
Chapter 9

Entity games: competition, coexistence, collaboration



The continuum equations of Part I are a statistical treatment of vast ensembles of molecules. That field accounting underpinned the construction of discrete, macroscopic convective entities in Part II, from updraft *parcels* to *thermals* or *bubbles* to multi-bubble *cells*, and on into multicellular and even gappy (partly cloudy) *systems* (chapters 7-8).

This chapter explores whether a statistical view of ensembles of entities can add new appreciation or useful tools -- a *parameterized* view, in the scientific or engineering senses of that term, respectively (Arakawa 2004). The engineering problem is to design better parameterization *schemes*: low-cost convection emulators to run inside scale-truncated (coarse grained) digital atmosphere models for weather and climate. Scientific goals could include formulating field equations for *moist dynamics*, encompassing the *expected* (mean and perhaps also stochastic) actions of ensemble of convective entities. If statistics govern tails as well as means, perhaps extremes could be anticipated as well.

All scales convect in a Fourier sense, with each wavenumber exploiting $[Qb]$ as its total energy source and $[wb]$ as its KE source (section 3.6). In this view, the scales are merely *superimposed*: added together as a set of orthogonal basis functions, independent and non-interacting. In this logical framing of decomposition, the upward motion in any given convective cloud is *composed of* both high-wavenumber motions expressing runaway parcel buoyancy, plus lower wavenumbers expressing more persistent updrafts of broader scale. The *modulation* discussion in section 3 teaches us the fundamental ambiguity or non-uniqueness of this additive view (section 3.4, exercise 3.8.3). Perhaps broad updrafts play their role through *modulation* of instability in the sense of section 8.1, while their own heat and moisture budgets are satisfied by *rectification* of the net condensation implied by the positive definiteness of precipitation^a.

In physical space, positive definite precipitation is produced both by progressively deepening cumuliform updrafts and by associated stratiform clouds and precipitation (Houze 1997). Together, these elements of convective cloud systems function as a gappy or stochastic condensation scheme for larger-scale updrafts. They perform this function just by being entities whose air parcels obey the local laws of motion and thermodynamics, but in the presence of larger-scale or ambient profiles of b , q , etc. that subtly shape their evolutions rates and fates.

A causally chained combination of these elemental entities (shallow convection spawning deep convection spawning stratiform precipitation) appears to act as a basic *building block* of larger-scale flows. However, the sub-parts of the building block (shallow, deep, stratiform) are sometimes stunted (e.g. shallow only), or systematically extended (e.g. with delayed transitions) or elongated or retracted (e.g. by the longevity of stratiform entities), according the phase of larger-scale waves they occur within (those ambient profiles of b , q , wind shear, etc.). The summed effect is a nearly self-similar (building-block-shaped) structure

^awith vertical heating profiles modestly reshaped by eddy fluxes, as in Yanai et al. (1973)

on larger scales (Mapes et al. 2006), remarkably -- or perhaps unsurprisingly, if basis sets limit the shapes of their combinations.

Some scale dependence of the larger-scale waves does occur, through aspect ratio. For instance, with tropospheric depth being the same for all horizontal scales, longer wavelengths require more inertia (imparted by the PGF) and kinetic energy (generated by $[wb]$) in the long horizontal flow branches, and suffer more surface drag on those long wind fetches (Kuang 2011, 2012). The greater horizontal wind speeds implied by spatially broader patches of divergence (and the vorticity it generates) introduce a scale dependence to wind-induced surface flux. Both wind speed, and the distances traversed on a nonuniform Earth, shape horizontal advective tendencies, so the relative magnitude of fluctuations of a prominently advected quantity like q , compared to b which is shaped mainly by vertical displacements, increases systematically with disturbance wavelength or period (Mapes et al. 2006). The largest-scale tropical wave (the Madden-Julian Oscillation) is thought to be largely q -governed), and almost of a qualitatively different character than the higher-frequency waves (thought to be more b -governed by a vertical dipole mode, as detailed in earlier chapters; Kiladis et al. 2009 is a good starting point for more detailed readings).

Another prominent and entirely different *category* or *manifold* of convective systems must be noted: moist convecting cyclones (noticed in sections 7.6 and 8.2). Bottlenecks to their spontaneous formation make these structures a distinct regime of behavior, lying past a developmental portal called *cyclogenesis* (e.g. Nolan 2007). Such rotational wind systems bring in a new fundamental scale: the *deformation radius* (the depth of the flow multiplied by $N/vorticity$, a ratio of order 10^2 in the ambient subtropics for merely planetary vorticity f). Despite this *minimum* scale of geostrophically balanced cyclones, the diversity of actual sizes is breathtaking (Fig. 9.1), and scientifically daunting.

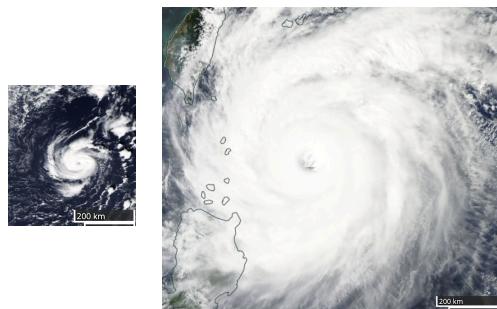


Fig. 9.1. Hurricane Pablo of 2019 (<https://go.nasa.gov/2BV4Rfs>) and typhoon Jelawat of 2012 (<https://go.nasa.gov/31ReAxH>). A same-length 200 km scale is in lower right of each image.

9.1 The dream of a governing statistical principle

Our daunting diversity of entities, including a *hierarchy* of overlapping classes thereof, invites analogies to disciplines like ecology or economics more than to the elegant and precise project of statistical mechanics, as foreshadowed in sections 0.5 and 3.7. In thermodynamics, entity interchangeability is fundamentally true: profound uncertainty principles of atomic physics correspond perfectly with expressions of our profound lack of knowledge (embodied in *maximum entropy statistical inference*) to beautifully explain the long-known empirical laws of thermodynamics (Jaynes 1957, Mohling 1982).

Some authors with a maximally physics-minded worldview have tried to reach beyond mere analogies, to treat planetary-scale flow dynamics as an *extension* of thermodynamic laws. For instance, a *maximum rate* optimum principle for entropy S can give real teeth to the Second Law's famously enigmatic inequality $dS/dt > 0$. Works reviewed in Paltridge 2001 and Ozawa 2003 offer arguments that nature may be somehow enforcing this optimization in the shaping of fluid motions on macroscopic scales, up to the planetary. But is there a compelling chain-of-causality reason to hope that the enforcers of thermodynamic laws (quantum mechanics, atomic attractions and repulsions) reach through the intervening scales to shape convection? Because gravity is the main force at play in convection, and remains unreconciled with the other forces in physics at the root, such a case for *supervenience* (discussed below) is far from obvious. Still, we would happily settle for a decent analogy if it comes with useful tools.

Several meanings of the word "entropy" are invoked in atmospheric science (reviewed in Liu et al. 2011). Its original 19th century sense reflected energy that is *unavailable for macroscopic work* in heat engines

(in contrast to *available* or *free* energy^b). This sense of entropy has clear implications for the *efficiency* of the gaseous "heat engines" of weather systems (Emanuel 2005 has a lucid, accessible discussion). Meanwhile, information-theoretic entropy, a scalar measure of the breadth of probability distributions best translated as *missing information* (Ben-Naim 2008; see exercise 3.8.1), obeys a calculus that seems to alchemically transform lack of information into knowledge, as mentioned above (Jaynes 1957). Might information entropy or free-energy accounting be repurposed usefully in our problem of understanding atmospheric flows that are full of convective entities?

9.2 Entropy of mini-macro scales, without the microscopic

How many convective entities inhabit Earth's skies? Her 500 million square km could hold about that many PBL *large eddies* spanning a typical PBL depth of order 1 km. Since the troposphere is roughly 10x deeper, deep convective "cells" are at least 100x fewer -- and far fewer than that, since their descent regions are far larger than the updrafts^c. Riehl and Malkus (1958) inferred that as few as 1500-5000 "hot towers" of aircraft-observed proportion and properties might do the necessary climatic job of deep convection, before satellites exposed the Earth to our gaze (Fleming 2007). That gaze and other modern tools have transformed thought at least once since then (Zipser 1971), but historical crust in conceptual space may yet remain to be scoured out with today's high-volume data firehoses, if correctly deployed.

Could a production-rate maximization principle about *thermodynamic* entropy, even if broadly correct, be so exacting that it constrains the configurations of mere millions of "mini" convective entities we would like to treat statistically? It seems vain to hope so, if

^bFree energy also invites wide-ranging analogizing and extension, for instance in neuroscience (http://en.wikipedia.org/wiki/Free_energy_principle); see Colombo and Wright (2018)'s eloquent summary and discussion why its "epistemic status is unclear."

^c The "slice method" of Bjerknes (1938) offers one classic way to reason why.

that entropy of *mini-macroscales* is heedlessly lumped in with the abyss of missing information about all $> 10^{18} \times 10^{23}$ air molecules^d. Even pooling convection data over time, or in the Reynolds notion of an ensemble of equivalent large-scale flows with different turbulent details, or with the numbers boosted by overlapping entities on a hierarchy of scales, the molecular missing information utterly dwarfs the number of possible convective weather configuration. Still, on the other hand, thousands or even mere hundreds of entities may obey statistics stable enough to build successful parameterization theories upon, especially if the finite-sample stochastic effects are viewed as part of the desired answer. Might there be some fruitful way to invoke some kind of entropy or free-energy principles in a statistical description of only those *mini-macroscales*, as an *analogy* to thermodynamics rather than an extension?

To realize such an analogy, we should specify what mini-macroscopic entities we wish to treat statistically, and discretize some abstract state space of their possibilities^e. A key step would be to postulate the existence of an analog for *temperature T*, or its energy equivalent $k_B T$ with Boltzmann's constant k_B . A broadband (many-scales) turbulent kinetic energy (*TKE*) seems an obvious candidate for this vigor of mini-macroscales. We must postulate that this *T*-like vigor field is well-defined and smooth on the desired large-macroscopic scales. That mini-scale vigor is both the rate at which trial-and-error motions are probing the combinatorics of complex (for instance, multi-cellular) entities in the unlikely corners of state space, and also the rate at which any such complicated structure, if frail, is shaken or stirred out of existence.

Such a *T* analog could be multiplied by an entropy measure of the unlikelihood of self-assembly of complex structures, like the $-T\Delta S$ term in Gibbs free energy or exergy^f change during a chemical transformation. Pursuing the analogy, that term should be added to an analog to a ΔH term measuring the energy that rare structures can access if they prove

^d kilograms of air, and molecules in a mole

^e information theory does not easily extend to continuous probability distributions

^f <https://en.wikipedia.org/wiki/Exergy>

more efficient at gravity's energy-supplying purpose of lowering the atmosphere's overall center of gravity. Just as the increment or decrement of such a $\Delta G = \Delta H - T\Delta S$ quantity^g determines whether a chemical process will occur spontaneously, might the spontaneous yet gradual evolution of *organization*^h in convective cloud fields could be fruitfully viewed in this chemical analogy? A self-catalyzing reaction in an open system consisting of just a modest number of molecules seems the analog, but must be generalizing for *rates* of transformation and energy production rather than begin and end *states*, befitting the nonequilibrium and open system dynamics of a convecting patch of air.

It complicates things that our "entities" are of a wide range of size or even categorical classes. One physics analogy that has been explored for convection is *critical phenomena* that exhibit *phase transitions* in their *order parameter* or field, generating a possibly very diverse size distribution of clusters or patches (e.g. Peters and Neelin 2006). But because *all scales convect* in the atmosphere (section 3.6)ⁱ, our entities overlap as a *hierarchy*, not merely a set of adjacent patches in space.

9.3 Hierarchical ecosystems: concepts and frameworks

Biology in general, and especially its largest and loosest agglomeration scales (ecology, and arguably social sciences as an extension thereof), must also contend with overlapping categories of entities. Sometimes individuals of one species interact importantly with whole *populations* of a smaller-scale species (from our microbiomes to our foodstuffs). Sometimes stochastic and even quite rare encounters with single individuals of comparable scale are crucial (mates, lions). Actually, for an omnivore with many vulnerabilities, the portfolio of consequential interactions may better be described in terms of higher-

^g https://en.wikipedia.org/wiki/Gibbs_free_energy

^h a concept for which we sorely need a meaningful definition and measure

ⁱ Not inevitably: compare the narrow range of sizes in the "mesogranulation" of solar photosphere convection (e.g. Rincon and Reitord 2018, Kessar et al. 2018).

abstraction-level *classes of populations* (foods; predators; infectious agents), with the specific species of protein or fangs or germs perhaps interchangeable from the point of view of an entity or population in question. How does quantitative ecology span these wide ranges of scales and even categories of interacting and overlapping entities?

9.3.1 Competition and cooperation in predator-prey models

A classic paradigm for categorical entity interactions in population ecology is the Lotka-Volterra equation^j. It describes the time evolution of the number n_i of individuals in a population of entities of category i , within some domain that must be specified, interacting with populations of entities of N other categories (indexed by j), here written following Eq. (9) of Nöber and Graf (2005):

$$d/dt(n_i) = F_i n_i + \sum_{j=1}^N K_{ij} n_i n_j \quad (7.1)$$

If the “food” or forcing coefficient F_i is positive, population i grows exponentially, in the absence of interactions (that is, if $K_{ii} = 0$). If diagonal terms K_{ii} are negative, purely negative interactions occur with other individuals of the same category i . Then $(F_i + K_{ii} n_i) < 0$ when the population gets too large, and the no-other-species-interactions solution asymptotes to $n_i = -F_i/K_{ii}$. More interesting possibilities lie with off-diagonal terms in K_{ij} . Depending on their signs, the system can express purely cooperative *mutualism* interactions ($K_{ij} > 0$), purely *competitive* negative interactions ($K_{ij} < 0$), or a mixed regime of *predator-prey* dynamics ($K_{ij}K_{ji} \leq 0$) that acts to redistribute the number of entities among categories.

A similar "master equation" for convective entities (Eq. 5 of Hagos et al. 2018) allocates "pixels" among entities of types i and j , emphasizing only the off-diagonal terms. With suitable re-weightings, the types could be measured in terms of more continuous stuff like an embodied energy resource or its production rate (biomass or metabolic vigor, in the

^j https://en.wikipedia.org/wiki/Competitive_Lotka-Volterra_equations

ecological problem), rather than by a number count. All of this can be normalized per square km of the Earth's surface, in either biological or convective terms, while recognizing that large organisms require a large enough domain to exist at all -- a problem that vexes scale separation thinking as discussed below.

All this is only accounting. The question is when and why and how exactly it is *useful* to define entities, and categories of entities, and feedbacks with *self* and with *fellow* category members, and interactions with the truly *other*. In other words, can tallying discrete entities in continuous air truly help us develop a predictive account or deeper appreciation of the natural phenomenon of convection?

The Lotka-Volterra equation discussion of Nuber and Graf (2005) continues: "The analogy to convective clouds is straightforward. The reason for convective clouds to form is convective instability ('food supply') ... each cloud type acts on its environment and tends to reduce instability. Therefore each cloud tends to reduce somehow the 'food-supply' for all other cloud types including itself." Ramirez et al. (1990) also adopt this utterly competitive view: "It is observed that the major contribution to the environmental stabilization comes from the drying of the planetary boundary layer induced by subsidence. The thermodynamic effect of nonprecipitating and precipitating convection is to reduce CAPE in the surrounding environment and hence reduce the conditional probability of further convection nearby. A new ... inhibition hypothesis states that, under completely homogeneous external conditions and assuming a spatially random distribution of cloud-triggering mechanisms, the spatial distribution of cumuli in the resulting cloud field must be regular, as opposed to either random or clustered, because cumulus clouds tend to reduce the available energy for convection, thereby inhibiting further convection nearby."

Such pure-competition views echo the ideas of Arakawa and Schubert (1974, AS74) framework of *cloud work function* (a CAPE to KE conversion, categorized by "cloud type" or size and even expressed in an interaction kernel like K_{ij}). A structural shortcoming flows foreseeably

from its ecological postulate of a univariate scarce resource: The largest permitted plume or bubble tends to dominate, in the absence of dispatcher conditionality or developmental limitations, as discussed in chapter 6. The implementers of parameterization schemes have cleverly battled this propensity ever since. The exquisitely elaborated but debatable physicality^k of the early masterpiece (Arakawa and Schubert 1974) has inspired generations. Another artifact of symbol-manipulation software enabled analytic theory, presented for its texture (a sense of complication level) rather than its face-value meaning requiring symbol definitions, is this majestic equation in Yano and Plant (2016, their equation 6.9):

$$\begin{aligned} K_{ij} = \int_{z_B}^{z_{Ti}} \eta_i \eta_j \left(-\alpha + \epsilon_i \tilde{\eta}_i \frac{\dot{\eta}_i}{\eta_i} \right) \left[\delta_j (s_{vj}^D - \bar{s}_v) + \frac{\partial \bar{s}_v}{\partial z} \right] dz + \\ \int_{z_B}^{z_{Ti}} \epsilon_i \dot{\eta}_i \eta_j \left[\tilde{b}_i + \tilde{c}_i - \frac{\tilde{\eta}_i}{1+\gamma} \left(\frac{\dot{\eta}_i}{\eta_i} \right) \right] \left[\delta_j (h_j^D - \bar{h}) + \frac{\partial \bar{h}}{\partial z} \right] dz + \int_{z_B}^{z_{Ti}} \epsilon_i \tilde{\delta}_i \tilde{\eta}_i \eta_j \left[\delta_j (s_j^D - \bar{s}) + \frac{\partial \bar{s}}{\partial z} \right] dz \\ - \int_{z_B}^{z_{Ti}} \epsilon_i L_v \tilde{\delta}_i \tilde{\eta}_i \eta_j \left[\delta_j (l_j^D - \bar{l}) + \frac{\partial \bar{l}}{\partial z} \right] dz \end{aligned} \quad (6.9a)$$

$$\begin{aligned} \left(\frac{\partial A_i}{\partial t} \right)_L = \rho_B \left[\tilde{a}_{iB} \frac{\partial}{\partial t} s_{viB} + (\tilde{b}_{iB} + \tilde{c}_{iB}) \frac{\partial}{\partial t} h_{iB} - L_v \tilde{\delta}_{iB} \frac{\partial}{\partial t} l_{iB} \right] + \left[\int_{z_B}^{z_{Ti}} \rho L_v \tilde{\delta}_i \left(\frac{\eta_i}{\dot{\eta}_i} \right) \dot{c}_0 dz \right] l_{iB} + \\ \int_{z_B}^{z_{Ti}} \left[\epsilon_i \tilde{a}_i \dot{\eta}_i \left(\frac{\partial \bar{s}_{vi}}{\partial t} \right)_L - \alpha \eta_i \left(\frac{\partial \bar{s}_v}{\partial t} \right)_L \right] \rho dz + \int_{z_B}^{z_{Ti}} \epsilon_i \dot{\eta}_i \left[\tilde{b}_i + \tilde{c}_i - \frac{\tilde{\eta}_i}{1+\gamma} \left(\frac{\dot{\eta}_i}{\eta_i} \right) \right] \left(\frac{\partial \bar{h}}{\partial t} \right)_L \rho dz + \\ \int_{z_B}^{z_{Ti}} \epsilon_i \tilde{\delta}_i \eta_i \left(\frac{\partial \bar{s}}{\partial t} \right)_L \rho dz - \int_{z_B}^{z_{Ti}} \epsilon_i L_v \tilde{\delta}_i \eta_i \left(\frac{\partial \bar{l}}{\partial t} \right)_L \rho dz + \int_{z_B}^{z_{Ti}} \epsilon_i L_v \tilde{\delta}_i \eta_i \frac{\partial}{\partial t} \left(\frac{\rho \sigma_i}{\epsilon_i M_i} \tilde{r}_i \right) \rho dz + \\ \int_{z_B}^{z_{Ti}} \epsilon_i L_v \tilde{\delta}_i \eta_i \int_z^{z_{Ti}} \dot{c}_0 dz' \rho dz + \dot{z}_{Ti} G_{Ti} + \dot{z}_B G_{Bi} \end{aligned} \quad (6.9b)$$

Is such an artifact a pinnacle of knowledge, or the outer edge of human-accessible meaning? Might more general possibilities about the whole problem have gotten lost behind by the specific but peculiar nature of the parameterization *scheme design* problem, attempting to minimize the bleeding from the scale-truncation wound forced upon us by a scale separation assumption in coarse-resolution numerical models?

^k Presented as a partitioning of *energy* (as clarified in Lord and Arakawa 1980), but the non-orthogonality of “cloud type” (plume radius) categories means that attempting to distribute physical energy over that basis set lacks true rigor.

9.3.2 A statistical thermodynamics analogy to ecology

One bio-physics attempt to frame and then model ecological succession (Würtz and Annila 2010) envisions an ecosystem of multiple entity types, each with its numbers of indistinguishable individuals. In that paper, a single ambient energy level (perhaps literally $k_B T$, in the view where ecosystems supervene on chemistry and physics¹). For each species of life (indexed by j or k), they measure an embodied or intrinsic Gibbs energy ΔG_{jk} , so that just as in chemical transformation the probability kernel for a spontaneous production of category or species j from species k is $e^{-\Delta G_{jk}/k_B T}$. The total probability for category j , accounting for the numbers of interchangeable (for this purpose) individuals N_k in each of the k necessary ingredients, and with a "degeneracy" g_{jk} that "numbers the k -ingredients that remain indistinguishable in the assembled j -species", is stated in Würtz and Annila (2010) as:

$$P_j = \left[\prod_k (N_k e^{(-\Delta G_{jk} + i \Delta Q_{jk})/k_B T})^{g_{jk}} / g_{jk}! \right]^{N_j} / N_j!$$

Ecosystems subsist on an external influx of free energy, whose projection on the jk -transformation process those authors measure as ΔQ_{jk} . Since this influx is "orthogonal...to the system's scalar potential energy differences [ΔG_{jk}] between j - and k - repository", they use an imaginary notation with i from complex analysis^m, rooted in the non-equilibrium thermodynamics of open systems (Annila 2016).

Those authors go on in describing this equation's logic: "The product N_k ... ensures that if any of the k -ingredients, e.g., food supplies, is missing, no j -species may appear in the ecosystem ($P_j = 0$)."
Furthermore, "the numerator is raised to the power of N_j because the

¹ The limitations of such "supervenience" arguments bear pondering again (Hoel 2018).

^m I confess to confusion about the level of rigor of this application of complex numbers.

process may combine the vital k - ingredients into any of the indistinguishable j -individuals in the population N_j . As usual, the division by the factorial $N_j!$ means that the combinatorial configurations of the j -entities in the ecosystem are indistinguishable."

Details aside, such an equation gestures at how discrete *categories of entities*, each with a *population*, can be accounted with some rigor, if a Gibbs free energy-like scalar measure can be devised to adjudicate their likelihood, through its ratio to some sufficiently well-defined background energy of random statistical motions ($k_B T$ or perhaps some *TKE* in our analogy, as discussed above).

Might such a formalism be useful for a game theory of interplay of overlapping diverse entities, from bubbles to cells to multicellular storms to gappy groups of them? Does the whole hierarchy of structures play a role in convection doing its teleological 'job', facilitating gravity's project of lowering the center of mass? Or might convection of such diverse scales and types actually have *multiple* jobs? After all, living things within ecosystems express more than a single purpose, exploiting the myriad mutually-contingent niches created by other living things. Even if an ecosystem obeys an optimum principle, like in the equation above for Würtz and Annila's (2010) univariate "energy dispersal" view of ecology, their solutions are inherently initial condition dependent. Rightly so, given the phenomena targeted for explanation: Earth's long-optimized ecosystems can be as distinct as Madagascar and Hawaii, despite similar gross energy and entropy inputs and outputs. Section 9.1's dream of a "governing" statistical principle may not actually be sufficient to predict behavior better, even if it were exactly true and well known!

9.4 Can we reject a random patterns null hypothesis?

What would a scientifically minded person, with the continuum equations of Part I and entity notions of Part II to draw on, expect the world to look like, before the first satellite gave us the grand overview we now take for granted? Fleming (2007) shares an image that illustrates

a striking poverty of multi-scale imagination (or at least its artistic depictionⁿ), even in relatively well-informed circles. Another intriguing essay on the satellite-age transition is Zipser (1970)'s "rise of the fourth school of thought." Although it is impossible to un-see the richness of texture, this author's surprisingly repeatable experience of surprise even after decades of immersion in data suggests that our brains have a hard time realistically remembering just how richly detailed the world is^o, in the face of temptingly simple ideas and conceptions.

Suppose we hypothesize that convection has *meaningful patterns*. To test this hypothesis, observational evidence must be shown to stand above some baseline or *null hypothesis* that convection is *meaningless or random*. Defining those null words well is half the battle in the quest for meaning. For instance, images of white clouds over a dark ocean differ obviously from those same dark and light pixels shuffled into maximum-entropy speckle patterns. But who cares?

A more sophisticated entity-based approach is to tally contiguous areas of whiteness or *cloud objects*. We can then ask if their size distribution deviates from some expectation that defines a random null hypothesis, like a pattern of gravel and rocks spilled onto a street from a great height.

One classic study with a well-defined stochastic model for its randomness baseline is Lopez (1977), whose results are adapted in the top panels of Fig. 9.1. The straightness of the plotting symbols (emphasized with line annotations) indicates the *empirical adequacy^p* of a stochastic model: the multiplication of many independent random numbers^q. However, physical interpretation is nonunique, an art of

ⁿ Leonardo Da Vinci's turbulence drawings provide an interesting counterpoint.

^o A visit to <https://worldview.earthdata.nasa.gov> is advisable before thinking too much.

^p <https://plato.stanford.edu/entries/constructive-empiricism/>

^q The *central limit theorem* states that a sum of many independent identically distributed (iid) random variables asymptotes to the normal (Gaussian) distribution. The log of a product of many iid variables becomes a sum, satisfying this theorem.

science rather than a science *per se*. Such multiplication might for instance embody the *independent random dispatch* of bubble-like entities whose superposition causes, in a multiplicative way, the cloud as a whole to reach a greater height. Additional plausible interpretations are discussed in Lopez's paper. Other properties of convective cloud entities (width, duration) also appear to follow such laws, adequately we can say, in this log-normal straightness measure of a curve between datapoints.

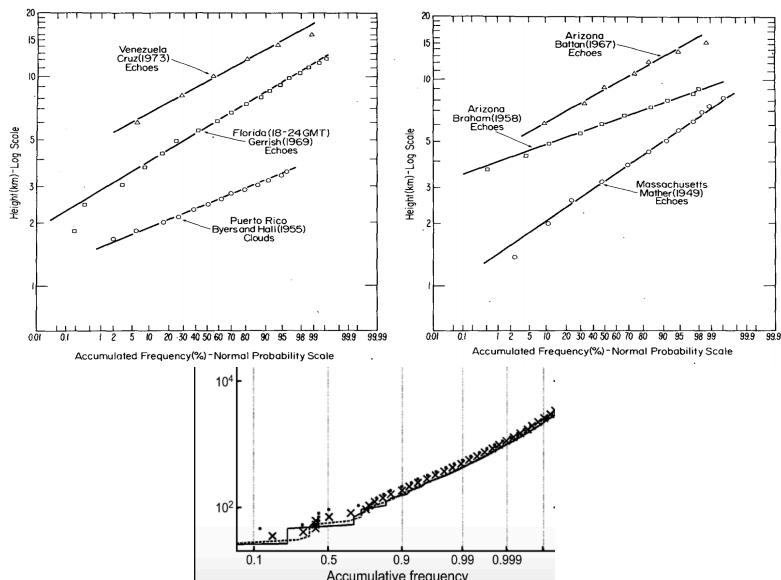


Fig. 9.2. Distributions of cumulus cloud height (top, adapted from Lopez 1977) and MCS anvil area (bottom, adapted from Yuan and Houze 2010), plotted against the cumulative normal function so that straightness of the arrangement of data symbols is indicative of nearness to a lognormal distribution.

In addition to the "inner" interpretive considerations of whether the plotted data is nearly straight on the graph, one must always consider an "outer" layer of interpretation as well: what conditional pre-screening or sampling lies upstream of the data depiction that lies before us (section 0.6)? Which scenes were excluded, and which others were declared equivalent enough to be statistically pooled? Do the results reflect patterns in the data, or are they subtly, tacitly enforced by the study's

own screening assumptions, rooted in its preconceptions or working hypotheses?

The question often comes to this: Would data generated in a hypothetical *randomworld*, embodying a null hypothesis of maximum entropy or some other expression of *lack of meaningful patterns*, appear differently from natural data in the plotting space, after being conditionally screened and filtered and pooled by the study's assumptions? If so, then the difference result is precisely as interesting (or not) as the *randomworld* is compelling in its construction, reflecting salient aspects of the data. Again this is an art-of-science, not a science.

To get beyond the unorganized (by selection) conditions upstream of the top panels of Fig. 9.1, the bottom panel shows the area of MCS high cloud objects observed in all-conditions satellite data over oceanic warm pool areas (Yuan and Houze 2010). This measure of convective entity size has a much wider dynamic range than Lopez's cumulus cloud height, bounded by the troposphere's depth. While the data curve is not as rigorously straight, again the lesson appears to be that a random multiplicative process, perhaps the combination of *independent* probabilities governing the development of larger entities, is not easy to reject as a null hypothesis for natural convective cloud fields.

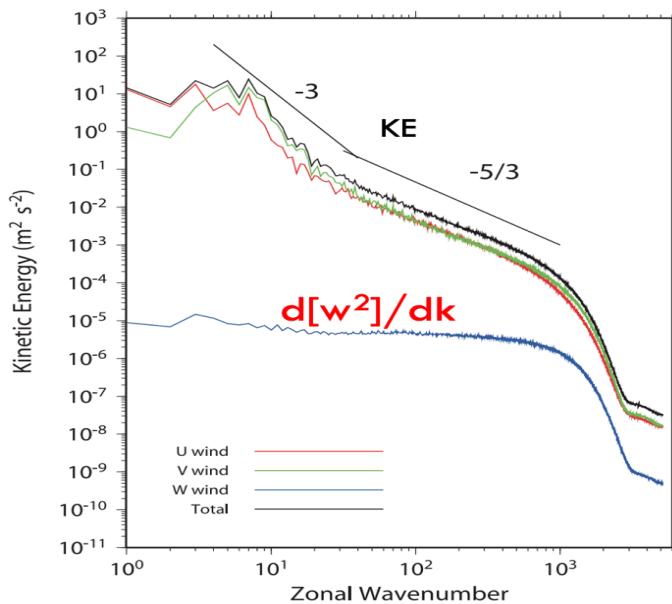
Overlap assumptions between vertical layers in cloud-radiation interaction may offer another useful toolkit (or at least analogy) for defining "organization" in deep convection. Even if a parcel dispatcher function is random, just as photons are everywhere, their probability of vertical travel can depends on the overlap of moisture and cloud patches at different levels. *Random overlap* is known to be drastically excessive for cloudy radiation, as even modest fractional cloudiness in each of many layers permits almost no photons to traverse a many-layer atmosphere. *Maximum overlap*, on the other extreme, means that the greatest cloud fraction at any single level is the total cloud fraction as seen by radiation, arresting too few photons. A meaningful concept of "organization" must lie in the space between these extremes (some

maximum-random overlap scheme), perhaps merely as a simple linear scale between the extremes (Mapes and Neale 2011).

9.5 Are entities more real than wavenumbers?

Continuing with the spirit of null hypotheses, to undercut our initial hight hopes of special importance and meaning for entities and their agglomerations, we must return to the spectral slope of Fourier decompositions (section 0.6). Although somewhat arbitrary^r, Fourier decompositon is powerful and may be viewed as a baseline for the question: Does discrete entity counting like in Fig. 9.2 pass the test of showing more about nature than about the method's definitions and assumptions?

One useful null hypothesis for non-organized (random) convection in Fourier space is the spectral slope of vertical motion w .



^r not quite: sinusoids are solutions to linearized versions of our equations (section 2.2.2)

Fig. 9.3. Spectral distribution of velocity variance with respect to spatial wavenumber, adapted from Terasaki et al. (2009).

A complete lack of correlation horizontally among updrafts appears as a *white* (or flat) spectrum of w variance with respect to horizontal wavenumber. As Fig. 9.3 shows, this null hypothesis is violated only modestly by the first generation of global nonhydrostatic cloud-resolving models. This finding is more evidence that organization of convection is a slight and subtle effect: the answer to section 9.5's titular question again appears to be *barely*, at best.

9.6 Lessons from and for large-scale modeling

9.6.1 Popcorn v. typhoons in storm resolving resolving models

Statistics have their limitations. Power (squared amplitude) spectra in Fourier analysis ignores the phase information that distinguishes a singular, isolated delta function (a highly organized situation) from utterly incoherent speckle patterns. We know of no tools to measure this phase information in spectral space: its meaning appears only in reconstructions (or back in the structure of un-deconstructed data) in physical space.

One early lesson of global cloud resolving models (Satoh et al. 2019) was that the climatic role of convection could be fulfilled adequately by either random "popcorn" convection or by a population of long-lived super-typhoons, based on whether the PBL scheme had low or high ventilation (mixing) at its top. This aspect of PBL turbulence was soon tuned toward realism, rightfully so in the enterprise of modeling, but the lesson remains: one a meaningful "game" or competition is between tropical cyclones and ordinary convection, in otherwise entirely realizable Earth-like atmospheres. Similar considerations animate the discussion of Zhao et al. (2012). More broadly, the texture differences between a suite of credible global cloud-resolving models are quite distinctive by eye, yet hard to characterize and not trivial to distinguish robustly from nature (the "Palmer-Turing test" in the DYAMOND model

intercomparison of Stevens et al. 2019). Our measurement tools seem to be simply inadequate as yet at separating the cosmetic from the essential.

9.6.2 Parameterization and its schemes

A parameterization *scheme* is a low-dimensional surrogate model, much simpler than a real subsystem, which produces almost the same gross or bulk behavior, by some carefully defined practical measures. Parameterization schemes for *scale-truncated numerical models* are a particular and peculiar example that has defined the problem space. Whether the scale truncation filter is coarse-grid area averages or horizontal spectral truncation, the missing flow motions that need to be reproduced by a scheme are (i) spectrally *blue* in the horizontal, meaning restricted to sub-grid scales or high wavenumbers; (ii) all-scale in the vertical dimension, a domain which even horizontally coarse models aim to resolve; and (iii) discretized to the model's time step, which is set by the fastest information transmission process between grid cells (usually a wave propagation speed), and therefore strictly local to a model grid column (so that convection must interact with its own "compensating subsidence"). For precipitating convection, there is also (iv) a net (filter-scale) conversion of vapor to heat, which rectifies directly onto large scales, but which depends on the small-scale vertical motions in point (i) so that it enormously amplifies the importance of errors in postulates about those parameterized motions.

The problem of designing such schemes has often overshadowed the underlying scientific problem of devising statistical theories of the whole convecting flow -- even while serving as a strong motivation (Arakawa 2004). The most profound difficulties of scheme design may actually lie in the time domain, followed by the vertical domain: these are the dimensions where we directly care about the scheme's explicit output (the tendencies it produces). Nonetheless, most mechanistic reckonings dwell inordinately on a scheme's inner assumptions and considerations about the *horizontal* scale truncation, assessing those considerations on arguably secondary virtues like the internal consistencies of what are clearly gross assumptions.

This problem is now frustrating its third generation of thinkers, in part because a decent level of success on the largest scales can be accomplished by rather simple ideas. For instance, *convective adjustment* is a concept directly in the time-height domain, local in the horizontal, and one that even mechanistic approaches can be re-interpreted in terms of (Arakawa 2004). The success of the brute-simple Betts-Miller (1984) convective adjustment scheme was humbling for much more painstakingly articulated schemes rooted in fluid-mechanical arguments about Part II's entities. Today such elaborated mass flux schemes (chapter 6) are back to being the leading approach, but only in the unabashedly engineered "bulk" sense of section 6.2. Final closures are usually still teleological claims of adjustment, albeit with a specified time scale whose calibration may be fitted to different grid truncation lengths ("scale-aware" is a lauded contemporary virtue in schemes).

Still there are longtime dilemmas^s in convection schemes, indicative of an ill-fitting or impoverished framework. For instance, entrainment is simultaneously too great for climate (cloud heights are too low in realistic profiles; and profiles are too unstable in realistic equilibria), yet too small for weather (deep convection occurs too easily in dry conditions). The deeper lesson is that deep convection is too *unconditional* in schemes. Opening up the environmental sensitivities of convection to overt barely-physical engineering, such as direct RH dependence (e.g. Hiron et al. 2016), improves performance but severs the literal connection back to convective entity dynamics as a source of essential constraints and claims of rigor. In this engineered approach, a plume becomes merely an adequate updraft-with-mixing framework which enforces the necessary integral constraints of the equation set, with the unfortunate but seemingly numerically inescapable virtue of time independence (a 1-dimensional calculation in z at each time step).

New pinnacles of elegant elaboration (Park 2014, 2019) perform comparably well with much cruder approaches, which is to say not yet

^s Mapes and Neale (2011)

breakthrough-level better. Meanwhile, equally elegant attempts to unify all of turbulence (e.g. Bogenschutz et al. 2018) or re-envision the whole dynamical solver to encompass cloudy and clear air volumes (Thuburn et al. 2018) have yet to sweep the field. Stochastic replacements for entrainment (Romps 2016) seem promising at this writing, but implementation ends up mattering to model performance as much as the appeal of mathematical formulations. Perhaps any scheme based on a too-rigid, too-specific framing will need external wrappings (stochastic or situational dependences like "effective" entrainment rates) to account for the myriad ways that natural convection escapes the grip of any one formulation as a definably-distinct entity in a well-defined mean environment.

Gross response functions (sensitivities and impacts) may provide the crucial top-level connection between natural convection and parameterization schemes. Diagnostic approaches that point to different values of scheme parameters for different phenomena (like large-scale convection waves or vacillations of different scales or frequencies, e.g. Adames et al. 2018, Wolding et al. 2019) are an indication that our scheme framing may still be collapsed into a too-small subspace of the real physical system. Convective clouds exist and behave without regard to whether they are part of short or long or high or low frequency waves, and indeed their responses are integral to *all* those larger scales simultaneously. Only by insisting that a scheme or surrogate model participate properly in the whole spectrum of overlapping entities and envelopes will we approach skill, and the underlying physical knowledge or appreciation we seek.

Finally, it must be reiterated that the study of severe storms and other particular situations, largely neglected in this statistical view, can also offer stark crises for our too-small conceptualizations that are inadequate to the mystery and power of convection in our atmosphere. Together the study of storms and statistics have many lessons yet to teach us.