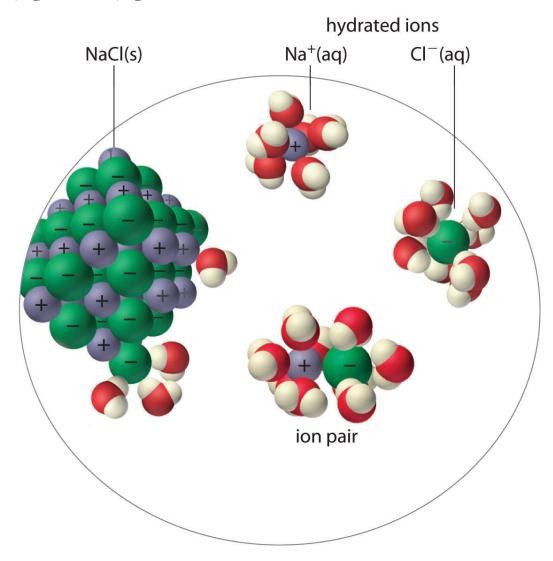
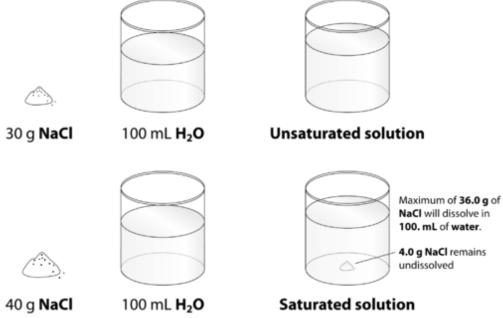
## The Solubility of Salts

✓ When a soluble salt dissolves in water, it dissociates completely into separate **hydrated** (or solvated or aquated) cations and anions that move apart from each other.

Ex: 
$$NaCl(s) \rightarrow Na^{+}(aq) + Cl^{-}(aq)$$

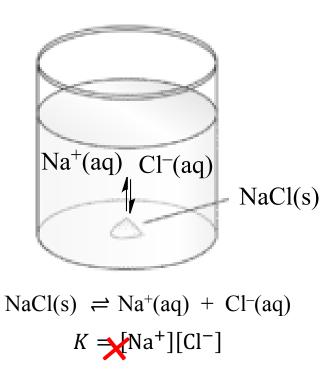


✓ Suppose that you have a beaker of water to which you add some soluble salt, stirring until it dissolves. You add more and that dissolves. You keep adding more and more salt, eventually reaching a point that no more of the salt will dissolve no matter how long or how vigorously you stir it. When this point is reached and no more solute will dissolve, the solution is said to be **saturated**, and the undissolved solute precipitates to the bottom.



At this point, the rate at which the ions dissociate into separate cations and anions and the rate at which the separate cations and anions combine to re-form the solid (the reverse of the dissociation reaction) become equal to each other and an equilibrium between the undissolved solid and the dissociated ions in the aqueous solution is established.

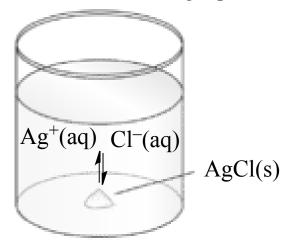
$$NaCl(s) \rightleftharpoons Na^{+}(aq) + Cl^{-}(aq)$$



- ✓ Unfortunately, this simple type of equilibrium expression cannot be used because the saturated solution for a soluble salt is not an ideal solution. The high concentrations of the ions in the saturated solution can form ion-pairs and larger clusters, so that the aqueous solution is far from ideal.
- Therefore, we restrict our attention to insoluble salts (or slightly soluble, sparingly soluble, or marginally soluble salts) for which concentrations of the solvated ions are so low that the interactions among solvated ions are negligible.

### **The Solubility Product**

The sparingly soluble salt AgCl(s) establishes the following equilibrium when placed in water.



$$AgCl(s) \rightleftharpoons Ag^{+}(aq) + Cl^{-}(aq)$$

$$K_{sp} = [Ag^+][Cl^-] = 1.6 \times 10^{-10} \text{ M}^2 \text{ at } 25 \text{ °C and } 2.2 \times 10^{-8} \text{ M}^2 \text{ at } 100 \text{ °C}$$
 (units are usually omitted)

- $K_{\rm sp}$  is called the **solubility product constant**.
- ✓ Note that [AgCl] is not included in the equilibrium expression because it is a pure solid.
- ✓ If no solid is present, the equilibrium expression is not valid.

✓ Other examples:

PbI<sub>2</sub>(s) 
$$\rightleftharpoons$$
 Pb<sup>2+</sup>(aq) + 2I<sup>-</sup>(aq)  
 $K_{sp} = [Pb^{2+}][I^{-}]^{2} = 7.1 \times 10^{-9}$   
Bi<sub>2</sub>S<sub>3</sub>(s)  $\rightleftharpoons$  2Bi<sup>3+</sup>(aq) + 3S<sup>2-</sup>(aq)  
 $K_{sp} = [Bi^{3+}]^{2}[S^{2-}]^{3} = 1.0 \times 10^{-15}$   
Al(OH)<sub>3</sub>(s)  $\rightleftharpoons$  Al<sup>3+</sup>(aq) + 3OH<sup>-</sup>(aq)  
 $K_{sp} = [Al^{3+}][OH^{-}]^{3} = 1.3 \times 10^{-33}$   
Ag<sub>2</sub>CrO<sub>4</sub>(s)  $\rightleftharpoons$  2Ag<sup>+</sup>(aq) + CrO<sub>4</sub><sup>2-</sup>(aq)  
 $K_{sp} = [Ag^{+}]^{2}[CrO_{4}^{2-}] = 9.0 \times 10^{-12}$ 

- ✓ The  $K_{\rm sp}$  values at 25 °C for many common ionic solids are listed in the following table.
- $\checkmark$  Notice that  $K_{\rm sp}$  values are all very small values because the solids are insoluble or sparingly soluble salts.
- ✓ One way to identify insoluble/sparingly soluble/marginally soluble/slightly soluble salts is from their  $K_{\rm sp}$  values. Another way is by using the **solubility rules**.

# Some Values for Solubility Product Constants $(K_{\rm sp})$ at 25 °C

Solute	Solubility Equilibrium	$K_{\rm sp}$
Aluminum hydroxide	$AI(OH)_3(s) \Longrightarrow AI^{3+}(aq) + 3 OH^{-}(aq)$	$1.3 \times 10^{-33}$
Barium carbonate	$BaCO_3(s) \Longrightarrow Ba^{2+}(aq) + CO_3^{2-}(aq)$	$5.1 \times 10^{-9}$
Barium sulfate	$BaSO_4(s) \Longrightarrow Ba^{2+}(aq) + SO_4^{2-}(aq)$	$1.1 \times 10^{-10}$
Calcium carbonate	$CaCO_3(s) \rightleftharpoons Ca^{2+}(aq) + CO_3^{2-}(aq)$	$2.8 \times 10^{-9}$
Calcium fluoride	$CaF_2(s) \rightleftharpoons Ca^{2+}(aq) + 2F^{-}(aq)$	$5.3 \times 10^{-9}$
Calcium sulfate	$CaSO_4(s) \Longrightarrow Ca^{2+}(aq) + SO_4^{2-}(aq)$	$9.1 \times 10^{-6}$
Calcium oxalate	$CaC_2O_4(s) \Longrightarrow Ca^{2+}(aq) + C_2O_4^{2-}(aq)$	$2.7 \times 10^{-9}$
Chromium(III) hydroxide	$Cr(OH)_3(s) \rightleftharpoons Cr^{3+}(aq) + 3OH^{-}(aq)$	$6.3 \times 10^{-31}$
Copper(II) sulfide	$CuS(s) \rightleftharpoons Cu^{2+}(aq) + S^{2-}(aq)$	$8.7 \times 10^{-36}$
Iron(III) hydroxide	$Fe(OH)_3(s) \Longrightarrow Fe^{3+}(aq) + 3OH^-(aq)$	$4 \times 10^{-38}$
Lead(II) chloride	$PbCl_2(s) \Longrightarrow Pb^{2+}(aq) + 2 Cl^{-}(aq)$	$1.6 \times 10^{-5}$
Lead(II) chromate	$PbCrO_4(s) \Longrightarrow Pb^{2+}(aq) + CrO_4^{2-}(aq)$	$2.8 \times 10^{-13}$
Lead(II) iodide	$PbI_2(s) \Longrightarrow Pb^{2+}(aq) + 2I^{-}(aq)$	$7.1 \times 10^{-9}$
Magnesium carbonate	$MgCO_3(s) \rightleftharpoons Mg^{2+}(aq) + CO_3^{2-}(aq)$	$3.5 \times 10^{-8}$
Magnesium fluoride	$MgF_2(s) \rightleftharpoons Mg^{2+}(aq) + 2F^{-}(aq)$	$3.7 \times 10^{-8}$
Magnesium hydroxide	$Mg(OH)_2(s) \Longrightarrow Mg^{2+}(aq) + 2OH^{-}(aq)$	$1.8 \times 10^{-11}$
Magnesium phosphate	$Mg_3(PO_4)_2(s) \implies 3 Mg^{2+}(aq) + 2 PO_4^{3-}(aq)$	$1 \times 10^{-25}$
Mercury(I) chloride	$Hg_2Cl_2(s) \Longrightarrow Hg_2^{2+}(aq) + 2Cl^-(aq)$	$1.3 \times 10^{-18}$
Mercury(II) sulfide	$HgS(s) \Longrightarrow Hg^{2+}(aq) + S^{2-}(aq)$	$2 \times 10^{-53}$
Silver bromide	$AgBr(s) \Longrightarrow Ag^{+}(aq) + Br^{-}(aq)$	$5.0 \times 10^{-13}$
Silver chloride	$AgCl(s) \Longrightarrow Ag^{+}(aq) + Cl^{-}(aq)$	$1.8 \times 10^{-10}$
Silver iodide	$AgI(s) \Longrightarrow Ag^{+}(aq) + I^{-}(aq)$	$8.5 \times 10^{-17}$

#### General Rules for the Solubility of Ionic Substances in Water

- 1. Nearly all salts of nitrates ( $NO_3^-$ ) and acetates ( $C_2H_3O_2^-$  or  $CH_3CO_2^-$ ) are soluble.
- 2. All salts of chlorides (Cl<sup>-</sup>) are soluble, except salts of Ag<sup>+</sup> ions, Hg<sub>2</sub><sup>2+</sup> ions and Pb<sup>2+</sup> ions. All salts of bromides (Br<sup>-</sup>) are soluble, except salts of Ag<sup>+</sup> ions, Hg<sub>2</sub><sup>2+</sup> ions and Pb<sup>2+</sup> ions. All salts of iodides (I<sup>-</sup>) are soluble, except salts of Ag<sup>+</sup> ions, Hg<sub>2</sub><sup>2+</sup> ions and Pb<sup>2+</sup> ions.
- 3. All salts of sulfates (SO<sub>4</sub><sup>2-</sup>) are soluble, except salts of Ba<sup>2+</sup>, Sr<sup>2+</sup>, and Pb<sup>2+</sup>. Salts of Ca<sup>2+</sup> ions, Hg<sub>2</sub><sup>2+</sup> ions, and Ag<sup>+</sup> ions are only slightly soluble.
- 4. All salts of group I metal ions and ammonium  $(NH_4^+)$  ions are soluble.
- 5. All salts of hydroxides (OH<sup>-</sup>) are insoluble, except for group I metal ions and ammonium (NH<sub>4</sub><sup>+</sup>) ions. Salts of Ba<sup>2+</sup> ions, Sr<sup>2+</sup> ions, and Ca<sup>2+</sup> ions are marginally soluble.
- 6. All salts of sulfides (S<sup>2-</sup>) are insoluble except those of group I metal ions, ammonium (NH<sub>4</sub><sup>+</sup>) ions, Ca<sup>2+</sup> ions, Sr<sup>2+</sup> ions, and Ba<sup>2+</sup> ions.
- 7. All salts of phosphates  $(PO_4^{3-})$  and carbonates  $(CO_3^{2-})$  are insoluble except those of group I metal ions and ammonium  $(NH_4^+)$  ions.
- 8. Most salts of chromates ( $CrO_4^{2-}$ ) are only slightly soluble except those of group I metal ions and ammonium ( $NH_4^+$ ) ions.

## Solubility and $K_{\rm sp}$

- The **solubility** of a solute is the quantity of the solute that dissolves in a certain amount of solvent. The **molar solubility** is the solubility in units of moles per liter ( $mol \cdot L^{-1}$ ).
- $\checkmark$  The molar solubility of an insoluble/sparingly salt can be calculated from  $K_{\rm sp}$ .

$$AgCl(s) \rightleftharpoons Ag^{+}(aq) + Cl^{-}(aq) \qquad K_{sp} = 1.6 \times 10^{-10}$$

(I) Initial concentrations (M):

Change in concentrations (*M*):

- -x +x +x
- (E) Equilibrium concentrations (M): -x = x

$$K_{sp} = 1.6 \times 10^{-10} = [Ag^+][Cl^-] = (x)(x) = x^2$$
  
 $x = 1.3 \times 10^{-5} \text{ M}$ 

 $\checkmark$  The amount *x* represents the concentration of AgCl(s) that dissolved (which is the molar solubility).

Molar solubility of AgCl in water =  $1.3 \times 10^{-5}$  M

#### Sample Exercise 18.4 Calculating Molar Solubility from $K_{sp}$

The  $K_{\rm sp}$  of CaF<sub>2</sub> is 3.9 × 10<sup>-11</sup>. Calculate the concentrations of Ca<sup>2+</sup> and F<sup>-</sup> ions and the molar solubility of CaF<sub>2</sub>(s) in pure water.

$$CaF_{2}(s) \rightleftharpoons Ca^{2+}(aq) + 2F^{-}(aq) \qquad K_{sp} = 3.9 \times 10^{-11}$$
(I) Initial concentrations (M):
$$- \qquad 0 \qquad 0$$
(C) Change in concentrations (M):
$$-x \qquad +x \qquad +2x$$
(E) Equilibrium concentrations (M):
$$- \qquad x \qquad 2x$$

$$K_{sp} = 3.9 \times 10^{-11} = [\text{Ca}^{2+}][\text{F}^{-}]^2 = (x)(2x)^2 = 4x^3$$
  
 $x = 2.1 \times 10^{-4} \text{ M}$   
 $[\text{Ca}^{2+}] = x = 2.1 \times 10^{-4} \text{ M}$   
 $[\text{F}^{-}] = 2x = 4.2 \times 10^{-4} \text{ M}$ 

Molar solubility of CaF<sub>2</sub>(s) =  $x = 2.1 \times 10^{-4}$  M

**Practice Exercise:** The  $K_{\rm sp}$  value of Al(OH)<sub>3</sub>(s) is  $5.0 \times 10^{-33}$  at 25 °C. Calculate its molar solubility in pure

water at 25 °C. (Answer:  $3.7 \times 10^{-9}$  M)

#### **Sample Exercise 18.5** Calculating $K_{sp}$ from Molar Solubility

Silver chromate, Ag<sub>2</sub>CrO<sub>4</sub>(s), is a red solid that dissolves in water to the extent of  $8.7 \times 10^{-5}$  mol·L<sup>-1</sup>. Calculate its  $K_{\rm sp}$ .

$$Ag_2CrO_4(s) \rightleftharpoons 2Ag^+(aq) + CrO_4^{2-}(aq)$$

- (I) Initial concentrations (M): 0
- (C) Change in concentrations (M): -x + 2x + x
- (E) Equilibrium concentrations (M):  $-2x x (x = 8.7 \times 10^{-5} \text{ M})$

$$K_{sp} = [Ag^+]^2 [CrO_4^{2-}] = (2x)^2(x) = 4x^3 = 4(8.7 \times 10^{-5})^3 = 2.6 \times 10^{-12}$$

**Practice Exercise:** Calculate the  $K_{\rm sp}$  value for bismuth sulfide, Bi<sub>2</sub>S<sub>3</sub>(s), which has a solubility of  $1.0 \times 10^{-15}$  in pure water at 25 °C. (Answer:  $1.1 \times 10^{-73}$ )