidity equation (often discussed as "Q_1 and Q_2" from the classic

Yanai et al. 1973). If M reflects ascent of air at water saturation, condensation $c = M\ d/dz(\ q_{sat}(T_{plume})\ )$, where $T_{plume}(z)$ itself is moist adiabatic ascent, Strictly, the *moist adiabat* is subject to the conditions of the plume's assumptions about microphysics, etc., and processes complications like freezing can be included in an M-proportionate way.

To be still more realistic, evaporation in precipitation-driven unsaturated downdrafts (section 5.2) can be grossly treated as a lesser, saturated *downdraft mass flux* M_d in an upside-down plume model, whose initial (top) conditions are related back to the precipitation produced in the updraft M. Finally, the *eddy flux* implied by these plumes can be computed from M and the other internal plume scalar properties like h, s, and q assumed to be well mixed across the plume. The difference of these from some larger-scale horizontal averaging (a *filter scale*) defines "eddy", as discussed in chapter 3.

This steady plume entity, characterized by its $M(z)$, is also the conceptual heart of *mass-flux cumulus parameterization* for coarse-grid models. The great classical formulation of this problem is in Arakawa and Schubert (1974)[a], whose Fig. 6 is adapted here as Fig. 6.1.

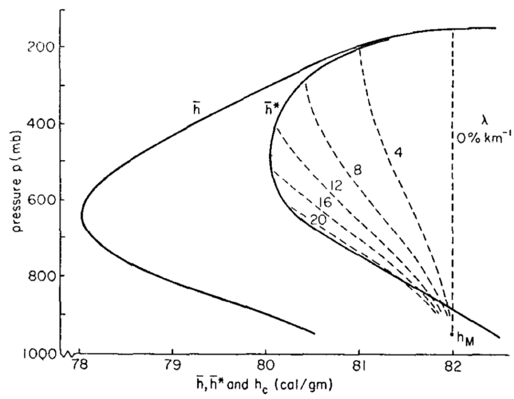

Fig. 6.1. Specific static energy plot like Fig. 4.1 showing how entrainment rate ε (called λ here) maps to the top height of plume buoyancy, in a simple sounding with no intermediate h_{sat} overhangs. Adapted from Fig. 6 of Arakawa and Schubert (1974).

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[a] although the reader should also notice Ooyama (1971), to be discussed more in Part III

Those authors articulated in symphonic detail the summation of plumes spanning a spectrum of different radii R (or equivalently, of entrainment rates indexed as $\lambda \sim 0.2/R$). Each steady-state plume was formally envisioned as an average over all stages of the life cycle of an ensemble of same-radius cumulus clouds, mapping the spectrum of ε (λ in Fig. 6.1) on to a spectrum of cloud top height (where lifted b first or finally vanishes[b]). Top and bottom conditions were detrainment of top mass flux M_t at cloud top[c], and lower boundary condition M_b at plume base height z_b (implying uniform mass convergence below z_b) was left as an undetermined variable to be used in a final, teleological *closure assumption* based on potential to kinetic energy conversion [wb]. That depends only on a plume's M and b profiles, since w and b are both assumed to be horizontally uniform within the plume.

Although steady state, Arakawa and Schubert's (1974) plumes exist only for one time step in an evolving flow. Fortunately for self-consistency, any filter-scale state variable changes implied in building and dismantling plume bodies at the beginning and end of the time step are negligible: in substituting *mean* conditions for Ooyama's (1971) formulation in terms of *environment,* the *small area approximation* for plume size is made[d].

While a plume is *kinematically* realizable (satisfying mass continuity, for instance), its assumptions about very thin layers of base convergence and top divergence are heedless of dynamical constraints in a stratified environment (discussed in Part III). Concerns about how steady-state plume assumptions meet timestep dependences in coarse-grid host models are yet again "as much craft as science", in the realm of *numerics* that is recently getting needed scientific attention (Gross et al. 2018). For present goals those issues are far afield, but the steady plume is so

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[b] Some elaborations detrain where w vanishes, in a steady w equation with source term b.

[c] Later elaborated by other authors to all altitudes, in *entraining - detraining plumes.*

[d] Finite plume fractional area coverage can be accounted for (Arakawa and Wu 2013), but arguably with logical shortcomings (Park 2014, Zhang et al. 2015).

influential conceptually, for instance as a framework for interpreting simulations (Kuang and Bretherton 2006) and observations (Labbouz et al. 2018), that an educated appreciation requires knowing its logic and vocabulary, including the rather practical roots of that logic.

6.2 "Bulk" plumes as pseudo-ensemble means

The *plume spectrum* picture elaborated above can be economized, and also made more practically tunable as an engineering construct, by envisioning a *bulk plume* with a single *M(z)* characterizing the integrated net vertical fluxes in a whole *ensemble* of plumes with different initial radii or other properties. When computational efficiency is prized, the practical simplicity gained is well worth the additional layer of complication (an integral, with additional assumptions) in the conceptual or analytic underpinning – especially when its apparent rigor is already quite compromised by tall stacks of idealizations and assumptions.

In a sense, doubling down on these compromises begins to liberate the reasoner into more information-theoretic or strategic thinking, like how best to load meaning (distinctions that make a difference) into the economy of available digital bins or degrees of freedom. Perhaps the ideal conceptual and mathematical framework would enforce only the most trusted conservation principles, and then be contrived to have exactly as many inputs and outputs as needed to represent all realizable convective cloud entities or fluxes. Examples might include the *transilient matrix* of Romps and Kuang (2011) or the ballooning area of machine-learned emulators (e.g. Beucler et al. 2019).

In code terms, the bulk plume is clearly a simplification rather than an elaboration, whose "chief merit is … an equation set with the same structure as that which describes each element" as reviewed by Plant (2010). One main shortcoming may be a required "ansatz" about microphysics, which motivated (as expressed in Wagner et al. 2011) the predator-prey model of Wagner and Graf (2010) that Part III will revisit. One prominent bulk plume is the equally-weighted integral over plumes in the Community Atmosphere Model scheme of Zhang and McFarlane (1995). While easy to critique in various ways, this scheme remains hard to beat in performance terms, especially after the entire model has been tuned around its behavior for years. It remains in CAM in 2019.

6.3 Entrainment dilemmas and alternative mixing models

How well can a plume that assumes thorough, instantaneous horizontal mixing represent the scalar quantities in a convective cloud, including the products of scalars that comprise flux terms? Must the crucial buoyant rising current [*bw*] that drives the kinetic energy be a uniform blend of all the properties of all the air that rises? Such a question focuses attention on entrainment (because detrainment does not affect the mixture that comprises the "train"). Actually, that spotlight may be misplaced: "despite the focus in the literature on entrainment, it appears that it is rather the detrainment process that determines the vertical structure of the convection in general and the mass flux especially" (De Rooy et al. 2014).

An early crisis for the entraining plume entity was its inability to reconcile aircraft-sampled dilution of cloud air with cloud external characteristics like height attained (debated in Warner 1970, Simpson 1971, Warner 1972, Simpson 1972). This cloud-scale dilemma has an echo in global model parameterizations as an unwanted trade-off between unrealistically easy development of deep convection (seen for instance in unrealistic predictions of frequent light rain, a problem militating for increased entrainment rates) vs. unrealistic dilution of the convection-lofted air entering the upper troposphere (militating for decreased rates). Such unwanted tradeoffs suggest an impoverished framing of the problem, in this case as a single value of a single parameter of overwhelming importance. The phenomenon of convective cloud field *organization* has been suggested as one minimalist way to expand the framework to 2 parameters (Mapes and Neale 2011).

Besides proving that instant horizontal mixing is unrealistic, aircraft observations can guide better treatments. Mixtures between two air bodies lie on a straight line in an abstract space of conserved quantities. This visual method has been used to diagnose the multiplicity of mixtures in cumulus clouds (Paluch 1979), using a q_t vs. h scatter diagram of data inside aircraft-sampled clouds. This technique helped distinguish the most *likely* outcomes of mixing (random samples by blindly-directed airplanes) from the "*lucky* parcels" that govern cloud top height, as illustrated lucidly in the extended discussion and diagram sequence in chapter 7 of Houze (2014).

Such observations led to conceptual models of *stochastic mixing followed by buoyancy sorting*, with air detrained at its altitude of ultimate (final) neutral buoyancy (Raymond and Blyth 1986, Kain and Fritsch 1990, Emanuel 1991). Innovation continues in stochastic treatments of mixing, trained on simulations rather than observations, such as Romps (2016) summarizing tracer studies like Romps (2010), and Böing et al. (2019). It remains to be seen whether statistical ensemble concepts, perhaps bolstered graphically by an updated soundings-analysis view like in chapter 4, can or should supplant simplistic modeling-driven plume notions in the field's culture of reasoning.

6.4 The whole convecting layer as an entity

The end point of our increasingly *bulk* models of convective *entities* is whole-layer models, from the humble but profound *mixed-layer* model (all variables well mixed in the vertical) to simple but fully realizable equation-set solutions like Rayleigh-Bénard convection (recently extended to moist air as the "Rainy-Bénard" system in Vallis et al. 2019). The well-mixed layer embodies the teleological claim that convection is infinitely efficient at its job of neutralizing instability, a useful concept called *adjustment* that can be retrofitted across the whole landscape of nominal conceptualizations of convection (Arakawa 2004). Key questions include whether and how adjustment applies across instability's multivariate *ingredients* like humidity vs. lapse rate of T.

6.5 Problems and solutions

6.5.1 *Entraining plume and the concentrations of scalars*

For the entraining plume (no detrainment), integrate (6.1) to show that $M = M_0 exp(\varepsilon z)$.

From the product of (6.1) with a specific (per unit mass) scalar h and its environmental value h_{env}, show that h (green curves in Fig. 4.1, 4.2) can be integrated upward using $h(z+dz) = \epsilon[h_{env}(z) - h(z)]$. Does this relationship depend on whether detrainment is allowed? Explain.

6.5.2 *Single column computations and convective adjustment*

A somewhat technical but illuminating exercise with the Community Atmosphere Model (CAM) from the National Center for Atmospheric Research (NCAR) is to install and operate a *containerized* version of its single column model, easily run on any modern laptop after installing the Docker container software (a light version of virtual machine technology that smooths over the difficulties of operating-system and library dependencies). While this URL will change with model versions (http://www.cesm.ucar.edu/models/cesm2/atmosphere/CAM6tutorial/ hard-wires the current model versions cesm2 and CAM6 at this writing), a sustained capability is anticipated, findable with Web-search skill. Simple starter questions could include: *How do the mean lapse rate and humidity profiles in equilibrium, and temporal variability, depend on the specified entrainment rate, the model's most important single disposable parameter*?