

Static energy profiles: a better view of soundings

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INTRODUCTION

Balloon-borne thermodynamic profiling began in the 1930s, measuring pressure, temperature, and relative humidity (initially using human hair). Moist-air thermodynamics was a mature enough science to compute tables of the various derived quantities tying these (relating wet bulb temperature to RH at a given temperature, etc.).

Early 20th century mathematical-physical sciences and engineering were accustomed to using ruled diagrams and slide rules for working with nonlinear functions. According to Fleming (2016), logarithmic pressure-temperature diagrams of Hertz were admired by his graduate student Bjerknes, a pioneer of atmospheric sciences. Large-format printed aerological diagrams allowed the important structure of the lower troposphere to be of adequate size for the eye, despite logarithmic pressure scales, even though most of the diagrammatic area is never utilized or is spent on depicting almost meaningless details of an extremely distorted view of moisture: dewpoint depression.

Today, large paper diagrams are scarce, while computation has become trivial. Thermodynamic theory has passed through high realms of elaboration and precision, and on into the more fertile ground of useful approximation. This allows our limited thinking and reasoning power to be spent going further into other areas and applications. A wave of atmospheric entropy budget studies at uncompromised precision found a few surprising gems of knowledge: for instance, that important irreversible entropy sources include turbulent dissipation behind raindrops at terminal velocity and the diffusion of vapor down its gradients (e.g. Pauluis and Held 2002, Romps 2008, Singh and O’Gorman 2016). But a scholar.google.com search on “atmospheric moist static energy budget” shows more results (over 25,000) than “atmospheric entropy budget”. Many of these MSE budgets are making more valuable conceptual advances, around more truly meteorological scale phenomena.

In support of this quiet advance (a pillar of the tropical meteorology progress noted in section 4 of Mapes 2016), an advanced diagram is offered here, showcasing the utility of moist static energy as opposed to entropy or potential temperature. While the nonlinearities in the formulas for the latter variables ($\ln()$ and $\exp()$ and power law functions) are weak, they nonetheless thwart the simple and quite adequately accurate ‘conservation of energy in different forms’ idea that is how meteorological researchers actually reason.

A brief review of the First Law of Thermodynamics and its wrangling into static energy forms is presented in section 1. The diagram’s many useful features are discussed in section 2. Section 3 tells how to access Python code and create this diagram for any sounding in the web-based sounding archive at the University of Wyoming.

1. THE FIRST LAW AND SPECIFIC STATIC ENERGY

The First Law of Thermodynamics for a Lagrangian parcel of air may be written:

$$C_p \frac{dT}{dt} = Q - \frac{1}{\rho} \frac{dp}{dt} \quad (1)$$

Here Q is a specific (per kg) diabatic heating rate (units: W/kg) by all physical processes: absorption or emission of radiation, chemical energy conversions like latent heat release, and conduction by contact with other matter. If mixing is added to the list of processes in Q , then the meaning of d/dt erodes, as it then means “following a parcel” within some scale-truncated version of the flow that excludes the turbulent scales of motion that are treated as mixing. The other symbols (p, T, ρ) are conventional: pressure, temperature, density.

Three-term equations are slippery and of limited use. For reasoning purposes, we hungrily seek an equation of the form

$$d(\text{heat content})/dt = 0 + J \quad (2)$$

for some useful definition of “(heat content)”. This definition may necessitate a unit conversion of Q to J . The zero on the right is included to indicate that we will often wish to truncate the equation and call it a “conservation law” – albeit one with complications, the sources and sinks J . Equation 2 is just like the equation for the mass (or specific mass, or mixing ratio) of a chemical tracer,

$$dq/dt = 0 + \text{Sources} \quad (3)$$

so essentially we wish to define “(heat content)” as if it were a tracer.

The traditional way to seek that – to battle the inconvenient third term in (1) -- is to divide by T and use the ideal gas law on $1/(\rho T)$ to make an equation of the desired form, with (heat content) = entropy. Entropy is an intuitively challenging quantity, with a nonlinear definition involving logarithms of T and p . These customarily are combined into $C_p \ln(\theta)$, where potential temperature θ is defined by nonlinear quantity $T(p_0/p)^{(R/C_p)}$. If latent release is treated theoretically using the Clausius-Clapeyron relation, an analogous procedure defines pseudoadiabatic or reversible “equivalent” potential temperatures, θ_e . These temperatures can all be depicted on a temperature-pressure diagram, often logarithmic in p , and “skewed” to visual taste since there is no formal orthogonality to thermodynamic quantities.

Unfortunately, nonlinear functions like $\ln()$ in entropy or p^k or $\exp()$ thwart intuition – and needlessly so, as we will see, for graphical and conceptual understanding of the limited range of conditions in Earth’s atmosphere. Of course, conservative thermodynamic accounting is needed for any time-integrating software model. Further, if observational data need to be assimilated into that model, then it must use a complete treatment of thermophysics (including laboratory data on H_2O , not ideal gas approximations), to at least the accuracy of observational instruments. But all this gains us no better understanding of moist meteorological phenomena: to the contrary, much is obscured by it.

A much more informative approach in the atmosphere is to define a $Z_h(p)$ coordinate, which satisfies the hydrostatic approximation $dZ_h/dp = -1/\rho g$ for the density profile of a local atmospheric column. In practical terms, this hypsometric height is computed from a radiosonde’s fundamental measurements and is taken as our best estimate of its altitude. With this coordinate substitution, (1) becomes:

$$C_p \frac{dT}{dt} + g \frac{dZ_h}{dt} = 0 + Q \quad (4)$$

Next, if the latent heat release part of Q is pulled out as L times dq_v/dt , the specific loss rate of vapor to condensation, then:

$$C_p \frac{dT}{dt} + g \frac{dZ_h}{dt} + L \frac{dq_v}{dt} = 0 + Q_{\text{non-condensatic}} \quad (5)$$

Our path toward the desired form for the LHS seems clear, along with the necessary approximations: *if the coefficients on the LHS are treated as constants*, then (heat content) = *dry static energy* s is conserved for unsaturated adiabatic processes, and (heat content) = *moist static energy* h is conserved for saturated or unsaturated but otherwise adiabatic processes:

$$ds/dt = 0 + Q \quad ; \quad s = C_p T + g Z_h \quad (6)$$

$$dh/dt = 0 + Q_{\text{non-condensation}} \quad ; \quad h = C_p T + g Z_h + L q_v \quad (7)$$

A simple linear sum of different types of energy (heat, height, and latent) is conserved. Energy is conserved, by may transfer between these 2-3 different containers (forms). How lucid!

Furthermore, the constant-coefficient sums also imply that mixing is linear: 50% mixtures of s and h fall halfway between the two input values, and likewise for fractions other than 50%. Again, how lucid!

Where has the rigorous worry about entropy gone, for instance the irreversible sources in the Second Law? It got buried in the approximations – but they are quite decently accurate approximations for meteorology, as we can see graphically. Textbooks such as Emanuel (2004) and primary literatures such as Davies-Jones (2009) are available to revisit rigor, if desired.

For present purposes, we merely enumerate several approximation challenges: (1) What exactly is q_v ? Is it a mixing ratio (mass of vapor per unit dry air) or a specific humidity (mass of vapor per unit moist air)? Relatedly, (2) What exactly is C_p ? Is it the heat capacity of dry air, or of an air+water mixture? Relatedly, (3) What is the value of L ? It is a function of temperature and thus of height in the atmosphere. Finally, (4) What exactly is Z_h ? Its calculation requires an entire density profile. Many of these approximations tend to cancel out, mercifully and not coincidentally: For instance, the different heat capacities of vapor and liquid water are precisely why L varies with temperature, so the neglect of both effects is probably better than the neglect of either. For present purposes, we take q_v as a mass mixing ratio, C_p for dry air, L at 273K, and hypsometric heights $Z_h(p)$ computed using virtual temperature. In the software (section 3), readers can play with these approximations and see their small effects if desired.

Finally, one more crucial fact must be noted by rewriting h in terms of sonde-observed variables:

$$h = C_p T + g Z_h(p) + L q_{\text{sat}}(T, p) \times RH \quad (8)$$

Here the functional dependence of $q_{\text{sat}}(T, p)$ is exposed. The crucial fact is that it is *a monotonically increasing function of T* (albeit an upward-curved curved one – here is where the irreducible Clausius-Clapeyron nonlinearity resides). The implication is that *if a hypothetically-lifted air parcel that is saturated has an h value exceeding the saturated h value of the environment, then the parcel is warmer*, and therefore positively buoyant.

The exact value of buoyancy (an vertical force per unit mass) is not linear in this ($h_{\text{parcel above LCL}} - h_{\text{sat of environment}}$) quantity, but the relation is monotonic, with a common zero value that is accurate even when one accounts for virtual temperature effects. In light of the severe approximations of parcel theory with respect to real convection and convective cloud processes -- undiluted parcel lifting with approximations about condensate retention and freezing, buoyancy as only one part and for many parcel geometries a minority of the vertical acceleration equation (e.g. Morrison 2015) – it would be silly to become too concerned about the precise quantitative

value of this hypothetical buoyancy. For this same reason, it seems counterproductive from a true convection-deducing point of view to fixate on the proportionality of parcel CAPE to area on the log-p axis of the traditional skew-T diagram: That slim virtue comes at a terrible cost of clarity (size of distances on the diagram) in the all-important lower troposphere relative to upper altitudes.

2. THE ENERGY-MASS DIAGRAM AND ITS INTERPRETATION

A pressure vertical coordinate makes vertical distance on the diagram proportional to mass (rather than to pseudo-height as in a log-p coordinate). Since our quantities (s , h) are energy per unit mass, area on the diagram is proportional to total energy in an atmospheric layer or column (units: J m^{-2}). This proves useful when visualizing the thermodynamic fluxes that govern tropospheric climate, as well as processes that govern the fates of local convective clouds.

Figure 1 shows our specific static energy \times mass (pressure) diagram, for a random summer afternoon in almost-tropical Miami, Florida.

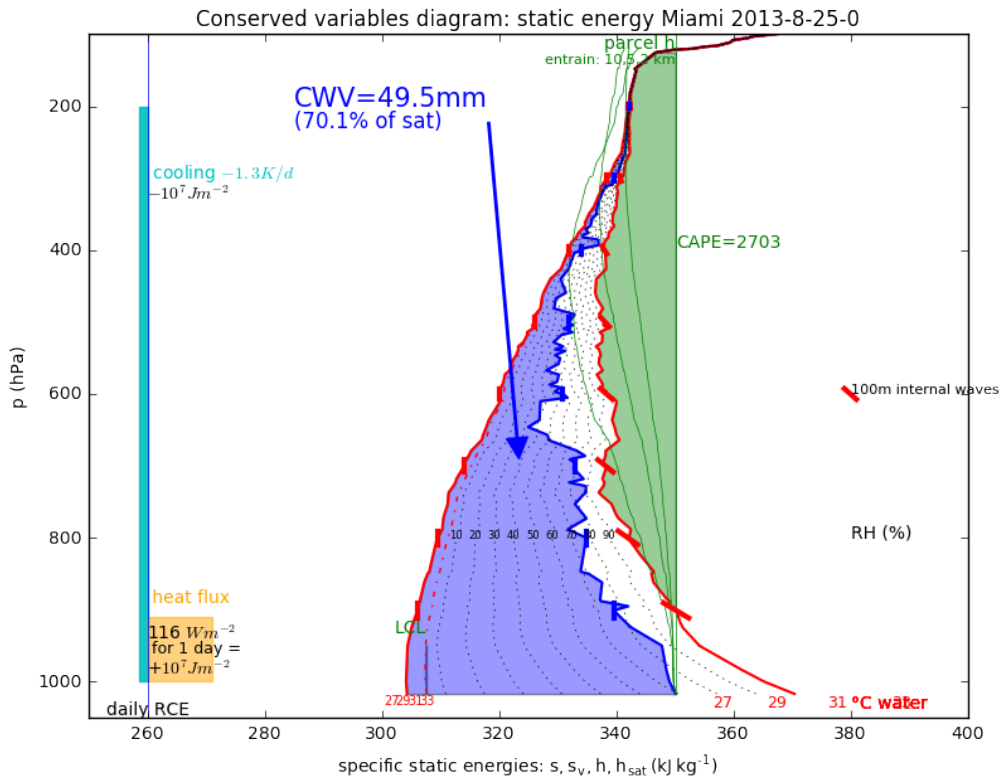


Figure 1: Specific static energy profiles vs. pressure (mass). Static energies from left to right are: dry s (red solid), virtual s (red broken, using T_v in the s formula), moist h (blue), and saturated h (rightmost red solid). Relative humidity rulings are dotted (labeled at 800 hPa altitude at right). Blue fill area is proportional to $L \times \text{CWW}$. The four green profiles are h for lifted parcels with entrainment rates of 0 (vertical) and 100% per (10 km, 5 km, 2 km) for which mixing acts as a linear tug of the parcel curve toward the h profile at each level. A purple wedge (absent here) below the green LCL annotation indicates the (usually negative) buoyancy of a lifted parcel in the unsaturated part of its ascent. Thick stubs on s , h , h_{sat} profiles every 100 hPa show the effect of hypothetical $\pm 100\text{m}$ adiabatic displacements in the air column, comparable to typical internal wave amplitudes and useful for envisioning their statistical effect on lifted-parcel buoyancy. Red number annotations at bottom show the s and h_{sat} of the molecular boundary layer over surface water with temperatures near the sounding's surface temperature, relevant for surface fluxes (see text). Annotations at left, around a vertical axis centered on 260 kJ/kg, indicate typical daily

diabatic increments in Earth-like radiative convective equilibrium (RCE), with equal and opposite cyan and orange areas each representing 10^7 J m^{-2} of daily static energy changes by radiation (cyan, approximating clear-air cooling) and surface flux (orange square, if distributed over a 100 hPa layer).

A variety of state variables and physical processes in the atmospheric column can be visualized and estimated using the curves and annotations of Fig. 1.

1. Red curves at left show s and its virtual counterpart s_v , which must monotonically increase with height in statically stable air columns. A well-mixed planetary boundary layer (PBL) near the surface will have constant s , q , and thus s_v . The latter is actually the buoyancy variable for the unsaturated convection that mixes a PBL heated from below.
2. The blue curve $h(p)$ adds to the $s(p)$ profile, weighting the vapor profile $q(p)$ by its latent heat content, a much larger effect than the virtual effect s_v . The PBL is now seen to be not quite well-mixed in q , an unsurprising result since the PBL entrains considerably drier (lower- h) air above its top.
3. The $h_{\text{sat}}(p)$ curve, red at right, is another depiction of the temperature profile $T(p)$, without any reference to RH. The capacity of air to hold moisture is measured by the gap between the red curves. RH is a linear scale between these, as indicated by the equally-spaced dotted rulings at 10% spacing, numbered along with the “RH (%)” label at the 800 hPa altitude.
4. The blue fill area is proportional to (L times) Column Water Vapor (CWV), a term we prefer to the older “Precipitable Water” (PW), a term that may derive from rough coincidence of typical CWV values with typical event totals in rain gauges. Note that 1 mm of CWV or PW (liquid equivalent if condensed) is equal to 1 kg m^{-2} .
5. The buoyancy of a parcel of air lifted from the surface may be assessed below the lifted condensation level (LCL) by comparing a vertical line (where s_v is conserved) to the slope of the ambient s_v curve. The LCL is at the base of the “LCL” annotation that abuts the s_v curve.
6. Above the LCL, the sign of buoyancy for a lifted surface parcel is assessed by comparing the vertical green line to the red $h_{\text{sat}}(p)$ curve. If buoyancy is present, the CAPE is computed behind the scenes and annotated next to the green fill area. However, the green fill area is not strictly proportional to CAPE, as the $h_{\text{parcel}} - h_{\text{sat}}$ distance has a different relationship to buoyancy at different altitudes, and p is not proportional to the distance over which work is done by the vertical buoyancy force. Given the severe approximations of parcel theory, the depiction here is quite adequate for any qualitative deductions or estimations about real moist convection’s likelihood and properties in the region represented by a balloon-sampled profile.
7. Thick line stubs every 100 hPa show the effects of adiabatic vertical displacements of $\pm 100 \text{ hPa}$ in the air column, for instance by internal gravity waves. Such displacements are common over the areas typically interpreted as being indicated by the happenstance location of a balloon sounding. In particular, the horizontal component of these stubs on the $h_{\text{sat}}(p)$ curve is an indicator of how sensitive lifted-parcel buoyancy is to such displacements. This gives a glimpse of how the statistics of partly-cloud cumulus skies may depend on the vigor of other sources of adiabatic oscillations in the air column or sampling errors if the balloon were launched a few minutes or a few km away. It also tends to underscore again the adequacy of the not-quite-CAPE-proportional green fill.

8. Mixing is linear on the diagram. For instance, the green curves indicate h profiles for a rising parcel characterized by a constant entrainment rate (expressed as % per km of ascent, as is traditional in cumulus cloud theory). Mixing can be viewed as a tug to the left on the rising parcel's h_{parcel} value, proportional to its distance from the blue curve and to the entrainment coefficient.
9. Since mixing is linear, the Bowen ratio of surface flux (over water) can be visualized directly, from the viewpoint of a linear aerodynamic bulk flux formula. If surface flux is viewed as a mixing between lowest-level air in the column and the saturated molecular boundary layer in equilibrium with a water surface at that pressure, then sensible vs. latent heat fluxes can be eyeballed as the ratio of distances on the diagram. Four integer temperature values (in C) are annotated by centered red numbers at the bottom of the diagram, bracketing the sounding's lowest air temperature (27, 29, 31, 33 C in this case).
10. Typical diabatic fluxes that overturn the troposphere are indicated on an axis centered at left. The cyan area depicts a sink of -10^7 J m^{-2} , in the form of a constant cooling of -1.3 K/d , a fairly typical value for clear-air radiation in tropical or temperate regions. The s , s_v , and h curves would move to the left by the width of this cyan rectangle each day, if this cooling acted alone. Surface flux is deposited in the lowest micron of air, but is rapidly redistributed by mixing up through the PBL depth. As a convenient possible realization, a surface energy flux of 116 W m^{-2} integrated over 1 day (an energy source of $+10^7 \text{ J m}^{-2}$, equal and opposite to the radiative cooling sink) is depicted over a 100 hPa layer (orange square). Note that the area on the diagram should be much more squat and wide in a stable boundary layer. The h curve is moved to the right by the total surface flux, while the s and s_v curves are moved to the right only by the sensible heat flux. The effect of this sensible flux on $h_{\text{sat}}(p)$ is not easily visualized, but that doesn't matter so long as this flux distributed only over a mixed PBL, because then it will have no consequential effect on any lifted parcel's buoyancy, whose estimation the only reason for the h_{sat} curve's inclusion. Surface latent heat flux (evaporation) has no effect on near-surface h_{sat} .

For comparison, Figure 2 shows the same sounding depicted using potential temperatures. None of the desirable properties used to annotate Fig. 1 (linear mixing, proportionality of blue fill to PW, ...) strictly holds for these nonlinear derived quantities. And yet, the basic information content, including the lifted-parcel buoyancy indicator, is essentially indistinguishable. This is because the nonlinearities involved are quite weak (or quasi-linear) over the range of values occurring in Earth's troposphere. *There is nothing meteorologically significant to be gained by operating in the obfuscated space of nonlinear thermodynamic formulas.*

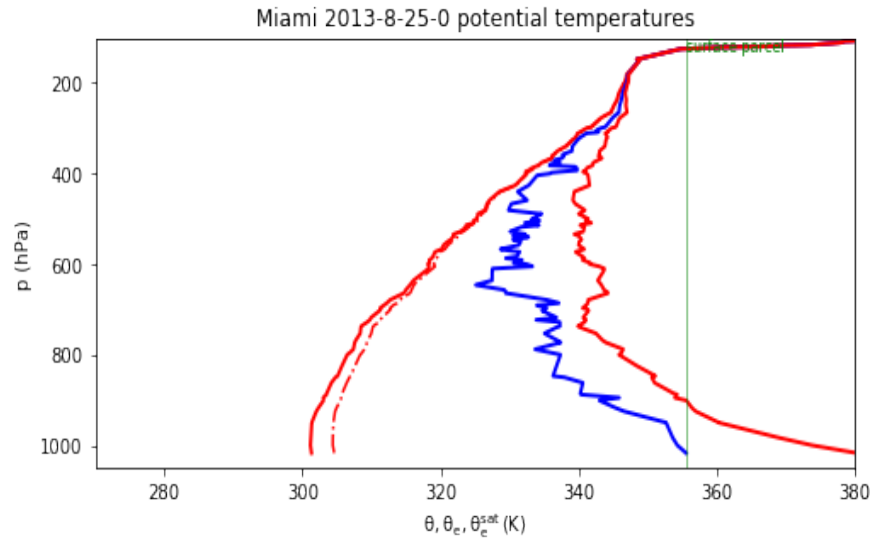


Figure 2: Potential temperatures version of Fig. 1, using more exact but nonlinear functions for potential temperature θ (red at left), equivalent potential temperature θ_e (blue), and its saturated value (red at right). These are related to entropy through further nonlinear functions (logarithms). Broken red line is virtual potential temperature, θ_v .

3. FREE PYTHON SOFTWARE

Thanks to the MetPy Python package from Unidata (<https://unidata.github.io/MetPy>), readers can easily and freely reproduce the diagram above for any sounding in the archives and near-real time stream of the University of Wyoming (see <http://weather.uwyo.edu/upperair/sounding.html> for conventional plots and data). Our plotting code is easily adapted to any input profile arrays of the set of needed variables $\{p, T, Z_h, RH\}$.

The Python code for these diagrams is embedded within a Jupyter Notebook, available at <http://www.tinyurl.com/EnergyMassSounding>. A Jupyter Notebook is a human-friendly document with embedded executable code cells, just as journal papers have embedded mathematics. This is a subtle change of emphasis from the paradigm of code with documentation embedded in it, although these are mutually transformable with reformatting.

Users begin by installing the free Anaconda (or miniconda) distribution of Python and Jupyter. Users must then install the (free) MetPy and Siphon packages from Unidata. To do this, simply type `conda install metpy` and `conda install siphon` in a terminal.

After downloading the Notebook above, users open it for operation by typing `jupyter notebook` in their terminal application, within a directory containing the Notebook. A browser tab will pop up, with a file tree. After opening the notebook, code cells can be executed with shift-return, and edited (e.g. to select another date-time or another site code from the Wyoming sounding data repository). Code cells at the top of the notebook define the plotting code, including colors, annotations, data sources, etc. It can be saved as a Python code for batch processing.

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