

Evolution of mesoscales in a countable configuration space



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1. Introduction

QUESTION: How do *unlikely but efficient* mesoscale organizations of convection evolve from more *likely but random* configurations of convective updrafts, in a newly convecting fluid?

Evolutionary reasoning principles:

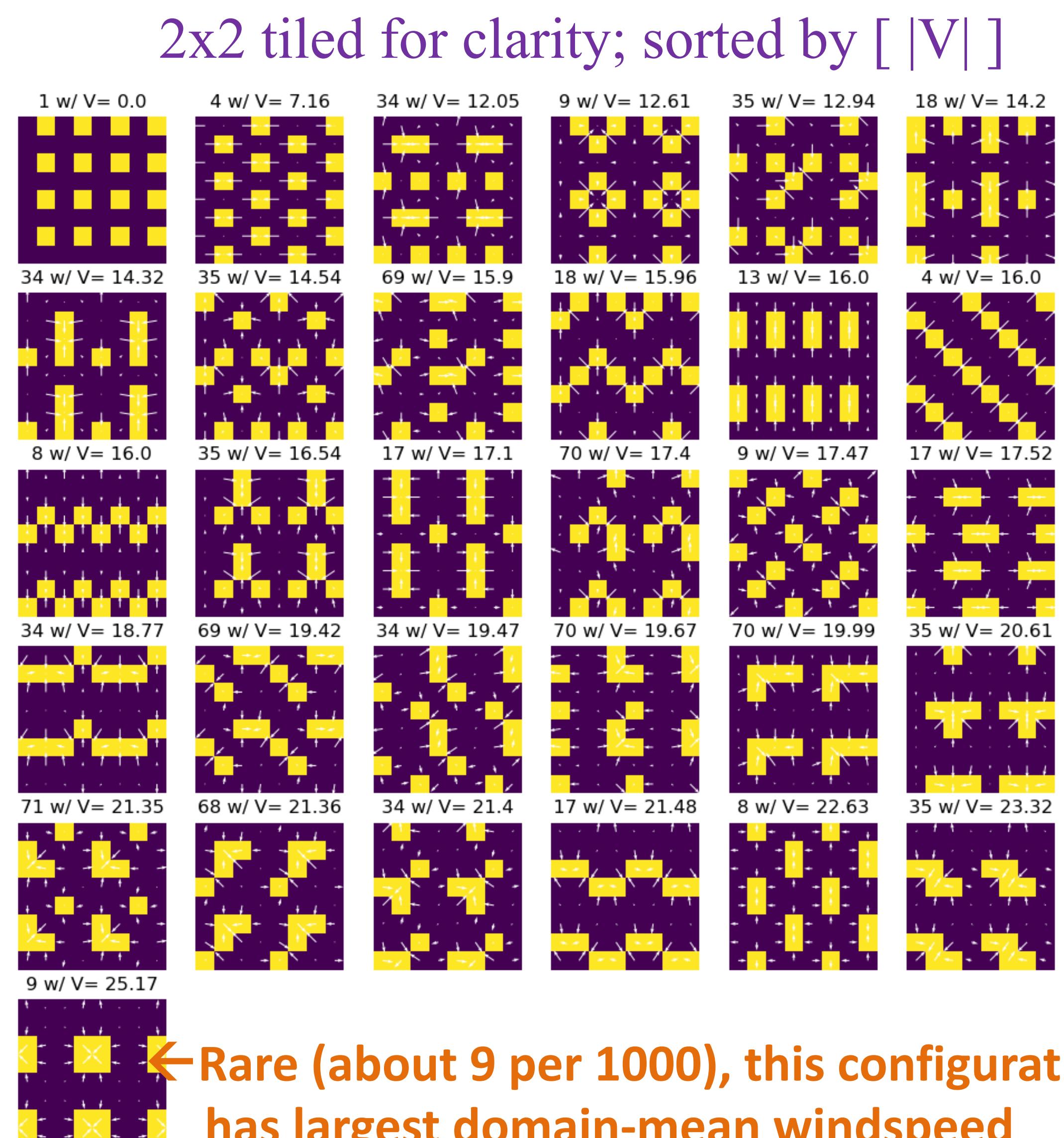
- Consider an abstract space of possible configurations of convective cells (here, $w=1$ updrafts)
- Vast majority of these are un-special; nearly redundant functionally
- A few are ‘fitter’ at persistence and/or reproduction (propagating)
- Hypothesis: Some config. *adjacency* defines a *gradient* of ‘fitness’ for evolution to climb
- Numerical experiments: how fast can a system discover its ‘fitter’ = ‘organized’ configurations?

2. Simplest, enumerable convection-like configuration space:

- * **4x4 lattice**, with **4 w=1** “updrafts” at every time. Enforce mean $[w]=0$.
- * Velocity potential $\chi = \text{ifft}(\text{fft}(w)/k^2)$ for total wavenumber k . $\mathbf{V} = \text{grad}(\chi)$.
simple numpy on unstaggered grid; $V=0$ for checkerboard w !

- **Randomly generate** configurations: thousands $16*15*14*13 = 43680$
- Distinguish **equivalencies** by domain-mean $[\mathbf{|V|}]$, and $[\mathbf{N}_4\text{neighbors}^2]$
2 things that shape Prob(w @ t+1) pattern: $F_{\text{sc}} \propto |\mathbf{V}|$; Entrainment $\propto (4-N_4)$

per random 1000 of **31 distinct** 4x4 4-cell configs



Simple combinatorics so far.

Interest begins with a temporal evolution law!

Define probability $P_w(x,y, t+1)$.
Top 4 P_w locations get $w=1$.

Iterate.

Do “winners” emerge?
How fast? By what pathways?

3. Time evolution: a game with rules + randomness

1-timestep jumps $\rightarrow 31 \times 31$ transition probability matrix **TPM**
Probability for time t+1, based on configuration at time t:

$$P_w(x,y, t+1) = W w(x,y, t) + R \text{ Noise} + E N_{4\text{neigh}}(x,y, t) + F |\mathbf{V}(x,y, t)|$$

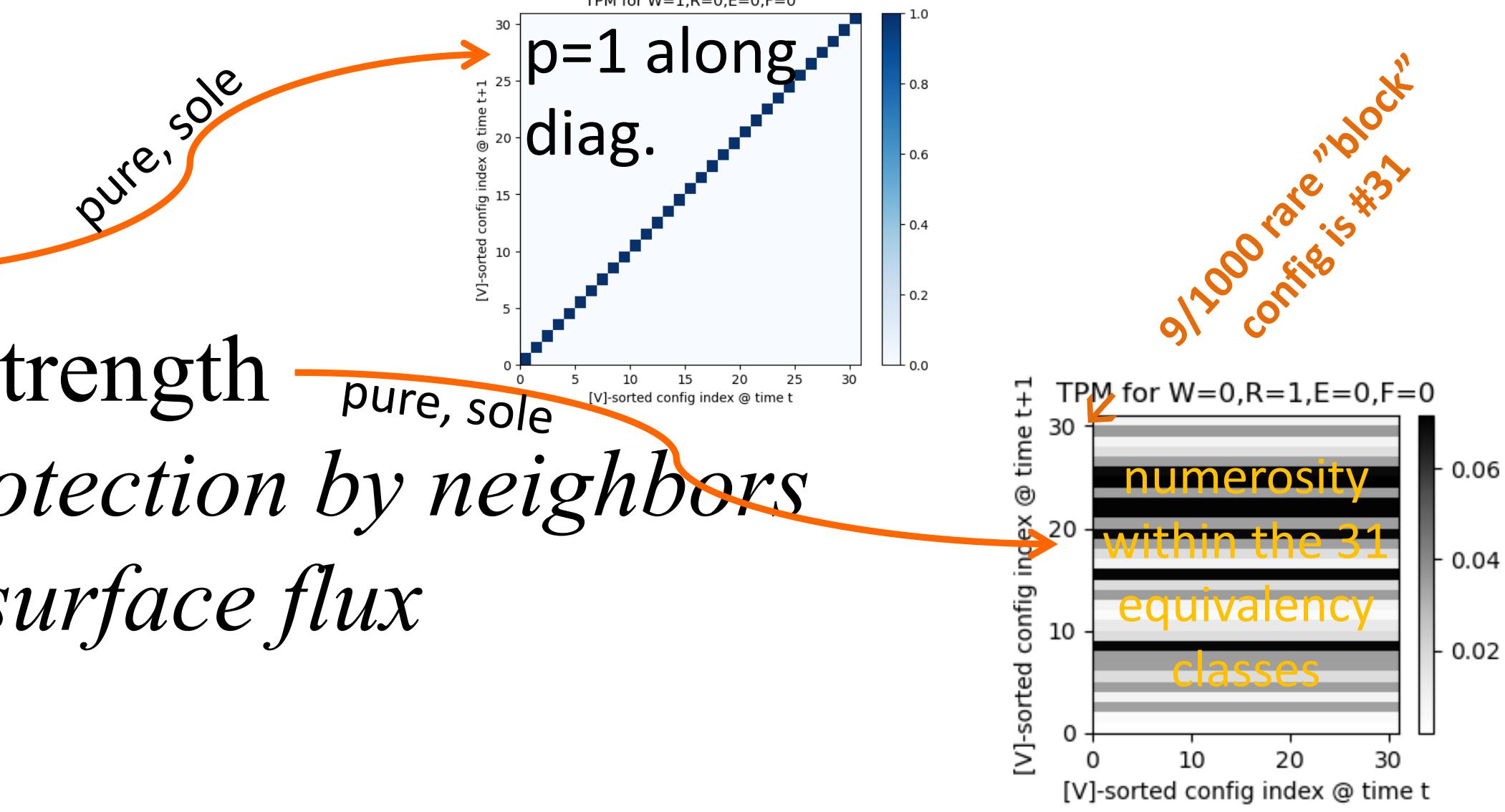
Parameter space: **W,R,E,F**

W governs persistence,

R governs random noise strength

E governs entrainment protection by neighbors

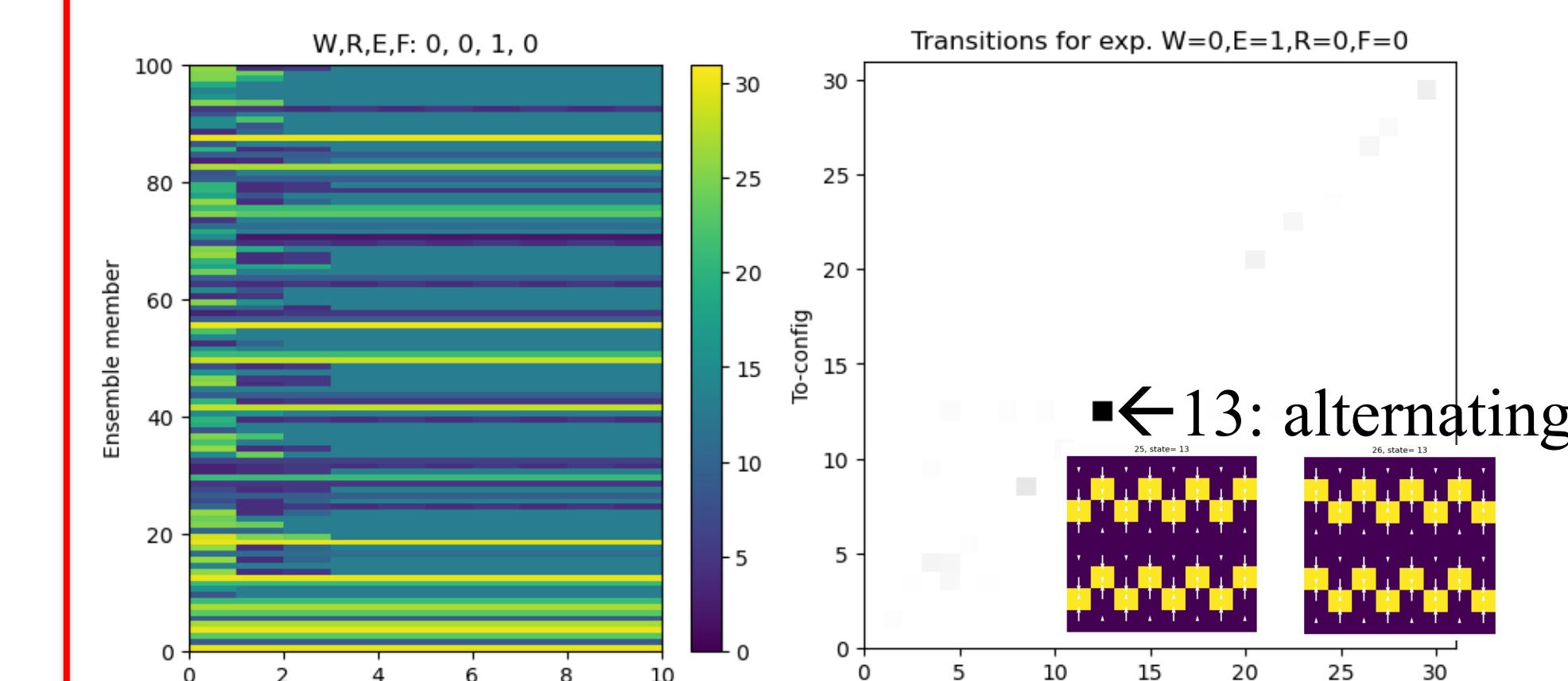
F governs $|\mathbf{V}|$ -dependent surface flux



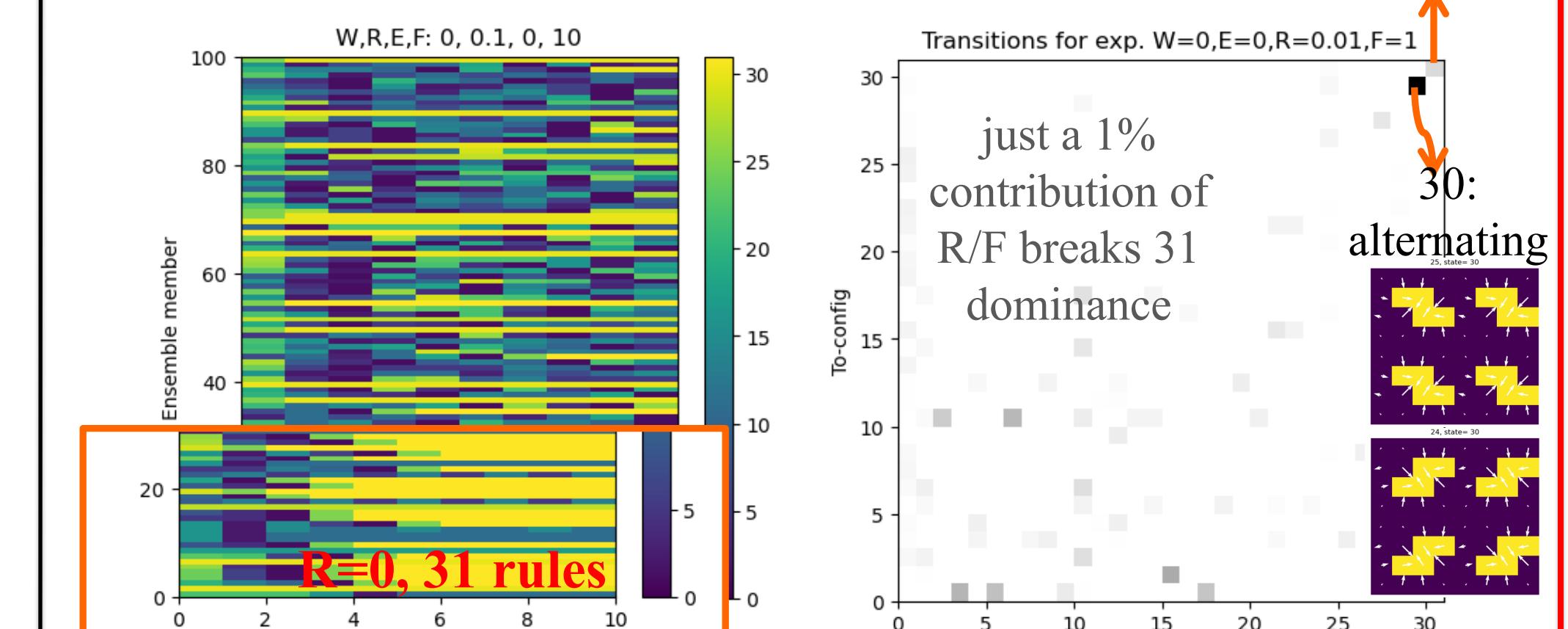
E,F (nonlocal) most interesting for contingent evolution...

1. Construct $P_w(x,y, t+1; W, R, E, F)$
2. Initialize 4 $w=1$ cells randomly, iterate for NT time steps
3. Repeat step 2. NENS times to see I.C.-robustness of results

E effect: $P_w \propto N_{4\text{neigh}}$



F effect: $P_w \propto |\mathbf{V}|$



4. The evolutionary conceptual project

- Even for 4 cells in a 4x4 periodic grid, configuration space is size 31. For 5 in 5x5, > 300, and on from there.
- Welcome to combinatorics, and life! **Pleiotropy**. How can back-propagation of evo. selection work??
- Absurdly many *configurations* (e.g., DNA) \rightarrow several *traits* \rightarrow binary fitness selection (survival, reproduction)
- Unlikely but efficient configs can only discover themselves by *pathways* in vast config space. Can we map it?
 - In biology, key is meso GRNs (Gene Regulatory Networks).
 - In convection, meso state networks?

References:

Mapes (2025, ArXiv & JAS in press): Evolutionary theory of convective organization (but this work is not there!)