OPTIMIZATION AND ANALYSIS OF EARTH AND MARS MOLNIYA ORBITS

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INTRODUCTION

Molniya orbits are highly elliptical orbits that were first developed by the Soviet Union in the 1960s for use in their communication satellite system.[1] Due to launching from regions with high latitudes and requiring a large inclination change, it can be very costly to launch satellites from Russia into Geosynchronous Equatorial Orbits (GEO). However, a GEO orbit is a very useful flight path for a satellite due to its distinct advantage of having the same orbital period as the rotation rate of the Earth. This means that when placed in orbit, the satellite will maintain over the same point on the Earth (or its central body of attraction, if not the Earth) at all times. This is a very useful feature for communications, imaging, and many other scientific and real-world applications.

To design a flight path that resulted in a similar objective as GEO orbits, the Molniya orbit was designed with high eccentricities and inclinations to achieve an orbit that could easily be launched from high altitudes and result in a large apogee altitude. This results in a flight path that spends a large portion of its time near apogee, over the desired target ground location, that moves quickly through perigee (at "lightning" speed) to return to the desired location quickly for communications and imaging purposes. The orbit got its name from the Russian word Molniya, meaning lightning, which characterizes the speed of the spacecraft through perigee and the orbit shape that is similar to that of a lightning bolt.

A disadvantage of the Molniya orbits is the low perigee altitude caused by the highly eccentric shape of the ellipse. This can be troublesome when considering the atmospheric drag that acts on the spacecraft at low altitudes, as well as the perturbation due to the Earth's oblateness (J2) that acts with greater magnitude with an inverse relationship to the distance from the central body. The effect of the Earth's J2 perturbation is well studied and known to cause a drift in the argument of perigee, ω, and the right ascension of the ascending node (RAAN), Ω. This perturbation would require constant stationkeeping maneuvers to maintain the orientation of the Molniya orbits and are cleverly resolved by solving for inclinations that "freeze" the argument of perigee and minimize the drift rate of the RAAN.

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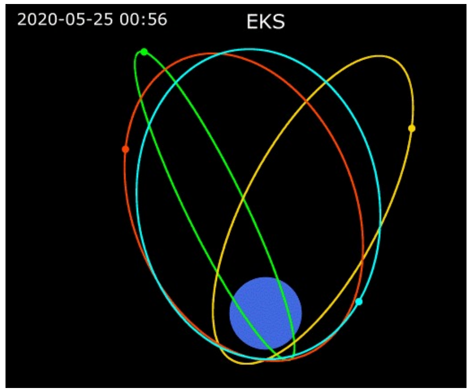

Figure 1: Monliya Orbits for the "Edinaya Kosmicheskaya Sistema" (a Russian early warning satellite system): Kosmos 2510 (red), Kosmos 2518 (yellow), Kosmos 2541 (cyan), Kosmos 2546 (green)[2]

Problem Statement

In this assignment, we were tasked with optimizing the orbital elements of two Molniya orbits such that the chosen orbit "freezes" the argument of perigee and minimizes the drift rate, $\dot{\Omega}$, of the RAAN. For a given orbital period, T, and minimum perigee altitude, r_p, we then must determine the optimal eccentricity and inclination that satisfies the aforementioned requirements. Lastly, we will look at the effects of a given Molniya orbit by propagating the state of a spacecraft using two-body dynamics, including the J2 perturbation, to analyze how its orbital elements change as a result of the J2 perturbation with the given initial conditions.

EARTH BOUND MOLNIYA ORBIT

The first problem that we will consider is that of an Earth-orbiting Molniya orbit with specified parameters given in Table 1.

$\boldsymbol{T}$ (Orbital Period)	$r_{p_{min}}$ (Minimum Perigee Altitude)
$\frac{1}{3}$ Day = 28800 seconds	600 km

Table 1: Specified parameters of the Earth-bound Molniya Orbit

From these parameters, we can derive the necessary semimajor axis for our orbit using the equation for the orbital period of an ellipse,

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}. \tag{1}$$

Rearranging for a we obtain,

$$a = \left(\left(\frac{T}{2\pi}\right)^2 \mu\right)^{\frac{1}{3}} = 20307.4006\text{km}. \tag{2}$$

Next, we must determine the appropriate eccentricity, e. Considering that we wish to achieve an orbit that allows for a spacecraft to spend long durations of time near apogee, in order to be positioned above the target ground location for long periods of time, the trivial answer is to maximize our eccentricity. Choosing the maximum eccentricity pushes the apogee distance out as far as possible, slowing the spacecraft down. Given the minimum perigee distance, r_p we can then solve for the appropriate eccentricity,

$$e = 1 - \frac{r_p}{a} = 0.65637. \tag{3}$$

Finally, we can then solve for the required inclination through the equation for the rate of change of the argument of perigee, knowing that we must "freeze" the argument of perigee,

$$\dot{\omega} = \frac{3}{4} \cdot n \cdot J2 \left(\frac{R}{a}\right)^2 \frac{5\cos^2 i - 1}{\left(1 - e^2\right)^2} = 0 \tag{4}$$

where,

$$n = \sqrt{\frac{\mu}{a^3}}. \tag{5}$$

Finally, since everything but the inclination is already determined, and therefore a constant, we simply need to find the roots of the equation,

$$5\cos^2 i - 1 = 0. \tag{6}$$

It is easy to see that the roots of the trigonometric function lie at,

$$i = \cos^{-1}\left(\pm\sqrt{1/5}\right). \tag{7}$$

We now have two choices for the inclination, $i = 63.435°$, and, $i = 116.565°$. Using the equation for the drift rate of the RAAN,

$$\dot{\Omega} = -\frac{3}{2} \cdot n \cdot J2 \cdot \left(\frac{R}{a}\right)^2 \frac{\cos i}{\left(1 - e^2\right)^2} \tag{8}$$

we obtain the following values,

$$\dot{\Omega}(63.435°) = -4.8246\text{e-08 rad/s} \tag{9}$$

$$\dot{\Omega}(116.565°) = 4.8246\text{e-08 rad/s}. \tag{10}$$

Considering that these values are identical in magnitude, we would then simply need to choose the direction that we want the RAAN to drift to determine the desired inclination of our Molniya orbit. This process results in the elements desired for our orbit seen in Table 2.

a	e	i
20307.4006 km	0.65637	63.435° or 116.565°

Table 2: Optimized orbital parameters for the Earth-bound Molniya orbit.

MARS-BOUND MOLNIYA ORBIT

For a Molniya orbit about Mars, we can apply the same approach from equations [1-8] with the given parameters specified in Table 3.

T (Orbital Period)	$r_{p_{min}}$ (Minimum Perigee Altitude)
88775 seconds	400 km

Table 3: Specified parameters of the Mars-bound Molniya Orbit

The optimized orbital elements for our Mars-bound Molniya orbit can be seen in Table 4.

a	e	i
20446.678239 km	0.814640	63.435° or 116.565°

Table 4: Optimized orbital parameters for the Mars-bound Molniya orbit.

The computed orbital elements for the Molniya orbit about Mars result in the following RAAN drift rates,

$$\dot{\Omega}(63.435°) = -2.261\text{e-}08 \text{ rad/s} \tag{11}$$

$$\dot{\Omega}(116.565°) = 2.261\text{e-}08 \text{ rad/s.} \tag{12}$$

MOLNIYA ELEMENT VARIATION

As previously discussed, one of the great discoveries of the Molniya orbits was the ability to "freeze" the argument of perigee while achieving very low RAAN drift rates. This, along with a cleverly chosen orbital period and eccentricity, allows for a spacecraft placed in one of these orbits to remain over a desired location on the Earth's (or other central body's) surface for prolonged periods of time. The highly eccentric orbit pushes the apogee distance to great lengths that slow the motion of the spacecraft during the desired regions of the true anomaly. This facilitates imaging, communications, and other tasks that require the spacecraft to be located in a specific location for a long duration.

We will analyze the change in the initial orbital elements of the specified Molniya orbit, given in Table 5, to better understand how these orbits yield favorable characteristics for mission design. To do so, the initial state of the Molniya orbit is propagated, using MATLAB's *ode89* numerical integrator, for a perturbed two-body dynamical system including the effect of the Earth's J2 spherical harmonic.

a	e	i	ω	Ω	M_0
26600 km	0.0.74	1.10654 rad	5°	90°	10°

Table 5: Initial osculating elements of the specified Molniya orbit to be studied.

The second-order equations of motion for a J2-perturbed two-body system can be written using the method of special perturbations,

$$\ddot{\boldsymbol{x}} = -\frac{\mu}{r^3}\hat{\boldsymbol{r}} + \boldsymbol{p_{J_2}} \tag{13}$$

where $\boldsymbol{p_{J_2}}$ is the acceleration due to the J2 perturbation written in cartesian coordinates as follows,

$$\boldsymbol{p_{J_2}} = \frac{3}{2}\frac{J_2\mu R^2}{r^4}\left[\frac{x}{r}\left(5\left(\frac{z}{r}\right)^2 - 1\right)\hat{\boldsymbol{i}} + \frac{y}{r}\left(5\left(\frac{z}{r}\right)^2 - 1\right)\hat{\boldsymbol{j}} + \frac{z}{r}\left(5\left(\frac{z}{r}\right)^2 - 3\right)\hat{\boldsymbol{k}}\right] \tag{14}$$

We can then write this as a system of first-order equations,

$$\frac{\boldsymbol{d}}{\boldsymbol{dt}}\begin{Bmatrix}\boldsymbol{r}\\ \boldsymbol{v}\end{Bmatrix} = \begin{Bmatrix}\boldsymbol{v}\\ \boldsymbol{a}\end{Bmatrix} = \begin{Bmatrix}\boldsymbol{v}\\ -\frac{\mu}{r^3}\hat{\boldsymbol{r}} + \boldsymbol{p_{J_2}}\end{Bmatrix} \tag{15}$$

Integrating this system of first-order equations, for a time span of 100 days, results in the trajectory seen in Figure 2. In a perfect two-body dynamical system, the Molniya orbit would be periodic. However, including the J2 perturbation, and cleverly choosing our orbital parameters, we can see the spacecraft revolutions forming a band as the RAAN drifts over 100 days. Converting the Cartesian coordinates of the trajectory to its keplerian orbital elements (Figure 3), it is clear that the initial elements chosen succeeded in producing the Molniya orbit objectives.

The J2 perturbation causes short-period variations for all of the elements. Typically for J2 perturbed two-body dynamics, we would expect to see a drift in the argument of perigee and the RAAN. However, as previously discussed, by choosing our orbital elements carefully we were able to "freeze" the argument of perigee and achieve $\dot{\Omega} << 1$. While the argument of perigee appears to drift throughout the trajectory, the total change is less than $0.1°$ for the entire 100-day duration $(1.093e - 08°/s)$. This is likely caused by a lack of precision for the given inclination as well as integration tolerance issues.

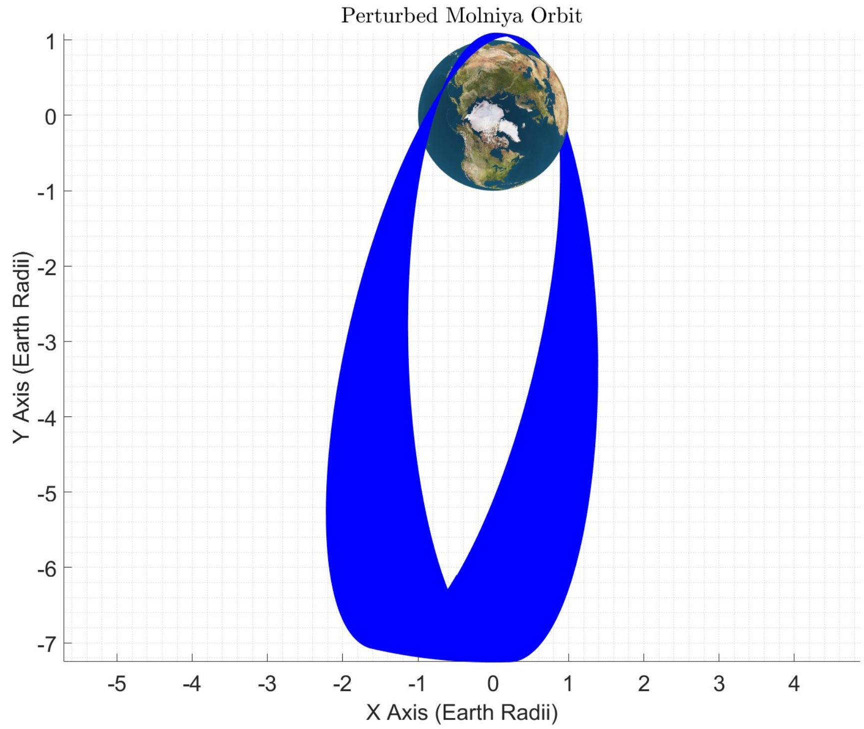

Figure 2: Perturbed Molniya orbit over a 100-day timespan in the Earth-centered-inertial frame

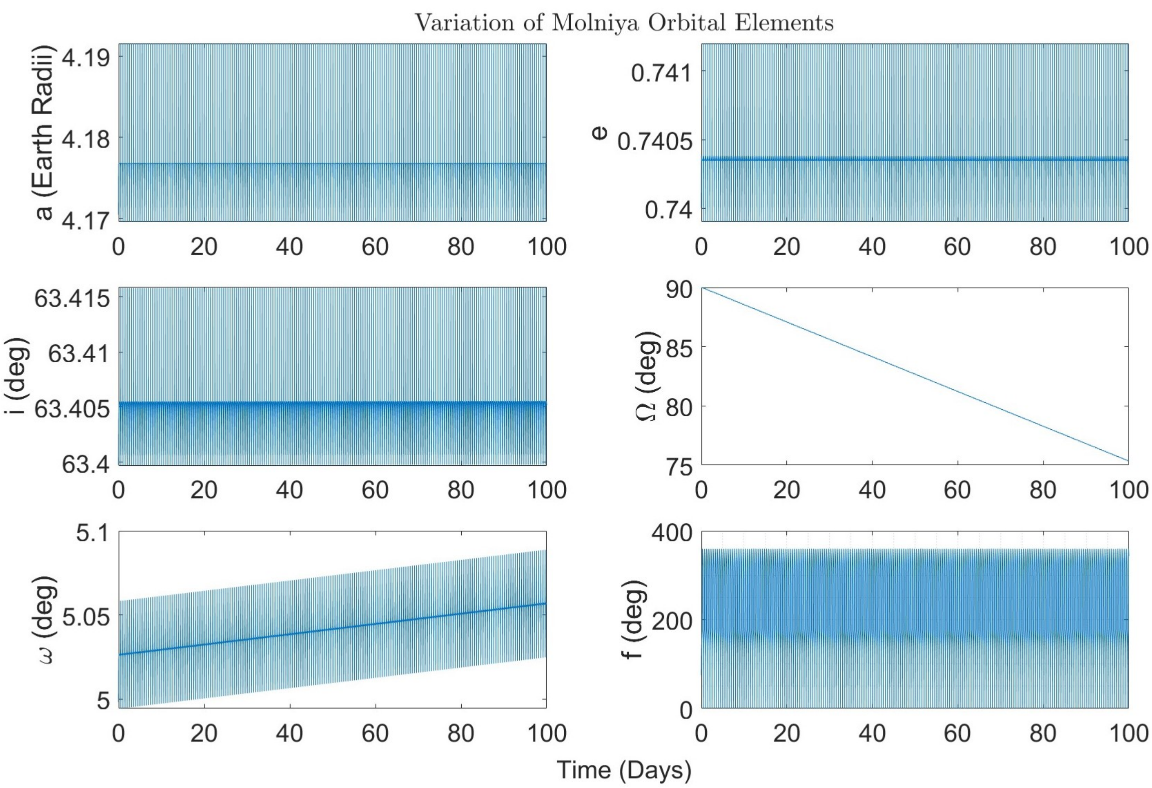

Figure 3: Orbital elements of the Molniya orbit throughout the 100-day trajectory.

GITHUB

Please find the code related to this assignment at the following link to my GitHub repository: https://github.com/brianpatrick3/Advanced-Orbital-Mechanics. In order to run the code properly, please make sure you have the CSPICE toolkit for MATLAB (mice) downloaded. If you do not, please go to the Homework1 folder and run the *setup.m* script (this script will automatically download and unzip CSPICE) and add the mice toolkit to the path.

REFERENCES

[1] Marcel J. Sidi. *Spacecraft Dynamics and Control: A Practical Engineering Approach*. Cambridge Aerospace Series. Cambridge University Press, 1997.

[2] Wikipedia. Animation of eks orbit around earth — Wikipedia, the free encyclopedia, 2023. [Online; accessed 18-March-2023].