STA4821 **Summer 2021**

Show all your work. No credit will be given for guessing.

<u>Prob. 1</u>

Let the random variable X denote the number of times that the number 6 shows when a dice is rolled 120 times.

(a) Find the probability that the number 6 never shows.

(b) Using the Gaussian approximation, find the probability that the random variable X lies between 15 and 25.

Prob.2

(a) There are 10 problems on a certain exam. Students are instructed to answer a random selection of 5 of the problems. What is the probability that a student who can figure out the solution to 7 of the problems will score

(i) 100% on the exam?

(ii) at least 80% on the exam?

(b) The Computer Science Department of a certain university has 100 students and offers three programing courses; Python, R and C Programing. All the students are eligible to register for any of the courses. There are 28 students in the Python class, 26 in the R class, and 16 in the C class. There are 12 students in both Python and R classes, 4 students in both R and C classes, and 6 in both Python and C classes. There are 3 students in all three classes.

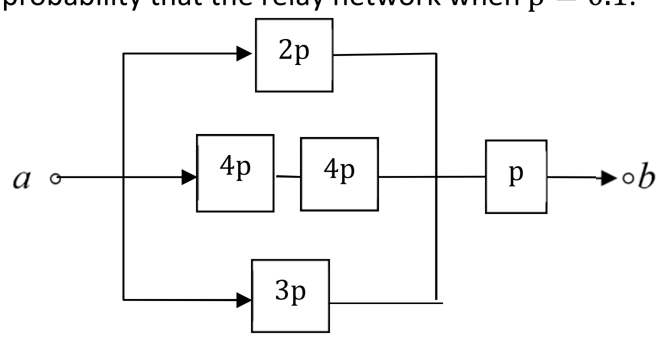
(i) Find the probability that a randomly selected student is not in any of the classes.

(ii) If two students are selected randomly, what is the probability that at least one of them is taking a programing class?

<u>Prob.3</u>

(a) In a certain community, 0.1% of the population is infected with a disease. Suppose that a laboratory test to detect the disease has the following statistics. Let
A = event that a tested person has the disease.
B = event that the test result is positive.
If it is known that $\mathrm{P(B|A)} = 0.99$ and $\mathrm{P(B|\overline{A})} = 0.005$, what is the probability that a person has the disease given that a person has the disease given that the test result is positive?

(b) The relay network in the diagram below operates if and only if there is a closed path of relays from points a to b. Assume that the relays fail independently and the probability of failure of each relay is related to p as shown.
(i) Find the probability that the relay network works as a function of p.
(ii) Find the probability that the relay network when $\mathrm{p} = 0.1$.



Prob.4

(a) Let the discrete random variable X be the even outcome when a fair die is rolled once. Then the probability mass function (pmf) of X is given by

$$P(X = k) = \begin{cases} \dfrac{1}{3}, & k = 2,4,6 \\ 0, & \text{otherwise} \end{cases}$$

Evaluate the mean and variance of the random variable X.

(b) Now, let X be a discrete random variable that take only even numbers less than or equal to an even number n, i.e.,

$$P(X = k) = \begin{cases} \dfrac{2}{n}, & k = 2, 4, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

Evaluate the mean and variance of the random variable X. Verify that your result agrees with that in (a) when $n = 6$.

(c) More generally, let the random variable X be a discrete even number in the interval (a, b), where a and b are even integers. i.e.,

$$P(X = k) = \begin{cases} \dfrac{2}{b - a + 1}, & k = a, a + 2, a + 4, \dots, b \\ 0, & \text{otherwise} \end{cases}$$

Evaluate the mean and variance of the random variable X. Verify that your result agrees with that in (b) when $a = 2$ and $b = n$.

Hint: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ and $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{\substack{k=0 \\ k\text{ even}}}^{n} k = n(n+1)$ and $\sum_{\substack{k=0 \\ k\text{ even}}}^{n} k^2 = \frac{2n(n+1)(2n+1)}{3}$

Prob.5

A random variable has probability mass function (pmf) given by

$$P(X = x) = \begin{cases} Ax^{-2}, & x = 1, 2, \ldots, 4 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the constant A and the probability mass function, $P(X = x)$.

(b) Find $P(2 \leq X < 4)$.

(c) Find and sketch the cumulative distribution function, $P(X \leq x)$.

(d) Find the standard deviation of the random variable X.

Prob.6

A random variable X has the probability density function given by

$$f_X(x) = Ae^{-2|x|}, \text{ for } -\infty < x < \infty$$

where A is a positive constant.

(a) Find the constant A.

(b) Find and sketch the cumulative distribution function of the random variable X.

(c) Find the mean and variance of X.

(d) Determine the probability that X is within two standard deviations of its mean, i.e., $P(-2a < X < 2a)$.