Lecture 12b – Green-Ampt model for estimating infiltration rates

Learning Outcomes

- Be able to draw and interpret graphs and carry out calculations related to infiltration rates using the following methods:
  - Horton equation (empirical)
  - Green-Ampt model (simplified conceptual model)

- Be able to explain briefly the instrumentation we can use to measure soil properties, soil moisture content, and infiltration rates

Infiltration models

Scale matters!

Watershed scale

- “Upscaled” versions of local scale models
- Empirical and semi-empirical models related to infiltration capacity (rainfall excess, Kostiakov, modified Kostiakov, **Horton**, Horland)

Local scale

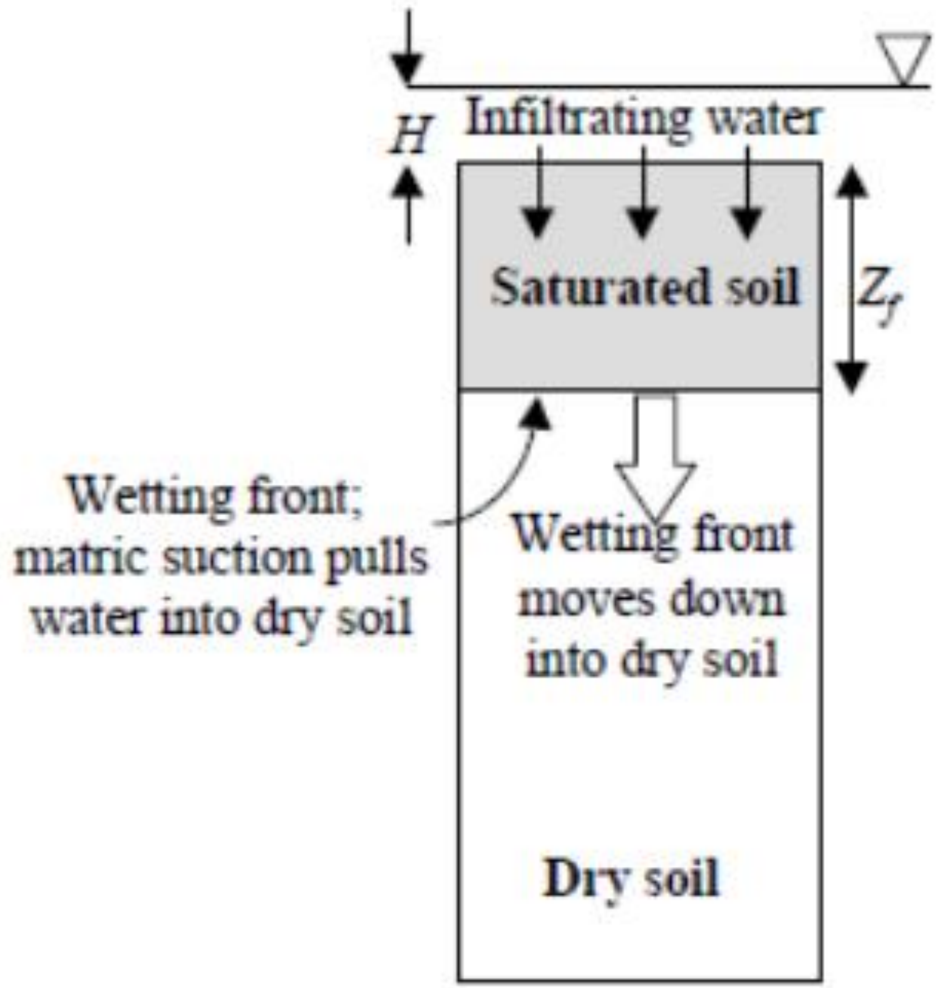
- Darcy’s Law with conservation of mass
- Approximate solutions to Richard’s equation (Philip, Smith-Parlange, Broadridge)
- Conceptually simplified models – **Green-Ampt models**

Conceptual model: Green-Ampt model

Based on fundamental physics with some simplifications:

- Wetting front not often sharp
- Soil above wetting front may not be saturated

More realistic = Richard's equation but too complex for this class!!



Conceptual model: Green-Ampt model

$$f = -K_s \frac{dh}{dz}$$

$$f = -K_s \frac{h_f - h_o}{Z_f}$$

$$f = K_s \frac{|\psi_f| + Z_f}{Z_f}$$

Where f = infiltration rate

h_o = hydraulic head at soil surface

h_f = hydraulic head at wetting front

ψ_f = suction/tension head at wetting front (cm)

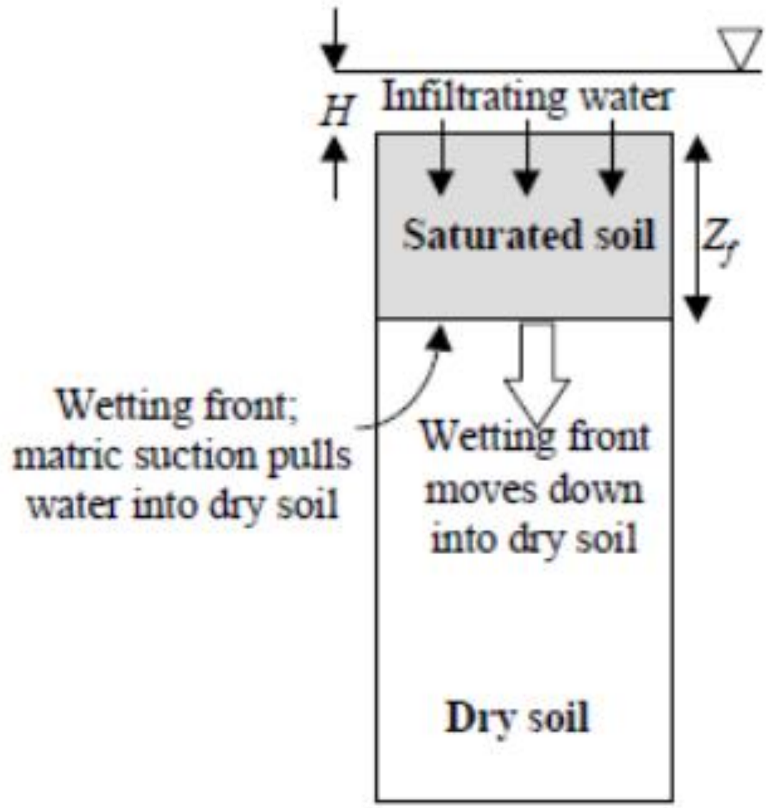
K_s = saturated hydraulic conductivity

$$F = Z_f(\theta_s - \theta_i)$$

Where F = cumulative infiltrated water (cm)

θ_i = initial moisture content

θ_s = saturated moisture content



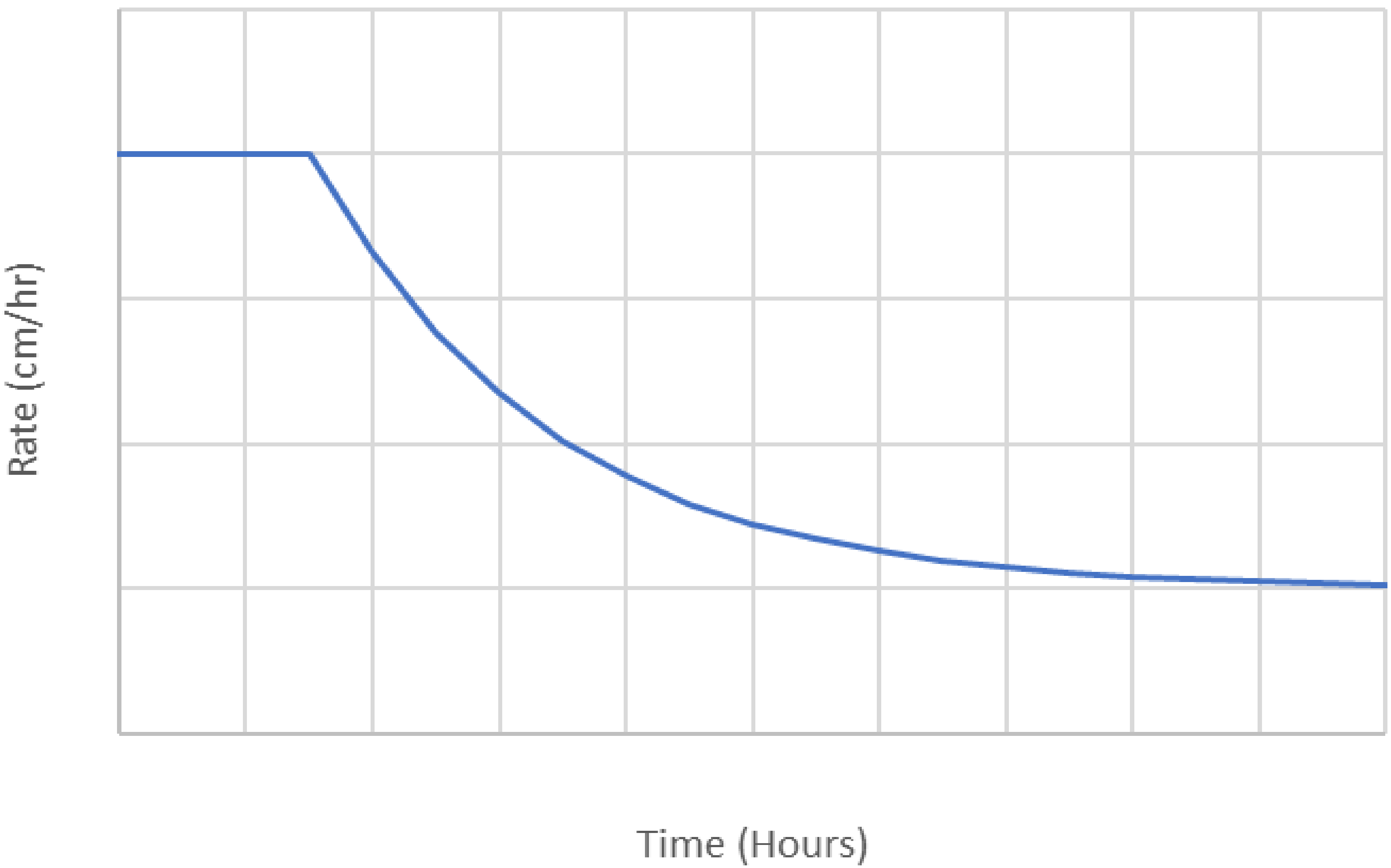
Conceptual model: Green-Ampt model

Where P = rainfall rate (cm/hr)
t_p = time of ponding (hr)
F_p = amount of water that infiltrates before ponding occurs (cm)

$$t_p = \frac{F_p}{P} \qquad F_p = \frac{|\psi_f| K_s (\theta_s - \theta_i)}{P - K_s}$$

Before ponding ($t \leq t_p$) $\quad f(t) = P$

Reminder: Rainfall infiltration



Conceptual model: Green-Ampt model

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Before ponding ($t \le t_p$) $\quad f(t) = P$

After ponding ($t > t_p$) $\quad f(t) = K_s + K_s \frac{|\psi_f|(\theta_s - \theta_i)}{F}$

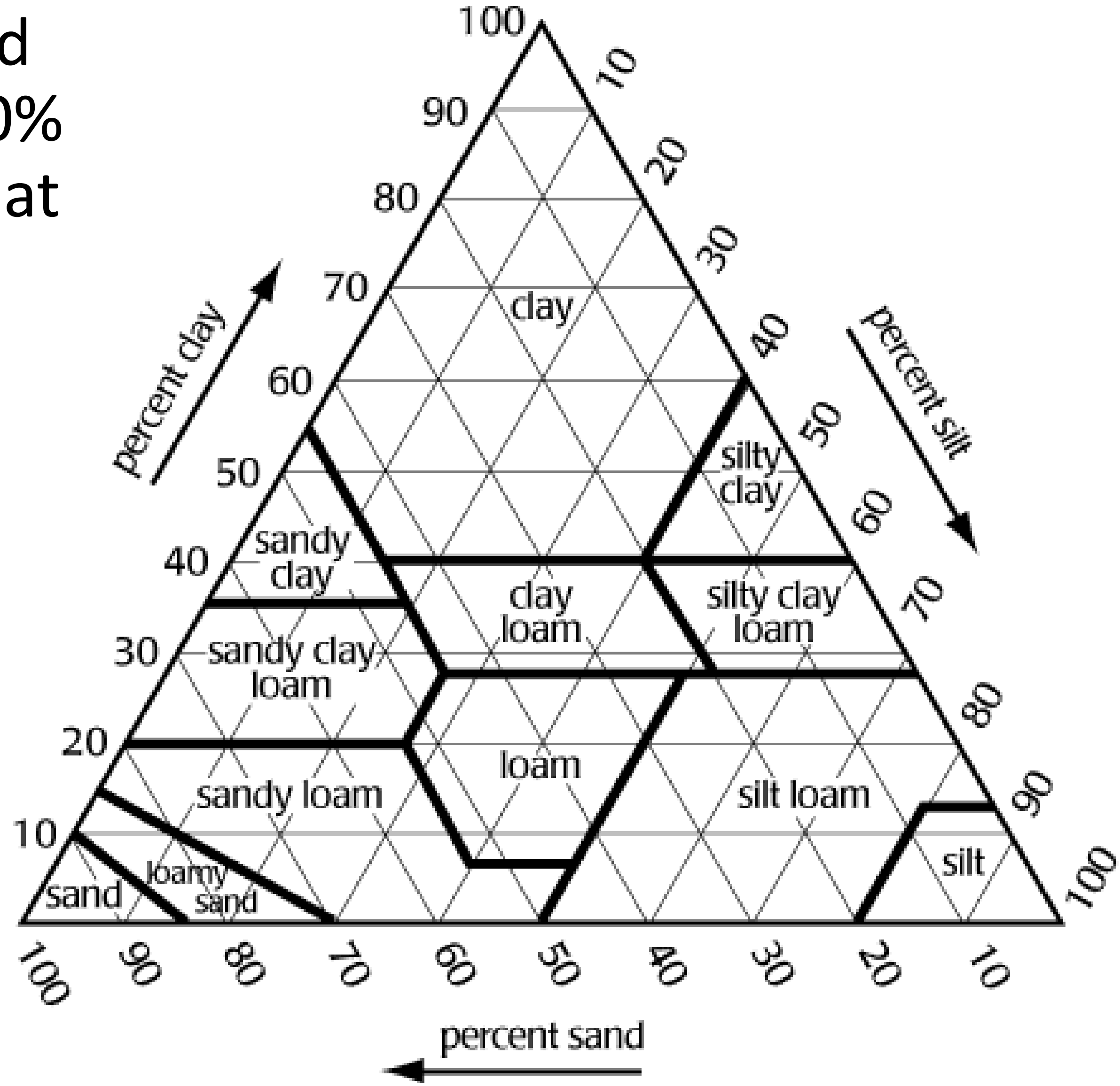
Given that f = dF/dt, we can write the calculate t for a given F (when $F>F_p$)

$$t = t_p + \frac{1}{K_s}\left[F - F_p + |\psi_f|(\theta_s - \theta_i)\ln\left(\frac{|\psi_f|(\theta_s - \theta_i) + F_p}{|\psi_f|(\theta_s - \theta_i) + F}\right)\right]$$

Conceptual model: Green-Ampt model

I sieve my soil and find that it is 50% sand, 20% silt, and 30% clay. What type of soil is it?

a) Clay loam
b) Sandy clay loam
c) Silt loam
d) Loam
e) Clay



Conceptual model: Green-Ampt model

- What is the K_{sat} (cm/hr) and suction head (cm) for your soil?

Texture	Porosity n	Residual Porosity θ_r	Effective Porosity θ_e	Suction Head ψ (cm)	Conductivity K (cm/hr)
Sand	0.437	0.020	0.417	4.95	11.78
Loamy Sand	0.437	0.036	0.401	6.13	2.99
Sandy Loam	0.453	0.041	0.412	11.01	1.09
Loam	0.463	0.029	0.434	8.89	0.34
Silt Loam	0.501	0.015	0.486	16.68	0.65
Sandy Clay Loam	0.398	0.068	0.330	21.85	0.15
Clay Loam	0.464	0.155	0.309	20.88	0.10
Silty Clay Loam	0.471	0.039	0.432	27.30	0.10
Sandy Clay	0.430	0.109	0.321	23.90	0.06
Silty Clay	0.470	0.047	0.423	29.22	0.05
Clay	0.475	0.090	0.385	31.63	0.03

Conceptual model: Green-Ampt model

A rainfall event lasts for 2 hours with a 0.8 cm/hr intensity. The soil θ_i = 0.25 and θ_s = 0.50.

What is the total runoff and infiltration (cm) from the rainfall event?

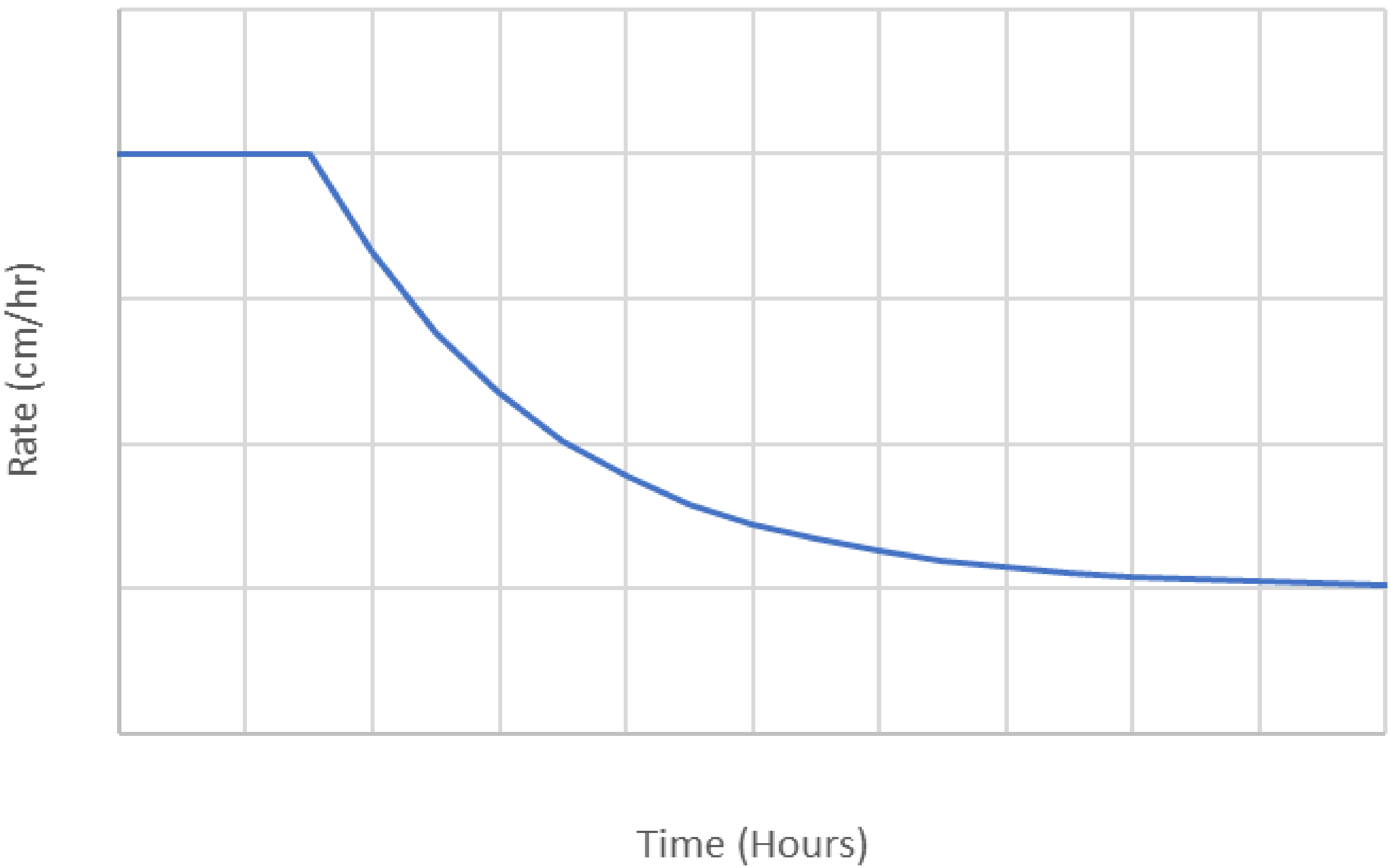
When does runoff begin?

What is the infiltration rate at the end of the storm?

Step 1: Calculate t_p and F_p

$$t_p = \frac{F_p}{P} \qquad F_p = \frac{|\psi_f| K_s (\theta_s - \theta_i)}{P - K_s} \qquad t = t_p \text{ and } P > K_s$$

Reminder: Rainfall infiltration



Conceptual model: Green-Ampt model

Step 2: Calculate t and f using below equations for randomly chosen F values

F (cm)	t (hrs)	f (cm/hr)
0	0	0.8
1.26 (=Fp)	1.57 (= tp)	0.8
1.35	1.69	0.757
1.45	1.83	0.715
1.57	2.00 (rain stopped)	0.672

Before ponding

For F<Fp , t = F/P

For t<tp , f(t) = P

After ponding

For F>Fp ,

$$t = t_p + \frac{1}{K_s}\left[F - F_p + |\psi_f|(\theta_s - \theta_i)\ln\left(\frac{|\psi_f|(\theta_s - \theta_i) + F_p}{|\psi_f|(\theta_s - \theta_i) + F}\right)\right]$$

For t>tp ,

$$f(t) = K_s + K_s\frac{|\psi_f|(\theta_s - \theta_i)}{F}$$