Outline

x.0 Chapter 0

Overview: our convecting atmosphere (Back to Outline)

Convection is a means for gravity to lower the center of mass of a fluid body, by bringing relatively denser air as low as possible, under realizability constraints such as mass continuity. That statement is *teleological*, expressing the nature of convection by *the job it does.*

Convection is the motion that occurs when displaced parcels in a fluid experience a component of gravitational force that aligns with their displacement. Work is done (force times displacement), generating macroscopic kinetic energy (a coherent component to the motions of all the parcel's molecules). That kinetic energy is quickly redistributed by a pressure field that adjusts at the speed of sound to enforce mass continuity. This description is *mechanistic*, explaining convection by *how motion is imparted to air*, using a teleological account of pressure.

A deep appreciation of convection must encompass both of those fundamental viewpoints (holistic and mechanistic), and others as well. Before diving into this book's multi-threaded effort to convey such a tapestry of appreciation, this chapter 0 summarizes some key overarching facts and phenomenology. It also declares (if not quite defines[a]) some key terms (in *italics*) for later use, and frames the book's overall account of atmospheric convection, with touch points to conventional wisdoms and customary accounts of the field.

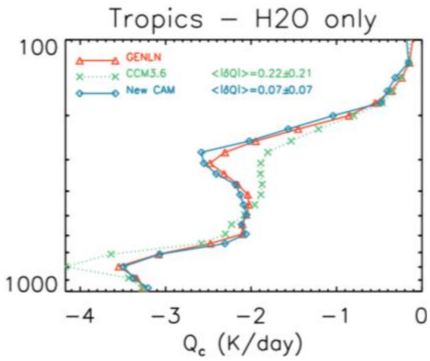
A state of possibility for convection is called *gravitational instability*, although there is more than one kind (section 0.4). Earth's atmosphere is perpetually *convecting* because it is perpetually *destabilized* (section

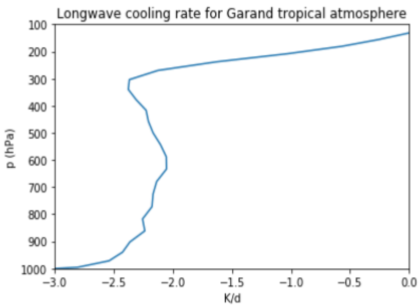
[a] See Glossary, or web sources, or ask; vocabulary tests are a suggested teaching tactic.

0.1). Although ultimately radiatively driven, instability is redistributed by air motions (some or all of them "convective", by various definitions of that term), through changes to vertical profiles of density or of variables that make up some *potential density* (the density an air *parcel* would have, under a hypothetical vertical displacement to a reference altitude or pressure level). Our main interest is in *turbulent* vertical convection, although *laminar* synoptic-scale upglide and downglide of warmer and cooler airmasses also comprise a *thermally direct* circulation deserving of the name *slantwise convection*[b].

x.1 Sun-heated surface, IR-cooled air, H_2O's 2 height scales

Sunlight reaches Earth's surface because air and water are nearly transparent in the visible part of the electromagnetic spectrum[c]. But without centuries of hard science, no observant person would know quite how air cools by emitting *longwave (infrared)* radiation (Fig 0.1).





[b] Discussed nicely in the late parts of chapter 7 of Wallace and Hobbs (2006).

[c] This defines *visible*, since our eyes evolved to see through air and water. The spectral narrowness of (https://en.wikipedia.org/wiki/Electromagnetic_absorption_by_water) is so special that it bears on life's evolution: our watery Earth and 6000K Sun are very fortuitous, albeit unsurprising by the https://en.wikipedia.org/wiki/Anthropic_principle.

Fig. 0.1. Clear-sky radiative cooling profiles for averaged conditions in the tropical troposphere. Left: Radiative cooling rates: from Old (CCM3.6, green) to New (CAM, blue), an improvement toward the trusted reference (GENLN, red). Adapted from Fig. 2 of Collins et al. (2004) which also explores some simulated climate impacts from the change. Right: the most up to date treatment (from calculations by Pincus et al. 2019 with input profiles not quite identical to those at left). *Shortwave* (solar) clear-air heating profiles are similar in shape, and nearly equal at noon, so the daily average or longitudinal average around the Earth is often about (1 - 1/π) or 70% of the longwave profiles shown (Mapes and Zuidema 1996).

Reasoning about radiation is possible, despite the complexities of the underlying physics. Purely radiative equilibrium, given air's composition, would feature upper-level temperatures (below the sun-heated ozone layer) so cold that, if they existed, world-scouring convection would erupt. But convection has been doing its job for eons, so instead we should think of convection as *maintaining*[d] the *troposphere*[e] at a temperature far warmer than its radiative equilibrium value. This warmth is an *ultimate cause* account of why air cools radiatively. We can also consider the *proximate cause*[f]: what molecules are emitting the radiation that cools air? That account is more useful for reasoning about the much larger impacts IR-active (polyatomic) gases, especially H_2O whose concentration varies much more than does the always-large gap between actual temperature T and the fiction of radiative equilibrium.

Clouds affect the troposphere's radiative cooling profile profoundly in any local column (illustrated below), and substantially on average (e.g. Kato et al. 2018), but the cooling of clear air and its peculiarly top-heavy profile remain key drivers of the *deep convection* of the entire troposphere. In the time averaged climate, convection (broadly construed) acts to supply energy equal to the emitted amount, level by level, drawing ultimately on energy from *sensible and latent heat flux* imparted by conduction (diffusion) to air that literally touches the Earth's skin.

[d] yet another teleological account of convection's "job"

[e] *Tropos* is from Greek, *overturning*. The Tropics are the sun's turning latitudes.

[f] https://en.wikipedia.org/wiki/Proximate_and_ultimate_causation

Strong infrared cooling near the 300 hPa *pressure level* in Fig. 0.1 is remarkable[g] because water vapor concentrations there (temperature about -35C, altitude about 10km) are only about 1% of surface values. This top-heaviness of infrared cooling requires deep convection to have a top-heavy heating profile on average, which cannot be achieved through the simple condensation of water: there is simply too little vapor mass available at those cold upper-level temperatures. In order to do their job in the heat budget, then, deep convective updrafts must carry heat bodily, by being considerably warmer than their interstices. This in turn implies their rarity. In contrast, shallower convective motions can wield latent heat alone to efficiently keep the lower-tropospheric temperature *lapse rate* nearly *neutral* (or *moist adiabatic)*. The foregoing is a stronger version of the inference of *hot towers* by Riehl and Malkus/Simpson (1958, 1979) from observed *moist-conserved variable* profiles -- inferences which, because of their blending of heat and moisture information, could also be satisfied by merely moist towers (implying a penetrative *eddy flux* of water).

Such warm updrafts are quite buoyant, making deep convection (on average) vigorous in the upper troposphere. Lightning and hail (*rimed ice*) are two consequences of upper level vigor, rooted partly in this rotation-band emission by the H_2O molecule. That upper-level cooling also has a litany of other consequences for tropical tropospheric dynamics on Earth (Mapes 2000). Modeling such top-heavy cooling requires, at a minimum, a second *spectral band* in the infrared (e.g. Fig. 2 of Vallis et al. 2018). The consequences of this top-heaviness for the atmosphere's *general circulation* and climate (merely glimpsed in Collins et al. 2004) are much less well studied than the basic first-order impacts of tropospheric cooling, for instance from *gray radiation* idealizations.

[g] This intense *cooling to space* occurs in a distinctive rotation band of the H_2O molecule.

x.2 Top-down vs. bottom-up convection

Top-down convection is simplest to explain, because the displacement-parallel force doing work is gravity, preferentially drawing down air's denser parcels. In *bottom-up* convection, the mean gravity-balancing *hydrostatic* part of the broader-scale pressure field preferentially pushes warmer lighter parcels upward. Despite this extra logical step for the positive (upward) sign, the *buoyancy* force is essentially symmetric. A more important asymmetry between bottom-up and top-down depends on whether convection's vertically accelerating parcels are formed by the horizontal gathering of intensely warmed low-level air, or of intensely cooled air aloft. A convenient measure of this asymmetry, indicative of what is driving convection, is the mean cube of vertical velocity *w* in the convecting layer, a statistic called *skew*. When up and down occur equally, the result is *symmetric* convective turbulence[h].

On Earth the thermal driving by radiation never stops, although periodic variations of solar heating offer us daily and seasonal signals we can repeatably observe in convection fields, and study in composite detail for clues to mechanisms. Important but less *predictable* is the convection-contingent effect of clouds as a feedback on the radiative driving itself, producing coupled (rather than simply forced) phenomena that challenge our understanding at the frontier. The hope of predicting complex *unforced variability* pays the bills for all science in this field.

Cloud feedbacks on radiation are utterly dominant in the challenging *open vs. closed cell* problem in *cloud-topped boundary layers* (CTBL, Fig. 0.2). The *albedo* of Earth, crucial for its energy budget and thus its mean temperature, hinges largely on whether convection is top-down (whiter areas in Fig. 0.2, *closed cells*) vs. bottom-up (*open cells*, where the blue ocean shows through). But that question hinges on the clouds themselves (overcast vs. clearer skies, respectively). The CTBL thus

[h] Symmetric flow may also indicate *decaying* turbulence, perhaps generated by convection (work done) in the past, but now merely air's inertia playing itself out.

exhibits a *hysteresis*, existing in either of two stable, long-lasting *regimes* selected by accidents of history of each patch of air.

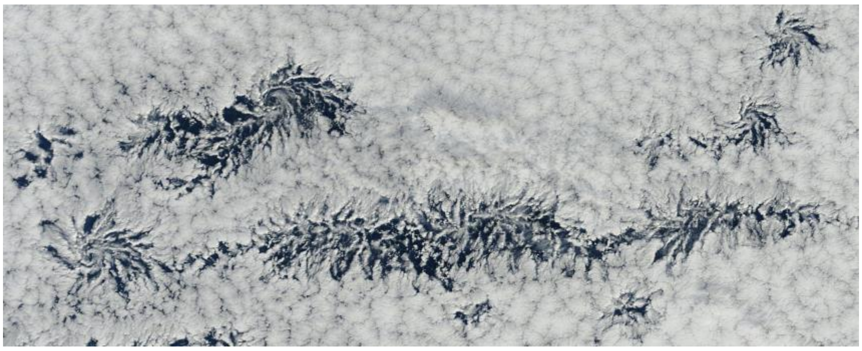


Fig. 0.2. Visible image of a stratocumulus cloud deck over ocean, with pockets of open cells (darker areas) where bottom-up convection prevails, amid closed cells (whiter areas) where convection is top-down. From http://worldview.nasa.gov.

The radiation driving top-down convection is intense cloud-top cooling, illustrated in the aircraft observations of Fig. 0.3 in and above a stratocumulus cloud deck. Cloudy radiation is complicated, so observations are emphasized here (dots and crosses), supported by calculations (solid curves). The net heating rate depends on radiant energy *flux convergence*, the negative of the vertical derivative of *net flux* (upward minus downward, in the *two-stream approximation*). The right panel shows longwave (IR) flux L, while the left panel shows midday shortwave S (which of course vanishes at night). Arrows denote upward vs. downward flux, measured on the belly and top of the aircraft respectively. The term $-d/dz(L\downarrow)$ is clearly the predominant effect (left curve in right panel): the absence of downwelling IR radiation is the important "to space" part of the useful *cooling to space* approximation.

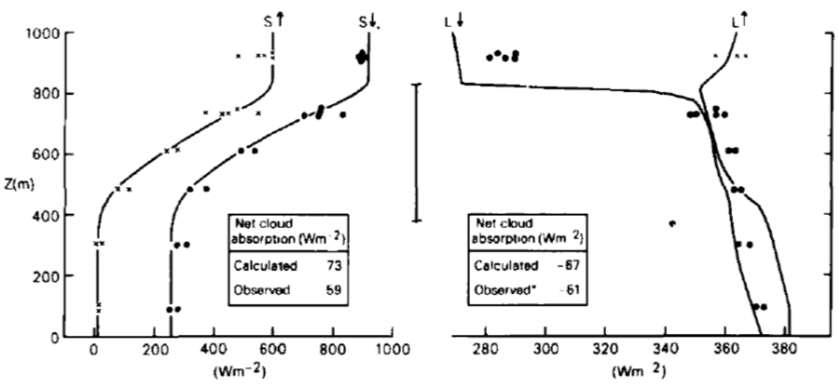


Fig. 0.3. Solar (S, left panel) and longwave (L, right panel) radiative fluxes, with upward and downward components (the *two-stream* approximation) indicated by arrows ↑↓, for a low-latitude stratocumulus cloud deck at midday. Curves are computed estimates, while symbols show aircraft observations. Adapted from Fig. 8 of Nicholls 1984.

x.3 More asymmetry: saturated drafts in stratification

In addition to top-down vs. bottom-up forcing, *moist (cloudy) convection* (see Stevens 2005 for an excellent broad survey) has other distinctive mechanisms that govern its up-down asymmetry (skew of w).

1. Latent heat released by water condensation upon ascent contributes a displacement-proportional term to parcel warmth and thus to the vertical buoyancy force. This term is partly compensated by *stratification* of the environment, but the latent heating term predominates in buoyant cumulus clouds, by definition of buoyant. A corresponding effect also occurs in downdrafts, if small-droplet condensed water is available to evaporate as rapidly as condensation occurs in supersaturation.

2. Mixtures of cloudy and clear air can be denser than either constituent. Specifically, the mixing fraction that yields exactly-saturated air has the greatest density of any mixture. This cooling upon mixing is called *buoyancy reversal*, and it can drive downdrafts whose close adjacency to cloudy updrafts can drive shear instability and thus momentum-forced turbulence, which itself feeds back into the mixing process. What scales of motion this effect drives is a little unclear: a density change happens when mixing occurs down to a scale at which cloud droplets evaporate into unsaturated air (fractions of a second later). But at that point, the gross density of moist air parcels at every larger scale is affected: when a 1 km turbulent ball of air with internal filaments of cloudy and unsaturated air finally, truly mixes, its kilometer-scale bulk density changes!

3. Radiative impacts of cloud particles, especially in “window” spectral bands in which air is transparent, give moist convection a feedback mechanism on radiative destabilization. Besides the intense infrared cooling of cloud tops in Fig. 0.3, cloud shading over land can dramatically weaken the surface heating that drives bottom-up convection.

Environmental stratification in fields of buoyant cumuli (effect 1 above) can be best understood teleologically: convection's job is to drive the embedding unstable layer toward *neutrality*. Its efficacy at that job drives the environmental density (by mechanisms clarified later) toward the updraft's profile, a *moist adiabatic* profile appropriate to the convection-governing buoyant draft[i].

There is no laboratory analog for effects 1 and 2 – gradual buoyancy release in updrafts within stratified environments, and mixing-dependent negative buoyancy on the flanks. Mixing-dependent chemical heating or bubble releases can be contrived, but the modest depth of laboratory tanks provides too small a pressure range for latent heating-like processes. Electrical heating grids can be driven by digital imaging, but the finite resolution makes this more akin to numerical software modeling than to true low-viscosity laboratory turbulence.

x.4 Conditionality of moist convective 'instability'

The word “stable” is ancient, from proto-Indo-European *to stand*. A ball on a hill is the standard trope of *instability*, the state of being not stable. Shear instability in fluids is often like this: the slightest perturbation will cause waves to grow, and eventually to fold or roll up into vortices. The *smaller-scale response to a larger-scale state of instability plus a sufficient initial perturbation* lacks a crisp name, unfortunately. Such a

[i]Real convection is never governed by *undilute reversible* or *pseudoadiabatic* updrafts, often used to define approximate *moist adiabats*. A *zero-buoyancy plume* model is often used to reverse-engineer more realistic moist adiabats for climate-change studies.

response always has detailed characteristics (like a preferred wavelength or size), akin to the instability hill having a steepest side, perhaps with a narrow chute where the ball is mostly likely to fall, or is least likely to return from if it does.

A ball in a *dimple* on a hill (like in a volcano crater) expresses a *finite-amplitude instability*, like the explosive potential of gunpowder or a dry forest lacking only an ignition source. The *lifted-parcel instability* common to moist convection is like this, but asymmetric: only upward-displaced parcels reach saturation and engage latent heat release. The common name *conditional instability* represents this situation, but ambiguously. The "conditions" between a stable situation and a runaway or spontaneous continuation of motion may be surmountable, or might be wishful thinking ("if only the state were unstable").

Combustion requires all three of fuel and oxygen and ignition. Moist convection also hinges on three main *ingredients*[j] of actual, functional instability (that is, the condition whose response is actual convection). These include moisture abundance, the *lapse rate* of temperature, and finite-amplitude disturbances, which we will meet again as the Ooyama (1971) *dispatcher function*. As a result, there is no single, univariate measure of the problem that moist convection's job is to clean up or "neutralize". Unfortunately, terminology in this area is littered with historical cruft (Sherwood 2000, Schultz et al. 2000). For instance, the term *potential instability*[k] can be useful for predicting the turbulence of forcibly far-lifted cumulus updrafts, or for explaining *mammatus* on descending *anvil cloud* bases. But if its definition is used uncritically as a mindless formula, dry air in midlevels can be mistaken as somehow 'good for convection', a naive error sometimes seen in climate literature.

[j] This term is an example of a purposefully *vague* term of art (or jargon): search *ingredients-based forecasting* to find sometimes passionate debates around its use.

[k] Worse yet, equated as definition 2 of the enormously general terms "convective instability", "buoyant instability", and "thermal instability" in the authoritative http://glossary.ametsoc.org.

Ball-on-a-surface instability, analogized as parcel-minus-environment density difference, are too crude to be quantitative. Mass continuity (enforced by pressure, chapter 1) connects everything in realizable flows, complicatedly for large air displacement distances. The only hope of sensible meaning is for users of the word *instability* to carefully define it for specific scales and even a specific purpose, often a teleological one.

x.5 Unlikelihood, fitness, and the ecology of convection

When some patch of atmosphere exhibits conditional instability, and fluctuations have produced exceedances of the conditional's conditions, then what? For realism of imagination, look at a summer sky or some satellite imagery[l] before reading books. Clearly an account of convection requires multi-scale descriptions, often lumped debatably as questions of convective *organization*[m]. Truly organized entities (a meaningful assembly of parts) are *unlikely* to form randomly and for no reason, but may be preferentially favored by *natural selection* if their structural details boost their energy efficiency or some other measure of the Darwinian logical concept of *fitness* in the situation.

With the word "unlikely", *probability* necessarily enters our discourse, as our only hope of making any generalizable sense about convection's many complicated forms. The *distribution or density* of probability over a set of possible *situations*[n] is called *likelihood,* notated as *P(situation)*. But how can we enumerate (and thus assign probability to) "situations"? This is an epistemological opportunity, a chance to align our feeble brains' thought patterns with infinitely complex nature, through a human-friendly choice of description framework. Seeking probabilistic *claims* and *narratives (accounts)* about nature puts us in the excellent company of other sciences, with access to their fruitful tool

[l] such as https://worldview.earthdata.nasa.gov/, https://rammb-slider.cira.colostate.edu

[m] from Greek *organon,* tool or instrument, implying a teleological function to the parts

[n] probability theory's *events* like the landing of coins or dice are combined in *situations*

kits. The mathematical notation of probability is not as fussy as *reductionism's* mechanistic calculus and algebra accountings of the specifics of fluid transport, for instance (as marched through in Part I). Instead, the challenges are logical and conceptual, ultimately accessed via words and even metaphors -- but those can have pitfalls. Careful thought is required.

Statistical mechanics thrillingly illuminated the droll *thermodynamics* of earlier centuries that brought us steam engines, albeit with a mental undertow for some of its practitioners[o]. Its re-interpretation of the thermodynamic *entropy* as the logarithm of the number of *indistinguishable microstates* of a *system* ("situations") is profound. Information theory (Shannon 1948) generalized this logarithmic form beyond the atomic and molecular realm where distinctions are literally unknowable by quantum mechanical *uncertainty principles*. In this broader view based on log-probability[p], *informational entropy*[q] is a quantification of our ignorance, or of our strategic or tactical refusal to draw certain distinctions, not a new law of nature per se. Might principles from statistical mechanics be brought to bear on the problem of atmospheric convection?

Perhaps convection's patterns can be viewed as *exploitation strategies of an energy resource* by *unlikely structures*, echoing evolutionary biology whose distinct plant or animal species and individuals our clouds and storms are sometimes likened to, sometimes for merely cosmetic reasons. Its cousin discipline ecology, and parallel social sciences like economics, are rooted in similar ideas. These fields wrangle with the framing issues around definition-dependent and overlapping multiple-scale (*micro and macro*) *entities* and *systems*, as we must. Ecological and

[o] Ludwig Boltzmann in 1886 wrote: “... natural science appears completely to lose ... the large and general questions; but all the more splendid is the success when, groping in the thicket of special questions, we suddenly find a small opening that allows a hitherto undreamt of outlook on the whole” (Annila 2019). He hanged himself in 1906.

[p] embodying the *frequentist interpretation* of probability as an enumeration

[q] best rendered verbally as *missing information,* as argued in Ben-Naim (2008)

economic[r] principles like *succession* and *competition* and *survival* which rule our environmental worldviews and daily lives provide, at the very leastk, a rich trove of metaphors we can draw on for reasoning.

Information theory quantities like *mutual* and *transfer information* are becoming genuinely valuable in convection-adjacent fields (e.g. Ruddell and Kumar 2009). These are calculable from tractable and even simple formulas, once the right strategic framings are laid out, treating various types of entities as macro and micro, distinguished or interchangeable. This is a new flourishing in the somewhat orphaned discipline of statistics, whose problematic 19th - 20th century culture is fascinatingly reviewed at a popular science level in Pearl and McKenzie (2018). Their mission is to revitalize statistics' broken relationship to *causality*, its vital link to the ultimate goals of science (and of the hope for non-stupid artificial intelligence). Those quests require not mere measurement and quantification, but robust methods of *predictive understanding*, somewhat beyond this book's goal of deep and cross-linked appreciation.

Back in the convection problem, a competition between neighboring air *parcels* (on many overlapping scales) is the central process of cumuliform convection. Pressure is one weapon in this competition, favoring the narrow; but mixing is another, which favors the broad (Part II). Natural selection by gravity relentlessly favors the fittest, but the resource defining fitness consists in more than one ingredient, so 'fitness' is continually redefined by the whole ecosystem, just as in biology. Does selection apply strictly to individual "entities" (however defined), or to whole groups of them plus their "environments", perhaps in the *situations* nomenclature above? Part III revisits these questions.

In biology, each generation of form (*ontogeny*) echoes a much longer-term *phylogeny*, shaped by competitions over the vast time scales of DNA's continuity (Dawkins 1989). In contrast, convective cloud fields develop anew from chaos each time instability becomes available for

[r] both from the Greek root *oikos* "house, dwelling place, habitation"

flow structures to exploit. We observe (and would like to measure usefully) how complex multi-cellular storm entities gradually rise to predominance, perhaps through a succession-like process -- whether viewed as competitive, cooperative, or exploitative. The resulting wild beauty we observe in storms can inspire curiosity for a lifetime, even as the droll wisdom of probability and averages and asymptotes and integral constraints "govern" it all, via inescapable but complicated enforcement mechanisms that may be a fool's errand to try to understand in detail.

If neighboring parcels feel the same displacement-reinforcing force, motions driven by gravity ("convective" motions in a broad sense) can be broad and persistent enough to engage the flywheel of horizontally rotational momentum on our spinning planet: this is *synoptic-scale* convection. If neighboring parcels feel differential force, it generates shear, whose momentum instabilities (another class of problem, beyond our titular scope but still powered by convection) drives smaller-scale turbulence. Together all these processes fulfil gravity's grand mission, subject to the realizability constraints of the laws of motion (such as mass continuity). Part III will revisit the prospects of *extremal principles* as a handle on this beautiful mess, but first we must unpack that innocuous phrase "we observe".

x.6 Observability, cognitive biases, and scope selection

In addition to natural selection as a force in nature, *cognitive selection* shapes all discourse and even thought, and so necessarily shaped this book. What underlies this moment of my writing, or your reading?

Is Earth's wondrously life-sustaining climate, with atmospheric convection bringing water to its lands, an 'unlikely' coincidence of factors? No, it is an *inevitable* coincidence of those factors, because the entire question is asked downstream of the condition that we are here to observe the situation. This is cosmology's *anthropic principle* (footnoted above), brought down to Earth. More mundane versions of this same cognitive pitfall (with facets that could be variously called *pre-screening* *bias*, *unarticulated conditional sampling*, *survivor bias*, *confirmation bias*) cast shadows throughout our corpus of knowledge about the world (as evoked in the Preface). An area of great advances in contemporary thought, long known but exposed so starkly by the large number of enumerated instances made easy in the data age, is the naming and thus spotlighting of these cognitive biases and distortions themselves. Everyone who has discussed politics at a family meal knows that nonunique or even wildly divergent worldviews can develop, even in perfectly-rational *Bayesian learners*, when fed on differently pre-screened *information diets*. These are now the classic paradoxes of *artificial intellegence safety*, one of humanity's direst new concerns.

In convection science, one major observation bias is that condensed water reflects electromagnetic radiation (light, radar waves, etc.). It doesn't help that our brains are built to *identify* (literally: assign identity to) contiguous opaque entities in the air we inhabit, assigning them a sense of longevity (only sometimes valid for cloud systems) and *heft*. It is worth recalling that a buoyant cloud is actually a relative void of mass compared to the invisible air around it, by definition of buoyant. If these feel like obvious truths you feel intuitively when looking at the sky or radar data, you have a better mind than mine. What are the consequences for our science?

This field has matured from case-study description to generalized knowledge over about 2-3 generations. But an account of "convection" focusing mainly on its opaque parts is incomplete, no matter how many instances are accumulated. For instance, airplane-window views have a visibility bias against overcast areas 100s of km in size, arguably underpinning simplistic early views of the *cumulus parameterization problem* that my PhD advisor and his generational cohort made a career of contrasting (e.g. Houze and Betts 1981) with the new data gushers of radars and satellites. Today the discourse may almost have overcompensated, as the acronym MCS (for *mesoscale convective system*) is sometimes taken with excessive specificity – it is a mere umbrella term, slightly less vague than its predecessor term "cloud clusters" but still based largely on blob size in satellite or radar imagery.

Without functional measures of the importance of mesoscale contiguity or structural patterns (Mapes 2019), might cosmetic categorizations and labels like "scale" and "organization" be of little or even negative utility scientifically?

Internal structure of convective storms sampled by aircraft also has biases, toward some combination of interest (justifying the expense and effort) and safe flying conditions, as thoughtfully discussed in Ludlam and Scorer (1953) for instance. Aircraft sampling was consciously partitoned by quartiles of size of satellite-observed deep cloud blobs in one large field campaign (Lukas and Webster 1992, Mapes and Houze 1992), although in practice the logistics of aircraft targeting are challenging, and such a blob-size measure may be more of a convenient metric than a profoundly important one.

x.7 The pull of interests: extremes vs. large scales

The biggest divide in the science of atmospheric convection is between the mission of improved prediction and warning against hazardous local storms (Brooks et al. 2018), and the mission of larger-scale weather and climate prediction and projection, which involves the *moist dynamics* of *convecting flows* on larger scales.

In the first kind of science, the importance filter for what gets studied is a cost function based on life and property damage. That function is extremely steep with respect to storm characteristics like local intensity and stationarity or coverage of areas especially on land. In that version of convective meteorology, it is perfectly rational to focus on tails of the tails of the distribution *P(situation)*, without dragging along a whole science of gentler instances. In the second kind of science, the expected mean value of latent heat release and eddy fluxes by convection are an integral over *P(situation)* dominated by its middle, emphasizing typical or likely phenomenology.

With this book's mission of imparting "appreciation", extremes are not the main focus, but are noticed where their dynamics are truly distinctive, like in *supercells*. But there I have nothing new to add to the many passionate treatises on those exciting phenomena (e.g. Doswell 2001, the lavishly well illustrated Markowski and Richardson 2010, Bluestein 2013, Trapp 2013). Students and many researchers tend to be excited more by storms than statistics, but this book leans more to the latter.

A major driver of the multiscale study of convecting flow is *cumulus parameterization*, an engineering effort with underlying genuine science (as reviewed in Arakawa 2004). The aim is to devise a computationally fast, simple *surrogate model* that can deliver unbiased horizontal averages of the heat, moisture, and momentum tendencies produced by the whole ensemble of convective entities in larger-scale grid boxes, as a function of the gridbox-averaged[s] conditions. This enterprise could hardly be more different from storm interests, yet this book must touch on both, and the governing equations (Part I) and basic elements of reasoning ("cells", Part II) do overlap considerably. Again this book cannot begin to compete with the great tomes on that topic (e.g. Plant and Yano 2015), and will seek merely to impart an appreciation of principles (in Part III).

[s] and perhaps of other moments of a statistical treatment of sub-gridbox scale fluctuations

Part I: Essentials of the fundamentals

x.0 Chapter 1

Keeping track of stuff in space (Back to Outline)

x.1 Units for space, time, and stuff

Before understanding must come labeling and accounting. Mass, momentum, and energy ("stuff"[t]) are conserved by isolated entities or systems, but we also care about different places in the world so we must account for transport. We also care about different categories of stuff (air vs. water mass, heat vs. latent (chemical) energy, motions of different scales, etc.) so we must account for transfers between the categories. In our equations, these accounting terms will often outnumber and out-complicate the few key terms expressing the fundamental laws of physics.

Space, time, and mass are measured here in *Systeme Internationale* (SI) units, based originally on our ten fingers, Earth, and water.[u] Space is in *meters* (m), devised as the Earth's equator to pole distance divided by 10^7 to be human scale. A *kilogram* is the mass of a cubic meter of water (1 *metric ton*), divided by 10^3 to, again, fit human bodies and commerce[v]. Time is the contentious domain. The Earth gives us *days*, which could be divided by 10 or 100 for finger-counting convenience. Clocks with decimal face labels were manufactured in 18th century France, but never caught on. After all, the six-related numbers of traditional time have their

[t] Quantities which are *extensive* upon aggregation, or *conserved* quantities.

[u] Replacing old traditions, in the 18th century French Revolution's radical rationalizing.

[v] That volume of water is also called the *liter*.

own Earth-related numerology (almost-12 months and almost-360 days in a year). In the end SI retained the *second*: a day divided by 86400 (60 x 60 x 24). Latitude in degrees (10^7m / 90^o = 111.111 km per degree) and subdivisions like nautical miles (1/60 degree) also carry this six-related history.

Temperature's Celsius scale is also about water and tens: 1 ^{o}C = 1 K = (water's boiling point minus freezing point)/100. This is not quite as fundamental, since the boiling point depends on atmospheric pressure. It is a remarkable coincidence that the weight of a 1 m^2 column of Earth's atmosphere (1 bar or atmosphere of pressure) happens to be so near the weight of a 10 m column of water, making surface pressure nearly 10^5 (the global mean is 101325) in the compound SI unit of Pascals (1 Pa = force/area = 1 N m^{-2} = 1 kg m s^{-2} m^{-2} = 1 kg m^{-1} s^{-2}).

Precision science has quietly replaced these original Earth and Water motivations with more fundamental quantum profundities as the root of official SI, without most people even noticing (the kilogram was redefined in 2019). But our 10 fingers still rule numbering in all but the time domain.

x.2 Conservation of the most fundamental "stuff": mass

In a given volume of space (let's say 1 m^3 for definiteness), the enclosed mass is customarily labeled ρ, a *mass density* whose inverse is called *specific volume*, $\alpha=1/\rho$. The rate of change of mass in a cubic meter of space is pure spatial accounting: it equals the net inflow of mass into the volume. Physical sources and sinks are zero except for tiny Einsteinian $E=mc^2$ effects. The flow of mass through a two-dimensional (2D) area, like the square face of a 3D cubic volume, is measured by a *flux*. The units of mass flux [kg m^{-2} s^{-1}] embody its meaning better than any further words can elaborate.

Mass flux is $\rho\boldsymbol{V} = \rho\vec{\mathbf{V}}$ in symbols[w], using bold face and optionally arrows for emphasis for vectors like velocity $\boldsymbol{V} = \boldsymbol{i}u + \boldsymbol{j}v + \boldsymbol{k}w$, in a Cartesian (x, y, z) coordinate system with its unit vectors $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ and velocity components (u, v, w).

Readers should verify that the units of $\rho\boldsymbol{V}$ are indeed a flux [(kg m^{-3}) (m s^{-1}) = kg m^{-2} s^{-1}]. Notice with that same units awareness that velocity $\boldsymbol{V}$ is a *volume flux;* and is also a *specific momentum* (where "specific" means *per unit mass*). A feeling for different interpretations of the same quantity is essential to fully appreciate the equations of convection: a firm physical grip is needed on the mathematical symbols and their units, but not too tight or exclusive.

Logic tells us that for the truly conserved stuff called mass, its *rate of change* in a volume equals the *net mass inflow*. That is called the *convergence of mass flux*, the negative of *divergence*[x]. Translating that into math, with subscripts denoting partial derivatives along the axes in (x, y, z, t) space,

$$
\begin{aligned}
\rho_t &= conv\left(\rho\vec{\mathbf{V}}\right) = -div\left(\rho\vec{\mathbf{V}}\right) = -\vec{\boldsymbol{\nabla}}\cdot\left(\rho\vec{\mathbf{V}}\right) \\
&= -(\rho u)_x - (\rho v)_y - (\rho w)_z + source\ terms \\
&= -(\rho u)_x - (\rho v)_y - (\rho w)_z + 0 \qquad (1.1)
\end{aligned}
$$

A diagram with a cube (a *control volume*) and flux arrows is sometimes used to illustrate this logic, but seems unnecessary to repeat here. Since mass is truly conserved, *source terms* are zero, except for negligible relativistic conversions to energy via Einstein's famous $E=mc^2$.

x.2.1 *Aside on mathematical expression culture*

Notice that the structure of (1.1) embodies an ambitious spirit:

[w] See Table of Symbols and Notation.

[x] Early meteorology texts simply used the word *vergence*, letting the sign reflect its sense.

$$something\ we\ want\ =\ complications\ we\ must\ face\ to\ get\ it$$

In the case of a *prognostic equation* like (1.1), what we want is to know the future, so we put that on the left-hand side (LHS). Let us call this the *grasping form* of equation writing, in contrast to a *majestic equality* like 0 = (sum of all terms). Mathematically, these are isomorphic: not different in any meaningful way, just in form, by custom. But equations do have a *brainfeel*[y] in addition to their content. To feel a difference, transform (1.1) into the equivalent majestic equality:

$$0 = (\rho u)_x + (\rho v)_y + (\rho w)_z + (\rho \dot{t})_t \quad (1.2)$$

In the first 3 terms on the RHS, velocity components in space ($u = \dot{x}$, $v = \dot{y}$, $w = \dot{z}$, using Newton's dot notation for univariate time derivatives) measure the spatial journey of moving matter, in units of meters traversed per second of elapsed time. In (1.2) an analogous quantity $\dot{t}$ measures the journey of matter through time, in seconds traversed or endured per second of elapsed time. Since both are seconds, $\dot{t}$ is unitless and equal to unity. But was no appreciation gained?

x.3 Conservation of specific (per unit mass) other stuff:

x.3.1 *Specific momentum and its physical source terms*

Having established the flux convergence form for mass (1.1), other budgets follow straightforwardly by accounting other intensive properties of air on a *specific* or *per unit mass* basis. For instance, to get the budget equation for momentum, simply multiply the mass flux by specific momentum, which (as noticed above) is velocity, and then reconsider the sources and sinks. In the vertical or **k** direction, along which air's position change $\dot{z} = w$ (vertical velocity) as above,

[y] This term is motivated by "mouthfeel," a food descriptor distinct from taste or nutrition.

$$
\begin{aligned}
(w\rho)_t &= -\vec{\nabla}\cdot\left(w\rho\vec{\mathbf{V}}\right) \\
&= -(w\rho u)_x - (w\rho v)_y - (w\rho w)_z \\
&\quad + w\ source\ terms
\end{aligned}
\qquad (1.3)
$$

The big difference from (1.1) is nonzero source terms on the RHS. Newton calls such momentum sources *forces,* in the famous equation **F** = m**a** that earned him the honorific SI unit for force (N = kg m s^{-2}).[z] Acceleration could be called *specific force,* but that doesn't add much sense: we don't really think of acceleration as semi-conserved "stuff" like momentum.

Two forces (momentum sources) are needed to appreciate convection:

(1) *Gravity*[aa] $-\boldsymbol{k}\rho g$
(2) The *pressure gradient force* $-\vec{\nabla}p$

What is this "pressure" *p*? Most students learned it as *force per unit area,* implying units $N/m^2 = Pa$ (Pascals). However, those units would also identify it as a *momentum flux,* with flux units of (stuff) $m^{-2}s^{-1}$ = (momentum) $m^{-2}s^{-1}$ = ($kg\ m/s$) $m^{-2}s^{-1}$]. But it is a strange flux: directionless, or imparting momentum in all directions at once. Since *p* is a scalar field, not a vector field, the differential operator measuring the net flux into a cube of space (the flux difference from one side to the other) is not the convergence of the directional vector flux field $\rho\vec{\mathbf{V}}$ as above, but rather the *gradient* of the scalar field *p*. Notice also that the calculus concepts of "gradient" and "convergence" are only well defined over a spatial length scale where the continuum approximation is valid (smoothing over the lumpy molecular nature of matter).

[z] Is force a flux of some definable “stuff”? Not really: 1 N = 1 (kg m^3 s) m^{-2} s^{-1}, so the implied unit of "stuff" being transported (kg m^3 s) makes no sense to this author’s brainfeel.

[aa] Strictly speaking, "gravity" in meteorology is Newton's *gravitational force* plus a small force due to the Earth's rotation which bulges the equator out a little relative to a sphere.

In a steadily rotating coordinate system, where air we call "motionless" is actually accelerating, we must add to (1.3) a third corrective term, the fictitious Coriolis force per unit mass -2$\vec{\boldsymbol{\Omega}} \times \vec{\mathbf{V}}$ where $\vec{\boldsymbol{\Omega}}$ is the coordinate rotation vector. For simplicity we will neglect the vertical component of the Coriolis force (which is tiny compared to gravity), and retain only the horizontal Coriolis force based on the *Coriolis parameter f = 2*$|\vec{\boldsymbol{\Omega}}|$ *sin(latitude)* in our Earth-tangent Cartesian (x,y,z,t) coordinates.

Gathering the considerations above for every cubic meter of space, we are up to 4 equations including 5 unknowns (*u,v,w,p,ρ*):

$$
\begin{aligned}
(\ \rho)_t &= -(\ \rho u)_x \ - (\ \rho v)_y - (\ \rho w)_z \quad + 0 && \text{(1.4a)}\\
(\rho u)_t &= -(u\rho u)_x - (u\rho v)_y - (u\rho w)_z \quad -p_x + \rho f v && \text{(1.4b)}\\
(\rho v)_t &= -(v\rho u)_x - (v\rho v)_y - (v\rho w)_z \quad -p_y - \rho f u && \text{(1.4c)}\\
(\rho w)_t &= -(w\rho u)_x - (w\rho v)_y - (w\rho w)_z \ -p_z - \rho g && \text{(1.4d)}
\end{aligned}
$$

Notice that each p in (1.4 b-d) could be moved inside the parentheses of a flux term, emphasizing its role as a flux. But pressure's momentum flux is transmitted across the borders of a spatial box by elastic collisions of the molecules outside the border with those inside it. This is new physics, *different in character* from pieces of matter carrying their properties with them as they cross a border. Border crossing by the smallest pieces of matter (molecules) is part of the flux terms like $u\rho w$ in (1.3). That molecule-borne flux acts as a *diffusion* process: *the convergence of a flux of stuff that is proportional to the gradient of that stuff* (revisited in section 3.1). That process is called the *viscosity force* when it is pulled out of the flux transport terms and moved to the RHS as a "source" term for equations like (1.3)[bb].

Without density variations, mighty gravity is indiscriminate in (1.4d), and with its pull equal on all air there can be no *convection* in the sense of our title. We therefore must allow ρ to vary a little so that the gravity force -$\boldsymbol{k}\rho g$ can do *work*. But that variable ρ drags us into

bb The *Navier-Stokes equations* for fluid dynamics have viscosity pulled out in this way.

thermodynamics (Chapter 2). Specifying ρ is called an *equation of state*. Before we tackle that complication, we can already gain a key insight into the nature of pressure from the simplest case, ρ = constant (Problem 1.1). Readers should understand the resulting lesson well: *Continuity is the Law, Pressure is the Enforcer, and* ***F****=m****a*** *is its Mechanism*.

x.3.2 *Other specific stuff: humidity and 'heat content'*

To close our four-equation but five-variable set (1.4) we need an equation for ρ. Although recognizing *specific momentum* as another name for velocity made momentum budget construction very direct (1.4 b-d), no similar trick of invoking 'specific density' can help us because density per unit mass is not physically incisive[cc]. We need another tactic, another physical law. The ideal gas law comes to hand, but that brings in temperature T. Since T is indicative of warmth or heat, we need a quantity that measures the "stuff" that heat is made of: an energy perhaps, although that will lead us on to more abstract stuff called *entropy*.

For cloudy convection, we also need to keep track of the mass of water, which we can measure as a *specific humidity q*, so you know what to do: replace *w* with *q* in (1.3) and replace the physics of the *w sources* term with the physics of *q sources*. It's a trivial extension to the notation to budgets of categorized water (specific cloud water, specific rain water, specific ice water, etc.). However, one subtle bookkeeping issue in this exercise needs to be noticed.

x.3.3 *Specific X, or mass mixing ratio of X?*

There is a subtle difference between *specific water vapor mass* q_v and *water vapor mass mixing ratio*[dd] r_v. Both have water vapor mass in the numerator, but mixing ratio has the mass of *dry air* rather than *total mass* in the denominator. To get more precise, one must decide if ρ in (1.1)

[cc] That would be the inverse of *specific volume* α = 1/ρ, which is just density ρ itself!

[dd] We avoid the chemistry term *concentration*, which seems ambiguous.

stands for dry air density or for total mass density. It seems tempting to use total mass, but then the velocity w is harder to define: in rain, it would have to be a mass-weighted mean of the wind velocity of gases and the fall velocity of particles. The complications explode. Alternately, the third step in (1.1) stating *mass sources = 0*, which is truer for dry air mass, could be revised slightly to account for precipitation falling out. The complications explode differently.

Strictly speaking, this book accepts (1.1) as if for dry air, yet keeps the word *specific* for brevity. Terms of art and their mathematical cousins (symbols) are often thought of as hyper-specific. But their greatest power can actually lie in being vague, refusing to draw distinctions that are inessential to a line of reasoning[ee]. Our plain symbol ρ papers over the complications intentionally, eliding the slight distinction between q_v and r_v as measures of water content. If your goal is to mathematically frame a numerical model that will be integrated over long times, requiring an equation set that obeys integral conservation laws punctiliously even as the various types of "stuff" are shuttled among many spatial boxes and categories, then such decisions must be strictly defined and adhered to. Symbols, subscripts, and small terms will proliferate. You should work from a longer, fussier book. If such a model must quantitatively assimilate absolutely calibrated observations, it must also use fully *accurate* as well as precisely conservative thermodynamics, coordinates, and conservation laws. Here, since our goal here is to facilitate appreciation of conceptual essentials, such details of rigorous fundamentals will be elided freely.

x.3.4 *Advection and the material derivative*

Notice (Problem 1.4.1) that you can distribute the derivatives in (1.4b) by the chain rule, divide by ρ, and use (1.4a) to rewrite it without approximation as:

$$u_t = -uu_x - vu_y - wu_z - \alpha p_x + fv \quad (1.5)$$

[ee] An example is the term *hydrometeor* for any falling condensed water object.

This *advective* form of transport terms on the RHS is fully general, and tempts us to interpret *advection* as being as valid as flux and its divergence. The sense of advection is to *look upwind: those air properties are coming toward you, so conditions at your location will soon be like that unless source terms on the RHS intervene*. Defining a special new *total* or *Lagrangian* time derivative $du/dt = u_t + \boldsymbol{V} \cdot \boldsymbol{\nabla} u = u_t + uu_x + vu_y + wu_z$, the set (1.4) becomes:

$$d\rho/dt = -\rho \boldsymbol{\nabla} \cdot \boldsymbol{V} \quad (1.6a)$$
$$du/dt = -\alpha p_x + fv \quad (1.6b)$$
$$dv/dt = -\alpha p_y - fu \quad (1.6c)$$
$$dw/dt = -\alpha p_z - g \quad (1.6d)$$

In Chapter 2 we will need to invoke thermodynamic laws learned from interrogating a kilogram of air trapped in a piston in a laboratory. To use these laws in our fluid equations, we need to equate laboratory *time derivatives referring to a unit mass of air* (denoted with Newton's univariate derivative notation like $\dot{T}$)[ff] to this Lagrangian total derivative *dT/dt* as defined above. One way to see the equivalence is to notice that if we truly had the field or function $T(t,x,y,z)$ – temperature everywhere forever – we could extract the temperature history of an arbitrary moving parcel of unit mass as $T(t, x_p(t), y_p(t), z_p(t))$. Using the chain rule to extract all the temporal changes in the function's argument,

$$dT_p/dt = \partial T/\partial t + \partial T/\partial x \cdot \dot{x_p} + \partial T/\partial y \cdot \dot{y_p} + \partial T/\partial z \cdot \dot{z_p}$$
$$= T_t + uT_x + vT_y + wT_z$$

which can be equated to $\dot{T}$, the time-only changes for an air parcel that is trapped in the laboratory chamber.

[ff] reviewed in https://en.wikipedia.org/wiki/Notation_for_differentiation

x.4 Problems:

x.4.1 ***Show the steps from 1.4b (flux) to 1.6b (advection) forms of the budget equations for the u component.***

x.4.2 ***Repeat the problem above for v and w, and gather terms to show that the advection of vector momentum can be written as*** $-u\vec{V}_x - v\vec{V}_y - w\vec{V}_z = -(\vec{V} \cdot \vec{\nabla})\,\vec{V}$

x.4.3 ***Set density to a constant ρ_0 and simplify the set (1.4) maximally in that case.***

What phenomena could this *incompressible fluid* (constant density) equation set describe? In other words, how can an incompressible fluid move, and why would it? What could drive motion, at what scales, and how could that motion decay?

What would be the speed of compression (sound) waves in such a fluid? That is, if boundary conditions jiggle one edge of an incompressible body of fluid, how soon is the motion transmitted to the other side?

x.4.4 ***Using the simplified set of component equations with constant density ρ_0 from 1.4.3, transform the majestic equality form of the mass continuity equation (0 = terms) into a grasping equation of the form:***

what I want (pressure) = complicated effort needed to construct it

Hint: differentiate mass conservation in time, momentum conservation equations in space, and substitute the latter into the former.

Subscripts for partial differentiation will save many redundant hand motions in this process. You may use the symbolic inverse $\boldsymbol{\nabla}^{-2}$of the Laplacian operator $\boldsymbol{\nabla}^{2}$ in the final answer, even though solving it is a nontrivial job (Chapter 3).

Interpret the result in your own words, elaborating on this summary: *Continuity is the Law, Pressure is the Enforcer, **F** = m**a** is its Mechanism.*

x.5 Solutions:

Solution to 1.4.3:

Substituting constant $\rho=\rho_0$. dividing by ρ_0, and negating the sign in (1.4a),

$$\begin{aligned}
0 &= u_x + v_y + w_z \\
u_t &= -(uu)_x - (uv)_y - (uw)_z \quad -\pi_x + fv \\
v_t &= -(vu)_x - (vv)_y - (vw)_z \quad -\pi_y - fu \\
w_t &= -(wu)_x - (wv)_y - (ww)_z \ -\pi_z - g
\end{aligned}$$

where we have introduced the pressure variable $\pi = p/\rho_0$.

What phenomena could this incompressible fluid equation set describe?

If a body of such fluid were initially at rest, the only forces that could drive coherent (larger than molecular) motions within it are coherent momentum sources applied at its boundary. Without density variations, gravity cannot discriminate and nothing worth the name "convection" can occur. Pressure can push divergent (irrotational) internal flows, like the motions inside a water balloon that make some part bulge out when another part is pressed in. If groups of molecules at the fluid's boundary are somehow given a coherent momentum tangential to the boundary, like by a *stress* we could call "friction", they could diffuse that momentum inward (viscosity)[gg]. Such viscous forces could create internal shear, which could break down due to shear instabilities into smaller-scale fluid motions that could transport momentum still deeper into the fluid, so that eventually the fluid could contain all sorts of turbulent motions. The energy of such motions would decay into heat (molecular motions) by internal viscous dissipation (diffusion of momentum down its gradient). The speed of sound (compressional waves) in an incompressible fluid is infinite: with constant density $\rho_t = 0$, the medium is infinitely stiff and any motion or vibration is transmitted throughout the body instantly.

Solution to 1.4.4:

Differentiating the *u* equation in *x*, the *v* equation in *y*, and the *w* equation in *z*, and summing them,

$$\begin{aligned} [u_{tx} &= -(uu)_{xx} - (uv)_{yx} - (uw)_{zx} \quad -\pi_{xx} + f v_x] \\ +[v_{ty} &= -(vu)_{xy} - (vv)_{yy} - (vw)_{zy} \quad -\pi_{yy} - f u_y - u f_y] \\ +[w_{tz} &= -(wu)_{xz} - (wv)_{yz} - (ww)_{zz} \quad -\pi_{zz} \; - g] \end{aligned}$$

Using mass continuity to see that the left side is zero since $\rho_t = 0$, and packing up terms into a vector form,

$$0 \; = \; -\boldsymbol{\nabla} \cdot [(\boldsymbol{V} \cdot \boldsymbol{\nabla})\boldsymbol{V}] \;\; -\boldsymbol{\nabla}^2 \pi \quad + f\zeta - u\beta$$

where ζ is the vertical component of relative vorticity $\zeta = v_y - u_x$, $\beta = f_y$ is the latitudinal gradient of the Coriolis parameter, and parentheses are carefully used to make the result depend on no notation beyond the familiar vector dot product and the vector differentiation operator $\boldsymbol{\nabla} = \mathbf{i}\,\partial/\partial x() + \mathbf{j}\,\partial/\partial y() + \mathbf{k}\,\partial/\partial z()$.

Hungrily solving for π, using the symbolic inverse of $\boldsymbol{\nabla}^2$,

$$\pi = \; \boldsymbol{\nabla}^{-2}[-\boldsymbol{\nabla} \cdot [(\boldsymbol{V} \cdot \boldsymbol{\nabla})\boldsymbol{V}] + f\zeta - u\beta]$$

[gg] The vorticity films at http://web.mit.edu/hml/ncfmf.html have excellent illustrations.

Continuity is the Law expressed by the equation for pressure. Since flow divergence is zero, to maintain the (assumed) $\rho=\rho_0$ forever ($\rho_t=0$), pressure must intimately cancel the divergence of each and every other field of force (terms on the RHS). In this case those divergent forces are (i) the "inertial force" implied by the advection of momentum in the inner square brackets, (ii) the inward- or outward-directed Coriolis force on horizontally swirling flow, and (iii) the divergent Coriolis force on zonal flow *u*, when the Coriolis parameter varies with latitude. Other forces like viscosity could be easily added.

Pressure is the Enforcer of mass continuity. Pressure does that job[hh] in the momentum equations, and *F = ma is the Mechanism* of that enforcement. From this standpoint, the common exercise in dynamics courses of calculating flows with the pressure field taken as a given is not very sensible: pressure is a cleanup force, the *last* force *logically*, the one that adjusts to intimately respond to the divergence of all the other forces, almost instantly (strictly, at the speed of sound). Here that sound speed is infinite, which is unrealistic, but even in air it is much faster than all the other information-transmitting waves in meteorological flows. Therefore, the point remains relevant: *Continuity is the Law, Pressure is the Enforcer,* ***F*** *= m**a** is the enforcement mechanism.*

[hh] This is *teleology*, the explanation of things in terms of the purpose they fulfil.

x.0 Chapter 2

Good enough equations (Back to Outline)

Budgets in boxes (Chapter 1) got us the mass continuity and momentum equations (1.4 and 1.6), with pressure and gravity as the sources (forces). The molecular part of flux convergence will soon (chapter 3) be treated as a diffusion and moved to the right-hand side as another force. Coriolis forces, although sometimes called "fictitious", are as real as the ground is stationary on our rotating planet. The important distinction among forces is not real vs. fictitious, but non-negotiable vs. *adaptive*.

Pressure is the adaptive force, as Problem 1.4.2 (solution 1.5.2) showed, adjusting itself instantly (in the incompressible fluid), or really at the speed of sound, to negate the divergence of all of the other forces in a fluid whose basic density structure varies little with time. In this chapter we will see that negating the divergence of gravity gives p a *hydrostatic* component that keeps air from collapsing into a puddle, negating divergent Coriolis forces gives p a *geostrophic* component, negating *buoyancy* (defined below) gives a component important for cumulus dynamics, and *dynamic pressure* arises to negate the divergence of the advection of momentum (sometimes called *inertial force*) in moving air. That problem (solution 1.5.2) left us with 4 equations for 5 unknowns (u,v,w,p,ρ), and a teleological sense of what pressure is.

To close our equation set more satisfyingly, we need equations to define density more interestingly than a constant, allowing it to vary horizontally so that gravity can drive convection.

x.1 Good-enough thermodynamics of moist air

x.1.1 *Density and the ideal gas law*

To close the set, we need an equation predicting the density of air. The ideal gas law springs to mind: $\rho = 1/\alpha = p/(RT)$, but that doesn't solve the problem. For one thing, it introduces a new unknown (RT), so now we will need additional equations. Also it is nonlinear (a quotient), so taking a time derivative to get the needed prognostic equation yields 2 terms. Using Newton's dot notation for the time rate of change in 1 kg of ideal gas trapped in a laboratory chamber, our grasping form predicting ρ is:

$$\dot{\rho} = (\dot{p} - \dot{T}/T)(RT)^{-1} \qquad (2.1a)$$

or perhaps more usefully we can grasp at the rate of change of T:

$$R\dot{T} = p\dot{\alpha} + \alpha\dot{p} \qquad (2.1b)$$

Here some complications must begin, which we will seek to minimize.

x.1.2 *Virtual temperature, density temperature*

The *gas constant R* quantifies the partition of energy between the kinetic energy of molecules (proportional to T) and an *elastic energy* implicit within a body of compressed gas $p\alpha$. R is defined on a *per mole* basis, not a *per unit mass* basis. Since H_20 (molecular weight 18 kg/mole) is lighter than Earth's mixture of N_2 and O_2 ("dry air"; average molecular weight 29), with a ratio between these of 0.622 we will see again, R for moist air depends on humidity. That dependence is fairly slight for Earthly humidity values, so meteorology's custom to hide the complication is to use R to denote the truly constant dry-air gas constant R_d, and then elaborate T with the subscript v for *virtual temperature* T_v =

$T(1 + 0.602\ q_v)$[ii], the temperature at which dry air would have the same bulk density as the actual *moist air*[jj] we are talking about.

x.1.3 ***First Law: internal energy and the quest for warmth***

Air's density depends on *warmth*, a kind of energy measured by *T*, so an energy budget or conservation law (the *First Law of Thermodynamics*) is surely our route to a *T* equation for a trapped kilogram of air in an insulated laboratory piston. But that air can still export energy, by doing one kind of *work*: expanding its volume *V* against the pressure force, as expressed by the term $pdV/dt = p\dot{\alpha}$. The conservation of energy added to a trapped gas with a controllable (perhaps electric) heater must therefore take the conceptual form:

$$heating\ rate\ Q = d/dt(internal\ energy) + work\ air\ does$$

Internal energy is defined by the above conservation statement, and is *T* times the *heat capacity at constant volume* C_v:

$$Q = C_v\dot{T} + p\dot{\alpha} \qquad (2.2)$$

Three term equations are uncomfortable: our brainfeel wants an equation with a single-term RHS, something more like:

$$heating = d/dt("some\ stuff\ called\ warmth\ involving\ T")$$

Substituting (2.1) into (2.2) doesn't help -- still there are 2 RHS terms:

$$Q = (R + C_v)\dot{T} - \alpha\dot{p} = C_p\dot{T} - \alpha\dot{p} \qquad (2.3)$$

We need a trick. Two approaches suggest themselves.

[ii] Derivation is in Wikipedia or many textbooks.

[jj] Inclusion of the mass of suspended particles is sometimes telegraphed by using *density temperature* T_ρ, but anyone who bothers to worry about T_v obviously wants total density.

Trick 1: Divide by temperature. Why? Because it works:

$$Q/T = C_p\dot{T}/T - \alpha\dot{p}/T = C_p\dot{T}/T - R\dot{p}/p$$

Since the coefficients are constant (C_p and R), they commute through the time derivative operator. From pure calculus, $T^{-1}dT/dt = d/dt(\ln T)$, so we may write a conservation law in our desired form involving *d/dt(stuff)*, but only for confusing stuff called *specific entropy S*:

$$\begin{aligned} Q/T = \dot{S} &= d/dt(C_p \ln T - R \ln p) \\ &= d/dt(C_p \ln\theta) \qquad (2.4) \end{aligned}$$

What is this specific entropy *S*? Do the units (J kg^{-1} K^{-1}) satisfy any intuition? No, because "degrees" of temperature are not fundamental (Ch. 1). What does it mean to divide a heating rate (energy input rate) by T in degrees? Deeper understanding is available in statistical mechanics (e.g. Ben-Naim 2013). If the vigor of molecular motions had been expressed in energy units instead of temperature's arbitrary equal ticks along a tube of mercury (degrees), then entropy would be a dimensionless quantity. This dimensionless entropy measures the *missing information* in our bulk thermodynamic description $\{p,T,\rho\}$ of the zillions of molecules in that kg of air. Specifically, it is the *log of the number of redundant microscopic states* that would yield the same bulk thermodynamic state $\{p,T,\rho\}$. That information is measured in bits if the log is base 2 (see chapter 0 and its Problem at the end).

Meteorology has invested heavily in *potential temperatures* like $\theta = T(p_0/p)^{R/C_p}$ in (2.4), expressing *the temperature a parcel of air would have after it undergoes various hypothetical processes* (in this case, adiabatic compression to a reference pressure p_0). Generations of students from the pre-computer age learned to navigate the mild nonlinearities of Earthly moist thermodynamics by following gently curved lines (such as *adiabats* or *isentropic lines* with constant S) on large paper *aerological diagrams*, labeled with the axes of thermodynamics' laboratory observables: pressure and temperature. An entropy version of the diagram (the *Tephigram*, shorthand for *T*-ϕ since ϕ

was used for entropy) has its pressure axes woozily not-quite parallel to the paper edges. In the USA this lost the textbook-mediated generational popularity contest to pseudoadiabatic or skewed-T[kk] by log-p diagrams. Despite the end of oversized-paper based science, the ease of evaluating mildly nonlinear functions in the computer age, and with the rise of tropical meteorology with its emphasis on quantitative humidity for moist convection (section 4.1 below), these anachronistic diagrammatic treatments still pervade the literature. Students should learn them (MetEd[ll]), but this book would gain nothing from covering them.

Trick 2: As an alternative to the entropy approach of Trick 1, define a new labeling function for pressure, the *hypsometric height* $Z_h(p)$ whose increments satisfy the hydrostatic force balance relationship $\alpha dp = gdZ_h$. Why? Because it works. Unlike α in (2.3), *g* is time independent, so it commutes through the time derivative in (2.3) like in Trick 1, giving our desired form, *d/dt ("some kind of warmth stuff") = heating*:

$$Q = \dot{s} = d/dt(C_pT + gZ_h) \qquad (2.5a)$$

This *s* is called *dry static energy* (per unit mass). It has the same useful property of potential temperature θ for reasoning: it is conserved during vertical motions of an unsaturated parcel. That makes it uniform with height in any well-mixed layer, which immediately gives the *dry adiabatic lapse rate of temperature* $\Gamma_d = -g/C_p$ = -10 K/km (taking $dZ_h/dz = 1$, an excellent approximation). Dividing $\dot{s}$ by C_p to get a warming rate at constant pressure is straightforward, unlike the use of $\dot{\theta}$ which -- despite units of K/s -- must be converted using the nonconstant profile factor (T/θ) to get a true warming rate $\dot{T}$ for density-change purposes, our larger goal.

[kk] Not statistical skew (chapter 0.2), but merely a graphical skew, a non-orthogonal axis that minimizes *one* of the diagram's failures to match the importance of distinctions to the size of the corresponding feature on the paper.

[ll] Free course materials at https://www.meted.ucar.edu/training_module.php?id=225

Turning back to that larger quest, we can easily recast (2.5) as a grasping equation for the rate of change of T, ready to append to the set (1.6):

$$\begin{aligned} dT/dt &= Q/C_p - g/C_p(\dot{Z}_h) \\ &= Q/C_p - g/C_p(Z_{h_t} + \boldsymbol{V} \cdot \boldsymbol{\nabla} Z_h) \quad \text{(2.5b)} \end{aligned}$$

But wait a minute! We indulged in some sleight of hand in Trick 2. For Z_h to depend only on *p*, so that its increment has only one term $gdZ_h = \alpha dp$, α must also depend only on p. In other words, we tacitly assumed that ρ *is approximated by a reference density profile that is a function only of altitude*. This approximation is sufficient for all this book's further purposes, and in fact is needed later for other reasons, but we backed into it.

This approximation eliminates all but vertical advection from the advection term in (2.5b). Before we say goodbye to the horizontal component, notice that its form $-\boldsymbol{V} \cdot \boldsymbol{\nabla}(gZ_h)$ can be reinterpreted in terms of meteorology's usual pressure-coordinate equations as *kinetic energy gained by flow down the PGF* (problem 2.4.1). This interpretation is one glimpse of the fact that *the key simplification of a static energy treatment is neglect of kinetic energy (Betts 1979)*[mm]. Fortunately, the kinetic energy of winds on Earth is far less than that of molecular pandemonium (thermal energy), so the simplification gained by Trick 2 is well worth its small cost in accuracy. Turning good-enough (2.5) into a prognostic equation for the *thermal buoyancy force b* will soon require a reference density profile anyway (section 2.2), so this is just a head start.

In the shorthand for convective dynamics implied by one-dimensional soundings analysis (section 4.1), a balloon-measured ρ profile is used as the reference to sensibly define Z_h=*Z*=*z*. The assumptions in that case are numerous and conceptual: they imply an oversimplified vertical momentum equation (buoyancy only) for the ascent of hypothetical pure lifted "parcels" within an infinitely broad "environmental" airmass whose

[mm] Relatedly, MSE minus CAPE is the true lifted-parcel conserved variable (Romps 2015).

conditions the balloon's tiny barometer, thermometer, and hygrometer are supposed to have measured. Neglect of kinetic energy is far from the most dubious of these idealizations, so (2.5) is more than good enough.

x.1.4 *Latent vs. 'diabatic' heating and moist adiabaticity*

Equation (2.5a) says that s is *conserved for adiabatic flow* (for which $Q=0$) even when air changes pressure by moving vertically. If we take the *latent heat of vaporization* L as constant[nn], the part of Q due to condensation minus evaporation $L(c-e) = L\, dq_c/dt = -L\, dq_v/dt$ can be moved to the right-hand side. This almost counts as a third Trick:

$$Q_d = \dot{h} = d/dt(C_pT + gZ + Lq_v) \qquad (2.6)$$

This h is called *moist static energy* (per unit mass), interpreted very readily as the sum of a warmth measure called *enthalpy* C_pT plus geopotential energy (equating Z to Z_h, the *hydrostatic* approximation) plus latent heat energy. Equation (2.6) says that *h is conserved for* $Q_d=0$, even when air changes pressure by moving vertically *with or without condensation and evaporation*. Such flow is sometimes called *moist adiabatic*, leaving Q_d to represent the genuinely 'diabatic' heating rates like radiation, heat conduction from the surface, and the small viscous dissipation of wind kinetic energy[oo].

The $T(p)$ or $T(Z_h(p))$ profile that results from such saturated but otherwise adiabatic vertical flow is called a *moist adiabat.* One fine distinction is a *reversible adiabat* in which the condensed water is carried along, adding its thermal inertia in C_p and keeping an ascending, cooling parcel slightly less cold; vs. a *pseudoadiabat* in which condensate is instantly precipitated out. Obviously natural convective cloud updrafts must lie

[nn] Its slight increase with T is compensated by the heat capacity of condensate, in the relevant no-free-lunch proof that perpetual-motion heat engines are impossible. Neglecting both (taking L constant and C_p for air only) is a good approximation pair.

[oo] Also "apparent" heating or moistening by small scale eddy flux convergence, if "d/dt" in (2.6) is defined to include apparent advection by large scales only (chapter 3).

somewhere in that range, depending on highly situational microphysics. We will later meet a more teleological or functional definition of a moist adiabat: the neutral stability profile toward which real moist convection adjusts the atmosphere. That functional definition is useful for understanding how thunderstorms will respond to climate change for instance (Singh and O'Gorman 2015, Romps et al. 2015).

x.1.5 *Static energy vs. entropy vs. potential temperatures*

Entropy (Trick 1 above) was derived without approximation. It connects to the rest of thermophysics, and can be extended to very different gases on strange planets. Entropy budgets and the Second Law are real, and have their profundities (e.g. Pauluis and Held 2002, Romps 2008) and some surprises[pp]. But in practice, most scientists really think with the brainfeel of conservation of some form of warmth-energy. Most costly to intuition is the formal nonlinearity of entropy's *ln*() function, or the *exp*() function that moist or "equivalent" potential temperature formulas bristle with. The clarity of $h = C_pT + gZ + Lq$, with its lucid self-evident meaning, could never be regained by Taylor expansions and discarding higher-order terms to formally linearize the ln() and exp() functions.

For soundings analysis (lifted parcel games, section 4.1), a side by side graphical comparison can be easily made between static energy, entropy, and potential temperature diagrams (chapter 4 computer exercises). The resulting almost-identical graphic (aside from axis units) indicates that static energy is clearly good enough, given the other uncertainties around mixing and microphysics and other situational effects in convection. Even for numerical model design, static energy can be used as part of a well-considered *suite* of approximations. At that equation-set level of consideration, having a conserved quantity is the point, not the precise form of that quantity. The popular System for Atmospheric Modeling (SAM, Khairoutdinov and Randall 2003) simulates impressively Earth-like convection and even planetary-scale circulations (Bretherton and

[pp] For instance, turbulent dissipation in the wakes of raindrops falling at terminal velocity is the main dissipative source of entropy in the atmosphere (Pauluis et al. 2000).

Khairoutdionv 2015), using static-energy thermodynamics with one additional term for the latent heat of freezing ("frozen" MSE). Framing inaccuracies are rarely the biggest problem in modeling[qq], except perhaps for precision models that must quantitatively assimilate absolutely calibrated observations across very different regimes (polar to tropical).

x.2 Good-enough fluid dynamics

We need a better density equation to close our mass and momentum budgets (1.4) and (1.6). Let's return to that project.

x.2.1 *Gravity becomes buoyancy, PGF is univariate*

Just as the linear sums of static energy make moist thermodynamics clear, linearized approximations for Earth's gentle convection can also open up powerful intuitive access to fluid dynamics. Begin by expressing density and pressure fluctuations as differences from a *reference* profile, $\rho \equiv \rho_0(z) + \rho'$ and $p \equiv p_0(z) + p'$. We already accepted the need for a reference density profile below (2.5b), but here we must choose it. The word *buoyancy*[rr] and its sign will have the most meaning if we choose $\rho_o = \bar{\bar{\rho}}$, with double overbars *the horizontally averaged density across the whole domain* over which we are building an accounting system for the flow: the whole Earth, or a cloud model with cyclic domain, or some conceptual "airmass" (defined by a balloon sounding) within which we are considering convection.

Since pressure's job is to enforce mass continuity in a fluid (like an atmosphere or ocean) where the basic density structure is not very time-dependent, a corresponding $\bar{\bar{p}}$ is the *hydrostatic* reference p_0 profile needed to hold up the mean density profile in the face of the mean

[qq] A fascinating case is Williamson et al. (2015, doi:10.1002/2015MS000448) exposing a longstanding hidden error in the energy formulation of a popular atmosphere model. The fix incurred a sizable T bias, soon compensated away in tunings and adjustments.

[rr] An ancient concept: see https://en.wikipedia.org/wiki/Archimedes_principle

gravity force $\bar{\bar{\rho}}g$. Level by level, $\bar{\bar{p}}_z = -\bar{\bar{\rho}}g,$ making $\bar{\bar{p}}(z)$ nearly linear for nearly-incompressible liquids in the *Boussinesq*[ss] *system*, and roughly exponentially decreasing for air. The necessary constant of integration for defining p springs from the *non-exploding planet approximation*: the entire fluid is not vertically accelerating. Using this hydrostatic pressure $\bar{\bar{p}}(z)$ as a vertical axis in graphs is a way to relate atmospheric (gaseous, exponential in z) data to the wonderfully clear Boussinesq equations in xyz coordinates, used preferentially in this book.

Adding the hydrostatic relation 0 = $\bar{\bar{p}}_z/\bar{\bar{\rho}} + g$ to (1.6d) yields:

$$dw/dt \; = -p'_z/\bar{\bar{\rho}} \; - g\rho'/\bar{\bar{\rho}} = \; -\pi_z + \hat{b} \qquad (2.7)$$

where $\hat{b} = g(\rho'/\bar{\bar{\rho}})$ is the *buoyancy force per unit mass,* and we have simply redefined the symbol $\pi = p'/\bar{\bar{\rho}}$, retaining the important spatially varying part of *p* but setting aside the mean hydrostatic $\bar{\bar{p}}(z)$.

Expressing the pressure gradient force (PGF) as the gradient of a single scalar π makes it vanish from vorticity equations (Problem 2.4.2). Likewise, horizontal derivatives of single scalar $\hat{b}$ retain the meteorologically important part of the untidy, distracting *baroclinic*[tt] term $\vec{\boldsymbol{\nabla}}\rho \times \vec{\boldsymbol{\nabla}}p$ in unapproximated fluid dynamics.

Buoyancy $\hat{b}$ can further be approximated as *thermal buoyancy* $b = gT_v{}'/\bar{\bar{T}}_v \cong gT'/\bar{\bar{T}}$. The use of virtual temperature T_v here helps emphasize that density is the true reference state and the essential variable whose perturbation matters, but for notational simplicity the *v* subscript is often left off, and for many purposes the virtual effect can be neglected. Thermal buoyancy neglects some untidy product-of-primes terms, and also the slight effect of p' on ρ', as revealed by a simple ideal gas law substitution into $b = g\rho'/\bar{\bar{\rho}}$. Fortunately, there are no

[ss] Named for https://en.wikipedia.org/wiki/Joseph_Valentin_Boussinesq

[tt] The spinup of synoptic cyclones in meteorology is unhelpfully called "baroclinic", even though it is mostly vortex stretching: circulation increases even on isobaric surfaces.

meteorological phenomena in our scope (Earth's very-subsonic convection) for which thermal buoyancy isn't good enough.

With (2.7) thus simplified by replacing $\hat{b}$ with b, we can finally bring a dT/dt equation extracted from the First Law (2.5b) or (2.6) into the b equation, to close a 5-equation set capable of expressing thermal convection. Applying the *total derivative d/dt* to the definition of *b,* with virtual effects hidden to clarity the z subscript for differentiation, gives

$$\begin{aligned} db/dt &= (g/\bar{\bar{T}}\,)\; d/dt[T - \bar{\bar{T}}(z)] \\ &= (g/\bar{\bar{T}})\;[dT/dt - w\bar{\bar{T}}_z] \qquad (2.8) \end{aligned}$$

Substituting the First Law (2.5b) into the first term, and again equating $Z_h(p)=Z=z$ (the hydrostatic approximation, plus a sensible choice of a zero origin for both geopotential height Z and the z coordinate)[uu], gives our desired prognostic (grasping) equation for b:

$$\begin{aligned} db/dt &= (g/\bar{\bar{T}})\;[Q/C_p\; + w(\Gamma_d - \bar{\bar{T}}_z)] \\ &= g/(C_p\bar{\bar{T}})\;[Q\; - w\bar{\bar{s}}_z] \\ &= Q_b - wN^2 \qquad (2.9) \end{aligned}$$

The first term reflects the *production or source of buoyancy by thermal energy addition rate Q*. The second term combines the vertical advection of T plus adiabatic warming/cooling by compression when air changes pressure. In the second line, that combination is expressed as the effect of the area-averaged *stratification* or *static stability* $\bar{\bar{s}}_z$. Since that quantity is positive definite for a stably stratified fluid, it is expressive to package it as a squared frequency of free vertical parcel oscillations, $N^2 = g\bar{\bar{s}}_z {C_p}^{-1}\bar{\bar{T}}^{-1} = g\bar{\bar{\theta}}_z(T/\theta)\bar{\bar{\theta}}^{-1}$ (units s^{-2}; the second equality is Problem 2.4.4). Virtual effects can be re-inserted for precision by simply inserting T_v and s_v for T and s or θ in all the forms above, so we are glad not to have carried them in the algebra.

uu Deriving (2.8) and (2.9) is a useful student exercise, with a few illuminating steps.

Gathering the above, we have achieved equation set closure, except for the heating rate *Q*. By substituting $\rho = \bar{\bar{\rho}}(z\ only) + \rho'$ and linearizing to retain only the *thermal* buoyancy b and neglect ρ' otherwise, and using good-enough static energy (Trick 2, Eq. 2.5) for the First Law, our equation set (1.6) has become five equations in 5 unknowns (*u*,*v*,*w*,*π*,*b*):

$$
\begin{aligned}
0 &= -\nabla \cdot (\bar{\bar{\rho}}\boldsymbol{V}) && \text{(2.10a)}\\
du/dt &= -\pi_x + fv && \text{(2.10b)}\\
dv/dt &= -\pi_y - fu && \text{(2.10c)}\\
dw/dt &= -\pi_z + b && \text{(2.10d)}\\
db/dt &= -wN^2 + Q_b && \text{(2.10e)}
\end{aligned}
$$

This set is called *anelastic* when $\bar{\bar{\rho}}$ is a function of z only, and *Boussinesq* when $\bar{\bar{\rho}}$ is a constant and can be erased from (2.10a) entirely.

The power of a closed set of PDEs has to be elicited to be fully appreciated: These equations for local relationships "*govern*" (or can describe) all possible flows or "*solutions"*, so they are not very discriminating. Let's march through just one simplest solution (for *internal or buoyancy waves*, sometimes called *gravity waves*). Other derived properties of this equation set are in Problems below.

x.2.2 *Ubiquitous simplest motions: buoyancy waves*

How many complications can we remove from the set (2.10), making it tractable to "solve" for non-trivial flows that vary in time? This question defined the art of fluid dynamics before computers. Waves are our cleanest time-dependent flow paradigm, with well-developed applied math strategies like complex number methods. The stripped-down simplicity of such a problem doesn't make the results irrelevant: quite the contrary! It means that such waves are *ubiquitous, almost no matter what happens with all the various complications we ignore and neglect and approximate away*.

(1) Let us neglect advection by replacing d/dt with $\partial/\partial t$. Physically, this assumes the fluid is at rest other than the wave motions, which remain

small enough not to advect their own scalars importantly (another *linearization)*. For simplicity, (2) neglect north-south flow and gradients, working only in the x-z plane. (3) make the Boussinesq approximation ($\bar{\bar{\rho}}$ constant). That is more valid for liquids, but Boussinesq results can be made relevant to the atmosphere by simply plotting atmospheric data on graphs with hydrostatic pressure as the vertical coordinate[vv]. For clarity (4) neglect the Coriolis force, which can be added in later fairly easily. Finally (5) take N^2 to be a constant; it is about 2π/(10 minutes) in the tropical troposphere. With these assumptions, we have the simplest possible set:

$$0 = u_x + w_z \qquad (2.11a)$$
$$u_t = -\pi_x \qquad (2.11b)$$
$$w_t = -\pi_z + b \qquad (2.11c)$$
$$b_t = -N^2 w \qquad (2.11d)$$

To analyze these equations, we combine them into a single higher-order equation for w. Pressure π can be eliminated with a vorticity approach (see problem 2.4.2) by differentiating and subtracting u and w as *(2.11b)*$_z$-*(2.11c)*$_x$ to prognose $\eta = (u_z - w_x)$:

$$\eta_t = -b_x \qquad (2.12)$$

That leaves b_x as the key right hand side term: *buoyancy gradients are a torque in the vertical plane*, valid even if the reference state for b is offset from local conditions[ww]. Equation (2.12) also shows that *b* and *w* are 90^o out of phase in these propagating waves, so that the *vertical flux of b is zero in internal buoyancy waves*. In other words, they are specifically *not* a form of convection: on a b-w scatterplot, waves trace a circle, contributing nothing to the 'buoyancy flux'[xx] (covariance *[b'w']*) which Problem 2.4.2 shows to be the source of all kinetic energy.

[vv] One phenomenon the Boussinesq equations cannot represent is the breaking of upward-propagating bouyancy waves as density lapses, important in the middle atmosphere.

[ww] *Effective buoyancy* (Jevanjee and Romps 2015) brings this virtue to the w equation.

[xx] a misleading use of the word "flux": b is not a conserved quantity being transported.

Differentiating (2.12) again in time to eliminate b with (2.11d) leaves only u and w in the equation. More differentiating allows substitution of (2.11a) to eliminate u, giving a single equation for w:

$$w_{xxtt} + w_{zztt} = -N^2 w_{xx} \qquad (2.13)$$

This is clearly a wave equation, because its equality requires that double derivatives act like a negative sign (N^2 being positive), characteristic of sine and cosine functions or their elegant combination in the complex exponential wave form $w = We^{i(kx+mz+\omega t)}$. Here k is horizontal *wavenumber* (2 x 3.14159/wavelength), m is vertical wavenumber, and ω is the wave's frequency (2 x 3.14159/period). The wave form satisfies (2.13) *if and only if* $\{k, m, \omega\}$ are related in a special way called the *dispersion relation*:

$$\omega = N\,[k^2/(k^2 + m^2)]^{1/2} \qquad (2.14)$$

How can this wave theory result be reasoned with?

Waves are everywhere in the atmosphere and ocean, often satisfying the dispersion relation as *free solutions*, far from their forcing and damping processes. The true character of observed wave fields must then be explained by the driving and decay mechanisms neglected in (2.11) -- thermal forcing Q_b, or boundary conditions like flow over mountains, or mechanical forces) -- as well as by reflection and filtering and screening effects, such as non-constant N_2 profile features and wind shear also neglected in (2.11). Expressing these phenomena tractably with equation sets like (2.10) is a mathematical-physics field unto itself (*e.g.* Sutherland 2010), but a few more words here can help launch your efforts in Problem 2.4.5 and discussions about convection-environment interactions in Parts II and III.

Buoyancy waves exist only between the high frequency limit N (vertical parcel oscillations, problem 2.4.4) and the low frequency limit of purely horizontal motions. The latter limit is set by the pendulum-day period of horizontal *inertial oscillations* if the Coriolis force is reintroduced to (2.11). Within this frequency range, anything goes -- so what do we expect to observe? That depends not on what is possible in the majesty of the governing equations, but on *what waves have drivers in our world*[yy].

The irreversible net buoyancy source implied by deep convection's latent heat bombs create motions near the low-frequency limit. In that case, vertical lengths are set by the depth of the heating, interactively as wave dynamics shape environmental *b* and thereby the local or effective buoyancy force felt by convective updrafts (chapter 6). That leaves horizontal wavenumbers to be determined by the dispersion relation (see Mapes 1993, 1997 for examples and implications). Temporal fluctuations of convective heating also drive oscillatory waves that propagate up out of the troposphere (e.g. Fritts and Alexander 2003). Beyond these convection-related examples are the wind-over-mountain oscillatory forcings at the heart of classic gravity wave work (Web-search imagery on *lenticularis* or *lee waves*). Mixed cases include convective clouds acting like moving mountains with possibly sheared wind blowing across the tops that can sometimes add an asymmetric aspect to cobweb-like cap or *pileus* clouds sometimes seen around cumulonimbus clouds, or the gorgeous newly named wavy yet somewhat turbulent cloud type *asperatus* (web-search for time lapse videos).

Buoyancy waves are everywhere, even where there aren't cloud droplets to mark them visibly, as revealed by stratigraphy in LIDAR observations (chapter 8, Parsons et al. 2019). Again, the simplicity of the derivation above means that internal buoyancy are specifically NOT an extraordinary or exotic phenomenological claim requiring extraordinary evidence, a historical blind spot (literally) in meteorology, where such clear-air (invisible) and transient motions have often been underappreciated, ignored, or even consciously disbelieved.

[yy] filtered by what we are *able* to observe (section 0.6, Fritts and Alexander 2003).

x.3 Good-enough moisture and microphysics (barely)

The First Law was the core of the prognostic b equation (2.10e) that closed the equation set, this chapter's main goal. To treat latent heating effects or virtual effects in Q_b, we need to carry a sixth equation for q_v. Condensation of that vapor can be folded into a *moist adiabatic* view of thermodynamics, as in section 2.1.4, by specifying the saturation condition for water (when *relative humidity RH* exceeds 100%). This "moist-adiabatic" approach is clearer than leaving condensation in with truly 'diabatic' parts of Q_b, because phase change largely cancels $-wN_2$ in (2.10e) in airmasses (like much of the tropics) whose lapse rate $T(z)$ and thus N_2 is maintained near a state of neutrality[zz] by cloudy convection.

The *saturation vapor pressure* $e_s(T)$ for water is called the *Clausius-Clapeyron relation*, a name easily searched. Its theoretical form, involving an exponentiated negated squared inverse of T, offers little intuitive insight, while real applications use calibrated formulas fit to laboratory steam tables of not-quite-ideal H_2O gas. The important concept for reasoning is merely that $e_s(T)$ is *monotonically increasing, and concave upward* (increasing at an increasing rate)[aaa]. This smooth curve, and the *ln*() in entropy that makes tephigrams appear a bit woozy, are the gentle nonlinearities that forced practical meteorological thermodynamics to be taught graphically in the pre-computer era[bbb].

Thermal buoyancy in convective clouds hinges on $\Delta T = T_{parcel} - T_{environment}$. Moist adiabatic ascent conserves h, so (2.6) = 0. For saturated air, $q_v = q_s(T,p) = 0.622\ e_s(T)/p$. Then (2.6) becomes a transitive equation determining $T(p)$, but not as an analytic formula with any helpful clarity. Nevertheless, the *sign* of ΔT and thus of b can be assessed by comparing h of a saturated updraft to h_s of the environment.

[zz] This is a *teleological* understanding of convection at the largest scales.

[aaa] Anyone pondering climate should also know its slope at Earthly T: about 7% per K.

[bbb] On oversized diagrams, to compensate for the graphic-design weakness of a poor correspondence between distances on the paper and the importance of differences.

Specifically, if at a given pressure $h_{parcel} = C_p T_{parcel} + gZ_h(p) + Lq_s(T_{parcel}, p)$ exceeds $h_s = C_p T_{env} + gZ_h(p) + Lq_s(T_{env}, p)$, it follows immediately that $T_{parcel} > T_{env}$, because $e_s(T)$ is an increasing function. A lifted-parcel buoyancy assessment diagram based on h and h_s is covered in Chapter 4.

Beyond its sign, the actual *value of* ΔT and thus of thermal buoyancy is complicated, not *linearly proportional* to $h(T_{parcel}, p, RH = 100\%) - h_s(T_{env}, p)$ because the upward-curved shape of $e_s(T)$ makes the proportionality depend on T, and $q_s(T,p)$ depends a little bit on p too. The Clausius-Clapeyron equation for $e_s(T)$, easily studied in Wikipedia, allows a mathematically closed approximate equation set for moist convection to be formulated (Vallis et al. 2019), but reproducing it here would not especially illuminate our way forward. Since computing a quantitative value of b for a convective cloud updraft involves understanding mixing and accounting for condensed water, writing down such equations here adds little to graphical methods (chapter 4.1).

The time has come for a terribly brief mention of the microphysics of condensed water particles, and even less about the nuclei (*aerosol* particles) they form on. Whole textbooks and careers address the subject, which is very important for electromagnetic observations at both optical and radar wavelengths (section 0.6), including aspects of visual beauty, and has some impacts on the details of how cloudy convective processes play out. The albedo of the Earth (discussed around Figure 0.2) depends sufficiently much on microphysics that our delicate planetary heat imbalance hinges on the details (as well as on myriad other processes)[ccc]. Because humans control significant sources of aerosol, that area has drawn an almost unreviewably large amount of minutely detailed study.

For present purposes (an appreciation of convection), a short list of principles may get us perhaps surprisingly far in terms of reasoning power, without delving into all that.

[ccc] a condition called overdetermination, https://en.wikipedia.org/wiki/Overdetermination

- Condensation of liquid droplets when humid air is lifted past its saturation level occurs very efficiently, far below RH = 101% and within fractions of a second. Nucleation determines only how many, and thus how large, are the resulting droplets.
- Evaporation of liquid cloud droplets (droplets small enough to have negligible fall velocity) is comparably efficient, again keeping RH very close to 100% upon air descent, with time delays very short compared to the life cycles of convection's entities (subject of Part II).
- A bulk process called *autoconversion* expresses the conversion of small cloud droplets to larger precipitation drops, defined as drops with a fall velocity through air that is important for subsequent evolution (>100 μm, roughly). This time scale is minutes, significantly long compared to meteorological convective air motion processes. It takes longer in cloud condensation nuclei (CCN) rich (polluted) airmasses, but it is difficult to be more specific because...
- Falling drops *collect* cloud droplets in a rapid positive feedback that transforms the condensed water field dramatically, leading to runaway precipitation development.
- Evaporation of precipitation-sized drops is slow enough that RH is often far below 100% in descending rain shafts, whose air descent rate is itself driven by the evaporative cooling of those drops, which requires RH substantially below 100%. Such *unsaturated downdrafts* are important to boundary-layer air properties under convection, and to convection-convection interactions (Part III), but it is difficult to be more specific because the details depend on rain shaft geometry, the distribution of drop sizes, and ambient T and RH.
- Freezing of liquid water releases another kind of *latent heating*, about 1/6 the strength of condensation of the same mass of water.
- Latent heat of freezing is significant to thermal buoyancy *b*, and can be delayed for a meteorologically significant time (minutes or even hours) by lack of ice nuclei. Only at -40C does water reliably freeze without nuclei. But it is difficult to be more specific, because the availability of ice-nucleating particles is very situational, and ...
- Ice particles nucleate additional ice formation very efficiently, in a runaway feedback (like precipitation development above) called *ice*

multiplication, by various mechanisms. It is difficult to be more specific because these are very situational.

- Air that is saturated with respect to liquid (RH = 100%) is supersaturated with respect to ice.
  - For this reason, a rare ice particle can rob mass from more numerous neighboring supercooled cloud droplets by vapor diffusion. Initial formation of a few ice particles is thus a shortcut to having fewer and larger condensate particles, initiating runaway precipitation processes (collection, *riming*) and changing cloud optics importantly.
- Ice particles grow to precipitation size by vapor deposition (becoming snowflakes) and by particle collisions (becoming aggregate snowflakes or liquid-rimed particles like graupel or hail). It is difficult to be more specific because ice particle shape (*habit*) is involved, along with many other situational factors.
- Precipitation-sized particles of ice, like liquid, evaporate so slowly that unsaturated downdraft dynamics are complex and situational.
- Melting of precipitating begins just above 0 C, without the nucleation complications of freezing, but for large particles it can take a meteorologically significant time or fall distance (as surface hail in summer makes obvious, for instance).

This concludes the chapter. We walk away an adequate equation set:

$$0 = -\boldsymbol{\nabla} \cdot (\bar{\bar{\rho}}\boldsymbol{V}) \tag{2.15a}$$
$$du/dt = -\pi_x + fv \tag{2.15b}$$
$$dv/dt = -\pi_y - fu \tag{2.15c}$$
$$dw/dt = -\pi_z + b \tag{2.15d}$$
$$db/dt = -wN^2 + Q_{b\_latent}(\mathrm{q}_v) + Q_{b\_diabatic} \tag{2.15e}$$
$$d\mathrm{q}_v/dt = e - c \tag{2.15f}$$

The representation of water phase changes as $Q_{b\_latent}(q_v)$ and condensation minus evaporation (c-e) is still merely symbolic, but a sufficient basis was presented in moist static energy reasoning to understand the key graphical rising-parcel b illustrations in chapter 4.1. Microphysics principles are too situational to bother crystallizing into equations here, but are also hopefully "good enough" in your mind to proceed with sensible reasoning.

x.4 Properties of an equation *set*: problems and solutions

For this problem, the Boussinesq set for a stratified rotating fluid (2.10) may be written with a horizontal "frictional" mechanical force **F** = $\boldsymbol{i}F_x$ + $\mathbf{j}F_y$ for completeness, as:

$$\begin{aligned} 0 &= u_x + v_y + w_z \\ u_t &= -uu_x - vu_y - wu_z - \pi_x + fv + F_x \\ v_t &= -uv_x - vv_y - wv_z - \pi_y - fu + F_y \\ w_t &= -uw_x - vw_y - ww_z - \pi_z + b \\ b_t &= -ub_x - vb_y - wb_z - N^2 w + Q_b \end{aligned} \qquad (2.16)$$

As seen in Problem-Solution 1.4.3-1.5.3, continuity (the Law) can be expressed as giving us the pressure (the Enforcer), with $\beta = f_y$

$$\pi = \boldsymbol{\nabla}^{-2}[-\boldsymbol{\nabla}\cdot[(\boldsymbol{V}\cdot\boldsymbol{\nabla})\boldsymbol{V}] + (f\zeta - u\beta) + div(\boldsymbol{F}) + b_z]$$

x.4.1 *Energetics of a horizontally unbounded atmosphere*

a. Multiply the 4 prognostic equations above by *u,v,w,b* respectively to generate prognostic equations for $KE = (uu+vv+ww)/2$ and $PE = bb/(2N_2)$.

Interpret these: where and how is KE generated? How is it transported away from there to other regions of the fluid?

b. Integrate the energy equations over a whole atmosphere, after transforming from advective to flux so you can express the fact that there are no lateral boundary fluxes (valid for a *cyclic* or *unbounded* air body like a spherical planetary atmosphere, even though we are using Cartesian coordinates for convenience). Also use the fact that w=0 at the top and bottom. Show your work and explain the logic. With brackets [] denoting the integral, show that

$$[KE]_t = [wb] - \boldsymbol{F}.\boldsymbol{V}$$
$$[PE]_t = [Jb] - [wb]$$
$$[PE + KE]_t = [Jb] - \boldsymbol{F}.\boldsymbol{V}$$

where $J = Q_b/N_2$

Interpret the terms: what drives fluid motion, what damps it? You may need to postulate a simple assumption about **F** (whose symbol should evoke *friction*). What kind of a steady state achieved in the motions of this atmosphere, deep into its history and over long time scales when the time rate of change of global averages is small?

How do fluid shear instabilities within the fluid fit into this framework?

x.4.2 *Vorticity equation for nondivergent motions*

Since the curl of a gradient vanishes in 3D vector calculus, expressing the PGF as the gradient of a single scalar like above allows it to be eliminated cleanly to form a vorticity equation. Derive at whatever mathematical detail you are assigned, but with the line of reasoning made clear, one of the *horizontal* components of vorticity like in (2.12). The subscript notation for derivatives is a real hand saver for the proliferating advection terms (since the derivative of three products creates 6 terms).

Emphasize especially: What is the role of the *horizontal gradient of buoyancy*?

x.4.3 *Stokes' theorem and vorticity patch reasoning*

In the *xy* plane, write Stokes' theorem, also known as the Circulation Theorem (pure math; look it up), for *every* circle of radius *R* centered on an isolated small circular patch of vertical vorticity $\zeta = \zeta_0$ of radius *r*, with $\zeta=0$ elsewhere. In this way, show that the tangential component of flow associated with ("induced by") every isolated element of vorticity decays with distance proportional to R^{-1}. With this rule, the flow from arbitrarily complicated arrangements of vorticity can be constructed by superposition of many localized vortices. (Try a web search on "contour dynamics" to see the power of this result, and share what you find.)

Interpret the result: Does it violate causality somehow that changing vorticity at one point or *r*-sized local patch instantly "induces" changes in the flow out to any arbitrary larger distance *R*? Think carefully about what was specified in this problem.

x.4.4 *Stratification*

a. Confirm the derivations of equations (2.8) and (2.9).

b. Show that $N^2 = g\bar{\bar{s}}_z {C_p}^{-1}\bar{\bar{T}}^{-1} = g\bar{\bar{\theta}}_z(T/\theta)\bar{\bar{\theta}}^{-1}$ as stated below (2.9).

c. Show using the w_t equation with b as the only force that N is the frequency of sinusoidal *buoyancy oscillation* of a parcel displaced vertically in a stratified fluid. You may be helped by examining Wikipedia or another source on the *Brunt–Väisälä_frequency*.

x.4.5 *Buoyancy waves*

a. Derive equations (2.11) - (2.13), following the text's meaning. Show that the simplified longwave limit ($k << m$) is the same answer you get from making the hydrostatic assumption at the outset by setting the left hand side of (2.11c) to zero.

b. Discuss applications (situations) in which the forcing mechanism of waves determines every combination of TWO of the set $\{k,m,\omega\}$, leaving the third to be determined by the dispersion relation as a condition for the existence of buoyancy-wave motions. Find and share online videos that illustrate the forcing and dispersion of these types of internal waves.

x.0 Chapter 3

Accounting scales of motion (Back to Outline)

Fluids are fascinating because they contain a lot of different motions at the same time, evocative of the potentialities of mind. Motions of different *sizes* are especially intriguing, like vigorous little scrambles (a dragon! no, a peacock!) caught up in sky-sized swirls, all drifting by on continental or planet-sized currents. These raise profound and important questions about predictability and knowability. To address them rationally, we need bookkeeping tools to decompose motion into scale components. Chapter 1 reviewed how to account for stuff (including momentum and kinetic energy) moving in space. Now we wish to keep track of motions in the abstract domain of *size*, whose vastness requires us to use logarithmic *scale*. The tools are crude, which will entice us into entity characterization in Part II, but valuable ideas exist along the way.

x.1 One size cut: molecular vs. macroscopic

Borders between regions of space are crossed by individual careening molecules, and also by coherent masses of them moving together. These cases should be distinguished, because the continuum limit of calculus only applies safely to the latter. In addition to contributing discrete little fluxes of mass, molecules carry specific stuff like momentum and energy from conditions at their place of 'origin' (last contact with other matter) to the place of their next collision, where that stuff is rapidly shared out in a new *neighborhood*, a term that properly implies the spatial averaging inherent in continuum reasoning.

Logic (not detailed here; think) tells us that the net flux of stuff by numerous random bidirectional exchanges of molecules act as a *flux* *down the gradient, proportional to the strength of the gradient.* Such a flux is called *diffusive*. Treating the proportionality coefficient as constant, the time-tendency called *diffusion* (the *convergence of diffusive flux*) is thus positively proportional to the *Laplacian operator*, $\boldsymbol{\nabla}^2 = \boldsymbol{\nabla} \cdot \boldsymbol{\nabla}$, which we also met in the pressure equation (problem 1.4.4 and solution 1.5). This diffusive molecular flux (called *viscosity* in the case of momentum) is separated from the flux by fluid motions in the *Navier-Stokes equations* of fluid dynamics. Specifically, $\nu\boldsymbol{\nabla}^2 u$, $\nu\boldsymbol{\nabla}^2 v$, $\nu\boldsymbol{\nabla}^2 w$ get pulled out of the flux terms in equations (1.4 b,c,d), respectively, and added to the RHS, leaving the flux by coherent groups of molecules to be treated as advection in equation sets like (2.10) and (2.15).

x.2 Another cut: large-scale flow vs. small "eddies"

Do *fluid blobs smaller than the flow we care about* (that is, the "eddies" in whatever stream we are pondering) act like molecules? That is, as they cross a border we care about, between regions we distinguish -- both ways, because of mass continuity -- can their effect be represented as a *diffusive flux* of specific stuff? If so, hooray! We simply reuse the argument of section 3.1, enhancing[ddd] molecular viscosity ν with an *eddy diffusivity*, called *eddy viscosity* when momentum is the stuff being transported. In that case, the Laplacian's curvaceousness-feeling derivatives must be redefined to have a broader span, based on *the size of those flow features we care about*. These personal words emphasize that scale separation is a tool for our reasoning, not a property of nature.

Specifically, the words *mean conditions* and *neighborhood* from section 3.1 should be redefined to denote a broader averaging operation or *spatial smoothing filter* (denoted by overbars $\bar{u}, \bar{v}, \bar{w}$) that just preserves, with acceptable blurring, the features we care about. We then write eddy

[ddd] or *replacing*, since eddy flux is orders of magnitude larger for Earthly convection.

quantities, representing all the smaller scales removed by the filter, with a prime[eee]:

$$u' \equiv u - \bar{u} \qquad (3.1)$$

Such a definitional equation for all variables can be substituted into the equations in flux form, and the terms involving products of primes (sub-filter-scale *eddy flux* terms) moved to the right hand side. For instance, for u, (2.15b) can be elaborated in this way as:

$$\bar{u}_t = -\bar{u}\,\overline{u_x} - \bar{v}\,\overline{u_y} - \bar{w}\,\overline{u_z} - \overline{\pi_x} + \overline{fv} \\ + \overline{(u'u')_x} \; + \overline{(v'u')_y} \; + \overline{(w'u')_z} \qquad (3.2)$$

Since the powerful Law of mass continuity applies to both large and small scales, the overbar terms (transport by smooth, large-scale, *filtered* motions) can be converted from flux form back to the *apparent advection* form in (3.2), as in problem 3.10.1[fff].

What has been gained from this exercise? Nothing yet, until the eddy terms on the RHS are treated somehow, but one rigorous scale cut has been made, and the messy eddy flux terms are its notational cost. Only an intrepid few feel inspired to write comparable bookkeping for *higher order* expansions of RHS terms (e.g. André et al. 1976), or make another size cut with a second smoothing filter operation (e.g. Tan et al. 2018).

For those interested in planetary-scale flows, a typical horizontal filter scale is Earth's size divided by about 1000 (for our visual screens) or 100 (fitting our brain's social hardware[ggg]) or 10 (for counting on our fingers). Such a filter means that all of the white parts of several or many

[eee] The prime was also used in chapter 2 where we only *cared about* the whole-fluid horizontal mean denoted by the special averaging operators $\bar{\bar{\rho}}$ and [KE] in problem 2.4.1.

[fff] Averaged prime terms like $\overline{\bar{u}u'}$ vanish only if the averaging is done over *fixed limits of integration*, not for a sliding boxcar or convolution smoother that leaves $\bar{u}$ a continuous function. For this reason, x,y,z differentiation subscripts are kept under the bar.

[ggg] https://en.wikipedia.org/wiki/Dunbars_number.

convective clouds are implied in the definition of "eddy", from which we then want to know only a *net effect:* the flux of stuff across the borders of regions of sizes we care about.

The atmosphere invites a peculiar application of this averaging nomenclature, in which the bar reflects only a *horizontal* averaging filter, retaining full dependence on vertical position. Models built around this mathematics retain far more vertical resolution than most viewers of the fields "care about" examining. But users want skillful horizontal flow evolution over time, which requires that filter-scale motions and physics (including sub-*horizontal*-filter-scale eddy fluxes) be computed accurately in the vertical. This is the strange realm of *convection parameterization* (confronted more in Part III), an engineering challenge that does also stand as an important and genuine test of our scientific understanding of convection (Arakawa 2004).

x.3 On deviations, anomalies, eddies, perturbations, etc.

Primes and overbars like (3.2) are sprinkled throughout the literature of fluid science, with various meanings. Customarily these are used for *time* averages and *deviations* from them, called *anomalies* or sometimes *fluctuations.* Square brackets and asterisks like [v^*u^*] represent *zonal* (east-west around the globe) spatial averages and *zonal eddy* deviations therefrom, in planetary-scale atmospheric general circulation work. In the *Reynolds average* interpretation, bars and primes are viewed as an *ensemble* mean plus individual *member or realization* deviations from that mean. Sometimes deviations are vaguely called *perturbations,* which makes the most sense if some agent changes or perturbs something else, like in an *experiment minus control* setting. In *time series* data analysis, primes and bars are sometimes interpreted as a proxy for spatial deviations or for Reynolds realizations, a view whose validity hinges on caveats like *stationarity* or its mathematical rabbit-hole cousin *ergodicity*. Wikipedia pages and literatures abound for all these terms and concepts, but here we merely emphasize that readers of the diverse literature should allocate enough mental capacity for thinking clearly about these conceptual distinctions, since the terms are sometimes used carelessly or interchangeably.

x.4 Fourier (spectral) decomposition and logarithmic *scale*

To make more than one or two size distinctions, we can leap to another kind of accounting: a *spectrum*[hhh] of size components such as sine and cosine waves (*Fourier harmonics*). Fourier analysis is one particular type of *Galerkin decomposition*, which divides complex structure into a weighted sum over a *basis set* of complete and orthogonal (mutually exclusive) basis functions. The advanced student should explore *wavelet analysis*, which illuminates the fundamental tradeoff or uncertainty principle between accounting for location vs. for scale.

To appreciate Fourier spectra clearly, the familiar words *wavelength* in space and *period* in time must be rethought as two pi (2 x 3.141...) times their inverses, called *wavenumber* in space and *frequency* in time. The abstractness of wavenumber (frequency) is lessened by measuring them in the units *number of cycles over a given distance* (or *cycles over a given time interval*), using our brain's counting faculty for wave crests or troughs. The word *amplitude* refers to the coefficient of each term, and its square (double the *variance* of each wavenumber component) is often called *power* (or *power spectral density*) for reasons that may be more historical than sensible: The units are variance (kinetic energy, in the case of velocity variance) per frequency bin width, not energy per time (true power from physics).

For finite data series (which Fourier analysis necessarily treats as repeating or periodic), the set of possible wavenumbers is discrete. Since these wavenumbers are "bins" over which we distribute conserved stuff like energy (variance)[iii], the amount of stuff falling in each bin depends on the widths (that is, the spacings) of those bins. The inverse dependence of wavenumber on wavelength (or frequency on period)

hhh The spooky revelation of secret colors in white light evoked *spectres* or apparitions.

iii See https://en.wikipedia.org/wiki/Parsevals_theorem makes the bins wider for long waves and narrower for short waves. For instance, when the fluctuations in a mesh of 4km discrete spatial grid boxes around the Earth's equator (10,000 points) are expressed as a power spectrum, all the variance in the wavelength range from 4000-40,000 km falls into just 10 wavenumber bins, while energy in the wavelength range 40-400 km is divided over 1000 bins. Partly for this reason, "power" spectra almost invariably tend to have much more "energy" in long wave bins.

Mathematically, the Fourier Transform and its inverse are linear and reversible, allowing us to jump back and forth between the *physical domain* (space and time) and *spectral domain* (wavenumber and frequency components) without any loss of information, with just a few pencil strokes, or now keystrokes. This is an essential power tool every science student should learn to appreciate and apply, if not to fully understand to its depths[jjj]. Computer exercises in section 3.9 encourage readers to play with Fourier analysis of imagery and sound.

To tame the vastness of the size or wavenumber domain, the *log* function suggests itself. In fact, logarithms are fundamentally what we mean by the word *scale* as opposed to *size*. We speak of *meter-scale eddies* whose size is about 1-10m but not 100m, while *kilometer-scale* clouds are about 1-10km across but not 100km. Scale is measured in *decades* (factors of 10), or sometimes more finely in *octaves* (factors of 2; but named for the mysteriously pleasing brainfeel of tones 1/8 of an octave apart in the ancient art of music on stringed instruments).

Any spatial or temporal data series can be *decomposed into* a sum of Fourier components. But does this mean that nature is truly, deeply, secretly *composed of* those components? Consider a short wave (with large wavenumber K), $\psi = A\ sin(Kt)$, whose amplitude A is *modulated by* a long wave with small wavenmber k: $A = C\ sin(kt)$ so that $\psi =$

[jjj] The world-changing Fast Fourier Transform (FFT) is a masterpiece of mid-20th century matrix factorization, where *NlogN* computations give the answer to a formerly N_2 sized question, *N* being the number of points in a series resolving *N/2* frequencies.

$C\ sin(kt)\ sin(Kt)$. This "modulation" of a small thing by a large thing might be quite physical and real, but Fourier analysis decomposes it into an *interference pattern* or *beating pattern* between two high wavenumbers near K, $\psi = C/2\ [cos(kt - Kt) - cos(kt + Kt)]$, with no long-wavelength energy involved at all. Which is the truer picture: the true scale *interaction* of "modulation", or the mere additive superposition of wave "interference"? AM and FM radio show us that both are useful descriptions, and their majestic equality means that mathematics is silent on which is really happening in the physical processes giving rise to the data. The lesson is that the categories of bookkeeping systems must not be mistaken for the realities they describe *even a complete and correct description may still be non-unique.*

x.5 Eddies, shear, and energy transfer across scale

x.5.1 *Downscale energy transfer: shear instability*

Whatever the interpretation of Fourier spectra, Parseval's theorem does give us an accounting framework for distributing energy over size or scale bins. Consider *shear instability* (the roll-up of sheets or filaments of vorticity into ball-like vortices). This process transfers conserved energy *downscale*, from larger to smaller scales, with no trace in the global kinetic energy budget's ultimate source term [*wb*] (Problem 2.4.1), at least for horizontal roll-ups.

x.5.2 *Upscale energy flux and convection-LS interaction*

Fluid eddies, unlike diffusive molecules, can carry momentum *up its mean gradient (up-shear)*, acting as a *negative viscosity* (Starr 1968). From a kinetic energy perspective, such eddies transfer energy from their own scale into the shear. This situation is common, since shear's effect on an eddy structure is to tilt it in such a way that the $\overline{(v'u')}_y$ or $\overline{(w'u')}_z$ terms in (3.2) then transfer momentum upshear. As long as there is some ongoing energy source for eddies (like buoyant convection), postulating this upscale flux process is not as exotic as it may seem at first blush. That local shear is always part of some larger-scale flow feature, but it may be of a vastly larger scale (like a planetary-scale jet stream), or simply a swirl one octave or decade larger than the eddy. In other words, this up-shear momentum flux need not be local in the scale domain.

x.5.3 *Triads and tunneling*

Detailed bookkeeping of kinetic energy transfers by the advection terms (substituting Fourier forms into the governing equations and rearranging) reveals that energy transfers across scale may occur in *resonant triads* of wavenumber (e.g. Bretherton 1969), based on frequency sum-and-difference formulas like in the beating vs. modulation example examined above and in problem 3.9.4. These "tunnels" for energy flux across the scale domain, transferring energy among well-separated wavenumbers, are one more way in which the *cascade* presumption about of multiscale flow may be full of holes.

x.6 The cascade fallacy in spectral energetics

The classical theory of turbulence (Kolmogorov 1941) shows that *if* a turbulent flow is driven at large scale (for instance, stirred by a huge spoon), and *if* the only dissipation is by diffusion at the molecular scale, and *if* the energy transfer in the intervening "inertial subrange" of scales is local (with only eddies of comparable size exchanging energy), *then* by purely dimensional analysis a logarithmic graph of the kinetic energy power spectrum must have a slope of energy vs. wavenumber of -5/3. This paradigm is called a *cascade*[kkk] because of its local-in-scale energy transfer assumption.

A cascade is only one particular type of waterfall, but such paradigms are sticky in the brain, especially in the absence of specific alternatives. When a gross statistic of observations (the approximate -5/3 slope)

kkk A *cascading* waterfall is one with many short drops from one pool to the next.

agrees with a mechanistically hazy (purely dimensional) prediction arising from a sweeping assumption, what is the weight of that evidence for the assumption?

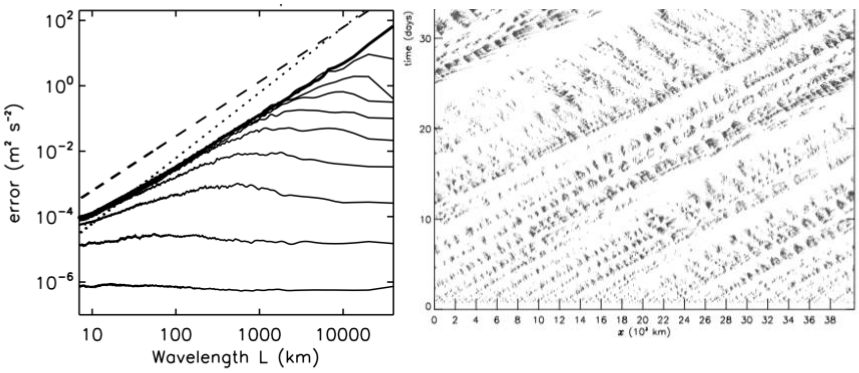


Fig. 3.1. Kinetic energy spectrum of a 2D (x-z plane) cloud model, with -5/3 reference slope (dashed) and -2 (dotted). The heavy line is the time-averaged spectrum, while lighter lines show the upward progression of "errors" (differences between two simulations after seeding the model with tiny uncorrelated random perturbations). Adapted from Fig. 5 of Mapes et al. (2008). Right panel: Surface precipitation time section for the first (spinup) month of the 40,000 km simulations analyzed at left.

Explicit simulations of deep moist convection, within the wavy stratified environment it creates and inhabits, exhibit a nearly -5/3 logarithmic slope to their energy spectrum at equilibrium with steady destabilization forcing. While global 2D wavenumber spectra on the sphere are hard to interpret, simulations with a more affordable 2D cloud model in the equator-like *x*-z plane fall extremely, compellingly close to that -5/3 slope across 5 decades of scale when multiscale structure is fully developed (Mapes et al. 2008; heavy line in Figure 3.1a).

However, the spectrally resolved kinetic energy budget of these simulations shows nothing like a simple cascade from a large forcing scale to a small dissipation scale. Instead, buoyant KE generation [*wb*] is important at every wavenumber. The kinetic energy generated by this gravity work is transferred from vertical to horizontal wind by pressure, and the horizontal wind energy is then lost to drag. The same is true in the spectral energy budget of a 3D Large Eddy Simulation (LES) of a cloud-topped boundary layer (Harm Jonker 2018, *pers. comm.*). The mechanisms of dissipation of kinetic energy differ somewhat between those two models, but the point is that source term *[wb]* it is broadband (active at many wavenumbers).

All scales convect is the overarching lesson of such spectral analyses of the [*bw*] term. This is true both for *initial energy growth* from small random fluctuations in a quiescent but unstable atmosphere, and for *difference-energy or difference variance growth* of initially small perturbations in predictability experiments on fully developed, steadily convecting flow. However, the low-wavenumber or long-wave energy (or squared difference magnitude in the predictability experiment) takes longer to *saturate* than the energy in high-wavenumber motions, because of the sloping ceiling of the -5/3 spectrum. For the initial growth problem, "saturation" means that the winds strengthen until dissipative processes match the buoyant generation. For difference growth, "saturation" means that predictability is lost: the difference energy has become as great as the background equilibrium spectrum of energy.

The word *growth* above meant the increase of energy *magnitude* at a given wavenumber; what Mapes et al. (2008) called *up-amplitude* growth. But there is also *up-scale* growth, meaning that *an increasing fraction of the energy is in the low-wavenumber end of the spectrum*. This up-scale energy progression is shown for difference growth in the upward-moving sequence of difference-energy spectra (thin lines) in Fig. 3.1.

The convenience of the cascade paradigm has led many into an under-appreciated cascade fallacy. Up-scale difference energy growth occurs in Fig. 3.1 not because energy flows through the smaller scales to reach the larger scales, but rather because the small scales simply saturate sooner, and at a lower energy level. Low-wavenumber motions (or differences thereof) can increase in magnitude for longer, and accumulate more energy (or difference energy), simply because their saturation 'ceiling' (heavy line in Fig. 3.1a) is higher. But why was that again? We have no adequate answer except the inadequate cascade paradigm of local-in-space energy transfer, as the foregoing explained in several ways.

For multi-scale convection in its stratified and thus wavy environment, the basic tenets of a cascade are simply inapplicable. Might there be other lines of reasoning, perhaps based on deeper unarticulated principles, behind compellingly clean (if rather abstract) spectral plots like figure 3.1 above, Fig. 4 of Wikle et al. (1999), and others? As lamented by Sun et al. (2017), "the authors have been unable to develop a simple explanation for why a −5/3 slope develops." The straightness in Figure 3.1 is surely no coincidence, but for all the formidable breadth and depth of modern turbulence theory (Alexakis and Biferale 2018, with the word *cascade* elevated to its title), and long-standing wave-spectrum saturation ideas in oceanography that might seem to be relevant (Munk 1980), we still lack a compelling paradigm or model for the multiscale *coupled* wave-convection problem (Bretherton 1969, Kiladis 2009).

x.7 Macro-description, entropy, information, and "systems"

The molecular vs. macroscale separation in section 3.1 expresses ideas from the field of *statistical thermodynamics*. Historically, that discipline matured only after the confirmation of atoms and molecules, greatly illuminating the older, more empirically grounded science of thermodynamics (e.g. Jaynes 1957). If temperature is sensibly translated into units expressing the energy of molecules (rather than arbitrary decimal "degrees" based on water's properties), then entropy is dimensionless, as a measure of pattern or probability properly should be. This dimensionless entropy is best translated as *missing information* (Ben-Naim 2008), in the Shannon (1948) sense of the word *information*, with units of *bits* that most 21st century reader will find intuitive. Specifically, S is a *functional* (an integrated, summarizing scalar) of the distribution of probabilities p_i that a set of molecules is in *microscopic state i*:

$$S = -\sum_i p_i log(p_i) \quad (3.3)$$

An example from Ben-Naim (2008) in exercise 0.8 makes this formula intuitive.

The words *missing information* indicate that entropy is a property of *our description* of nature -- that is, it is a conceptual tool, a tactic of scale separation between the "macro" scales we choose to represent explicitly and the "micro" scales we choose to describe only statistically, discarding information that is redundant or uninteresting, or simply assuming the least we can when information is unavailable. Might similar tools of description be borrowed for a statistical treatment (Part III) of large numbers of small[lll] "entities" in a fluid (Part II), just as eddy viscosity of section 3.2 extended the concept of true molecular viscosity from section 3.1?

Crises of description can spotlight real scientific issues. For instance, Fourier analysis leads to a crisis at small scales. Even in one dimension, there are ten times more Fourier wavenumber bins in the 10s of km scale range than the 100s of km scale range, and so on to smaller scales. Each bin is a *degree of freedom* (*DOF*), a container for information (the drawing of distinctions, measurable in units of *bits*) as well as for energy as discussed above. The limitlessly growing multiplicity of DOFs with smallness of scale presented a philosophical crisis for sensible thinking about macroscopic phenomena in early 20th century physics. This *ultraviolet catastrophe* of infinite energy in the infinity of small scales was an early clue to the existence of irreducible *quanta* as a floor to the bottomless pit of microstructure. Classical philosophy's *atomism* was finally proven out.

Above quantum theory's lowest basement floor of physicalism, there remains a philosophical (or descriptive) crisis in multiscale (micro to macro) reasoning. A strict version of reductionism insists that the most fundamental or real causality must lie at the smallest scale, with all

[lll] Much smaller than convecting flows, albeit much larger than molecules.

larger-scale entities being mere *epiphenomena* which are *supervenient* on the underlying microscale. This view leaves no role for macroscopic identity or agency, from convective cells to biological cells, neurons to behavior. While rather abstruse, supervenience subtly undermines causality discourse in all the macro-sciences, privileging the hardest of hard sciences (particle physics), which turns around and hands off its ultimate causality to quantum indeterminacy. There is no bottom, it is turtles[mmm] all the way down!

Of course, this is a crisis only of description: nature has no problem operating sensibly at macroscales. Fluid flow usefully challenges any and all attempts at defining tidy bounds of macroscale sense-making, with its scale-dependent time horizons of predictability and even knowability. Might our humble little corner of science offer a unique vantage point and testing ground for bulk accountings of microstructure in macroscopic weather? Exploring this possibility may require statistical accounting schemes in the middle ground between rigorous statistical thermodynamics with its practical infinity of profoundly indistinguishable molecules, and strained analogies in fields like biology where individuals have identites rooted in both life history and very long-term heritable structure.

Information theory might offer the necessary toolkit for this project of creating accounts of multiscale phenomena, with philosophy-of-science platforms that could span the supervenience abyss at levels usefully high above the molecular or quantum basement (e.g. Hoel 2018). Neuroscience seems to be at the leading edge of this exciting intellectual project, because its practitioners have all the ingredients: an interest in complex structure (biology), rooted in chemistry's utilitarian understanding of *entropy*, but also a functional interest (the teleological purpose of neurons) in processing of entropy's mathematical close cousin *information* (Ben-Naim 2008).

[mmm] evidently a Hindu image, https://en.wikipedia.org/wiki/Turtles_all_the_way_down

The core concept of statistical thermodynamics is the *system*. An *isolated system* exchanges no matter or energy across its *boundaries*, and the Second Law says that its *state* tends toward *equilibrium*. A *closed* system is a fixed set of matter which exchanges energy with its surroundings. An *open* system exchanges both matter and energy. By these exchanges it may exist in a state that remains far from equilibrium, indefinitely. At first blush, the *nonequilibrium thermodynamics of open systems* may seem almost like a post-modern parody of science. Since the only thing that really defines such an open "system" is provisionally labeling it as one, this sounds like another supervenience crisis: the study of arbitrary human-assigned labels, not of anything fundamentally real in nature. Until we realize that our very bodies and brains are precisely such systems! Might we usefully draw on the theory of such open systems as a bulk description basis for convective "systems" in the atmosphere?

The power of systems descriptions often lies in *integral principles* those systems putatively obey. For instance, the *principle of least time* or of *least action* in physics allow the computation of wave refraction and other complex behaviors, sidestepping the need for detailed local mechanistic descriptions. In thermodynamics, the Second Law's mere inequality that entropy does not decrease with time ($dS/dt \geq 0$) can be given real teeth if it is strengthened it to a principle of *maximum entropy production rate*. Might such a principle, applied to a descriptive treatment of small-scale motions about which our "missing information" is very great, gain us some useful traction on the problem of convection?

The principle of maximum entropy production rate has been invoked as an approach to atmospheric dynamics, as a direct *extension* of statistical thermodynamics to convective cells (Asai and Kasahara 1971) or the planetary scale mean atmospheric circulation (Paltridge 1978). The word "extension" emphasizes that these theories maximize the production rate of *thermodynamic entropy* -- the missing information in the zillions of redundant *molecules* in the atmosphere. Despite the fascination of applying a principle across such vast scale separations (see review in Liu 2011, inspirational mention in Palmer 2019), this elegant approach has arguably not yet unlocked the compelling predictive or even explanatory power one might hope for.

Perhaps an *analogy* to statistical thermodynamics, rather than such a direct *extension*, could be more fruitful. Such an analogy might invoke entropic assumptions (such as maximum missing information; e.g. Jaynes 1957) about statistically redundant but not identical puffs of air, without adding to it the vast underlying abyss of molecular entropy. Borrowing an analogy rather than extending thermophysics theory would embody the view that such principles are really tools of our description, not expressions of fundamental physics obeyed by convecting flow (and every other system) supervening upon an ultimate "basement" causality of molecular interactions.

Biological systems reasoning is sometimes of this analogy type. For instance, *ecosystem succession* is a familiar and powerful concept in that field, and might seem to be a relevant analogue for atmospheric convection developing and organizing (or 'evolving') over land on a summer day. Both can be viewed as problems in which an available energy resource (an energy *flow*, ultimately sunshine) can be processed (assisted, but also lived off of) by open "systems" (entities) which compete in a trade space of energy inefficiency vs. structural complexity.

In this trade space, "organized" systems may be optimal for exploiting their specific niche, but are complicated (unlikely to form spontaneously) and perhaps highly contingent on other systems (trophic levels). These compete for existence against simpler, general-purpose structures that can spontaneously or more readily develop (i.e., more "cheaply" in terms of some required resource), but may be less efficient at the "job" of consuming or transmuting energy. It is doubtful that a whole ecosystem has a teleological "job"[nnn] that can be expressed as an optimum principle around some simple scalar like energy: the diversity of ecosystems shows that at the very least such an optimum must be wildly non-unique. But the "job" of convection in lowering the center of gravity of a bottom-heated body of fluid seems clearer, and more likely to be relatable to the key local force (buoyancy) that drives the process.

These ideas will be revisited in Part III, in an attempt to apply them to the larger-than-molecular convective flow entities of Part II. While no great breakthrough in cumulus parameterization emerges straightforwardly from the pure ideas and definitions of information theory, these ideas comprise a distinct facet of this chapter's topic of "accounting scales of motion", and point to incompletely explored and possibly promising avenues for progress or greater appreciation.

x.8 Problems and solutions

x.8.1 *Scale separation (large scale vs. eddy)*

Derive (3.2) from (3.1), noting the footnote there.

x.8.2 *Spectra of spatial data (your photograph)*

View the Jupyter notebook Spectra.ipynb from this Github page: (*URL*). Using Jupyter-Python (easily installed as explained at *Unidata's python page URL*), operate the notebook to replicate its figures. Replace the photograph with your own, and adjust code there to explore how Fourier analysis in 1 or 2 horizontal dimension works to decompose an energy-like quantity (*variance of brightness* in an image domain).

x.8.3 *Modulation vs. beating*

a. In a Jupyter notebook or other coding and exposition environment, illustrate the modulation vs. beating (interference) interpretations at the end of section 3.4.

b. Read about and explain in your own words the difference between AM and FM radio.

x.8.4 *Multiscale solutions to fluid equations*

Show and explain the truth of this statement from Palmer (2019):

"Although the Navier–Stokes equations cannot be solved directly, they have certain symmetry properties ... One of these is a scaling symmetry: if u(x,t) is the velocity field and p(x,t) is the pressure field associated

$$u_\tau(x,t) = \tau^{-\frac{1}{2}} u\left(\frac{x}{\tau^{1/2}}, \frac{t}{\tau}\right) \qquad (1)$$

and

$$p_\tau(x,t) = \tau^{-1} p\left(\frac{x}{\tau^{1/2}}, \frac{t}{\tau}\right) \qquad (2)$$

with a solution to the Navier–Stokes equations, then so are:
For another approach, see section 2.2 of Lovejoy and Schertzer (2014).

x.8.5 *Missing information, measured in bits*

Shannon (1948) information, the negative of the dimensionless entropy, has units of *bits*. This unit should feel familiar or even visceral in the digital age, where most of us pay money for data plans. To illustrate it, consider the Missing Information denoted by H in Ben-Naim (2008), a book which also offers many additional exercises and deeper discussion. H is a *functional*, a sum or integral over an underlying distribution, in this case a probability distribution *P(situation)*. Consider this example from Ben-Naim (2013) section 3.1:

Situation: One stone is in one of 8 identical boxes arranged in a line.

Question: How many bits of information are missing from this incomplete description of the situation?

a. Compute the missing information H from the formula
$H = -\sum_i p_i \log_2 p_i$ for the 8 equal probabilities. The unit is bits, because the log is base 2.

b. Explain an optimum strategy to learn which box contains the stone from someone who knows, by asking the smallest possible number of yes-no questions (so that each answer is 1 bit of information). Can you do it in any less than the number of bits from a? Does the inefficiency of other possible but nonoptimal strategies change what your intuition would properly call the "missing information" (MI) in the description of the *situation*? Should it? Is the term MI well chosen?

Part II: Entities and elements of convection

x.0 Chapter 4

The buoyancy of lifted air parcels

(**Back to Outline**)

How does buoyant air ascend through ambient air while satisfying the prime constraint of fluids, mass continuity? Messily, say time-lapse videos of cumulus clouds[ooo]. We need massive complexity-reduction compromises like the concept of discrete *entities* to bring convection into our five-fingered grasp. Buoyant clouds are opaque like animals, even though they are actually less substantive than their invisible environments. We must get what inspiration and information what we can from the observability of their 'skin', while recognizing the illusion of cloud identity and heft as a cognitive hazard -- but also a possibly useful tool, special cognitive hardware we can bring to bear. Sometimes they really are distinct and long-lived and impactful, even deserving of names in the case of hurricanes.

Our only rigorous basis so far for explaining an updraft is Newton's law for vertical acceleration (2.10d, 2.14d), whose b term is the ultimate driver of all fluid motion (problem 2.4.1).

x.1 Graphical analysis for moist thermo and probability

If a kilogram of air lifted to lower pressure can achieve positive buoyancy relative to its environment, convection is possible. There are two challenges to saying any more with the field equations of Part I. First

[ooo] for instance, http://www.atmo.arizona.edu/?section=weather&id=cloudBest

is the unfinished business of predicting buoyancy with moisture folded in as a moist adiabatic process, leaving only the truly diabatic effects in (2.14e) as the necessary externalities of forcing and complications. Section 4.3 said only that $e_s(T)$ is monotonic and upward-curved, not how to reason with that fact, and a complicated formula doesn't actually help: a graphical way to express the curvature is clearest.

A second challenge to an entity-based viewpoint is how to express the vast ambiguous multiplicity of entities, and bracket the complexities that make it silly to take our little elements (parcels and plumes) too literally. Dressing their depictions in probability (a graphical *spaghetti* of *ensembles* of distinct but equally-plausible realizations) helps keep the mind from fixating inappropriately.

Customarily, a one-dimensional *sounding* or profile $T(p)$, $q(p)$ is usually taken as both the environment and the source of a lifted parcel's properties (from the lowest altitude, or the level of greatest h, or some mixture averaged over some convection inflow layer). Data from point sensors on a balloon, released at a particular place and moment and rising slantwise through the sky over an hour, are an imperfect fit to those assumptions, so *representativeness error* sets a useful floor on how fussy to be about precision. The situation-dependent processes of mixing and microphysics also add diversity to outcomes. Even a gridded model's column fails to really be either the environment (for fine grids: parcels don't rise vertically through their own narrow column as sampled at a prior instant) or the parcel (for coarse grids: because gravity naturally selects the *best* air for convection, not the average).

In short, an ensemble of parcels rising and mixing and churning through an environment full of statistical fluctuations must be envisioned. Graphics can help cement this appropriate diversity in the viewer's mind, leaving the necessary assessment of likelihood and weighting of these plausible parcel fates to sensible reasoning. Perhaps better illustrations may leaven a culture of sometimes silly fixation on numerical values of extremely crude indices like undiluted-parcel *convective available potential energy (CAPE)* or *convective inhibition* *energy (CIN)*. These indices badly underestimate the role of elevated moisture, a factor also wildly distorted in customary *aerological charts* of the pre-computer age, labeled as temperatures vs. pressure and thus expressing moisture as the steeply T-dependent *dewpoint depression*.

x.2 Conserved variables in lifted air

Our only rigorous basis so far for explaining an updraft is Newton's law for vertical acceleration (2.10d, 2.14d), with its b term as the ultimate driver of all fluid motion (problem 2.4.1). We therefore begin with a graphically supported understanding of the thermodynamics of b for rising parcels of air in a quasi-uniform unstable airmass, before turning to the mass-continuity problem of realizable ascending motions.

Air parcel density (or T_v) at any pressure can be easily computed using conserved variables (h and total water mixing ratio q_t), by a quick iterative solution of the mildly nonlinear saturation condition deciding whether $q_t > q_{sat}(T,p)$. These variables are only "conserved" in the absence of surface contact, radiative heating, and precipitation (section 2.1.5), leaving those phenomena to be assessed sensibly by the reasoner. Figure 4.1 shows profiles of the specific static energies s, h, h_s for a Miami summertime sounding[ppp]. The reader should appreciate all of the following reasoning-relevant aspects of the diagram, as evaluated in the Problems and Exercises of section 4.5.

[ppp] Created with the Python (pip) package mseplots-pkg, which rests on thermodynamics in the MetPy package from Unidata. Try the entropy and potential_temperature keywords to see how little these complications of nonlinearity matter on Earth compared to the above ambiguities of the whole 'parcel' conceptualization, as illustrated in Exercises below.

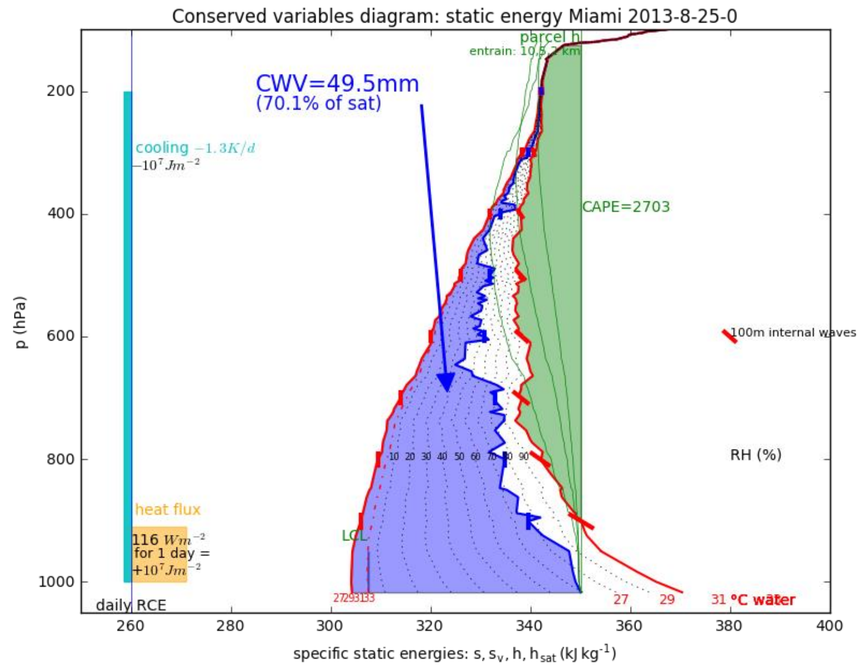


Fig. 4.2. Specific static energy profiles vs. pressure (mass). Static energies from left to right are: dry $s=C_pT+gZ$ (red solid), virtual s_v (red broken), moist $h=s+Lq_v$ (blue), and saturated $h_s=s+Lq_s(T,p)$ (rightmost red solid). Relative humidity RH in 10-90% are dotted (labeled at the 800 hPa level). Blue fill area is proportional to L times column water vapor. Below the lifted condensation level (LCL), buoyancy of a lifted parcel is assessed by the s_v curve. Above the LCL, its sign is indicated by the gap between h_s (red) and h_p for lifted near-surface air parcels (green curves, with entrainment rates of 0 (vertical) and 100% per (10 km,5 km,2 km), which act as the strengths of a linear pull of the rising parcel curve toward the h profile at each level. Thick stubs on *s, h, h_{sat}* profiles every 100 hPa show the effect of hypothetical +/- 100m adiabatic displacements in the air column, comparable to typical internal wave amplitudes, for envisioning their statistical effect on lifted-parcel buoyancy. Red number annotations at bottom show the *s* and h_{sat} of the molecular boundary layer over surface water with temperatures near the sounding's surface temperature, relevant for viewing surface fluxes as another distance-proportional mixing process. Annotations at left centered on 260 kJ/kg indicate typical daily diabatic increments for Earth-like radiative convective equilibrium (RCE), with cyan and orange areas representing 10^7 J m^{-2} of daily static energy changes by radiation (-1.3 K/d, compare Fig. 0.1) and surface flux (orange square, if distributed over a 100 hPa layer).

1. The vertical coordinate is hydrostatic pressure, so its increments are proportional to mass ($\Delta p_{hyd} = g\ \Delta m$).
2. The horizontal coordinate is specific energy (units: kJ/kg).
3. Because of 1 and 2, area on the diagram is proportional to energy.
4. Relative humidity RH (the hygrometer measurement) is proportional to the distance between the two indicators of the thermometer measurement (s *and* h_{sat}), as ruled by the dotted lines labeled 10-90%.
5. The filled blue area is column-integrated water vapor or *precipitable*[qqq] *water vapor* multiplied by latent heat coefficient L.
6. Unsaturated air conserves s and s_v during vertical displacements, so the stability of unsaturated layers is assessed by the slope of the dashed red s_v curve below the *lifted condensation level* (LCL). A well-mixed layer like the *planetary boundary layer* (PBL) here has vertical s and s_v curves, because absolute instability rapidly mixes a layer if s_v ever decrease with height.
7. An undilute surface parcel lifted through the troposphere conserves h (thick vertical green line) even after condensation commences. The vertically integrated buoyancy of this parcel (assuming instant precipitation of condensate) is labeled with MetPy's pseudo-adiabatic undilute parcel CAPE, which is not quite strictly proportional to the green fill area (nor to any aspect of a realistic convective process; avoid fixating on the number).
8. Lifted buoyancy of any *saturated* parcel (above its LCL) is indicated by the horizontal distance between the green h_p and red h_{sat} curves, because at any given altitude $h_p - h_{sat} = C_p(T_p – T_{env}) + L[\ q_{sat}(T_p) - q_{sat}(T_{env})\ ]$ is monotonically (albeit not linearly) related to $T_p – T_{env}$, as explained in section 2.3.
9. Horizontal mixing with the environment (entrainment) pulls any lifted saturated parcel (green curves) toward the environmental h (blue curve) with a strength linearly related to horizontal diagram distance between the curves.

[qqq] This old term for condensed puddle depth in mm or kg m^{-2} (section 1.1) may have stemmed from rough coincidence of its typical values with typical rainstorm totals.

10. Surface flux can be viewed as a mixing process with the microlayer of air that is in thermodynamic equilibrium with the surface. Over water, that microlayer is saturated at the water temperature. If that water temperature is known, latent and sensible heat fluxes (respectively) are proportional to the respective distances between the sounding's lowest-level air values and the water temperature values (red number annotations at the bottom of the diagram are centered on the values).
11. The effect of small adiabatic vertical displacements in the environment (such as by the buoyancy waves of section 2.2.3) is indicated by whiskers on the *s*, *h*, and h_{sat} curves. These are vertical on the s and h curves (since both are conserved), but are sloped on the h_{sat} curve, expressing the nonlinear saturation relationship $e_{sat}(T)$.
12. Energy added to the mixed-layer PBL by surface fluxes (proportional to area on the diagram, by point 3.) will be spread over a layer whose depth is defined by the area added, and by the condition of verticality of s_v upward from the surface. *Sensible heat flux* SHF increases *s* alone, while the sum of SHF plus *latent heat flux* (LHF) moves the *h* curve to the right (adding to the area encompassed to the left of the h curve). These principles are sufficient to distribute any given sensible and latent surface energy input over a mixed-layer PBL, such as on a morning sounding for anticipating convection on a summer day.

A few other points are worth noting:

- The relative smallness of water's virtual contribution to buoyancy, compared to its latent heating impacts, can be seen graphically by the relative distances s_v-s and h-s.
- There is no fundamental area relation for the distance that a diabatic temperature warming like SHF moves h_s to the right. The h_s curve is the locus of the main nonlinearity, the curvature of $e_s(T)$, and is the reason that a diagram instead of equations is needed for reasoning.
- It is not possible to read absolute *T* off the diagram; annotation along the s curve for the 0C level would be a

useful addition to respect the importance of ice, although the nucleation dependence of freezing complicates its use in reasoning. Qualitatively, the latent heat of freezing adds about 15% to the value of L measured by the h_{sat}-s distance.

x.3 Parcel diversity, dilution, and detrainment profiles

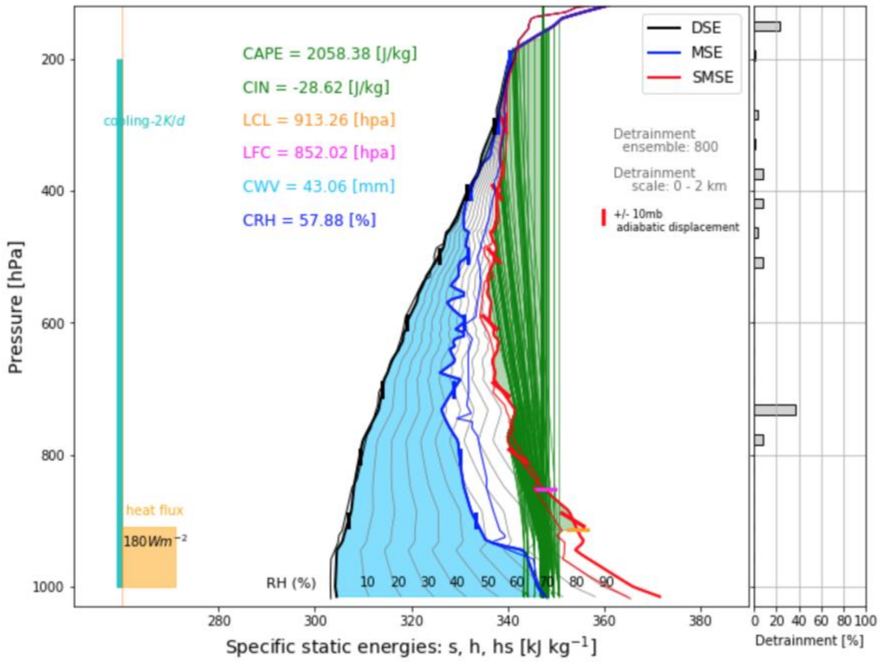


Fig. 4.2. As in Fig. 4.1, but with some statistical features. Surface parcels are sampled randomly from an assumed distribution (loosely based on aircraft data over a tropical ocean) of h values centered on the balloon's data value. Each is then subject to an arbitrary fixed set of different entrainment rates. The bar plot at right shows the resulting histogram of topmost $b>0$ altitudes for members of that parcel ensemble, a crude estimate of cloud "detrainment" (chapter 6) layers that might be produced by natural variability in a sky of which the balloon data is a representative sample. A second balloon sounding one day later is also overlaid, to illustrate the size of day to day variations at this location and season.

Figure 4.2 shows a similar diagram with an ensemble of thermodynamically possible lifted parcels (green curves). These were created with two assumptions (adjustable as keywords in the Python call):

1. Parcel diversity. The balloon is assumed to have sampled the center of a distribution of PBL *h*. Based on aircraft data at low levels over a warm tropical ocean (Kingsmill and Houze 1993), 20 more samples are drawn randomly from a normal distribution with a standard deviation of 2 kJ/kg[rrr].
2. Mixing diversity. For each parcel, a fixed *ad hoc* distribution of 5 entrainment rate coefficients is considered, ranging from the 1 km^{-1} value typical of shallow convection (chapter 6) to much smaller values that permit nearly-undilute ascent to great heights.

Together these give 100 parcels, whose altitudes of topmost positive buoyancy (summarized in the bar chart at right) are a crude estimate of the profile of likely detrainment altitudes (and hence perhaps of cloud layers one would expect to see in the sky). Sky photographs paired with soundings suggest some relevance to these computations, although feature altitude is hard to estimate visually. A computer exercise invites you to explore this relationship for any place and time, for instance where you can find photographs near a sounding site.

Layers where h_s increases more rapidly tend to cap the ascent of larger numbers of parcels in the distribution. In other words, from a broad distribution of *h* values in buoyant parcels, more will detrain preferentially into stable layers. This is a key mechanism forming a logical basis for a teleological interpretation of convection’s job: adjusting the atmosphere toward a moist adiabat (Betts and Miller 1984, Bretherton and Smolarkiewicz 1989, Bretherton 1993).

Assembling an ensemble of credible parcels into a realizable convective event is not simple. The work done by buoyancy (*b* integrated over height) must be positive for viability, but mass continuity must hold.

[rrr] 1 kJ/kg in *h* is about 1K in Kingsmill and Houze's θ_e; this example is narrower than their distributions, and so is a very conservative estimate of PBL diversity in general.

x.4 Problems and computer exercises

x.4.1 *Jupyter notebook of MSE plot*

View and obtain the MoistStaticEnergy.ipynb sounding plotting notebook at github.com/brianmapes/ShortCourse. Install Jupyter, and add the Siphon, MetPy, and mseplots-pkg Python packages (instructions in the notebook). Select a date, time, and location of interest from the Wyoming sounding archive, with interesting convection (maybe where you have an interesting sky photograph or time-lapse or satellite image). Edit the notebook and execute it (in Jupyter) to display your sounding and interpret it. Are there any discernable differences between static energy plot (with all of the linear sum's clear properties listed above) vs. the entropy or potential-temperatures displays (keywords in calls) whose nonlinear formulas make those properties less obvious? Consider how graphical features in the diagram correspond to the corresponding wiggles in the traditional skew-T display.

x.4.2 *Traditional sounding indices for convection*

Consider the many traditional sounding indices, for instance at
https://www.spc.noaa.gov/exper/soundings/help/index.html or
http://weather.uwyo.edu/upperair/indices.html.
How do the various measures fit into the matrix below?

Indicator of →	Likelihood of convection occurring	Expected (mean or typical) convective outcome or impact	Worst-case hazard the conditions might indicate as possible
Physically based (a crude model of some lifted-parcel scenario)			

Empirically based (calibrated with historical data)			

x.4.3 *IDV conserved-variable display*

Install the free IDV software from Unidata, version 5.6 or later (https://www.unidata.ucar.edu/downloads/idv/nightly/index.jsp). While online, under the Bundles menu, select Weather→Real Time Weather→Mapes Current Weather→CONUS-jet-vort-xsec-sounding. Wait while the data loads. In the Dashboard window, under Field Selector, find the "Latest NCEP…" model data in the Data Sources area at left. In its Fields (3D→Derived sub-menu), select Conserved Sounding and then click Create Display.

Drag the little squares around for the Sounding (a skew-T log-p plot) and the Conserved Variables Probe (a potential temperatures based version of Fig. 4.1). Examine corresponding satellite imagery (on the Web, or in the IDV you can find these in the Data Choosers tab, Sat & Radar → Images area at left).

Move the probes to the same point, somewhere relevant to observed convection, and interpret these different depictions of the same vertical thermodynamic profiles. Which graphical features (distances between curves on the screen) correspond most informatively to distinctions you can see in the convection field? Discuss how.

x.0 Chapter 5

Kinematically possible convective entities (Back to Outline)

x.1 Thermals, bubbles, starting plumes, plumes

Coherent structures are common in fluids. Bird watchers and glider pilots know that convective ascent is often concentrated in *thermals*sss (Ludlam and Scorer 1953), even in clear air, which can be formalized a bit as *bubbles* (Scorer and Ludlam 1953). Long-lived visible *plumes* over fires and smoke stacks are another ancient visual paradigm of turbulent ascent, amenable to controlled study in laboratory tanksttt. A *starting plume* (Turner 1962) is the combined concept: the response to a localized buoyancy source that is initiated and then maintained. A thorough history and discussion is in chapter 7 of Houze (2014).

Besides observation (indoors and out) and description, thinkers before the computation age had only mathematical study of the dynamical PDEs to fall back on for defining quantitative frameworks. One long-known analytic solution, specific but necessarily idealized, was the vortex ring or spherical vortex (Reynolds 1876, Levine 1959). Such entities are occasionally seen in the skyuuu, but they require rare initial and boundary conditions, just as skilled lips are required to blow smoke rings. Still, they can serve as a *paradigm* for more realistic and ragged entities, useful for instance as a framework for statistical composites of data from turbulent simulations (like in the fruitful debate between Sherwood et al. 2013 and Romps and Charn 2015).

sss Glossary of Meteorology: "A discrete buoyant element…small-scale rising current…"

ttt along with less-relevant vertical *jets* driven by initial momentum rather than buoyancy

uuu e.g. Fig. 8.28 of Markowski and Richardson (2011); the Web surely has many examples

A second equation-based approach is very different, based on concepts of *similarity* or *self-similarity* and depending only on the scale-independent properties of the fluid set (see problem 3.9.4). An admirable high culture in fluid dynamics prizes formal agnosticism equal to our ignorance. Often this is expressed in purely units-based reasoning, or the assertion that flow can only depend on fundamental *nondimensional parameters*[vvv]. This approach leads to deceptively simple final formulas, like the empirical result that the geometric slope of conical-shaped turbulent plumes is about 0.2 on average, as measured for instance on long-exposure photographs (Morton et al. 1956). Since it lacks any physical scale, such a number is expected to generalize directly from 1m deep water tanks to 1km deep air layers.

Although such conical turbulent flows are evidently realizable over their layer of similarity, that is not a complete and closed mass circulation. More elaboration is certainly needed the inevitable terminus at the wide end (like the starting plume concept). Synthesizing such overly-idealized bubble and incomplete plume entities into mathematical models for use at larger scales is a craft as much as a science. Might the time average of a sequence of bubbles act like a plume? An opinionated review of the historical literature and the rather blurry and *ad hoc* synthesis in common use today is in Yano (2014).

Below any given rising bubble, air from the turbulent *wake* is inevitably drawn up into the core, such that a bubble turns itself inside out as it 'rolls' upward a distance about equal to its diameter (e.g. Sànchez et al. 1989). It is better for the entity's success if that wake air is buoyancy-enriched, either by the continuing *b* source under a starting plume, or by moist or otherwise enriched air mixtures left behind from an earlier process. That "earlier process" can include the *shedding* or *erosion* process on the sheared and turbulent outer flanks of the bubble itself, which created the material that makes up the turbulent wake.

vvv https://en.wikipedia.org/wiki/Buckingham_pi_theorem

Another possible "earlier process" is enhancement by the dead bodies of prior bubbles that have stalled and mixed out, enriching the area in ingredients for future bubble buoyancy. Indeed, laminar motions (perhaps driven by a prior bubble) can lift the background moisture gradient, at least in a transient way, as the crest of an internal wave. All of this steers toward the view that convection is an inherently multi-bubble process or "thermal chain" (Morrison et al. 2019). Aircraft-reported humidity enhancements around cumulus clouds (Perry and Hobbs 1996) are thought to indicate cloud-caused moistening, but might natural selection and conditional sampling be also at play? Perhaps successful clouds[www] only "get by with a little help from their friends", including ancestors.

While research on simple entities continues today, and some essential aspects are important to appreciation itself, and to later reasoning, the problem of *entity combinatorics* is so central that it may set a limit on how much concern about precision is warranted in the elemental realm.

One more equation-based approach should be mentioned: the *two-fluid* approach, in which the total flow (satisfying mass continuity) is split into two 'flavors' of air, like convective and environmental (Thuburn et al. 2018, Tan et al. 2018). Bookkeeping equations for the two categories can be written from the pure logic of accounting stuff in space, yielding terms that distinguish the motion *of the boundary* between the labeled flavors of air, distinct from the motion of air across that (moving) boundary. Many complications ensue: a useful definition of boundaries is tricky, but can be devised and calibrated from study of iso-surfaces of some scalar in an uncategorized reference simulation. While fully rigorous, this is at bottom another accounting tool, so its insight and utility depend entirely on how it is deployed.

[www] Perry and Hobbs selected on criteria of "well isolated," but also "active growth".

x.1.1 *Size, geometry, and buoyant updraft acceleration*

Understanding the rising of "parcels" in terms of buoyancy b alone (chapter 4) is badly incomplete. Realizable ascent of finite-sized fluid elements importantly involves *pressure drag*, which can be thought of as *pressure acting to enforce of the law of mass continuity* by pushing the air above a buoyant element out of the way and filling the space below as it ascends. The kinetic energy of all this ambient motion comes from a 'tax' on primary energy production by the product bw seen in problem-solution 1.5.1. In other words, the PGF must opposes buoyancy b. This energy required for of the rest of a realizable circulation is sometimes treated as a "virtual mass" (that is, extra inertia of a "parcel") as a factor of about 3, reducing acceleration in the too-simple $dw/dt = b$.

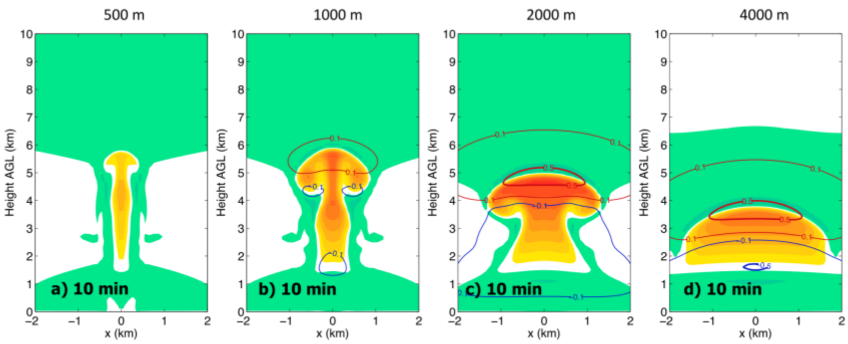


Fig. 5.3. Buoyancy (warm colors for positive values) and π_{buoy} from Eq. (5.1), ten minutes after releasing a bubbles in quiescent air. Narrower drafts feel less opposition from the BPGF, and have ascended further. Fig. 5 of Morrison and Peters (2018).

Pressure's pushback on convective elements can be quantified with our equation for π (problem 1.4.3), expanding its conceptual "forcing" term $div(\mathbf{F})$ for the complete equation set (2.10) in the Boussinesq case:

$$\begin{aligned}\pi = \boldsymbol{\nabla}^{-2}&[b_z - \boldsymbol{\nabla}\cdot[(\boldsymbol{V}\cdot\boldsymbol{\nabla})\boldsymbol{V}] + \boldsymbol{\nabla}\cdot\boldsymbol{F}_{Cor}] \\ &= \pi_{buoy} \quad + \pi_{dyn} \quad + \pi_{Cor} \qquad (5.1)\end{aligned}$$

The $\boldsymbol{\nabla}^{-2}$ operator is linear (allowing clean additive decomposition). However, it is *nonlocal* and *scale-selective*, bringing in spatial geometry considerations we can no longer ignore. Evaluating $\boldsymbol{\nabla}^{-2}$ involves two anti-derivatives, requiring additional information (constants of integration and/or boundary conditions), but it is useful as a symbolic matter to write it in this way.

It is helpful to write down seudo-code for a simple computational approach, on a periodic domain (codified as computer exercise 5.5.1), using the multi-dimensional Fast Fourier Transform **fft** and its inverse **ifft**, with total wavenumber $k_{tot} = [k^2 + l^2 + m^2]^{1/2}$ for spatial wavenumbers (*k*,*l*,*m*) in the (*x*,*y*,*z*) directions*:*

$$\pi_{buoy} = \boldsymbol{\nabla}^{-2}(b_{\boldsymbol{z}}) = \mathbf{ifft}[\, {k_{tot}}^{-2}\, \mathbf{fft}(b_{\boldsymbol{z}})] \qquad (5.2)$$

Figure 5.2 shows a toy two-dimensional calculation using (5.2) of the net vertical force b + BPGF, where BPGF is the vertical force per unit mass $-\partial_z(\pi_{buoy})$ for a hypothetical bumpy but assumed uniformly buoyant body (yellow contours over a periodically repeated cloud photo used to define the multiscale bumpy shape).

The feature shape and size dependence of the $\boldsymbol{\nabla}^{-2}$ operation makes the combined vertical force maximize in the growing tips of the tops (brightest reds in lower panel), with the gaps feeling a downward force that acts to enlarge the lobes (pure BPGF, since *b*=0 is specified there).

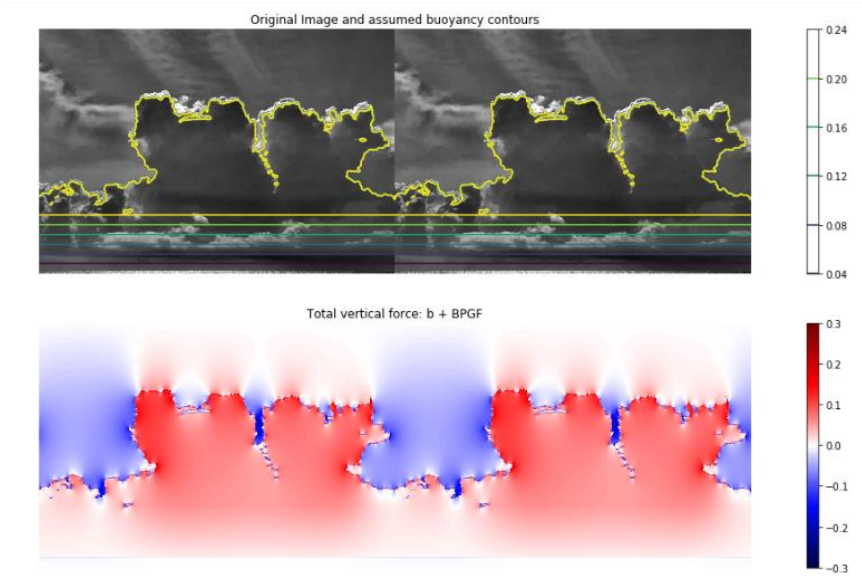


Fig. 5.2. Top: buoyancy (contours), assumed constant and positive in the main cumulus cloud bodies of the underlying photograph). Bottom: Resulting net vertical force b+*BPGF* computed from π_{buoy} evaluated by Eq. (5.2) for this two-dimensional periodic scene. Units are $m\ s^{-2}$ but values are arbitrary for this linear problem.

The distinctive texture of growing cumulus tops reflects a vital trade-off between opposition to b by the BPGF, which favors narrower entities (Figs. 5.1, 5.2), and vulnerability to mixing which disfavors too-narrow updrafts by thermodynamic destruction of b (including *buoyancy reversal*) as well as by viscous ‘friction’. Inside even a single cumulus cloud, myriad sub-entities are competing under these influences (along with nonlinear advection and the associated gradients of dynamic pressure π_{dyn}) to create the characteristic cumulus fractal shape, with bumps upon bumps upon bumps[xxx]. Many an observant outdoors person has made important life and death decisions based on a reading of cumulus top textures. Might this visual texture contain underutilized quantitative information for our science, perhaps extricable by the powerful new tools of machine learning trained on imagery?

[xxx] A humorous depiction is at https://xkcd.com/2185/

Dynamic pressure is a formidable puzzle, but some insight can be gained. A decomposition from the Boussinesq equations (problem 5.5.2) has a positive definite squared-deformation "splat" term, plus a negative definite squared-vorticity "spin" term:

$$\pi_{dyn} = -\boldsymbol{\nabla}^{-2}(\boldsymbol{def^2} - \boldsymbol{vor^2/2}) \qquad (5.2)$$

(2.131 of Markowski and Richardson 2011), with vorticity ***vor*** = $|\boldsymbol{\nabla} \times \boldsymbol{V}|$ and ***def**2* given in summation notation for Cartesian coordinates by

$$\frac{1}{4}\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2$$

The signs of squared terms in (5.2) make clear that *vortices of any rotation direction have negative-curvature[yyy] features such as pressure minima* in them, to do the job of holding parcels together against the divergent centrifugal force, while *colliding flows in any direction tens to have positive curvature like pressure maxima represent*, pushing back against the convergent "inertial force" of momentum advection.

Unfortunately, the common case of straight shear flow has equal but opposite contributions by ***vor*** and ***def*** as elementary kinematics tells us, even though it has no curvature or collision aspects *per se*. This cancellation makes the decomposition (5.2) unsatisfying in realistic complex flows. Might *curvature vorticity* be isolated usefully with further manipulation, as a way to set aside a residual shear vorticity that is spin-splat balanced, thereby defining some purer 'splat' term for interpretation? The possibility of making more satisfying causality accounts for events within the rigorous but slippery mathematics of Boussinesq or Anelastic fluid equations would be the greatly desirable outcome of such an analytic advance.

[yyy] *curvature* is the familiar meaning of a Laplacian, although it is a bit abstract in 3D

Another understandable dynamic pressure contribution is the *linear* interaction of a complex convective entity with a simple mean flow it is superposed upon. Such a component to π_{dyn} can explain how sheared wind impinging on a thermal retards its ascent and creates upshear-downshear asymmetry (Peters et al. 2019), or why right-moving storms are favored by curved wind shear hodographs in splitting supercells (section 8.4 of Markowski and Richardson 2011).

The non-*b* terms in *dw/dt* invite attempts at *parameterization* for some class of idealized updraft entities such as bubbles, based on prescribed shape or other assumptions. Uncertain coefficients in such parameterization *schemes* can be estimated by controlled numerical simulations (de Roode et al. 2012, Morrison and Peters 2018, Tian et al. 2019). The lengths to which this kind of frontal assault can now be carried are illustrated by Eq. (33) of Morrison (2017), too complex to explain in detail here but illustratively useful:

$$w = \left[\frac{2\mathrm{CAPE}([1/z^2 - 9L_v g k^2 L \Phi/(4c_p P_r R^2_{\mathrm{HMB}} \mathrm{CAPE})]\{4P_r R^2_{\mathrm{HMB}} z/(9k^2 L) - 16 P_r^2 R^4_{\mathrm{HMB}}/(81 k^4 L^2) \ln[9k^2 L z/(4P_r R^2_{\mathrm{HMB}}) + 1]\})}{1 + \alpha^2 R^2_{\mathrm{HMB}}/H^2 + 3k^2 L z/R^2_{\mathrm{HMB}}} \right]^{1/2}. \tag{33}$$

Ignoring parentheses and the second term in the denominator, we can recognize $w = [2\,CAPE]^{1/2}$ as the conversion of potential energy (CAPE) to vertical kinetic energy (*w2/2*) for an inviscid undiluted parcel, which certainly needs some correctives for realizability. While somewhat inscrutable, perhaps such a formal dependence on radius R could form the seed of a basis for assessing the competition among bubbles of different sizes, for a simplest ecology (competition among independent entities) in the competitive game considerations of Part III.

Does such mathematical detail become excessive at some stage, given the great idealization of the entity in the first place? Computers may change this doubt's answer. Software for symbolic math manipulation now makes such derivations reliable and trustworthy. While overuse of that convenience may seem to run counter to the goal of reader insight that once spurred such quests, analytic formulae have the great numerical virtues of economy and accuracy. Since formula translation into high-performance code is also becoming easier and more reliable[zzz], perhaps the old tools of theory could find a renaissance as simulation, testable against traditional brute force numerical grid computations (which can also be criticized as complicated and assumption-laden in their own way), and then implementable in genuinely predictive models.

x.2 Supercell updrafts

Summarizing the above, the vertical velocity equation is at the heart of updraft models, which are in turn at the heart of bubble or thermal theory in cumulus dynamics. The VPGF must be parameterized for such models, usually based primarily on the tractable buoyancy-induced pressure π_{buoy}. However, the dynamical pressure π_{dyn} can become so large that it predominates over the b force in shaping the evolution of a storm. Such is the case in *supercells.* Readers should study an excellent storm-oriented source like Markowski and Richardson (2011) for the loving detail that passionate researchers have assembled on this subject. Science aside, web imagery of supercell convection from storm chasers is abundant and amazing nowadays. Mesmerizing time lapse videos are worth the hours of study they will definitely seduce any reader of this book into staring at.

Ultimately such convection still depends on bw as its energy source, but energy can also be extracted from the strong mean shear flow that makes dynamic pressure competitive with b in such situations. Dynamic pressure arising from shear also creates surprising pathways for long-distance transports of energy across the sky, in the form of terms like $-V \cdot \nabla\pi$ from (2.15)[aaaa] that problem 2.4 conveniently integrated away. The nonlocality of energy, some of it in the form of potential energy as stable air is lifted to saturation, helps create the visually striking quality of laminar cloud structures among convective ones, giving severe and

[zzz] in the Julia computer language for instance

[aaaa] this looks like *advection of pressure,* but pressure is not "stuff" that can be advected -- instead this should be pronounced as *work done by flow down a pressure gradient.*

tornadic convective skies their distinctively spooky textures, reminiscent of sculpture galleries strewn with heavy cobwebs.

x.3 Downdrafts: condensation-evaporation asymmetry

Negatively buoyant downdraft entities are often envisioned as analogous to buoyant updrafts (bubbles, plumes, etc.). Visually striking *mammatus* clouds are perennial darlings of online image and video materials; search also for *asperatus*. Scorer and Ludlam (1953) postulated these to be an upside-down expression of potential instability on descending anvil bases, sparking long-running debates reviewed in Schultz et al. 2006). The distinctly modern phenomenon of downward lobes on jet condensation trails (Fig. 5.3) strike the eye as convective, but as Schultz and Hancock (2016) review, this phenomenon may be rooted in the vorticity dynamics of the aircraft wake rather than simply gravitational moist convective buoyancy considerations.

Fig. 5.3. Contrail with downward protuberances.

While the above examples are nearly moist adiabatic, a few oddities of evaporation in air + condensate mixtures must be recalled from section 2.3. *Buoyancy reversal* after the mixing of clear and unsaturated air is peculiar to cloudy convection. The fractional mixture that is just saturated (evaporating all the available condensate) has the greatest bulk density of all possible mixtures. However, this density increase occurs only when complete molecular-scale equilibrium is reached by the mixture! Mere turbulent folding (like the *dynamic entrainment* of wake ingestion into a rising bubble discussed above) is not yet mixing, nor is *turbulent entrainment* by shear instabilities on the flanks of narrow updrafts, although that process may lead more quickly to true mixing.

What is the fate of the extra-dense air created by buoyancy reversal? Squires (1958) predicted that it would perforate cumulus clouds with penetrative downdrafts from the mixing at cloud top, while Jonker et al.

(2008) emphasize a sheet of descent along cloud edge -- contributing more shear whose instability could drive further mixing. Conceptual models of mixing the subsequent sorting of mixtures are discussed in more detail in section 6.3.

x.4 2D entities: slabs, jumps, squalls

Equation-rooted entity models of 2D convection in steady state have been devised, featuring “jump” updrafts and “overturning” flow branches that satisfy mass continuity and other Lagrangian (d/dt) conservation laws along trajectories, which are the same as streamlines for steady flow (Moncrieff 1978, 2006). These entities have become paradigms or "archetypes" for squall momentum flux. Unfortunately, their virtue of rigor as solutions to the dynamical equations is nontrivial to weave into larger-scale flows. Their far-field asymptote of these 2D flows implies (or requires) a pressure difference that extends out to infinity.

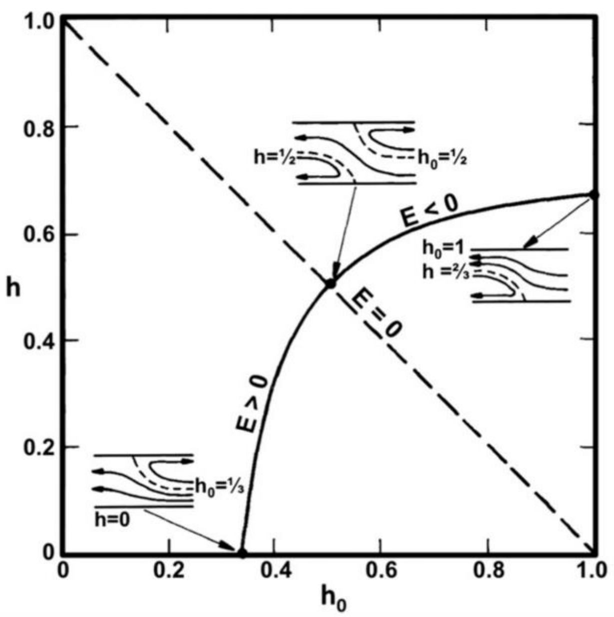


Fig. 5.4. Regime diagram for 2D steady jump and overturning flows bases in rear and front inflow altitudes. Adapted from Fig. 13 of Moncrieff and Lane (2015) where E is defined as a "Bernoulli number" measuring the net pressure drop across the system

The theory of atmospheric convection suffers from a lack of self-consistent, semi-quantitative models for mesoscale convection, in which the ‘dynamic entrainment’ implied by outside-the-updraft motions is reconciled with the main updraft's buoyancy and the $[wb]>0$ requirement for the system as a whole to be energy-generating. Just as the useful plume concept discussed above had to be capped with a bubble as a starting plume, "a craft as much as a science", partial models and partial considerations must be stitched together in *ad hoc* ways at the edges of mesoscale convective entities, whose relevance to realistic reasoning about convection are just as great as plumes and bubbles.

x.5 Problems and exercises

x.5.1 *Dynamic pressure derivation*

Derive Eq. (5.2). Using subscript notation for differentiation will save you many redundant hand motions. Can you spot the terms that go into (5.2)? Do any other decompositions suggest themselves, as discussed in that section?

x.5.2 *Jupyter notebook on inverse Laplacian*

Reproduce Fig. 5.2 from the Jupyter notebook at github.com/brianmapes/ConvectionShortCourse/BPGF.ipynb. Construct other 2D periodic buoyancy patterns, and use the notebook to solve for the implied π_{buoy} and associated BPGF.

x.5.3 *Coalescence and repulsion of buoyant updrafts in 2D*

Explore the effects of interactions by experimenting with the release times of 3 bubbles of thermal buoyancy *b*, in a 2D convective flow model representing an unstratified fluid. The Jupyter notebook at github.com/brianmapes/ConvectionShortCourse/ThreeBubbles.ipynb contains instructions. Optional: relate your work to early model work on multi-bubble clumping and mergers (Wilkins et al. 1976, Orville et al. 1980).

x.5.4 *IDV exploration of Giga-LES data*

Explore the 3D spatial patterns of spatial-eddy temperature (proportional to thermal buoyancy *b*), perturbation pressure, and other fields in three time steps (5 minutes apart) from a very large and high resolution (giga-LES) doubly-periodic deep convection simulation. *Capture images that illustrate the relationship between perturbation pressure and terms in (5.2), as best you can find them by focusing on regions where a single term likely dominates.*

Model details are as described at Khairoutdinov et al. (2009), but here the model was forced with domain-mean winds derived from the TWP-ICE program on Jan 20, 2006 over northern Australia (Glenn and Krueger 2017), leading to a ragged south-to-north propagating squall comprising several arc-like segments with more 3D structure. Use Unidata's free IDV software, on a computer with at least 8GB of available RAM (Before attempting this, set the IDV's memory allocation in Edit→Preferences menu, System tab, then restart the IDV.) Instructions and orienting images of the basic fields displays are at github.com/brianmapes/ConvectionShortCourse/GigaLES.ipynb.

x.0 Chapter 6

Mass 'trains': bulk flux and mixing (Back to Outline)

x.1 Plumes and entrainment and detrainment

The word *entrainment* (Stommel 1947, 1951) was used in chapter 4 for horizontal mixing of thermodynamic quantities: a coefficient with units of % per km, expressing how far the green h_{parcel} curves for ascending parcels in Fig. 4.1 are pulled toward the environmental h curve as they ascend a 1 km distance. This mixing-driven change in a conserved property is just one facet of a *steady-state plume* model, defined by its vertical *mass flux M(z)* by

$$dM/dz = M(\epsilon - \delta) \qquad (6.1)$$

where $\epsilon - \delta$ is entrainment minus detrainment (Yanai et al. 1973, de Rooy et al. 2013). Logically, these words imply a "train" with parcels embarking from and disembarking to a larger-scale "environment". Mass continuity requires that M must all detrain at the top of the plume, while M at plume base level (usually called *cloud base*, although it need not be at the LCL of surface air) is supplied somehow by horizontal convergence in the *sub-cloud layer*.

This crude entity model suffices as a framework for larger-scale flows in a convecting atmosphere, providing its M can be translated into a large-scale buoyancy source term Q_b in (2.10), or in its symbolically moist version (2.14) with a corresponding h-conserving term in a specific humidity equation (often discussed as "Q_1 and Q_2" from the classic Yanai et al. 1973). If M reflects ascent of air at water saturation, condensation $c = M\ d/dz(\ q_{sat}(T_{plume})\ )$, where $T_{plume}(z)$ itself is moist adiabatic ascent, Strictly, the *moist adiabat* is subject to the conditions of the plume's assumptions about microphysics, etc., and processes complications like freezing can be included in an *M*-proportionate way.

To be still more realistic, evaporation in precipitation-driven unsaturated downdrafts (section 5.2) can be grossly treated as a lesser, saturated *downdraft mass flux* M_d in an upside-down plume model, whose initial (top) conditions are related back to the precipitation produced in the updraft *M*. Finally, the *eddy flux* implied by these plumes can be computed from *M* and the other internal plume scalar properties like *h, s,* and *q* assumed to be well mixed across the plume. The difference of these from some larger-scale horizontal averaging (a *filter scale*) defines "eddy", as discussed in chapter 3.

This steady plume entity, characterized by its *M(z)*, is also the conceptual heart of *mass-flux cumulus parameterization* for coarse-grid models. The great classical formulation of this problem is in Arakawa and Schubert (1974)[bbbb], whose Fig. 6 is adapted here as Fig. 6.1.

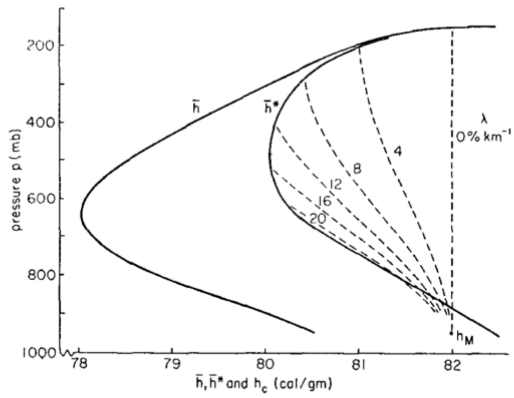


Fig. 6.4. Specific static energy plot like Fig. 4.1 showing how entrainment rate ε (called λ here) maps to the top height of plume buoyancy, in a simple sounding with no intermediate h_{sat} overhangs. Adapted from Fig. 6 of Arakawa and Schubert (1974).

[bbbb] although the reader should also notice Ooyama (1971), to be discussed more in Part III

Those authors articulated in symphonic detail the summation of plumes spanning a spectrum of different radii R (or equivalently, of entrainment rates indexed as $\lambda \sim 0.2/R$). Each steady-state plume was formally envisioned as an average over all stages of the life cycle of an ensemble of same-radius cumulus clouds, mapping the spectrum of $\varepsilon(\lambda$ in Fig. 6.1) on to a spectrum of cloud top height (where lifted b first or finally vanishes[cccc]). Top and bottom conditions were detrainment of top mass flux M_t at cloud top[dddd], and lower boundary condition M_b at plume base height z_b (implying uniform mass convergence below z_b) was left as an undetermined variable to be used in a final, teleological *closure assumption* based on potential to kinetic energy conversion [wb]. That depends only on a plume's M and b profiles, since w and b are both assumed to be horizontally uniform within the plume.

Although steady state, Arakawa and Schubert's (1974) plumes exist only for one time step in an evolving flow. Fortunately for self-consistency, any filter-scale state variable changes implied in building and dismantling plume bodies at the beginning and end of the time step are negligible: in substituting *mean* conditions for Ooyama's (1971) formulation in terms of *environment,* the *small area approximation* for plume size is made[eeee].

While a plume is *kinematically* realizable (satisfying mass continuity, for instance), its assumptions about very thin layers of base convergence and top divergence are heedless of dynamical constraints in a stratified environment (discussed in Part III). Concerns about how steady-state plume assumptions meet timestep dependences in coarse-grid host models are yet again "as much craft as science", in the realm of *numerics* that is recently getting needed scientific attention (Gross et al. 2018). For present goals those issues are far afield, but the steady plume is so

[cccc] Some elaborations detrain where w vanishes, in a steady w equation with source term b.

[dddd] Later elaborated by other authors to all altitudes, in *entraining - detraining plumes.*

[eeee] Finite plume fractional area coverage can be accounted for (Arakawa and Wu 2013), but arguably with logical shortcomings (Park 2014, Zhang et al. 2015).

influential conceptually, for instance as a framework for interpreting simulations (Kuang and Bretherton 2006) and observations (Labbouz et al. 2018), that an educated appreciation requires knowing its logic and vocabulary, including the rather practical roots of that logic.

x.2 "Bulk" plumes as pseudo-ensemble means

The *plume spectrum* picture elaborated above can be economized, and also made more practically tunable as an engineering construct, by envisioning a *bulk plume* with a single *M(z)* characterizing the integrated net vertical fluxes in a whole *ensemble* of plumes with different initial radii or other properties. When computational efficiency is prized, the practical simplicity gained is well worth the additional layer of complication (an integral, with additional assumptions) in the conceptual or analytic underpinning – especially when its apparent rigor is already quite compromised by tall stacks of idealizations and assumptions.

In a sense, doubling down on these compromises begins to liberate the reasoner into more information-theoretic or strategic thinking, like how best to load meaning (distinctions that make a difference) into the economy of available digital bins or degrees of freedom. Perhaps the ideal conceptual and mathematical framework would enforce only the most trusted conservation principles, and then be contrived to have exactly as many inputs and outputs as needed to represent all realizable convective cloud entities or fluxes. Examples might include the *transilient matrix* of Romps and Kuang (2011) or the ballooning area of machine-learned emulators (e.g. Beucler et al. 2019).

In code terms, the bulk plume is clearly a simplification rather than an elaboration, whose "chief merit is … an equation set with the same structure as that which describes each element" as reviewed by Plant (2010). One main shortcoming may be a required "ansatz" about microphysics, which motivated (as expressed in Wagner et al. 2011) the predator-prey model of Wagner and Graf (2010) that Part III will revisit. One prominent bulk plume is the equally-weighted integral over plumes in the Community Atmosphere Model scheme of Zhang and McFarlane (1995). While easy to critique in various ways, this scheme remains hard to beat in performance terms, especially after the entire model has been tuned around its behavior for years. It remains in CAM in 2019.

x.3 Entrainment dilemmas and alternative mixing models

How well can a plume that assumes thorough, instantaneous horizontal mixing represent the scalar quantities in a convective cloud, including the products of scalars that comprise flux terms? Must the crucial buoyant rising current [*bw*] that drives the kinetic energy be a uniform blend of all the properties of all the air that rises? Such a question focuses attention on entrainment because it is assumed to "dilute" (perhaps too strongly; Hannah 2017) the mixture that comprises the "train". Actually, that spotlight may be misplaced: "despite the focus in the literature on entrainment, it appears that it is rather the detrainment process that determines the vertical structure of the convection in general and the mass flux especially" (De Rooy et al. 2014).

An early crisis for the entraining plume entity was its inability to reconcile aircraft-sampled dilution of cloud air with cloud external characteristics like height attained (debated in Warner 1970, Simpson 1971, Warner 1972, Simpson 1972). This cloud-scale dilemma has an echo in global model parameterizations as an unwanted trade-off between unrealistically easy development of deep convection (seen for instance in unrealistic predictions of frequent light rain, a problem militating for increased entrainment rates) vs. unrealistic dilution of the convection-lofted air entering the upper troposphere (militating for decreased rates). Such unwanted tradeoffs suggest an impoverished framing of the problem, in this case as a single value of a single parameter of overwhelming importance. The phenomenon of convective cloud field *organization* has been suggested as one minimalist way to expand the framework to 2 parameters (Mapes and Neale 2011).

Besides proving that instant horizontal mixing is unrealistic, aircraft observations can guide better treatments. Mixtures between two air bodies lie on a straight line in an abstract space of conserved quantities. This visual method has been used to diagnose the multiplicity of mixtures in cumulus clouds (Paluch 1979), using a q_t vs. h scatter diagram of data inside aircraft-sampled clouds. This technique helped distinguish the most *likely* outcomes of mixing (random samples by blindly-directed airplanes) from the "*lucky* parcels" that govern cloud top height, as illustrated lucidly in the extended discussion and diagram sequence in chapter 7 of Houze (2014).

Such observations led to conceptual models of *stochastic mixing followed by buoyancy sorting*, with air detrained at its altitude of ultimate (final) neutral buoyancy (Raymond and Blyth 1986, Kain and Fritsch 1990, Emanuel 1991). Innovation continues in stochastic treatments of mixing, trained on simulations rather than observations, such as Romps (2016) summarizing tracer studies like Romps (2010), and Böing et al. (2019). It remains to be seen whether statistical ensemble concepts, perhaps bolstered graphically by an updated soundings-analysis view like in chapter 4, can or should supplant simplistic modeling-driven plume notions in the field's culture of reasoning.

x.4 The whole convecting layer as an entity

The end point of our increasingly *bulk* models of convective *entities* is whole-layer models, from the humble but profound *mixed-layer* model (all variables well mixed in the vertical) to simple but fully realizable equation-set solutions like Rayleigh-Bénard convection (recently extended to moist air as the "Rainy-Bénard" system in Vallis et al. 2019). The well-mixed layer embodies the teleological claim that convection is infinitely efficient at its job of neutralizing instability, a useful concept called *adjustment* that can be retrofitted across the whole landscape of nominal conceptualizations of convection (Arakawa 2004). Key questions include whether and how adjustment applies across instability's multivariate *ingredients* like humidity vs. lapse rate of T.

x.5 Problems and solutions

x.5.1 *Entraining plume and the concentrations of scalars*

For the entraining plume (no detrainment), integrate (6.1) to show that $M = M_0 exp(\varepsilon z)$.

From the product of (6.1) with a specific (per unit mass) scalar h and its environmental value h_{env}, show that h (green curves in Fig. 4.1, 4.2) can be integrated upward using $h(z + dz) = \epsilon[h_{env}(z) - h(z)]$. Does this relationship depend on whether detrainment is allowed? Explain.

x.5.2 *Single column computations and convective adjustment*

A somewhat technical but illuminating exercise with the Community Atmosphere Model (CAM) from the National Center for Atmospheric Research (NCAR) is to install and operate a *containerized* version of its single column model, easily run on any modern laptop after installing the Docker container software (a light version of virtual machine technology that smooths over the difficulties of operating-system and library dependencies). While this URL will change with model versions (http://www.cesm.ucar.edu/models/cesm2/atmosphere/CAM6tutorial/ hard-wires the current model versions cesm2 and CAM6 at this writing), a sustained capability is anticipated, findable with Web-search skill. Simple starter questions could include: *How do the mean lapse rate and humidity profiles in equilibrium, and temporal variability, depend on the specified entrainment rate, the model's most important single disposable parameter*?

Part III: Envelopes and larger-scale interactions

x.0 Chapter 7

Dispatch and survival in multi-cellular entities (Back to Outline)

The PDE sets of Part I like (2.15) comprise a very general knowledge about the dynamics of fluids, and Earth's moist air in particular, but that was only one level of appreciation. There we saw how *teleology* can express the essential, truest understanding of the nature of something: in that case, pressure. The micro-macro distinction (molecules vs. continuum) also gave us a clean paradigm for the more general concept of *scale separation* or *filtering* (chapter 3). Software PDE solvers (atmosphere models) are glorious tools, both at high resolution where they comprise a new source of data on convection's structure, and at low resolution as a demanding test of our understanding its function. But they are not quite understanding itself. The conceptual *entities* of Part II added something new.

Entities align with our mind's limitations, such as narrow discrete focus and finger-related limits on number appreciation. But they also activate its strengths, such as hierarchical reasoning with logarithms (a glorious flip-side of the apparent weakness of limited enumeration), and attentive attraction to extended dramas among characters (if not taken too literally). To get further, we need principles of entity *interactions*. Because our entity depictions are crude caricatures, compounding them in simple heaps will not sum up to right answers: our only hope is to seek *emergent principles of interaction*, perhaps drawing on other fields of scholarship, which all bear the same problems with *scale*.

x.1 Introduction: systems of cells of bubbles

Even shallow moist convection is observed to be frequently non-random in its spatial patterns (Zhu et al. 1992). Large-scale structures resembling flowers or fish skeletons (Rasp et al. 2019) develop especially when multi-bubble convective cells get deep enough to precipitate, and that process becomes overwhelmingly salient for deep convection. Still-larger scale structure is evident, but for the moment let us focus on contiguous *cloud systems.*

Let us label any multi-cellular entity governed by a positive feedback loop, with new cells spawned preferentially adjacent to prior cells, as a *system*. In the case of deep convection, a popular umbrella term is *mesoscale convective system* (MCS, Houze 2004). The adjective *mesoscale* typically refers to horizontal cloud or rain area contiguity larger than 100 km. Everything mesoscale is undeniably multi-cellular, and even cumulonimbus *cells* are multi-bubble entities as seen in Part II.

Like *thermal*, a descriptive term that motivated specific entity models which went on to title themselves after it, *MCS* is a catch-all that invites the *category error* of equating it with specific dynamical paradigms for its best-studied instances (like a *squall line*). One appropriately general and agnostic conceptual model of such entities is the *particle fountain* of Yuter and Houze (1995), firmly grounded in the electromagnetic observability (chapter 0.6) behind empirical descriptions, the first step to scientific appreciation of every phenomenon.

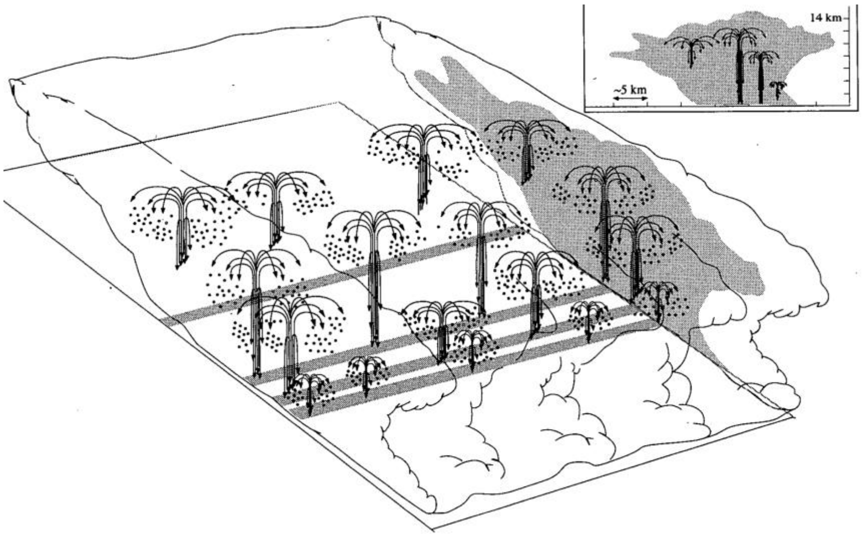


Fig. 7.1. Particle fountain model of the observed phenomenon of MCSs. Convective cells that are abundantly dispatched within a region, *for whatever reason*, expel ice crystals at high altitudes which merge into a contiguous area of cloud and falling hydrometeors. Adapted from Yuter and Houze (1995), who call this "an extension of bubble-based conceptual models".

A sizable cottage industry is devoted to further sub-categorizations of MCSs, in case studies and in conditionally sampled statistics of large samples from today's amazing global data sources like "spaceborne" (orbiting) meteorological radars. Many excellent sources focus on mesoscale processes and their impactful if arbitrary characterization distinctions (for instance, the titular scope of Trapp 2013). Here we only lightly survey such mechanisms for elementary appreciation, before steering discussion toward a combinatoric reasoning, which in this book is finally aimed more at expecting the typical than at explaining extremes (as admitted in section 0.7).

x.2 Dispatch probability, survival, and reproduction

Once discrete entities are defined, a counting scheme can be set up to keep track of them. A memorable formulation of this idea is the "dispatcher function" in section 4 of Ooyama (1971)ffff:

> If…the environment of all the bubbles is the same, … the properties of a bubble with any set of initial conditions [**s**] can be calculated by the [bubble] model … independent of similar calculations for other bubbles. … At a given time-step and at a given horizontal grid-point of the large-scale model, [the number distribution of] a statistical ensemble of bubbles with various initial states **s** … is denoted by N(**s**). To be precise, N(**s**) d**s** is defined as the number of bubbles, per unit time and per unit area, starting from initial states between **s** and **s**+d**s**, that is, between [starting altitude] p* and p*+dp*, [mass] m* and m*+dm*, etc. It seems appropriate to call N(s) a "dispatcher" function.

The assumed *independence* of dispatched entities here is an initial expression of agnosticism, not a strongly valid claim about nature. One aspect of a meaningful definition of *organization* will find expression in its modification.

The independence assumption envisions convective bubbles of different sizes as *competing* for a *common resource* (instability, measured in energy terms), in the form of an identical shared environment. Such an unconditioned competition tends to favor the largest (most undilute) bubbles that are permitted to exist, the most buoyant green curves on Fig. 4.2. A straightforward consequence and illustration is the longstanding syndrome in global models with parameterized deep convection that scheme-produced rainfall occurs too easily and thus too frequently.

Two main altitude layers are involved in the interactions and feedbacks that shape multi-cellular entities like MCSs: the *planetary boundary layer* (PBL), where effects tend to be quite local or tied to merely advectively moving airmasses, and the lower reaches of the

ffff in a "special issue" whose now-eliminated access difficulty may have limited readership overlying *free troposphere* where nonlocal wave propagation can also be important. Although upper-tropospheric dynamics often reaches down to *force* or at least *disinhibit* these convective events, especially on synoptic scales, the bottom-up nature of deep convection and the vertical confinement of water availability (Mapes 2001) make those upper-level influences arguably secondary in many cases, as asserted in chapter 0.

x.3 Near-field dispatch effects: impacts of the convected air

The dispatcher function's basic resource is conditionally unstable PBL air. Its mechanism of constructing larger (and thus more successful) bubbles is often the *gust front*, a sharp thermal gradient and gravitationally driven updraft above convergent surface flow at the edges of divergent *downdraft outflows*. These effects are often pooled in the term *cold pools*, because outflow air is comprised of cool air, chilled by the evaporation of precipitation. The production of precipitation particles, their fall and evaporation, and the descent of the chilled air cause a tens-of-minutes time lag of the effect of one convective cell on the next dispatch event.

So long as old downdraft outflow air can be spatially segregated from the source of rising air, for instance by simple horizontal offset in a squall translating with time, convection can thrive on the *nonuniformity* of its PBL, even if the large-scale *mean* PBL in the broad area containing the system becomes neutralized or stable. But a final end to runaway-dispatcher positive feedbacks (both gust front and disinhibition) must come when the entire PBL of an unstable airmass has been lofted in convective drafts, replaced with downdraft outflow that eliminates all instability. At that point, post-convective PBL air must gradually 'recover' its instability (in the form of h) via surface fluxes.

Over warm ocean where flux is proportional to wind speed, recovery has a characteristic distance scale of about 500 km. There the spatial pattern possibilities for this ceaseless game of outflow, lifting, and recovery on such a spatial scale are endless, even in the merely horizontal 2D domain with merely near-field interactions among cells (exercise 7.7.1). Convective cloud fields over warm oceans are almost never free of the long chains of history, so mesoscale structure is omnipresent, perhaps ‘saturated’ or equilibrated with its sources and damping in the spatially spectral language of chapter 3.

Over land, in contrast, dispatcher-scale causality can be reset overnight, so that the cloud field must begins from more homogeneous initial conditions the next day. This daily reset makes warm lands often a more informative laboratory for defining and testing ideas about *organization*, although the particularity of geography often makes process inferences from land study less generalizable than from the arbitrary spot of a ship at sea.

x.4 Mid-distance interactions: waves of low-level T'

In contrast to PBL-rooted dispatch, the lower free tropospheric layer often plays more of a screening role, culling survivorship of entities, at least in situations less organized than the slab jumps and squalls of section 5.4. This claim is supported for instance by a sign reversal near 900 hPa of the sensitivity of deep convective precipitation to domain-averaged T perturbations in an isotropic cyclic cloud-permitting model (CCPM), as shown in Fig. 7.2a.

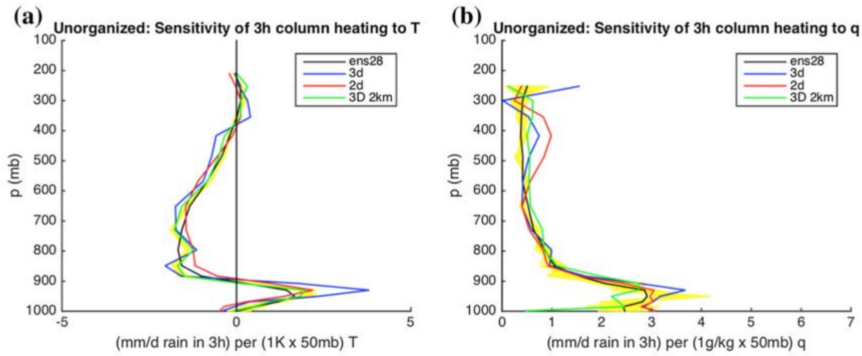


Fig. 7.2. Sensitivity (horizontal axis) of simulated rainfall over 3 subsequent hours to instantaneous perturbations of T and q at each indicated altitude (on vertical axis). The

results pertain to a cyclic convection-permitting model (CCPM) of about 2km horizontal mesh size, in a convecting state of statistical equilibrium with prescribed radiation-like cooling about the strength of Fig. 0.1 over surface water at 28C. Adapted from Fig. 5 of Mapes et al. 2017.

Since the sensitivity of such CCPM convection is nearly linear (Tulich and Mapes 2010), a clever matrix inversion technique (Kuang 2010) can be used to summarize it in the form of a time-independent data object, a *linear response function* matrix, of which Fig. 7.2 is an integrated (functional) summary. That result was derived from long simulations of isotropic, unsheared convection of quite modest intensity, in small domains containing only isolated or even intermittent cumulonimbus clouds -- arguably the closest we can bring model-realized convection to the ideal of randomly dispatched entities.

Strong positive sensitivity to PBL T and to q in Fig. 7.2 can be understood in terms of simple parcel buoyancy (Fig. 4.1). Above-PBL moisture sensitivity is also positive but smaller, with a nearly-uniform profile that implies the functional or *effective entrainment* in the CCPM's realized convective entities, although that information content is not easily extracted into a coefficient for a mixing scheme.

Negative sensitivity to lower free tropospheric T is consistent with the notion of *convective inhibition*, and its removal *disinhibition*, but is a far bigger and deeper-layer effect than the integrated negative buoyancy of an *undilute* lifted parcel (the traditional and rather trivial *CIN* definition lamented in chapter 4) can explain. Reasons for that negative sensitivity are further explored in Tian and Kuang (2019).

This negative sensitivity is thought to be crucial to the dynamics of convectively-coupled waves in the tropical atmosphere (Kiladis 2009) through mechanisms elucidated in Mapes (2000) and elaborated in Kuang (2008). It is also important in the near-field problem of MCSs, in combination with the more obvious gust front effects of the previous chapter (e.g. Fovell and Tan 1998), although the tens-of-minutes time lags among multiple effects, some of them transparent air, makes causality tricky to infer clearly in such disturbed and transient settings.

Stable stratification in the unsaturated air around moist convection makes inflows and outflows obey the dispersion relation of the *internal wave* (aka gravity or bouyancy wave) flow solutions considered in section 2.2.2. These effects are mathematically first-order but quite unintuitive, so my own road to belief and understanding was importantly supported by laboratory tank experiments (Fig. 1 of Mapes 1993, partially reproduced below). The invisibility and transience of internal waves remain an observational challenge to their outdoor appreciation, although their crests do create visible cloud features -- most famously high-amplitude and quasi-stationary lee *mountain waves,* but quite ubiquitous to the observant.

Vertically pointing water vapor LiDAR data offer a compelling new way to see the vertical displacements in the stratigraphy of the atmosphere, as shown in Fig. 7.3 from Barbados Cloud Observatory (Stevens et al. 2016). In addition to secular trends (in which low-frequency 'waves' are hard to distinguish from the advection sloping features), more periodic laminar features can be discerned, for instance with periods near 1 hour, and near the shortest possible period for internal waves, $2\pi/N$ (about 10 minutes in a moist adiabatic stratification, although shorter in a more stable inversion layer). Amplitudes are commonly about 100m (justifying the adiabatic displacement whiskers on Fig. 4.1), and vertical coherence of features can exceed 1 km.

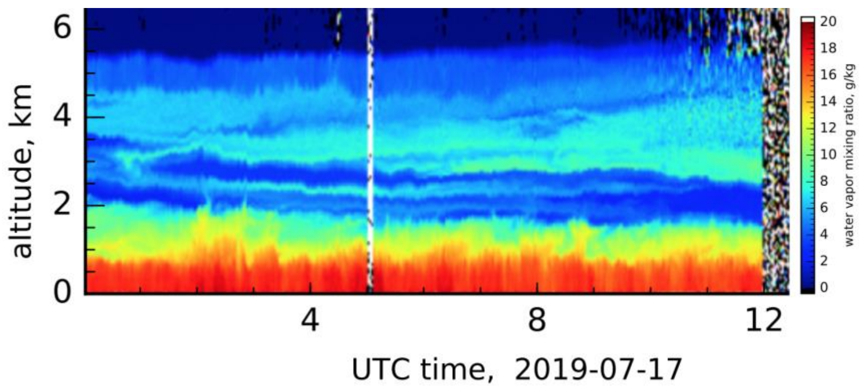


Fig. 7.3. Water vapor mixing ratio as detected by a clear-air Raman LiDAR system on Barbados on July 17, 2019. Adapted from quick-look imagery file http://bcoweb.mpimet.mpg.de/quicklooks/lidarql/RamanLidar-CORAL/lowResolution/co2019/co1907/coral_190717_0002_0000.pdf. A satellite view of the cloud field setting on that day may be perused at https://go.nasa.gov/2yihy1Z.

Only shallow convection was present in the area of Fig. 7.3, as evident from the low water vapor mixing ratio just above 2km as well as from the satellite imagery link in the caption. Deep convection can excite, and couple, internal waves that are much larger in terms of both scale (vertical and horizontal and temporal) and amplitude, sometimes so large that linearity is inadequate and the waves are called *undular bores* (e.g. Haghi et al. 2019).

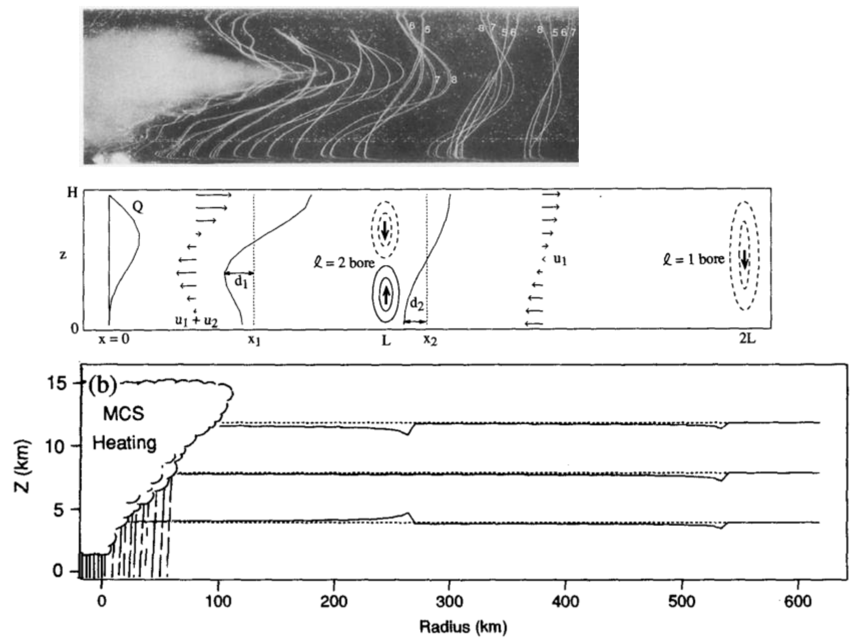


Fig. 7.4. Stratified environment flow and displacement responses to a localized convective entity (from Mapes 1993). The top panel (Fig. 1 there) is a photograph of dye lines in an almost two-dimensional (2D) tank, strobe lit at 5-8 buoyancy oscillation periods (5-8 x $2\pi/N$) after sudden release of dyed buoyant fluid at left. The spooky smoothness of far-field flow profiles is clear, as vertical wavenumbers separate themselves out horizontally. The middle panel is a schematic solution in slab 2D (x-z

plane) geometry for a buoyancy source of specified top-heavy shape that is maintained after sudden switch-on (Fig. 3 there). Horizontal displacements (d1 and d2) are the time integral of horizontal winds (arrows) whose pattern at any point evolves with the passage of the "bore" or wavefront features propagating to the right. The bottom panel (Fig. 4b of Mapes 1993) is for a similar buoyancy source in cylindrical (3D) geometry, and shows vertical displacements (solid) of initially horizontal (dotted lines) material surfaces.

A concentrated local heating like latent heat release in a MCS, forces adiabatic wave-like motions in its environment which spread the warmth horizontally by downward displacement of the stratification (bottom panel of Fig. 7.4), as part of a wave[gggg] response that travels at a speed proportional to vertical wavelength (section 2.2.2). Despite the multiple heating processes involved (phase changes, radiation, small-scale eddy heat fluxes), the far-field wave response really does respond to the decomposition of that total heating by vertical wavenumber, a dispersive *chromatography* effect that is spooky like any "spectral" phenomenon.

Although the deep-layer mean warming has been exported to a very broad area (over 500 km in 3 hours), Fig. 7.4 illustrates how a vertical dipole (wavenumber-1) component, expressing the difference between the top-heavy heating and the deep-layer "mode" of the wave solution, travels half as fast and thus has its effects confined to a smaller area in which they are therefore more intense.

Upward displacement and environmental cooling at low levels can occur (bottom panel) even though the imposed heating Q>0 at all levels. The resulting disinhibition contributes another near-field (or mid-distance) positive feedback to the enhanced dispatch effects from the previous section, acting to make convection locally gregarious (Mapes 1993) or to organize it into convectively coupled waves (Raymond 1983, Kiladis et al. 2009).

However, the inflow wind profile driven by top-heavy heating also has an unfavorable implication (near-field arrows in middle panel of Fig. 7.4). The inflow of low-*h* midlevel air is a mesoscale form of *dynamic entrainment* into the MCS-scale net updraft, like we encountered in

[gggg] The irreversible down-only wave front was called a *buoyancy bore by* Mapes (1993).

convective-scale entities of chapter 5. While this inflowing air will not necessarily be mixed into updrafts right away to reduce their buoyancy, its low *h* must somehow have an impact on longer time scales, as a *gross moist stability*[hhhh], conceptually a brake on the instability of the runaway dispatcher-plus-survivorship complex that governs system success.

Still-higher vertical wavenumbers in a buoyancy release (heating) event remain even nearer to their source, as wiggles in the density profile. In an area of repeated cell dispatch, the lifted-parcel buoyancy of subsequent convective updrafts is affected, just like the wiggles in h_{sat} in Fig. 4.2 that cap the buoyancy of members of the ensemble of green updraft curves causing spikes in the ‘detrainment’ histogram there. As discussed in section 4.3, this detrainment or *buoyancy sorting* mechanism makes such subsequent convective heating act to smooth out the wiggles, adjusting[iiii] the *T* profile in convection’s near field toward that convection’s effective moist adiabat. In other words, high vertical wavenumbers of convective heating are specifically damped by prior cell - subsequent cell interactions in these multicellular systems.

The exception that proves the rule of high-wavenumber damping in MCSs is the systematic thin (high vertical wavenumber) dipole T forcing by shallow melting at the 0C level plus the associated latent heat of freezing that occurs a little higher, at whatever altitude the nucleation of freezing places it on average (Mapes and Houze 1995). Dispersion by vertical wavenumber also shapes the temperature structure of sharp radiative cooling layers, making a warm layer at the base of dry layers. That warmth reduces the relative buoyancy of updrafts, and may specifically aid the longevity of such layers, by protecting them from penetration and moistening by convection from below (Mapes and Zuidema 1996).

[hhhh] a term adding to section 0.4's lament about "stability" words; e.g. Raymond et al. (2009)

[iiii] Bretherton and Smolarkiewicz (1989), Bretherton (1993), Mapes and Houze (1995)

One final illustration of the reality of spectral chromatography is shown in Fig. 7.5. Horizontal wind divergence computed from Doppler radar data is shown for two selected cases from the Lin and Mapes (2005) family of data sets. The right panel especially involved the clear passage of a nearly line-shaped[jjjj] wave of convective activity that is frequently observed to sweep from north to south in the Bay of Bengal during the summer monsoon (Fig. 4 of Webster et al. 2002). Although this is essentially a contiguous cloud system or "squall line", its propagation speed and spatial texture indicate that it may be better viewed as a convectively-coupled internal wave phenomenon. In addition to wavenumbers ½ and 1 (monopole and dipole heating profiles) from Fig. 7.4, the next-lowest wavenumber of the troposphere (3/2) can sometimes escape the event horizon of the high wavenumber damping in these multicellular MCS events. Again we see the spookily (spectrally) smooth wavelike shape that seems closer to mathematical dynamics solutions than to the turbulent mess of individual convective cells, reminiscent of the top photograph of Fig. 7.4.

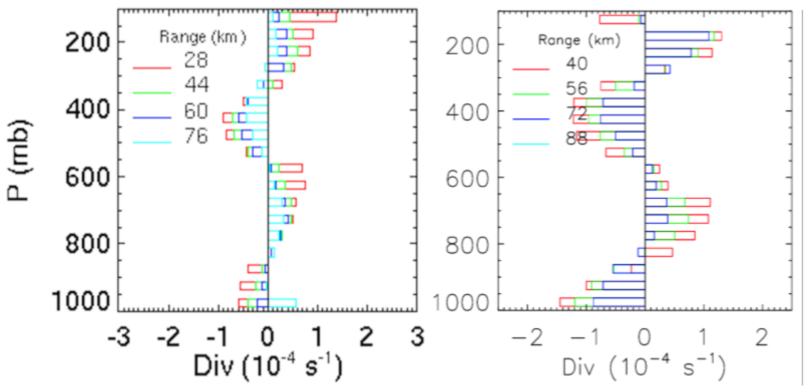


Fig. 7.5. Unusual wavenumber 3/2 profiles of horizontal wind divergence profiles in MCSs, measured by shipborne Doppler radars (as in Mapes and Lin 2005), averaged over 1 hour and over cylinders of air of the indicated radii (Range legends). Left: 1997-08-18, 22 UTC on the TEPPS campaign (Zuidema et al. 2006). Right: 1999-05-22, 16 UTC on the JASMINE campaign (more data in Fig. 3 of Zuidema and Mapes 2008).

jjjj https://atmos.washington.edu/~beth/MG/JASMINE/radar/19990522/19990522.html

Summarizing, the foregoing showed how the intimate interaction of updrafts with their predecessors' legacy (cold pools in the PBL, vertical displacements in the stratigraphy of the lower tropopshere) can lead to a very nonrandom dispatcher function and survival screening. These positive feedback mechanisms lead to multicellular “MCS” meta-entities that are at least two named scales bigger than the air “bubbles” that make up “cells”, although we begin to realize that a continuum view of scale would be closer to nature.

Do such meta-entities of convection still play the same simple teleological roles like *adjustment*, albeit perhaps toward a different target state or effective “moist adiabat”? Or do their additional realizability constraints, such as the significant inertia implied in horizontal flow branches, start to perform a whole different "job"? Might they even create new jobs that smaller-scale processes must perform, for instance by disrupting pre-existing balances?

x.5 Shear's help: focus, 2-dimensionality, supercell lift

Horizontal geometry has a continuum extending from 2D circles (ripples in a puddle, cold pools, etc.[kkkk]) to 1D linear structures which are the asymptotic sum of an infinitely-long aligned set of circles (by the https://en.wikipedia.org/wiki/Huygens–Fresnel_principle). In the middle ground of “one and a half dimensional” flow, arcs and line segments compete against these Platonic ideals of isotropic thermals and squall lines (section 5.4). How do such competitions play out?

Convection in shear tends to become arranged in more two-dimensional configurations, either across the PBL shear vector or along the midlevel shear (Robe and Emanuel 2001). Some near-field advantages of this concentration of updrafts into lines may include “mutual protection”[llll] of clouds from exposure to the dry environment (Randall and Huffman 1980), and boosting of the dispatcher function’s

[kkkk] Bretherton and Smolarkiewicz (1989) has the circular response to a mass source

[llll] a *behavioral* analogy to herd animals, but one honed through survivorship in past time

preferential production of larger updraft entities (Khairoutdinov and Randall 2006). More specifically, the downshear side of a cold-pool gust front is propped up taller and lifts inflow air especially deeply (Rotunno et al. 1988[mmmm]). Markowski and Richardson (2011) has lucid depictions of these mechanisms in particular weather situations. Individual thermals are also affected directly by shear through dynamic pressure effects (Peters et al. 2019).

How do convection of circular to linear geometry compare or compete? Using CCPMs, this question can be addressed. In one set of experiments measuring a CCPM's interaction with a larger-scale wave, varying convection's geometry by means of the periodic domain's aspect ratio, or with imposed shear (Riley 2010), reveal an intrigueing internal optimum. In a different kind of large-scale interaction harness, Anber et al. (2014) also found a nonmonotonic dependence on imposed shear: "For weak wind shear, time-averaged rainfall decreases with shear and convection remains disorganized. For larger wind shear, rainfall increases with shear, as convection becomes organized into linear mesoscale systems".

Might the favorability of "one and a half dimensional" convection be related to the vertical dispersion processes illustrated above? Deep or "gravest" vertical mode motions driven by buoyant convection consist of subsidence that is unfavorable for additional convection (although see chapter 8 for more discussion). Meanwhile the shorter vertical scales (higher wavenumbers) can be good for subsequent convection. Perhaps line segments are able to export their subsidence via quasi-3D dynamics, while the near-field and mid-field effects are concentrated by quasi-2D geometry, making them somehow more competitive than convection of either extreme.

The linear response function (sensitivity profiles) of Fig. 7.2 are also vastly different for long narrow domains that enforce squall-line geometry on the convection (see the other panels of Fig. 5 of Mapes et al.

[mmmm] a theory of extremes sometimes misinterpreted as a prediction of what is typical

2017), although the interpretive cautions there must be noted. The vapor climate of such squall-only atmospheres is drastically drier, so their increased moisture sensitivity is relative to that base state. The overall result may really indicate that squall convection can survive in much drier conditions (i.e. that it is *less* moisture-sensitive than isotropic convection).

x.6 Mid-distance interactions II: mesovortex effects

The mid-distance *T* effects on convective dispatch and survival discussed around waves in section 7.4 have direct analogs in balanced vortex flows (e.g. Raymond and Jiang 1990). Spectral dispersion is again involved, because different vertical wavenumbers achieve the force balance that defines a vortex on different scales or *deformation radii*. Tropical cyclone development stands as an important application for these effects, and a basis for defining organization in truly meaningful ways. Their longevity in a confined area makes vortices a much more tractable natural laboratory to see these effects than the transient waves of section 7.4. But cyclones also have additional dynamical complexities, such as different pathways horizontal transport of moisture (advection by the primary horizontally swirling flow), and frictionally controlled wind convergence in the PBL. A thorough discussion of these phenomena is beyond our scope here, not because it is uninteresting: quite the contrary, the literature is so large, and expressed in so many reasoning frameworks from practical to scientific, that a survey would run too long.

Monsoon depressions (Berry et al. 2012, Hurley and Boos 2015) are one notable phenomenon of convecting vortices receiving new attention from a broad fundamental perspective. But their scale (synoptic, bigger than dispatch-and-survival dynamics of contiguous cloud systems) shades into the next chapter's scale purview.

x.7 Problems and solutions

x.7.1 *Local rules, cellular automata, and the game of life*

A classic algorithmic game that is potentially relevant to convective cloud fields involves a two-dimensional grid of cells that can flip between two states (activated or suppressed), based on rules about the states of cells in its spatial neighborhood in the recent past. Examples can be searched under *cellular automaton* or *Conway's game of life* for instance. The analogy to convection could involve deep convection over a warm ocean. Downdraft outflow (cold pool) air masses are suppressed, until the passage of some recovery time (or distance, if a vector wind field were added to the game). But these also tend to activate new cell growth at their bounding gust fronts. Surprisingly complex emergent patterns can develop from repeated applications of simple deterministic rules within local space-time neighborhoods, while random number seeding and stochastic rules can further enhance the diversity of outcomes.

Locate a Jupyter notebook with the basics of such a computation, such as https://nbviewer.jupyter.org/github/gapatino/Cell-Automata-in-Jupyter-Notebook/blob/master/cellauto_jupyter.ipynb or a more advanced library package like https://gitlab.com/apgoucher/lifelib/tree/master. Devise rules based on scientifically plausible interactions in the PBL, perhaps motivated by published works such as Bengtsson et al. (2013) or others in its citation tree, and describe what patterns emerge. Can you make anything become resemble realistic convective cloud fields?

x.0 Chapter 8

Large systems from pooled far-field effects (Back to Outline)

Chapter 7 extended its cell dispatch-and-survival reasoning only up to the largest contiguous condensate-containing air bodies, limiting itself to “near field” and “mid-distance” interaction mechanisms. The resulting phenomena are still often viewed as discrete: the term *mesoscale convective system* (MCS) is essentially a generalization of the too-specific entity model of section 5.4, the 2D *squall line*.

Once the visual boundary of contiguous cloud is lost, definitions of systems or entities become more arbitrary. Reasoning falls back into scale-filtered continuum ideas from chapter 3, such as *wave packets*. The book’s march from continuous fields (Part I) to discrete entities (Part II) to sets of entities (Part III) must now take this half-step backward. Here the whole vast field of *large-scale meteorology* rears its head, deserving of the moniker *convection* only in the strained sense that [*bw*] is the ultimate energy source for all motion. Still, a book on atmospheric convection would be woefully incomplete without considering how such flows fit in the reasoning framework here. *All scales convect* was the key point of the discussion in section 3.6, and even a visible-cloud maximalist sees that large-scale motions shape cloudiness fields profoundly.

x.1 General nomenclature of interactions in systems

In partial differential equations (section 1.2.1), the temporal rate of change of a spatial field (local *tendency*, customarily on the “left-hand side” LHS) is equal to the sum of a set of *partial tendencies* (terms on the “right-hand side” RHS).

When we assign identity to a macroscopic *system*, some *pattern* or *entity* worth naming, its size necessarily lies somewhere between the largest scale (the domain mean value, wavenumber 0, making global temperature a science of its own) and too-small scales which we care about only in aggregate or statistically. Such an entity may be defined and measured variously. In this realm beyond contiguous clouds, it may be a pattern like a set of adjacent anomalous positive and negative patches (wave packet), which might be measured statistically as a contour of the variance in some filtered set of wavenumbers of the field. It has some history of past entity- or pattern-shaped tendencies that created it; and it contributes to the present partial tendencies of the field through its shaping of physical processes. Figure 7.1 illustrates the terminology and logic of temporal flow in this description.

In figure 7.1, the pattern or entity is depicted in the middle of the left-to-right dimension of decreasing scale (Fourier wavenumber for instance). The entity can inter-act with itself (lending it a meaningful identity persistence across time), or with other entities or patterns of similar and larger and smaller scales. In addition, the entity is subject to tendencies not labeled as other entities, which can be viewed and treated formally as background *forcing* (positive or negative) or *noise*.

If coherent entity-relative tendencies are *in phase* with existing entity-defining patterns (the parallel symbol $\parallel$ in Fig. 7.1), they make the entity *amplify* or *decay* according to sign. If they are in spatial quadrature (the perpendicular symbol $\perp$), they will cause entity *propagation* in space. If they are positive around some blob-entity's perimeter (symbol $\odot$), the entity will *grow* or *shrink*. If they are simply uncategorizable (denoted by $\not\approx$), we can only say that the entity or pattern will *evolve*, perhaps eventually causing a loss or change of identity.

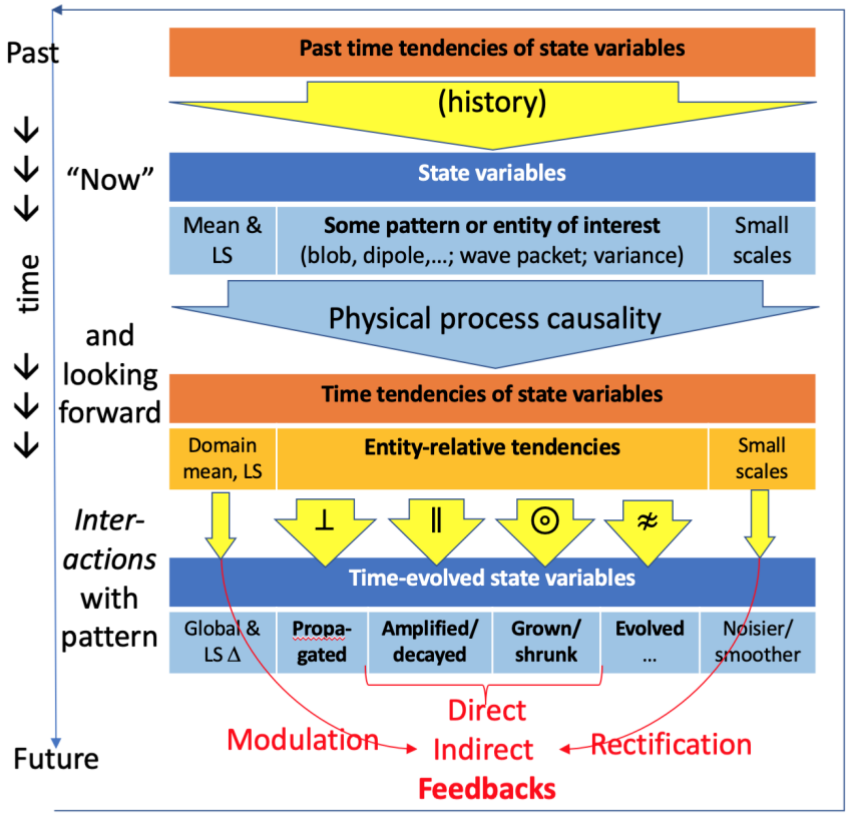


Fig. 7.5. Illustration of the cycle of time integration of partial differential equations, with large scales at left and small scales at right relative to the scale of an "entity" or pattern shown in the center. Time runs downward, but then it cycles back to the top as future becomes past for the central "Now". These relationships define the terms *tendency, state variable, pattern or entity, physical process, interaction, scale, propagate, amplify, decay, grow, shrink, evolve, noise vs. smoothness, feedback, modulation, rectification*, as narrated in the text. State variables are the *fields* in Part I's equation sets.

The word *feedback* is another word for *self-interaction*, referring to any process by which entity-caused tendencies affect the entity going forward. In addition to *direct* entity-scale feedbacks (to amplify, propagate, grow, evolve), larger-scale tendencies can *modulate* entities (like by destabilization in the case of convection). Smaller-scale tendencies (like shear-driven turbulence on the flank of a convective updraft) can have a net or *rectified* entity-scale effect, such as diluting that updraft and reducing its buoyancy. Such across-scale feedbacks are indirect, which can be just as real and important as direct feedbacks.

Modulation is a large scale's impact on a smaller scale. Besides the modulation of a bulk amount of instability (perhaps an available energy measure), modulation can operate in the difficulty of meeting the condition of conditional instability (a probability envelope of the dispatcher function, and/or a survival likelihood pattern for dispatched bubbles or cells). All of these enhancements of convection's success rate can be called *destabilization*, in the practical or functional sense of "instability" as *whatever conditions lead to convection.* As chapter 0 emphasized, no single scalar measure of (in)stability is sufficient: reasoners must allocate mental space for a few distinctions.

A classic illustration of modulated probability of achieving the condition of parcel instability is Fig. 8.1, adapted from Crook and Moncrieff (1988). The "large-scale convergence" in this diagram was forced by an artificial specified momentum source term in their model, perhaps akin to a wave process.

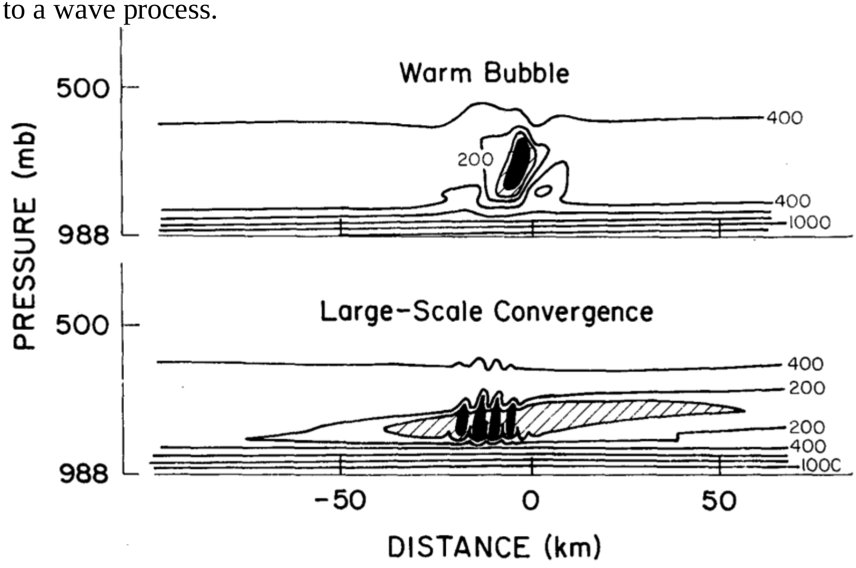


FIG. 7. The vertical distance (in meters) that air needs to be lifted to its level of condensation for the two different methods of initiation, warm bubble and large-scale convergence. The solid regions are regions of cloud, the hatched regions indicate air that has to be lifted less that 100 m.

Fig. 8.6. Figure 7 from Crook and Moncrieff (1988), illustrating the difference between a mere triggering event (in their case, a warm bubble) and a 100 km scale modulation of the conditional instability itself; mainly an easing of the conditionality, as expressed by the contours of how far a random air motion has to rise to become a cloud.

The above example emphasizes *positive* interaction, and is part of runaway dispatch-plus-survival aimed at MCS entities like chapter 7, but the same kind of effect can occur on larger scales in larger *statistical envelope* forms of convectively coupled waves (Kiladis et al. 2009). Other work emphasizes *negative* interactions, for instance in Arakawa and Schubert's (1974) competition for a scarce univariate resource by entities assumed to be independent, encountered in chapter 6 and revisited in chapter 9.

x.2 Deep tropospheric rotational entities

The dispersion of internal wave motions by vertical wavenumber was discussed in chapter 7 from the near and middle-distance perspective, emphasizing short vertical scales which remain behind. But the deepest, lowest wavenumber motions (or *gravest modes*) of latent heat release events spread and blend their influences over vast areas, to powerful effect. Large-scale dynamics reasoning gets quite far with so-called *intermediate complexity* models based on a single internal or *baroclinic* vertical mode of the troposphere, augmented perhaps with an external or *barotropic* mode and a planetary boundary layer that helps couple and damp these by incorporating frictional effects[nnnn]. Such models represent only the deep far-field effects of latent heat release events, pooled (in the sense of *superposition* or *constructively interference*) and turned by the Coriolis force to become the great jets and vortices that of atmospheric dynamics. The shear and deformation of those flows further blends (pools) evidence of the localized cloud "entity" influences that collectively help to drive and sustain them.

[nnnn] Examples include complete models (Neelin and Zeng 2000, named for a convective assumption in its framing), and mere “response” models (Lee et al. 2009).

At that scale, pooled heating from many latent heat releases and broader, smoother radiative cooling of clear areas may more fruitfully be viewed as an almost symmetric form of convective overturning on planetary scale, not just an aggregated view of the clouds and positively skewed vertical motions in the heated, rising branch of that circulation. Here is a *general circulation* view of the atmosphere's motion, usually considered separate from "convection" in the field's categorization of its topics, e.g. in books with convection relegated to sub-chapters (e.g. Vallis 2017).

Even though large-scale flows are primarily rotational (horizontal) on our rotating Earth, they can still modulate the ingredients for cloudy convection. For instance they can directly advect patterns and gradients of moisture crucially (Pritchard and Bretherton 2014). Also, the maintenance of horizontal force balances in rotational *primary circulations*, with their great longevity rooted in the conservation of vorticity, implies (teleologically) the existence of divergent *secondary circulations* as expressed in the classical *quasi-geostrophic* paradigm for synoptic-scale vertical motions in middle latitudes. Such vertical motions strongly modulate cellular and mesoscale convection through its gravitational instability resources, in addition to sometimes producing laminar clouds and precipitation in winter storms.

On planetary scales, flow evolution and weather are driven as much by momentum instabilities in these *flywheel*-like rotational kinetic energy reservoirs as by gravitational instability *per se*. For instance, tropopause-level or *cool core troughs* -- lobes or tentacles of the great polar reservoirs of vorticity and its thermally driven source field *potential vorticity* -- dominate the daily weather cycles of the midlatitudes and outermost tropics. Again their impacts include both direct advection by horizontal winds (especially across prevailing north-south planetary gradients), and the vertical motion in secondary circulations. Those motions are often *thermally direct* with [*bw*]>0, a form of convection in that sense (e.g. chapter 7 of Wallace and Hobbs 2006): *all scales convect* on the rotating sphere as well as in the flat models of section 3.5. An education in atmospheric convection is incomplete without understanding the above concepts and terms from large-scale meteorology, although elaborating them here is beyond our scope. Such processes in the subtropics are weaker than in midlatitudes, as are the Coriolis force and the north-south gradients, but since the mean state at low latitudes is closer to the conditions defining functional instability (and thus driving real convection), rotational momentum features such as *easterly waves* modulate tropical convective weather strongly.

As rotating tropical disturbances get smaller and stronger, they also engage frictional convergence as flow descends their surface pressure gradients into concave low centers. In addition, their intensifying wind speeds draw large fluxes of heat and moisture from the massive thermal reservoir of the upper ocean, down to 100 m or more where a mere 2.5 meters has the same heat capacity as the entire atmosphere. Such fluxes destabilize air columns strongly (adding many times the energy-area per day of the globally typical orange square on Figs. 4.1-4.2), and condensation and rainfall more than match that surface evaporation. The whole troposphere behaves as a nearly-saturated, nearly moist adiabatic mixed layer. At that point the most meaningful sense of "convection" is really on the scale of a cyclone and its "moat" of suppressed cloudiness often seen in satellite imagery, even though the internal cloud field is turbulently cumuliform when visibility permits glimpses of it.

Such tropical cyclonic systems are so long-lived and coherent, and so distinctively individual in character (for instance spanning more than an order of magnitude in diameter) that they genuinely deserve the human names we bestow on them. In a sense, tropical cyclones belong as another class of entity, following on beyond those section 5.4, whose competition and coexistence with more ordinary convection are part of the convective ecosystem dynamics to be considered in chapter 9.

(MORE CITATIONS NEEDED - REVIEWS? TEXTBOOKS?)

x.3 Negative entities: top-down radiative dry holes

A moist convecting atmosphere, full of bottom-up convection driven by broad radiative cooling of air and intense inputs of energy at the surface (section 0.2), is unstable on a larger scale. Columns of air dried by subsidence (that is, by negative vertical advection of *q*) continue to undergo radiative cooling, because moist air's longwave emissivity decreases only slowly (roughly logarithmically) with dryness. Meanwhile, cloudy convection is much more sensitive to *q*, so its existence can be quenched entirely in these dry columns.

The intensification and spread of such runaway radiatively-cooled dry downdrafts, if they are somehow protected from moistening by wind shear or other forms of lateral transport for many days, concentrates the cloudy convection in the humid interstices, where high clouds limit infrared cooling and wind convergence brings together the vapor gathered from evaporation in the broad dry areas. Based in part on our observability bias toward clouds (section 0.6), this phenomenon of their forced confinement has come to be called "self-aggregation". This is arguably the field's most recent example of nomenclature that misdirects, but it cannot be undone: the literature now runs into the hundreds of papers, already the subject of reviews (Mapes 2016, Wing et al. 2017, Wing 2019). Other mechanisms and additional details of the radiative cooling process have been extensively examined, mostly in highly idealized situations, although with some notable efforts to seek some evidence for the phenomenon's mechanisms in nature (Holloway et al. 2017).

x.4 Problems and solutions

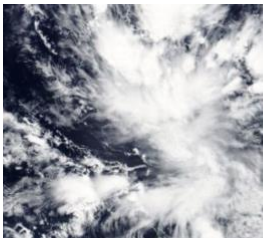
A balanced vortex problem. Frictional convergence vs. surface flux in moisture or MSE budgets (Rosenthal or Ooyama cyclone, and Emanuel's re-envisioning?).

Are the old classic models toys yet, that we can put into Jupyter notebook exercises?

exercise numbering

x.0 Chapter 9

Entity games: competition, coexistence, collaboration (Back to Outline)

The continuum equations of Part I are a statistical treatment of vast ensembles of molecules. That field accounting underpinned the construction of discrete, macroscopic convective entities in Part II, from updraft *parcels* to *thermals* or *bubbles* to multi-bubble *cells*, and on into multicellular and even gappy (partly cloudy) *systems* (chapters 7-8).

This chapter explores whether a statistical view of ensembles of entities can add new appreciation or useful tools -- a *parameterized* view, in the scientific or engineering senses of that term, respectively (Arakawa 2004). The engineering problem is to design better parameterization *schemes*: low-cost convection emulators to run inside scale-truncated (coarse grained) digital atmosphere models for weather and climate. Scientific goals could include formulating field equations for *moist dynamics*, encompassing the *expected* (mean and perhaps also stochastic) actions of ensemble of convective entities. If statistics govern tails as well as means, perhaps extremes could be anticipated as well.

All scales convect in a Fourier sense, with each wavenumber exploiting [*Qb*] as its total energy source and [*wb*] as its KE source (section 3.6). In this view, the scales are merely *superimposed*: added together as a set of orthogonal basis functions, independent and non-interacting. In this logical framing of decomposition, the upward motion in any given convective cloud is *composed of* both high-wavenumber motions expressing runaway parcel buoyancy, plus lower wavenumbers expressing more persistent updrafts of broader scale. The *modulation* discussion in section 3 teaches us the fundamental ambiguity or non-uniqueness of this additive view (section 3.4, exercise 3.8.3). Perhaps broad updrafts play their role through *modulation* of instability in the sense of section 8.1, while their own heat and moisture budgets are satisfied by *rectification* of the net condensation implied by the positive definiteness of precipitation[oooo].

In physical space, positive definite precipitation is produced both by progressively deepening cumuliform updrafts and by associated stratiform clouds and precipitation (Houze 1997). Together, these elements of convective cloud systems function as a gappy or stochastic condensation scheme for larger-scale updrafts. They perform this function just by being entities whose air parcels obey the local laws of motion and thermodynamics, but in the presence of larger-scale or ambient profiles of *b*, *q*, etc. that subtly shape their evolutions rates and fates.

A causally chained combination of these elemental entities (shallow convection spawning deep convection spawning stratiform precipitation) appears to act as a basic *building block* of larger-scale flows. However, the sub-parts of the building block (shallow, deep, stratiform) are sometimes stunted (e.g. shallow only), or systematically extended (e.g. with delayed transitions) or elongated or retracted (e.g. by the longevity of stratiform entities), according the phase of larger-scale waves they occur within (those ambient profiles of *b*, *q*, wind shear, etc.). The summed effect is a nearly self-similar (building-block-shaped) structure

[oooo]with vertical heating profiles modestly reshaped by eddy fluxes, as in Yanai et al. (1973)

on larger scales (Mapes et al. 2006), remarkably -- or perhaps unsurprisingly, if basis sets limit the shapes of their combinations.

Some scale dependence of the larger-scale waves does occur, through aspect ratio. For instance, with tropospheric depth being the same for all horizontal scales, longer wavelengths require more inertia (imparted by the PGF) and kinetic energy (generated by [*wb*]) in the long horizontal flow branches, and suffer more surface drag on those long wind fetches (Kuang 2011, 2012). The greater horizontal wind speeds implied by spatially broader patches of divergence (and the vorticity it generates) introduce a scale dependence to wind-induced surface flux. Both wind speed, and the distances traversed on a nonuniform Earth, shape horizontal advective tendencies, so the relative magnitude of fluctuations of a prominently advected quantity like *q*, compared to *b* which is shaped mainly by vertical displacements, increases systematically with disturbance wavelength or period (Mapes et al. 2006). The largest-scale tropical wave (the Madden-Julian Oscillation) is thought to be largely *q*-governed), and almost of a qualitatively different character than the higher-frequency waves (thought to be more *b*-governed by a vertical dipole mode, as detailed in earlier chapters; Kiladis et al. 2009 is a good starting point for more detailed readings).

Another prominent and entirely different *category* or *manifold* of convective systems must be noted: moist convecting cyclones (noticed in sections 7.6 and 8.2). Bottlenecks to their spontaneous formation make these structures a distinct regime of behavior, lying past a developmental portal called *cyclogenesis* (e.g. Nolan 2007). Such rotational wind systems bring in a new fundamental scale: the *deformation radius* (the depth of the flow multiplied by N/vorticity, a ratio of order 10^2 in the ambient subtropics for merely planetary vorticity *f*). Despite this *minimum* scale of geostrophically balanced cyclones, the diversity of actual sizes is breathtaking (Fig. 9.1), and scientifically daunting.

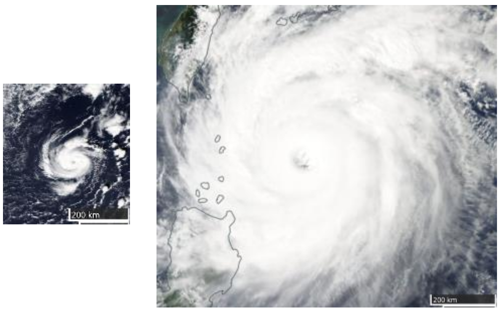


Fig. 9.1. Hurricane Pablo of 2019 (https://go.nasa.gov/2BV4Rfs) and typhoon Jelawat of 2012 (https://go.nasa.gov/31ReAxH). A same-length 200 km scale is in lower right of each image.

x.1 The dream of a governing statistical principle

Our daunting diversity of entities, including a *hierarchy* of overlapping classes thereof, invites analogies to disciplines like ecology or economics more than to the elegant and precise project of statistical mechanics, as foreshadowed in sections 0.5 and 3.7. In thermodynamics, entity interchangeability is fundamentally true: profound uncertainty principles of atomic physics correspond perfectly with expressions of our profound lack of knowledge (emboded in *maximum entropy statistical inference)* to beautifully explain the long-known empirical laws of thermodynamics (Jaynes 1957, Mohling 1982).

Some authors with a maximally physics-minded worldview have tried to reach beyond mere analogies, to treat planetary-scale flow dynamics as an *extension* of thermodynamic laws. For instance, a *maximum rate* optimum principle for entropy S can give real teeth to the Second Law's famously enigmatic inequality $dS/dt > 0$. Works reviewed in Paltridge 2001 and Ozawa 2003 offer arguments that nature may be somehow enforcing this optimization in the shaping of fluid motions on macroscopic scales, up to the planetary. But is there a compelling chain-of-causality reason to hope that the enforcers of thermodynamic laws (quantum mechanics, atomic attractions and repulsions) reach through the intervening scales to shape convection? Because gravity is the main force at play in convection, and remains unreconciled with the other forces in physics at the root, such a case for *supervenince* (discussed below) is far from obvious. Still, we would happily settle for a decent analogy if it comes with useful tools.

Several meanings of the word "entropy" are invoked in atmospheric science (reviewed in Liu et al. 2011). Its original 19th century sense reflected energy that is *unavailable for macroscopic work* in heat engines (in contrast to *available* or *free* energy[pppp]). This sense of entropy has clear implications for the *efficiency* of the gaseous "heat engines" of weather systems (Emanuel 2005 has a lucid, accessible discussion). Meanwhile, information-theoretic entropy, a scalar measure of the breadth of probability distributions best translated as *missing information* (Ben-Naim 2008; see exercise 3.8.1), obeys a calculus that seems to alchemically transform lack of information into knowledge, as mentioned above (Jaynes 1957). Might information entropy or free-energy accounting be repurposed usefully in our problem of understanding atmospheric flows that are full of convective entities?

x.2 Entropy of mini-macro scales, without the microscopic

How many convective entities inhabit Earth’s skies? Her 500 million square km could hold about that many PBL *large eddies* spanning a typical PBL depth of order 1 km. Since the troposphere is roughly 10x deeper, deep convective "cells" are at least 100x fewer -- and far fewer than that, since their descent regions are far larger than the updrafts[qqqq]. Riehl and Malkus (1958) inferred that as few as 1500-5000 "*hot towers*" of aircraft-observed proportion and properties might do the necessary climatic job of deep convection, before satellites exposed the Earth to our gaze (Fleming 2007). That gaze and other modern tools have transformed thought at least once since then (Zipser 1971), but historical cruft in conceptual space may yet remain to be scoured out with today’s high-volume data firehoses, if correctly deployed.

Could a production-rate maximization principle about *thermodynamic* entropy, even if broadly correct, be so exacting that it constrains the configurations of mere millions of “mini” convective entities we would like to treat statistically? It seems vain to hope so, if

[pppp]Free energy also invites wide-ranging analogizing and extension, for instance in neuroscience (http://en.wikipedia.org/wiki/Free_energy_principle); see Colombo and Wright (2018)'s eloquent summary and discussion why its "epistemic status is unclear."

[qqqq] The “slice method” of Bjerknes (1938) offers one classic way to reason why.

that entropy of *mini-macroscales* is heedlessly lumped in with the abyss of missing information about all > 10^{18} x 10^{23} air molecules[rrrr]. Even pooling convection data over time, or in the Reynolds notion of an ensemble of equivalent large-scale flows with different turbulent details, or with the numbers boosted by overlapping entities on a hierarchy of scales, the molecular missing information utterly dwarfs the number of possible convective weather configuration. Still, on the other hand, thousands or even mere hundreds of entities may obey statistics stable enough to build successful parameterization theories upon, especially if the finite-sample stochastic effects are viewed as part of the desired answer. Might there be some fruitful way to invoke some kind of entropy or free-energy principles in a statistical description of only those *mini-macroscales*, as an *analogy* to thermodynamics rather than an extension?

To realize such an analogy, we should specify what mini-macroscopic entities we wish to treat statistically, and discretize some abstract state space of their possibilities[ssss]. A key step would be to postulate the existence of an analog for *temperature T*, or its energy equivalent $k_B T$ with Boltzmann's constant k_B. A broadband (many-scales) turbulent kinetic energy (*TKE*) seems an obvious candidate for this vigor of mini-macroscales. We must postulate that this *T*-like vigor field is well-defined and smooth on the desired large-macroscopic scales. That mini-scale vigor is both the rate at which trial-and-error motions are probing the combinatorics of complex (for instance, multi-cellular) entities in the unlikely corners of state space, and also the rate at which any such complicated structure, if frail, is shaken or stirred out of existence.

Such a *T* analog could be multiplied by an entropy measure of the unlikelihood of self-assembly of complex structures, like the $-T\Delta S$ term in Gibbs free energy or exergy[tttt] change during a chemical transformation. Pursuing the analogy, that term should be added to an analog to a ΔH term measuring the energy that rare structures can access

[rrrr] kilograms of air, and molecules in a mole

[ssss] information theory does not easily extend to continuous probability distributions

[tttt] https://en.wikipedia.org/wiki/Exergy

if they prove more efficient at gravity's energy-supplying purpose of lowering the atmosphere's overall center of gravity. Just as the increment or decrement of such a $\Delta G = \Delta H - T\Delta S$ quantity[uuuu] determines whether a chemical process will occur spontaneously, might the spontaneous yet gradual evolution of *organization*[vvvv] in convective cloud fields could be fruitfully viewed in this chemical analogy? A self-catalyzing reaction in an open system consisting of just a modest number of molecules seems the analog, but must be generalizing for *rates* of transformation and energy production rather than begin and end *states,* befitting the nonequilibrium and open system dynamics of a convecting patch of air.

It complicates things that our "entities" are of a wide range of size or even categorical classes. One physics analogy that has been explored for convection is *critical phenomena* that exhibit *phase transitions* in their *order parameter* or field, generating a possibly very diverse size distribution of clusters or patches (e.g. Peters and Neelin 2006). But because *all scales convect* in the atmosphere (section 3.6)[wwww], our entities overlap as a *hierarchy*, not merely a set of adjacent patches in space.

x.3 Hierarchical ecosystems: concepts and frameworks

Biology in general, and especially its largest and loosest agglomeration scales (ecology, and arguably social sciences as an extension thereof), must also contend with overlapping categories of entities. Sometimes individuals of one species interact importantly with whole *populations* of a smaller-scale species (from our microbiomes to our foodstuffs). Sometimes stochastic and even quite rare encounters with single individuals of comparable scale are crucial (mates, lions). Actually, for an omnivore with many vulnerabilities, the portfolio of

[uuuu] https://en.wikipedia.org/wiki/Gibbs_free_energy

[vvvv] a concept for which we sorely need a meaningful definition and measure

[wwww] Not inevitably: compare the narrow range of sizes in the "mesogranulation" of solar photosphere convection (e.g. Rincon and Reiutord 2018, Kessar et al. 2018).

consequential interactions may better be described in terms of higher-abstraction-level *classes of populations* (foods; predators; infectious agents), with the specific species of protein or fangs or germs perhaps interchangeable from the point of view of an entity or population in question. How does quantitative ecology span these wide ranges of scales and even categories of interacting and overlapping entities?

x.3.1 *Competition and cooperation in predator-prey models*

A classic paradigm for categorical entity interactions in population ecology is the Lotka-Volterra equation[xxxx]. It describes the time evolution of the number n_i of individuals in a population of entities of category *i*, within some domain that must be specified, interacting with populations of entities of N other categories (indexed by *j*), here written following Eq. (9) of Nober and Graf (2005):

$$d/dt(n_i) = F_i n_i + \sum_{j=1}^{N} K_{ij}\, n_i n_j \qquad (7.1)$$

If the "food" or forcing coefficient F_i is positive, population *i* grows exponentially, in the absence of interactions (that is, if K_{ii} = 0). If diagonal terms K_{ii} are negative, purely negative interactions occur with other individuals of the same category *i*. Then ($F_i + K_{ii}\, n_i$) <0 when the population gets too large, and the no-other-species-interactions solution asymptotes to $n_i = -F_i/K_{ii}$. More interesting possibilities lie with off-diagonal terms in K_{ij}. Depending on their signs, the system can express purely cooperative *mutualism* interactions ($K_{ij} > 0$), purely *competitive* negative interactions ($K_{ij} < 0$), or a mixed regime of *predator-prey* dynamics ($K_{ij}K_{ji} \leq 0$) that acts to redistribute the number of entities among categories.

A similar "master equation" for convective entities (Eq. 5 of Hagos et al. 2018) allocates "pixels" among entities of types i and j, emphasizing only the off-diagonal terms. With suitable re-weightings, the types could be measured in terms of more continuous stuff like an embodied energy

xxxx https://en.wikipedia.org/wiki/Competitive_Lotka-Volterra_equations resource or its production rate (biomass or metabolic vigor, in the ecological problem), rather than by a number count. All of this can be normalized per square km of the Earth's surface, in either biological or convective terms, while recognizing that large organisms require a large enough domain to exist at all -- a problem that vexes scale separation thinking as discussed below.

All this is only accounting. The question is when and why and how exactly it is *useful* to define entities, and categories of entities, and feedbacks with *self* and with *fellow* category members, and interactions with the truly *other*. In other words, can tallying discrete entities in continuous air truly help us develop a predictive account or deeper appreciation of the natural phenomenon of convection?

The Lotka-Volterra equation discussion of Nober and Graf (2005) continues: “The analogy to convective clouds is straightforward. The reason for convective clouds to form is convective instability (‘food supply’) ... each cloud type acts on its environment and tends to reduce instability. Therefore each cloud tends to reduce somehow the ‘food-supply’ for all other cloud types including itself.” Ramirez et al. (1990) also adopt this utterly competitive view: “It is observed that the major contribution to the environmental stabilization comes from the drying of the planetary boundary layer induced by subsidence. The thermodynamic effect of nonprecipitating and precipitating convection is to reduce CAPE in the surrounding environment and hence reduce the conditional probability of further convection nearby. A new ... inhibition hypothesis states that, under completely homogeneous external conditions and assuming a spatially random distribution of cloud-triggering mechanisms, the spatial distribution of cumuli in the resulting cloud field must be regular, as opposed to either random or clustered, because cumulus clouds tend to reduce the available energy for convection, thereby inhibiting further convection nearby.”

Such pure-competition views echo the ideas of Arakawa and Schubert (1974, AS74) framework of *cloud work function* (a CAPE to KE conversion, categorized by "cloud type" or size and even expressed in an interaction kernel like K_{ij}). A structural shortcoming flows foreseeably from its ecological postulate of a univariate scarce resource: The largest permitted plume or bubble tends to dominate, in the absence of dispatcher conditionality or developmental limitations, as discussed in chapter 6. The implementers of parameterization schemes have cleverly battled this propensity ever since. The exquisitely elaborated but debatable physicality[yyyy] of the early masterpiece (Arakawa and Schubert 1974) has inspired generations. Another artifact of symbol-manipulation software enabled analytic theory, presented for its texture (a sense of complication level) rather than its face-value meaning requiring symbol definitions, is this majestic equation in Yano and Plant (2016, their

$$\begin{aligned} K_{\mathrm{ij}} &= \int_{z_B}^{z_{\mathrm{Ti}}} \eta_i \eta_j \left(-\alpha + \epsilon_i \tilde{\mathfrak{a}}_i \frac{\hat{\eta}_i}{\eta_i}\right)\left[\delta_j (s_{\mathrm{vj}}^D - \overline{s}_v) + \frac{\partial \overline{s}_v}{\partial z}\right]\mathrm{dz} + \\ &\int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i \hat{\eta}_i \eta_j \left[\tilde{\mathfrak{b}}_i + \tilde{\mathfrak{c}}_i - \frac{\tilde{\mathfrak{d}}_i}{1+\gamma}\left(\frac{\tilde{\eta}_{l_i}}{\hat{\eta}_i}\right)\right]\left[\delta_j (h_j^D - \overline{h}) + \frac{\partial \overline{h}}{\partial z}\right]\mathrm{dz} + \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i \tilde{\mathfrak{d}}_i \tilde{\eta}_{l_i} \eta_j \left[\delta_j (s_j^D - \overline{s}) + \frac{\partial \overline{s}}{\partial z}\right]\mathrm{dz} \\ &- \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i L_v \tilde{\mathfrak{d}}_i \tilde{\eta}_{l_i} \eta_j \left[\delta_j (l_j^D - \overline{l}) + \frac{\partial \overline{l}}{\partial z}\right]\mathrm{dz} \end{aligned} \tag{6.9a}$$

$$\begin{aligned} \left(\frac{\partial A_i}{\partial t}\right)_L &= \rho_B \left[\tilde{\mathfrak{a}}_{\mathrm{iB}} \frac{\partial}{\partial t} s_{\mathrm{viB}} + (\tilde{\mathfrak{b}}_{\mathrm{iB}} + \tilde{\mathfrak{c}}_{\mathrm{iB}}) \frac{\partial}{\partial t} h_{\mathrm{iB}} - L_v \tilde{\mathfrak{d}}_{\mathrm{iB}} \frac{\partial}{\partial t} l_{\mathrm{iB}}\right] + \left[\int_{z_B}^{z_{\mathrm{Ti}}} \rho L_v \tilde{\mathfrak{d}}_i \left(\frac{\eta_i}{\tilde{\eta}_{l_i}}\right) \dot{c}_0 \, \mathrm{dz}\right] l_{\mathrm{iB}} + \\ &\int_{z_B}^{z_{\mathrm{Ti}}} \left[\epsilon_i \tilde{\mathfrak{a}}_i \hat{\eta}_i \left(\frac{\partial \overline{\tilde{s}}_{\mathrm{vi}}}{\partial t}\right)_L - \alpha \eta_i \left(\frac{\partial \overline{s}_v}{\partial t}\right)_L\right] \rho \mathrm{dz} + \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i \hat{\eta}_i \left[\tilde{\mathfrak{b}}_i + \tilde{\mathfrak{c}}_i - \frac{\tilde{\mathfrak{d}}_i}{1+\gamma}\left(\frac{\eta_i}{\hat{\eta}_i}\right)\right]\left(\frac{\partial \overline{h}}{\partial t}\right)_L \rho \mathrm{dz} + \\ &\int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i \tilde{\mathfrak{d}}_i \eta_i \left(\frac{\partial \overline{s}}{\partial t}\right)_L \rho \mathrm{dz} - \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i L_v \tilde{\mathfrak{d}}_i \eta_i \left(\frac{\partial \overline{l}}{\partial t}\right)_L \rho \mathrm{dz} + \int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i L_v \tilde{\mathfrak{d}}_i \eta_i \frac{\partial}{\partial t}\left(\frac{\rho \sigma_i}{\epsilon_i M_i} \tilde{r}_i\right) \rho \mathrm{dz} + \\ &\int_{z_B}^{z_{\mathrm{Ti}}} \epsilon_i L_v \tilde{\mathfrak{d}}_i \eta_i \int_z^{z_{\mathrm{Ti}}} \dot{c}_0 \, \mathrm{dz}' \, \rho \mathrm{dz} + \dot{z}_{\mathrm{Ti}} G_{\mathrm{Ti}} + \dot{z}_B G_{\mathrm{Bi}} \end{aligned} \tag{6.9b}$$

equation 6.9):

Is such an artifact a pinnacle of knowledge, or the outer edge of human-accessible meaning? Might more general possibilities about the whole problem have gotten lost behind by the specific but peculiar nature of the parameterization *scheme design* problem, attempting to minimize the bleeding from the scale-truncation wound forced upon us by a scale separation assumption in coarse-resolution numerical models?

[yyyy] Presented as a partitioning of *energy* (as clarified in Lord and Arakawa 1980), but the non-orthogonality of “cloud type” (plume radius) categories means that attempting to distribute physical energy over that basis set lacks true rigor.

x.3.2 *A statistical thermodynamics analogy to ecology*

One bio-physics attempt to frame and then model ecological succession (Würtz and Annila 2010) envisions an ecosystem of multiple entity types, each with its numbers of indistinguishable individuals. In that paper, a single ambient energy level (perhaps literally $k_B T$, in the view where ecosystems supervene on chemistry and physics[zzzz]). For each species of life (indexed by *j* or *k*), they measure an embodied or intrinsic Gibbs energy ΔG_{jk}, so that just as in chemical transformation the probability kernel for a spontaneous production of category or species *j* from species k is $e^{-\Delta G_{jk}/k_B T}$. The total probability for category j, accounting for the numbers of interchangeable (for this purpose) individuals N_k in each of the *k* necessary ingredients, and with a "degeneracy" g_{jk} that "numbers the *k*-ingredients that remain indistinguishable in the assembled *j*-species", is stated in Würtz and Annila (2010) as:

$$P_j = \left[\prod_k \left(N_k e^{(-\Delta G_{jk} + i\Delta Q_{jk})/k_B T} \right)^{g_{jk}} / g_{jk}! \right]^{N_j} / N_j!$$

Ecosystems subsist on an external influx of free energy, whose projection on the jk-transformation process those authors measure as ΔQ_{jk}. Since this influx is "orthogonal...to the system's scalar potential energy differences [ΔG_{jk}] between *j*- and *k*- repository", they use an imaginary notation with *i* from complex analysis[aaaaa], rooted in the non-equilibrium thermodynamics of open systems (Annila 2016).

Those authors go on in describing this equation's logic: "The product N_k ... ensures that if any of the *k*-ingredients, e.g., food supplies, is missing, no *j*-species may appear in the ecosystem (P_j = 0)."

[zzzz] The limitations of such "supervenience" arguments bear pondering again (Hoel 2018).
[aaaaa] I confess to confusion about the level of rigor of this application of complex numbers.

Furthermore, "the numerator is raised to the power of N_j because the process may combine the vital *k*- ingredients into any of the indistinguishable *j*-individuals in the population N_j. As usual, the division by the factorial N_j! means that the combinatorial configurations of the j-entities in the ecosystem are indistinguishable."

Details aside, such an equation gestures at how discrete *categories of entities,* each with a *population,* can be accounted with some rigor, if a Gibbs free energy-like scalar measure can be devised to adjudicate their likelihood, through its ratio to some sufficiently well-defined background energy of random statistical motions ($k_B T$ or perhaps some *TKE* in our analogy, as discussed above).

Might such a formalism be useful for a game theory of interplay of overlapping diverse entities, from bubbles to cells to multicellular storms to gappy groups of them? Does the whole hierarchy of structures play a role in convection doing its teleological ‘job', facilitating gravity's project of lowering the center of mass? Or might convection of such diverse scales and types actually have *multiple* jobs? After all, living things within ecosystems express more than a single purpose, exploiting the myriad mutually-contingent niches created by other living things. Even if an ecosystem obeys an optimum principle, like in the equation above for Würtz and Annila's (2010) univariate "energy dispersal" view of ecology, their solutions are inherently initial condition dependent. Rightly so, given the phenomena targeted for explanation: Earth’s long-optimized ecosystems can be as distinct as Madagascar and Hawaii, despite similar gross energy and entropy inputs and outputs. Section 9.1's dream of a "governing" statistical principle may not actually be sufficient to predict behavior better, even if it were exactly true and well known!

x.4 Can we reject a random patterns null hypothesis?

What would a scientifically minded person, with the continuum equations of Part I and entity notions of Part II to draw on, expect the world to look like, before the first satellite gave us the grand overview we now take for granted? Fleming (2007) shares an image that illustrates a striking poverty of multi-scale imagination (or at least its artistic depiction[bbbbb]), even in relatively well-informed circles. Another intriguing essay on the satellite-age transition is Zipser (1970)'s "rise of the fourth school of thought." Although it is impossible to un-see the richness of texture, this author's surprisingly repeatable experience of surprise even after decades of immersion in data suggests that our brains have a hard time realistically remembering just how richly detailed the world is[ccccc], in the face of temptingly simple ideas and conceptions.

Suppose we hypothesize that convection has *meaningful patterns*. To test this hypothesis, observational evidence must be shown to stand above some baseline or *null hypothesis* that convection is *meaningless or random*. Defining those null words well is half the battle in the quest for meaning. For instance, images of white clouds over a dark ocean differ obviously from those same dark and light pixels shuffled into maximum-entropy speckle patterns. But who cares?

A more sophisticated entity-based approach is to tally contiguous areas of whiteness or *cloud objects*. We can then ask if their size distribution deviates from some expectation that defines a random null hypothesis, like a pattern of gravel and rocks spilled onto a street from a great height.

One classic study with a well-defined stochastic model for its randomness baseline is Lopez (1977), whose results are adapted in the top panels of Fig. 9.1. The straightness of the plotting symbols (emphasized with line annotations) indicates the *empirical adequacy*[ddddd] of a stochastic model: the multiplication of many independent random

[bbbbb] Leonardo Da Vinci's turbulence drawings provide an interesting counterpoint.

[ccccc] A visit to https://worldview.earthdata.nasa.gov is advisable before thinking too much.

[ddddd] https://plato.stanford.edu/entries/constructive-empiricism/

numbers[eeeee]. However, physical interpretation is nonunique, an art of science rather than a science *per se*. Such multiplication might for instance embody the *independent random dispatch* of bubble-like entities whose superposition causes, in a multiplicative way, the cloud as a whole to reach a greater height. Additional plausible interpretations are discussed in Lopez's paper. Other properties of convective cloud entities (width, duration) also appear to follow such laws, adequately we can say, in this log-normal straightness measure of a curve between datapoints.

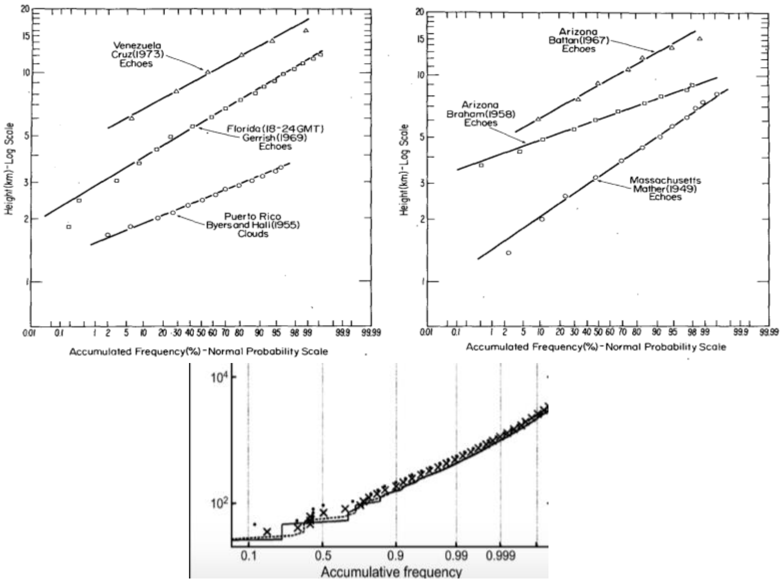


Fig. 9.2. Distributions of cumulus cloud height (top, adapted from Lopez 1977) and MCS anvil area (bottom, adapted from Yuan and Houze 2010), plotted against the cumulative normal function so that straightness of the arrangement of data symbols is indicative of nearness to a lognormal distribution.

In addition to the "inner" interpretive considerations of whether the plotted data is nearly straight on the graph, one must always consider an

[eeeee] The *central limit theorem* states that a sum of many independent identically distributed (iid) random variables asymtotes to the normal (Gaussian) distribution. The log of a product of many iid variables becomes a sum, satisfying this theorem.

"outer" layer of interpretation as well: what conditional pre-screening or sampling lies upstream of the data depiction that lies before us (section 0.6)? Which scenes were excluded, and which others were declared equivalent enough to be statistically pooled? Do the results reflect patterns in the data, or are they subtly, tacitly enforced by the study's own screening assumptions, rooted in its preconceptions or working hypotheses?

The question often comes to this: Would data generated in a hypothetical *randomworld*, embodying a null hypothesis of maximum entropy or some other expression of *lack of meaningful patterns,* appear differently from natural data in the plotting space, after being conditionally screened and filtered and pooled by the study's assumptions? If so, then the difference result is precisely as interesting (or not) as the *randomworld* is compelling in its construction, reflecting salient aspects of the data. Again this is an art-of-science, not a science.

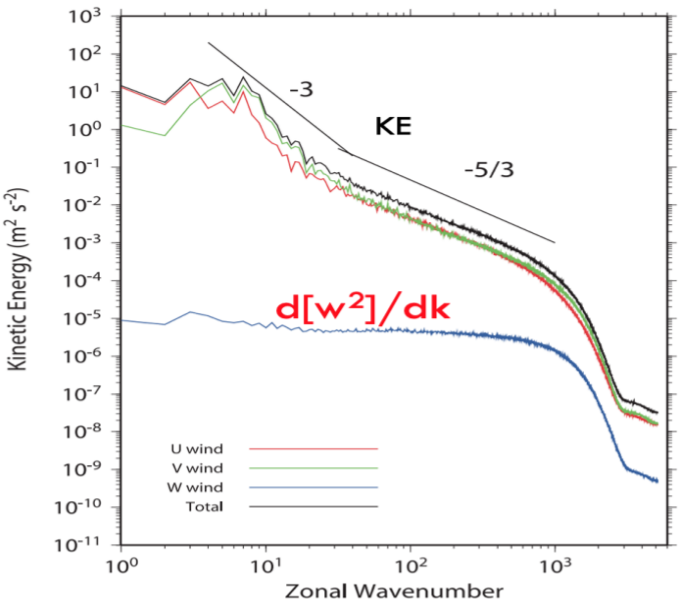
To get beyond the unorganized (by selection) conditions upstream of the top panels of Fig. 9.1, the bottom panel shows the area of MCS high cloud objects observed in all-conditions satellite data over oceanic warm pool areas (Yuan and Houze 2010). This measure of convective entity size has a much wider dynamic range than Lopez's cumulus cloud height, bounded by the troposphere's depth. While the data curve is not as rigorously straight, again the lesson appears to be that a random multiplicative process, perhaps the combination of *independent* probabilities governing the development of larger entities, is not easy to reject as a null hypothesis for natural convective cloud fields.

Overlap assumptions between vertical layers in cloud-radiation interaction may offer another useful toolkit (or at least analogy) for defining "organization" in deep convection. Even if a parcel dispatcher function is random, just as photons are everywhere, their probability of vertical travel can depends on the overlap of moisture and cloud patches at different levels. *Random overlap* is known to be drastically excessive for cloudy radiation, as even modest fractional cloudiness in each of many layers permits almost no photons to traverse a many-layer atmosphere. *Maximum overlap*, on the other extreme, means that the greatest cloud fraction at any single level is the total cloud fraction as seen by radiation, arresting too few photons. A meaningful concept of "organization" must lie in the space between these extremes (some *maximum-random overlap* scheme), perhaps merely as a simple linear scale between the extremes (Mapes and Neale 2011).

x.5 Are entities more real than wavenumbers?

Continuing with the spirit of null hypotheses, to undercut our initial hight hopes of special importance and meaning for entities and their agglomerations, we must return to the spectral slope of Fourier decompositions (section 0.6). Although somewhat arbitraryfffff, Fourier decompositon is powerful and may be viewed as a baseline for the question: Does discrete entity counting like in Fig. 9.2 pass the test of showing more about nature than about the method's definitions and assumptions?

One useful null hypothesis for non-organized (random) convection in Fourier space is the spectral slope of vertical motion w.



fffff no ons (section 2.2.2)

Fig. 9.3. Spectral distribution of velocity variance with respect to spatial wavenumber, adapted from Terasaki et al. (2009).

A complete lack of correlation horizontally among updrafts appears as a *white* (or flat) spectrum of *w* variance with respect to horizontal wavenumber. As Fig. 9.3 shows, this null hypothesis is violated only modestly by the first generation of global nonhydrostatic cloud-resolving models. This finding is more evidence that organization of convection is a slight and subtle effect: the answer to section 9.5's titular question again appears to be *barely,* at best. One caution to this interpretation is that is that power (squared amplitude) spectra in Fourier analysis ignore the phase information that distinguishes a singular, isolated delta function (arguably an extremely highly organized situation) from utterly incoherent random speckle patterns. We know of no tools to measure this phase information in spectral space: its meaning appears only in reconstructions (or all the way back in the physical-space structure of un-decomposted data).

x.6 Lessons from and for large-scale modeling

We have reached the farthest outpost of the book's scope, and inconclusively: none of the frameworks above inspire a frenzy of newly possible calculations that will answer a burning question. Perhaps the Preface's top goal of *appreciation* has been served, somewhat. At the very least, the limitations of available tools of mathematical accounting, as illustrated in the tiny or tidy paradigms of solvable physics that inform the supervenience worldview (*positivism, physicalism?*[ggggg]). In any case, the practical questions of weather and climate remain as challenges, unmoved, and offer a way to climb back down to the realms of current and future research.

[ggggg] philosophers of science surely have names for all of science's ruts of reasoning

x.6.1 *Popcorn v. typhoons in storm resolving resolving models*

One early lesson from the new tool of global models with explicit convection (Satoh et al. 2019) was that the climatic role of convection could be fulfilled adequately by either random 'popcorn' clouds or by a population of long-lived super-typhoons. The distinction seemed to depend on whether the turbulence scheme had more or less mixing (respectively) ventilating the PBL. This aspect of the model was soon tuned toward realism, quite properly in the enterprise of modeling, but the lesson remains: One meaningful "game" or competition in convection fields is between entities with vs. without strong rotational effects. Similar considerations animate the discussion of Zhao et al. (2012).

More broadly, texture differences between cloud fields in a suite of credible global explicit-convection models are quite distinctive by eye, yet hard to characterize formally, and nontrivial to distinguish robustly from nature (the "Palmer-Turing test" in the DYAMOND model intercomparison of Stevens et al. 2019). Our measurement tools seem to be simply inadequate as yet at separating the cosmetic from the essential.

x.6.2 *Parameterization and its schemes*

A parameterization *scheme* is a low-dimensional surrogate model, designed to approximately reproduce the same bulk sensitivities and impacts (comprising its *responses* or *interaction* characteristics) as a more complicated system. Parameterization schemes for *scale-truncated numerical models* have loomed over the problem space of convection for generations. Whether the scale-truncation filter takes the form of coarse-grid cell area averages or horizontal spectral truncation, the missing motions whose effects need to be mimicked by a scheme are (i) spectrally *blue* in the horizontal, meaning restricted *solely* to sub-grid scales or high wavenumbers to avoid "double counting" with effects of resolved motions; (ii) all-scale in the vertical dimension, a domain whose structure even horizontally coarse models aim to resolve well; and (iii) discretized to the model's time step, which is set by the fastest information transmission process between grid cells (usually a wave propagation speed).

A corollary of (iii) is that the motions are strictly local within a model grid column, so that convective entities must contend with their own "compensating subsidence", as in the Lotka-Volterra discussions quoted above. Meanwhile, the larger scales were long externalized as an unspecified "forcing" from outside the reasoning process, although today their sensible interactive parameterization is finally becoming commonplace (e.g. Daleu et al. 2016). Of course, points (ii) and (iii) are incompatible: convection's *vertical* development does not really take place in one larger-scales *time step*. These problems of scale-separating convection in the *temporal and vertical domains* may be the real heart of the problem, more important than fussy but temptingly straightforward micro-accountings of whether cloud features fall inside or outside the boundaries of horizontal grid boxes. After all, time and height are the domain dimensions where we directly care about the scheme's outputs (the tendencies it produces).

For precipitating convection, there is also (iv) a net (filter-scale) conversion of vapor to heat, which rectifies directly onto large-scale motions handled by the embedding fluid dynamical solver. Since this rectified heating depends on the small-scale vertical motions in point (i), it enormously amplifies the importance of errors in postulates about those parameterized motions, to exceed the fussy but temptingly straightforward micro-accountings of eddy fluxes by such motions.

The problem of designing convection schemes has sometimes overshadowed the scientific problem of devising statistical theories of convecting flow -- even while serving as a strong motivation[hhhhh] for the whole enterprise (Arakawa 2004). This problem is now frustrating its third or fourth generation of thinkers, in part because a decent level of success on the largest scales can be accomplished by rather simple ideas like *convective adjustment,* whether 'hard' or instantaneous (Manabe and

[hhhhh] the fuel for this author's entire career, it must be acknowledged

Strickler 1964) or 'soft' (with some finite timescale of relaxation). The success of the brute-simple Betts-Miller (1984) convective adjustment scheme was humbling for much more painstakingly articulated schemes rooted in fluid-mechanical arguments about Part II's entities. Today such elaborated *mass-flux schemes* (chapter 6) are back to being the leading approach, but in the unabashedly engineered "bulk" sense of section 6.2. Final closures are usually still teleological claims of adjustment, albeit with a time scale that may depend on the horizontal truncation length ("*scale-aware*" is this most contemporary virtue in scheme design).

There are longtime dilemmas[iiiii] in convections schemes, indicative of an ill-fitting or conceptually too-small framework. For instance, entrainment is simultaneously too great for climate (cloud heights are too low in realistic *T* and *q* profiles; yet those profiles become too unstable in realistic equilibria), and at the same time too small for weather (deep convection occurs too easily in dry conditions). The deeper lesson is that entrainment is too *unconditional*: a single prescribed value is a too-impoverished framework. Engineering improvements such as an RH dependence of the entrainment rate (e.g. Hirons et al. 2012) improves performance importantly, even as it strans literal connections back to convective entity dynamics, while rather general narratives of "organization" offer other ways to make entrainment contingent rather than constant (Mapes and Neale 2011, Ahn et al. 2019).

New pinnacles of elegant elaboration (Park 2014) perform somewhat better than much cruder approaches - which may be to say, not exactly breakthrough-level better. Might elaboration have reached its apex? Meanwhile, elegant attempts to unify all of turbulence (e.g. Bogenschutz et al. 2018), or even to re-envision the whole fluid dynamical solver to encompass two fluids (like cloudy and clear air volumes, Thuburn et al. 2018) have yet to sweep the field. Stochastic replacements for fixed entrainment (Romps 2016) seem promising at this writing, but implementation is as hard as formulation. The isolated cumulus entity simulations underlying newer surrogate-model fitting approaches may

[iiiii] the titular discussion point in Mapes and Neale (2011)

still suffer from the same dilemmas as surrogates framed around conceptual models of those same isolated structures. Perhaps any scheme based on any too-specific framing will need external wrappings (stochastic or situational dependences like "effective" entrainment rates) to account for the myriad ways that natural convection escapes the bounds of any tidy "entity" in any well-defined mean "environment".

Bulk response functions (sensitivities and impacts) might provide a set of measures that can connect convection to schemes. Diagnostic studies of both models and observations can aim to populate this shared domain of characterization, where reconciliation could occur -- unless dilemmas again indicate that the subspace size is too small. For instance, if different values of bulk parameters are diagnosed for different phenomena (like different frequencies in topical field-campaign time series), the framing is probably inadequate since convective clouds cannot know if they belong to the high vs. low frequency wave envelopes they simultaneously populate. Indeed, insisting on a consistent result for scheme parameters diagnosed at different scales and frequencies could be a very strong *basis* for the plausible validity of a parameterization.

Finally, severe storms and other particular situations, largely neglected in this statistical view, can also offer stark critiques and crises for conceptualizations inadequate to the mystery and drama of convection in our atmosphere. Together the study of storms and statistics have many lessons yet to be learned and reconciled and shared, in a fuller appreciation, perhaps achieved by you.

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