Network Analysis Report

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Facebook Network Analysis

This project is an analysis of data gathered by researchers at Stanford University and available at https://snap.stanford.edu/data/gemsec-Facebook.html under the "Public Figures" dataset.

The structure of the publicly available data is a list of connections between verified Facebook pages of 11,565 public figures, with a total of 67,114 mutual connections. This is relatively sparse then, with an average degree of about 6 out of 11,564 other nodes.

My goal is to fit a logistic regression to model the probability of an edge between any two nodes of the graph (that is, a connection between any two public figures). For any two nodes, call them u and v, we have $P((u, v)) = logit^{-1}(\beta_u + \beta_v)$

All analysis is done in R 3.5.1, including this report written using the RMarkdown format.

Data Cleaning and Transformation

Before fitting a model, I removed 34 rows which listed a node connected to itself. I also adjusted the node names from numeric indicators (0 through 11564) to characters (n0 through n11564). This was done to avoid potential errors with improper variable names. To avoid possible duplication, the data was also arranged such that "node_1" is always the lower numbered of the pair.

The larger data transformation task was adding information on the missing edges. The training data for the logistic regression needs to have every possible connection between nodes; the dataset from SNAP only includes the nodes that are in fact connected. With 11565 nodes, there are $\binom{11565}{2} = 66,868,830$ possible nodes. I filled those in, along with an indicator variable connection to show if that pair of nodes was connected (1) or not (0).

Due to the size of this full training set, it was not possible to fit the logistic regression through regular R packages on a laptop computer. Memory constraints will be discussed more later.

Testing

In order to build the regression model, I first worked with a much smaller dataset that I constructed. This allowed me to verify the model using an existing GLM function in R. The small test dataset has 13 nodes, labeled a-m, for a total of 78 possible edges, as such:

The design matrix has 78 rows of 14 columns; one column is the connection, the others are indicators of which nodes are involved in that potential edge

connection	a	b	c	d	e	f	g	h	i	j	k	l	m
1	1	1	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	1	0	0	0	0	0	0

From this matrix we can fit a logistic regression with the glm() function in R, and see the following coefficient estimates:

	beta
a	0.1377
b	0.1377
c	0.5256
d	0.1377
e	-0.6669
f	-0.6669
g	0.1377
h	-0.6669
i	0.1377
j	-0.2566
k	0.9218
l	-0.6669
m	-0.6669

This should generally be trustworthy, and so my implementation of logistic regression should give the same coefficient estimates.

Logistic regression is an iterative fit; we start with a set of initial estimates for the coefficients $\beta_1, \beta_2, ...\beta_n$ and then use the design matrix Z, the current coefficients $\beta^{(t)}$, the predicted probabilities $\pi^{(t)}$ and the true connections Y to update the coefficients and get a new set of estimates $\beta^{(t+1)}$ as such:

$$\beta^{t+1} = \beta^t + (Z^T W Z)^{-1}(Z^T(Y - \pi^t))$$

where W is a diagonal matrix with entries π^t

After several iterations, the estimates β should converge to final values. Running quite non-optimized code of this process for 10 iterations took approximately 1 second on my laptop, and got the following coefficient estimates:

names	Initial	One	Two	Three	Four	Five
a	0	0.1212	0.1373	0.1377	0.1377	0.1377
b	0	0.1212	0.1373	0.1377	0.1377	0.1377
c	0	0.4848	0.5248	0.5256	0.5256	0.5256
d	0	0.1212	0.1373	0.1377	0.1377	0.1377
e	0	-0.6061	-0.6656	-0.6669	-0.6669	-0.6669
f	0	-0.6061	-0.6656	-0.6669	-0.6669	-0.6669
g	0	0.1212	0.1373	0.1377	0.1377	0.1377
h	0	-0.6061	-0.6656	-0.6669	-0.6669	-0.6669
i	0	0.1212	0.1373	0.1377	0.1377	0.1377
j	0	-0.2424	-0.2564	-0.2566	-0.2566	-0.2566
k	0	0.8485	0.9204	0.9218	0.9218	0.9218
l	0	-0.6061	-0.6656	-0.6669	-0.6669	-0.6669
m	0	-0.6061	-0.6656	-0.6669	-0.6669	-0.6669

Although I did run 10 iterations, I've clipped the table as the results had converged by the 5th iteration, and match the glm() output to each decimal point.

The intial values were all $\beta = 0$, which corresponds to all predicted probabilities = 0.5. Alternatively I could have calculated βs such that all intial predicted probabilities were 0.4487, the proportion of connections in the training data. This shouldn't affect the eventual estimates of β but may impact how many iterations are required to converge.

Selecting a Larger Dataset

Now that I've verified my method, I'll move on to a larger dataset. When the entire design matrix can't fit into memory at once, what I'll need to do is split up the data into chunks of k rows, and for each chunk calculate $(Z^T WZ)$ and $(Z^T(Y - \pi^t))$, then sum up all of those so we can calculate $(Z^T WZ)^{-1}(Z^T(Y - \pi^t))$ and the new estimates $\beta^{(t+1)}$

I should be clear that at no point is $(Z^T WZ)^{-1}$ directly computed; that would be inefficient in terms of memory and processor usage. Instead, through a few steps using functions implemented in Fortran and C, the components of the resulting matrix $(Z^T WZ)^{-1}(Z^T(Y - \pi^t))$ are calculated without needing to directly know the invverse $(Z^T WZ)^{-1}$. As these functions are wrappers around Fortran and C functions, they're also quite fast relative to normal R code.

Although memory is no longer a relevant restriction (assuming relatively small size of chunks k), compute time still is. Early tests on the full dataset led me to estimate a computer time of several months for a single iteration. For that reason, my final analysis here is on a smaller subset of the data.

For the full 11565 nodes, I calculated the degree of each node (number of connections including that node). I then selected the 1200 most popular nodes (slightly more due to ties). That amount was chosen after several trials estimating time per iteration for various dataset sizes.

I decided to choose popular nodes instead of nodes at random because the overall data is quite sparse, as mentioned previously. Randomly choosing nodes resulted in a sample with many unconnected nodes that needed to be dropped, and often most remaining nodes only had 1 or 2 connections. Predicted probabilities would almost always be less than 0.5, so the model would predict no connections at all. Selecting high degree nodes gave a more interesting data set for analysis.

Fitting the large model

node_1	node_2	connection
n0	n100	0
n0	n1006	0
n0	n1009	0
n0	n1011	0
n0	n1014	0
n0	n10168	0

node_1	node_2	connection
n9985	n9992	0
n9985	n9993	0
n9985	n9994	0
n9992	n9993	0
n9992	n9994	0
n9993	n9994	0

As you can see from these snippets of my big sample, there still aren't a lot of connections overall. With 1286 nodes, 826,255 possible connections, only 18,967 are actually connected, about 2%

Each iteration of updating the β estimates took about 45 minutes on my laptop. I chose a chunk size of 643, primarily because the data would be split into exactly 1285 chunks with no need to worry about partially filled chunks.

	Initial	Iter 1	Iter 2	Iter 3	Iter 4	Iter 5	Iter 6	Iter 7	Iter 8	Iter 9
n0	0	-1.0273	-1.6493	-2.2774	-2.8969	-3.3424	-3.5040	-3.5198	-3.5199	-3.5199
n100	0	-0.9588	-1.4400	-1.7637	-1.9112	-1.9403	-1.9416	-1.9416	-1.9416	-1.9416
n1006	0	-0.9058	-1.2888	-1.4481	-1.4527	-1.4400	-1.4382	-1.4382	-1.4382	-1.4382
n1009	0	-0.9868	-1.5236	-1.9572	-2.2390	-2.3352	-2.3447	-2.3448	-2.3448	-2.3448
n1011	0	-0.7376	-0.8624	-0.7315	-0.6021	-0.5636	-0.5593	-0.5592	-0.5592	-0.5592
n1014	0	-0.9588	-1.4400	-1.7637	-1.9112	-1.9403	-1.9416	-1.9416	-1.9416	-1.9416

A list of the final estimates for all βs are at the end of this report.

Now that I have coefficients for the model, I can generate predictions for each pair of nodes; as per the model specification given earlier, the prediction is the probability that the two nodes are connected. If we want a binary prediction, we just round - anything above 0.5 turns into 1 = predicted to have an edge, anything below turns into 0 = predicted not to have an edge.

Here are the most and least likely predicted edges, according to the logistic regression:

node_1	node_2	connection	Probability	Binary_Prediction
n4093	n9690	1	0.6581725	1
n4093	n6400	1	0.6550922	1
n4093	n5638	1	0.6488171	1
n4093	n7912	1	0.6456208	1
n6400	n9690	0	0.6249133	1
n2099	n4093	1	0.6220865	1

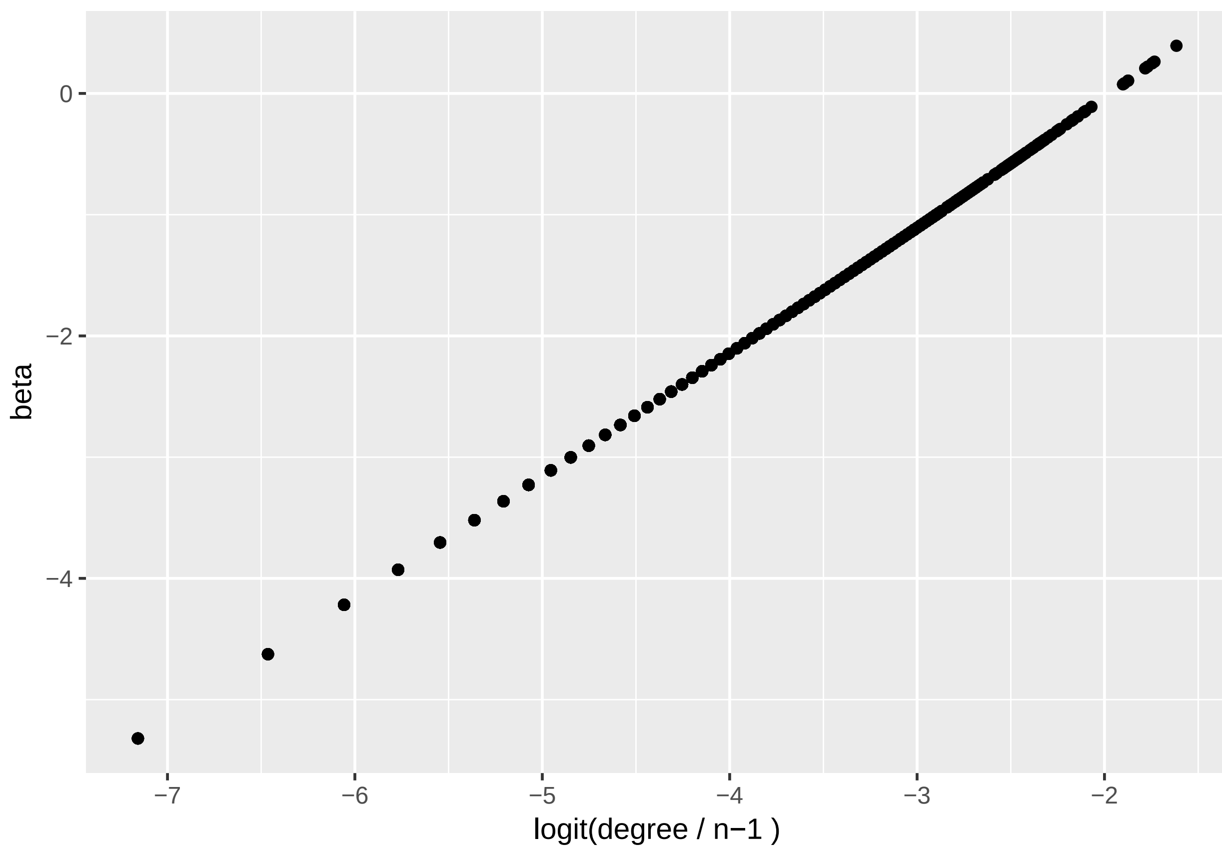
node_1	node_2	connection	Probability	Binary_Prediction
n11185	n8821	0	0.0000239	0
n1745	n4426	0	0.0000239	0
n1745	n8821	0	0.0000239	0
n3803	n4426	0	0.0000239	0
n3803	n8821	0	0.0000239	0
n4426	n8821	0	0.0000239	0

To summarize the correct and incorrect predictions, we can view a confusion matrix

	Predicted Edge	Predicted No Edge
Edge	63	18904
No Edge	28	807260

Overall, when the model predicts an edge it is correct 63 / 91 = 69% of the time. When the model predicts no edge, it is correct 807620 / 826524 = 97.712% of the time, for a combined accuracy of 97.710%. This only very slightly exceeds the 97.704% accuracy of the null model, predicting no edge in every situation. Overall, due to the severely imbalanced training set, it is safer to predict “no edge”, and thus the model has far more false negatives (real edges that were not predicted) than false positives (predicted edges that were not real).

For a different perspective on the logistic model in this situation, let’s look at the following plot



What’s clear from this is that the logistic regression is only going to predict edges based on the overall popularity of the nodes. If you really wanted to predict whether or not two people are friends, you’d probably be interested in a lot of information: if they have mutual friends, if they have common interests, if they live in the same area, if they’re close in age, etc. You probably wouldn’t just assume two people are friends because they both have a lot of friends (or that two people aren’t friends because they both don’t have many friends). There are a lot of intracacies to social networks that aren’t a part of this logistic regression. However, based on the large but simple input data, it does a good job of identify likely and unlikely connections nonetheless.

Conclusion

This was a very rewarding project for me to work on. I learned a lot about programming in R, much of which ended up not being relevant but that I may have use for in the future. The biggest takeaway for me in terms of optimizing code in R is that matrices are faster than dataframes (or tibbles, or data.tables), and of course as mentioned previously that using backsolve() and chol() is faster than inverting matrices.

It may be relevant to note that the method I implemented for this project could easily be modified to be run in parallel over multiple processors or machines. With each processor evaluating one chunk, efficiency should increase linearly with the number of processors (up to the number of chunks, but chunk size can be decreased). Fitting a logistic model to the entire 11,565 node dataset could be done in that way.

The source code for this analysis, along with several intermediate datasets I constructed, are available in a repository on my github page https://github.com/brianpclare/Network_Analysis

Full results

node	beta	node	beta	node	beta	node	beta
n0	-3.519911	n3307	-1.591482	n5722	-1.019750	n7929	-4.218261
n100	-1.941602	n3310	-0.789836	n5748	-2.146877	n7933	-3.228767
n1006	-1.438190	n3316	-2.734621	n5749	-1.200891	n7935	-1.591482
n1009	-2.344800	n3323	-3.703966	n5759	-0.847163	n7940	-3.928844
n1011	-0.559228	n3331	-1.801236	n5770	-2.400587	n7948	-1.438190
n1014	-1.941602	n3332	-1.261017	n5772	-2.060067	n7955	-2.400587
n10168	-4.218261	n3336	-3.519911	n5775	-3.109254	n7959	-3.002165
n1018	-3.364029	n3338	-2.146877	n5786	-3.228767	n7973	-2.816389
n1024	-1.564464	n3341	-2.905127	n5790	-2.291788	n7974	-2.291788
n1032	-1.676632	n3343	-0.414355	n5794	-1.979681	n7975	-1.125154
n10381	-5.320347	n3344	-1.834692	n5795	-2.734621	n7977	-2.658789
n1048	-2.459466	n3350	-1.538071	n5809	-1.512275	n7990	-1.391314
n10490	-4.218261	n3354	-2.291788	n5812	-1.941602	n7991	-0.922672
n105	-1.261017	n3355	-3.002165	n5814	-2.734621	n7992	-1.346254
n1053	-2.588073	n3359	-2.905127	n5824	-1.281755	n7998	-2.344800
n1060	-2.734621	n3367	-1.462359	n5843	-0.891923	n8003	-3.519911
n1078	-1.904801	n3371	-3.228767	n5844	-2.521812	n8007	-0.362440
n1084	-3.519911	n3374	-0.607105	n5858	-3.109254	n8016	-1.143646
n1087	-2.658789	n3375	-1.088997	n5876	-2.102603	n8020	-3.228767
n109	-2.905127	n3376	-2.102603	n5877	-0.669480	n8021	-3.703966
n11043	-3.928844	n3387	-3.519911	n5882	-0.547520	n8022	-2.734621
n1114	-3.364029	n3397	0.075138	n5896	-1.768758	n8030	-1.941602
n1117	-2.588073	n3399	-3.002165	n5902	-3.364029	n8035	-2.588073
n11185	-5.320347	n3401	-1.240635	n5904	-3.109254	n8036	-2.734621
n1126	-2.459466	n3403	0.105312	n5906	-2.459466	n8039	-2.291788
n1127	-1.462359	n3406	-2.459466	n5916	-3.928844	n8058	-2.816389
n1134	-1.834692	n3408	-0.302560	n5932	-1.487046	n8063	-3.519911
n1143	-1.538071	n341	-2.459466	n5934	-2.905127	n8064	-4.218261
n11438	-4.625462	n3414	-3.364029	n5943	-2.344800	n8065	-1.261017
n11459	-3.928844	n343	-2.400587	n5954	-3.109254	n8072	-0.656767
n1152	-3.109254	n3433	-0.708374	n5959	-1.768758	n8077	-0.789836
n11557	-4.625462	n344	-3.109254	n5962	-2.734621	n8099	-0.734963
n1156	-2.102603	n3468	-2.816389	n5978	-2.658789	n8125	-2.344800
n117	-2.905127	n348	-3.928844	n5988	-1.869190	n8142	-1.904801
n1171	-2.019133	n3483	-0.490454	n5996	-2.146877	n8147	-0.938334
n1175	-1.591482	n3491	-2.344800	n5998	-1.904801	n8166	-2.905127
n1177	-1.003033	n35	-1.438190	n6	-2.521812	n8169	-1.281755
n1179	-0.292820	n3508	-1.302864	n6002	-3.228767	n8172	-1.768758
n1181	-3.002165	n3524	-2.521812	n6007	-0.254504	n8173	-1.512275
n1186	-3.364029	n3527	-3.519911	n6026	-1.801236	n8194	-2.193044
n1187	-1.801236	n3539	-1.676632	n6027	-1.200891	n8200	-1.240635
n1189	-0.189785	n3550	-1.979681	n6034	-3.364029	n8201	-1.240635
n119	-2.734621	n3551	-3.002165	n6056	-1.125154	n8226	-1.512275
n1204	-2.905127	n3554	-0.861908	n6065	-1.053883	n8230	-1.676632
n1218	-1.801236	n3569	-3.364029	n6069	-0.619347	n8243	-2.400587
n122	-3.519911	n3582	-0.594974	n6081	-2.102603	n8244	-2.588073
n1220	-3.109254	n3587	-3.109254	n6084	-2.816389	n8247	-3.519911
n124	-2.344800	n3598	-1.619159	n6086	-2.588073	n8250	-2.291788
n1242	-3.519911	n3607	-3.109254	n6090	-5.320347	n8255	-2.459466
n1245	-3.928844	n3614	-2.521812	n6111	-3.002165	n8257	-2.588073

node	beta	node	beta	node	beta	node	beta
n1248	-2.344800	n3645	-0.535914	n6125	-2.588073	n8259	-3.109254
n125	-3.519911	n3666	-3.519911	n6127	-1.487046	n8269	-3.002165
n126	-1.346254	n3682	-0.970262	n6130	-3.228767	n8270	-2.658789
n1265	-2.102603	n3683	-2.102603	n6149	-2.241281	n8271	-2.291788
n1269	-2.241281	n369	-2.658789	n615	-2.459466	n8280	-2.102603
n1271	-2.459466	n370	-2.241281	n6155	-1.801236	n8288	-0.847163
n1286	-2.060067	n3723	-2.905127	n6156	-1.019750	n8291	-2.816389
n129	-4.218261	n3724	-1.619159	n6158	-1.564464	n8293	-0.922672
n131	-2.816389	n3725	-1.071314	n6165	-0.818178	n8301	-3.109254
n1310	-2.521812	n3731	-1.941602	n6168	-2.588073	n8305	-2.400587
n1324	-3.002165	n3732	-2.102603	n6171	-3.002165	n8324	-2.241281
n1325	-1.106941	n3735	-2.193044	n6183	-2.241281	n833	-2.521812
n1334	-1.162426	n3740	-0.189785	n6187	-1.801236	n8330	-2.588073
n1341	-1.391314	n3742	-1.125154	n619	-2.291788	n8340	-1.414515
n1352	-1.768758	n3747	-2.588073	n6203	-1.088997	n8342	0.082735
n1358	-1.869190	n3749	-2.291788	n6204	-2.400587	n8346	-3.228767
n1360	-2.193044	n3751	-2.658789	n621	-2.344800	n8351	-1.676632
n1361	-1.676632	n3760	-0.110526	n6210	-2.816389	n8356	-3.002165
n1369	-2.400587	n3767	-1.438190	n6211	-2.102603	n8358	-2.734621
n1370	-2.588073	n377	-1.619159	n6222	-1.737199	n8362	-3.228767
n1376	-2.734621	n3771	-2.734621	n6223	-3.109254	n8375	-3.928844
n139	-3.228767	n3783	-2.102603	n6232	-0.970262	n8381	-2.291788
n1395	-2.344800	n3795	-1.647530	n6237	-2.291788	n8382	-1.834692
n14	-1.564464	n3800	-3.002165	n6239	-1.324359	n8392	-0.619347
n1403	-1.706507	n3803	-5.320347	n6250	-3.228767	n8403	-2.734621
n1405	-1.737199	n381	-1.200891	n6251	-4.218261	n8417	-3.519911
n1407	-2.060067	n3825	-3.228767	n6262	-2.400587	n8422	-1.869190
n1408	-2.588073	n383	-3.002165	n6276	-3.228767	n8425	-2.344800
n1452	-3.519911	n3831	-1.801236	n6290	-3.228767	n8428	-2.816389
n1453	-4.625462	n3855	-1.019750	n6296	-2.816389	n8433	-2.905127
n1468	-3.109254	n3856	-3.228767	n6297	-2.291788	n8436	-0.938334
n1479	-2.905127	n3863	-2.734621	n6302	-1.302864	n8442	-2.400587
n1488	-3.703966	n3867	-3.109254	n6311	-2.816389	n8444	-1.088997
n1493	-2.400587	n3873	-2.658789	n6320	-1.737199	n8457	-1.768758
n1494	-2.521812	n3879	-1.869190	n6322	-2.588073	n8460	-0.468280
n150	-3.703966	n388	-2.588073	n6327	-1.737199	n8461	-2.658789
n1504	-1.768758	n3887	-2.734621	n6330	-2.588073	n8462	-2.588073
n1509	-2.241281	n3892	-1.706507	n6342	-2.291788	n8465	-0.512994
n1511	-1.302864	n3911	-3.109254	n6343	-1.979681	n8468	-2.816389
n1520	-2.658789	n3918	-2.816389	n6351	-1.979681	n8472	-3.109254
n1522	-1.979681	n392	-2.521812	n6356	-0.922672	n8479	-2.400587
n1529	-3.928844	n3921	-2.588073	n6360	-0.891923	n8488	-1.706507
n153	-2.905127	n3924	-2.588073	n6364	-1.768758	n8490	-2.459466
n1532	-0.762108	n3937	-1.564464	n6366	-3.002165	n8506	-2.459466
n1536	-2.102603	n394	-3.364029	n6374	-2.400587	n8519	-2.588073
n1542	-2.291788	n3946	-1.619159	n6379	-1.737199	n8529	-2.102603
n1549	-3.109254	n3949	-4.218261	n6393	-2.459466	n8530	-2.291788
n1551	-2.291788	n396	-2.344800	n6394	-0.818178	n8535	-1.979681
n1565	-1.904801	n3976	-2.658789	n6399	-2.658789	n854	-1.869190
n157	-2.816389	n3979	-2.291788	n6400	0.248397	n8541	-2.734621
n1571	-1.036698	n3980	-3.228767	n6401	-1.414515	n8552	-3.109254
n1579	-2.193044	n399	-2.459466	n6410	-1.538071	n8572	-1.368567

node	beta	node	beta	node	beta	node	beta
n1586	-2.734621	n4007	-2.102603	n6411	0.082735	n8573	-2.658789
n1587	-2.459466	n401	-1.391314	n6418	-2.102603	n8580	-1.801236
n1588	-1.941602	n4010	-2.291788	n6438	-3.519911	n8585	-2.193044
n1589	-0.582952	n4011	-1.941602	n6451	-1.737199	n8589	-1.676632
n1593	-1.941602	n4018	-1.979681	n6452	-2.344800	n8597	-2.658789
n1595	-1.591482	n4030	-2.060067	n6460	-0.876826	n8601	-2.521812
n1598	-2.658789	n4043	-0.938334	n6464	-2.459466	n8603	-2.816389
n1602	-2.905127	n4055	-2.521812	n6474	-2.816389	n8605	-1.834692
n1620	-2.816389	n4060	-2.734621	n6496	-1.979681	n8606	-2.146877
n1632	-2.459466	n4063	-3.364029	n65	-2.291788	n8613	-1.438190
n1633	-2.734621	n4069	-2.146877	n6503	-1.125154	n8625	-0.393358
n1644	-2.102603	n4072	-2.521812	n6532	-2.146877	n8634	-2.658789
n1648	-3.228767	n4078	-3.002165	n6534	-2.521812	n8637	-3.364029
n1662	-2.193044	n4079	-1.979681	n6546	-1.647530	n8653	-2.658789
n1667	-3.703966	n4086	-3.364029	n6548	-2.344800	n8655	-3.002165
n1677	-4.218261	n4093	0.393102	n6554	-3.364029	n8665	-1.106941
n1685	-2.146877	n410	-1.834692	n6555	-2.588073	n8669	-1.071314
n1687	-3.002165	n4110	-4.625462	n6559	-0.832588	n8677	-1.737199
n1710	-2.019133	n4119	-3.228767	n6562	-2.734621	n8678	-1.904801
n1711	-2.905127	n4120	-2.658789	n6573	-2.658789	n8681	-1.834692
n172	-1.941602	n4126	-3.002165	n6576	-3.228767	n8684	-2.658789
n1737	-1.281755	n4128	-0.154039	n6582	-0.734963	n8687	-1.261017
n1740	-3.703966	n413	-2.400587	n6590	-2.734621	n8689	-3.364029
n1741	-1.591482	n4133	-1.979681	n6596	-2.905127	n8691	-3.002165
n1745	-5.320347	n4141	-0.907203	n6605	-1.143646	n8704	-0.762108
n1746	-1.261017	n416	-2.658789	n6608	-1.904801	n8720	-1.538071
n1752	-3.703966	n4168	-0.524405	n6621	-2.241281	n8721	-3.364029
n1767	-2.734621	n4170	-3.002165	n6623	-2.019133	n8730	-2.816389
n177	-2.019133	n4174	-0.446457	n6625	-3.109254	n8732	-2.816389
n1771	-2.060067	n4182	-2.193044	n6639	-1.200891	n8735	-2.102603
n1776	-1.143646	n4196	-2.344800	n6641	-0.847163	n8741	-3.228767
n1781	-3.002165	n4202	-1.240635	n6656	-1.346254	n8743	-1.281755
n1808	-0.748464	n4203	-1.281755	n6657	-2.658789	n8746	-3.928844
n1829	-2.241281	n4226	-3.109254	n6679	-2.734621	n8750	-3.109254
n1837	-2.816389	n4232	-1.162426	n6684	-1.414515	n8752	-2.344800
n184	-0.571038	n4234	-3.364029	n6700	-2.816389	n8753	-3.109254
n1843	-3.228767	n4258	-2.816389	n6708	-2.459466	n8783	-2.521812
n1867	-3.364029	n426	-3.364029	n6709	-0.970262	n8785	-2.344800
n1868	-0.535914	n4265	-1.125154	n6711	-2.241281	n8786	-1.706507
n1885	-2.241281	n4267	-2.019133	n6723	-2.816389	n8787	-1.261017
n1897	-2.146877	n4268	-2.816389	n6728	-1.801236	n8790	-2.344800
n1912	-1.737199	n4282	-2.102603	n6733	-2.588073	n8794	-1.053883
n1914	-2.019133	n4284	-1.438190	n6747	-2.291788	n8798	-2.459466
n1921	-1.801236	n4285	-3.364029	n6749	-2.102603	n8801	-1.462359
n1932	-2.658789	n4292	-3.002165	n6774	-1.591482	n8805	-2.734621
n1934	-2.588073	n4295	-2.146877	n6783	-1.834692	n8813	-2.658789
n1947	-2.193044	n4297	-3.109254	n6795	-3.002165	n8821	-5.320347
n1949	-2.658789	n43	-2.193044	n6799	-2.588073	n8825	-1.979681
n1954	-1.737199	n4302	-2.102603	n6810	-2.459466	n8826	-2.588073
n1955	-2.816389	n4330	-2.734621	n6814	-3.002165	n8839	-2.193044
n1963	-0.803929	n4337	-3.002165	n6819	-1.302864	n8842	-2.658789
n1964	-2.521812	n4352	-0.876826	n682	-2.521812	n8843	-2.905127

node	beta	node	beta	node	beta	node	beta
n1977	-1.768758	n4353	-1.200891	n6848	-0.382976	n8854	-1.941602
n1981	-2.291788	n4354	-0.490454	n6853	-2.060067	n8859	-1.768758
n1984	-1.647530	n4359	-2.734621	n6860	-2.019133	n8862	-2.400587
n1986	-2.241281	n4367	-2.905127	n6867	-3.002165	n8874	-2.905127
n1995	-3.228767	n4368	-1.414515	n6869	-1.302864	n8876	-3.002165
n2	-0.403817	n4372	-1.834692	n6872	-2.734621	n8877	-1.368567
n2003	-2.521812	n4383	-1.487046	n6873	-3.364029	n8878	-2.588073
n2022	-2.146877	n4385	-2.816389	n6878	-2.816389	n8886	-3.364029
n2031	-3.519911	n4393	-2.905127	n6889	-2.193044	n8887	-0.986539
n2034	-2.588073	n4400	-3.364029	n6891	-2.019133	n8902	-3.002165
n206	-1.181504	n4403	-3.228767	n6893	-2.734621	n8903	-0.414355
n2073	-2.344800	n4418	-1.979681	n6898	-2.193044	n8906	-2.588073
n2075	-1.941602	n4426	-5.320347	n6899	-1.904801	n8909	-1.200891
n208	-2.816389	n4438	-1.869190	n6901	-1.591482	n8911	-2.734621
n2086	-0.938334	n4464	-1.676632	n6907	-2.905127	n8917	-2.400587
n2099	0.105312	n4473	-2.588073	n6910	-0.571038	n8926	-1.240635
n2115	-2.905127	n4482	-1.302864	n6922	-2.816389	n8930	-0.490454
n2127	-1.346254	n4488	-2.658789	n6926	-2.588073	n8935	-3.109254
n2129	-1.088997	n4489	-3.109254	n693	-1.869190	n8936	-2.734621
n2149	-2.816389	n4499	-2.241281	n6932	-1.162426	n8945	-2.816389
n2150	-1.414515	n4501	-3.364029	n6937	-2.588073	n8947	-1.564464
n2156	-2.060067	n4508	-3.228767	n6942	-1.979681	n8951	-1.538071
n2157	-2.816389	n4516	-1.869190	n6947	-2.344800	n8953	-2.291788
n217	-3.364029	n453	-2.658789	n6952	-1.003033	n8969	-0.226416
n2179	-2.658789	n4532	-2.816389	n6972	-3.109254	n8974	-0.832588
n2189	-2.060067	n4541	-2.816389	n6973	-1.737199	n8980	-2.400587
n2197	-1.591482	n455	-1.941602	n6974	-0.775897	n8988	-2.241281
n2203	-4.218261	n4556	-2.291788	n6978	-0.424974	n8994	-2.905127
n2211	-2.102603	n4557	-2.291788	n6980	-1.834692	n8997	-0.312367
n2212	-2.588073	n4581	-1.647530	n6987	-2.905127	n9014	-2.060067
n2224	-2.459466	n459	-1.391314	n6994	-0.775897	n9038	-1.512275
n2228	-2.588073	n4625	-2.400587	n6998	-2.658789	n9044	-2.102603
n2232	-3.928844	n4632	-1.438190	n7001	-3.109254	n9050	-2.588073
n2249	-2.400587	n4633	-2.241281	n7012	-0.535914	n9080	-2.241281
n2269	-3.228767	n4641	-2.816389	n7021	-1.979681	n9090	-2.459466
n2282	-2.241281	n4653	-2.521812	n7026	-2.905127	n9092	-0.217173
n2285	-1.979681	n4655	-3.519911	n7029	-1.834692	n9093	-1.647530
n2303	-2.291788	n4662	-2.658789	n7034	-2.060067	n9100	-1.647530
n2342	-2.019133	n4667	-1.512275	n704	-2.102603	n9104	-1.834692
n2352	-2.905127	n4682	-0.891923	n7050	-1.438190	n9117	-2.241281
n2364	-2.291788	n4684	-3.228767	n7053	-4.218261	n9118	-2.060067
n237	-1.487046	n4689	-2.060067	n7054	-2.734621	n9120	-3.228767
n2370	-2.459466	n4704	-1.591482	n7073	-2.588073	n9127	-2.734621
n2372	-3.703966	n4713	-1.941602	n7082	-3.109254	n9131	-2.816389
n2390	-1.261017	n4715	-2.193044	n7084	-2.060067	n9133	-0.631703
n2391	-1.737199	n4722	-1.088997	n7089	-1.462359	n9135	-3.002165
n2395	-2.734621	n4735	-2.344800	n7092	-1.019750	n9136	-3.002165
n24	-1.737199	n4761	-2.060067	n7102	-1.801236	n9170	-4.218261
n240	-2.344800	n4767	-1.462359	n7111	-2.521812	n9178	-2.146877
n2414	-2.588073	n4778	-1.125154	n7112	-1.706507	n9204	-2.658789
n2419	-1.391314	n4779	-1.941602	n7130	-3.928844	n9210	-3.109254
n2423	-3.109254	n4780	-2.734621	n7140	-0.446457	n9214	-1.036698

node	beta	node	beta	node	beta	node	beta
n2432	-0.876826	n4781	-1.979681	n7141	-2.658789	n9238	-3.519911
n2449	-2.019133	n4787	-2.102603	n7142	-3.109254	n9239	-1.869190
n2451	-1.564464	n4788	-1.003033	n7155	-3.002165	n9247	-2.241281
n247	-3.364029	n4789	-2.241281	n7158	-1.462359	n9253	-1.036698
n2473	-1.869190	n4792	-2.459466	n7164	-2.400587	n9258	-1.979681
n2480	-3.364029	n4796	-3.002165	n7172	-0.382976	n9278	-1.619159
n2500	-2.291788	n4799	-1.834692	n7173	-2.459466	n9288	-1.302864
n2506	-2.905127	n481	-3.519911	n7204	-0.424974	n9294	-2.588073
n2510	-2.816389	n4815	-1.904801	n7207	-3.002165	n9306	-2.734621
n2523	-1.979681	n4821	-1.240635	n7209	-1.768758	n9310	-2.734621
n253	-4.218261	n4850	-2.588073	n7214	-3.519911	n9312	-4.218261
n2531	-2.816389	n4862	-1.869190	n7217	-2.193044	n9331	-2.459466
n2532	-2.400587	n4864	-2.734621	n7237	-0.501678	n9337	-2.905127
n2535	-2.521812	n4865	-2.400587	n7255	-3.002165	n9341	-2.193044
n2549	-1.053883	n4867	-2.521812	n7265	-1.768758	n9345	-1.438190
n2574	-2.344800	n487	-3.228767	n7269	-2.193044	n9346	-3.364029
n2580	-3.364029	n4874	-2.459466	n7272	-2.344800	n9350	-2.193044
n2587	-2.019133	n4885	-2.588073	n728	-2.060067	n9352	-2.400587
n2602	-2.102603	n4889	-3.002165	n7286	-2.060067	n9360	-3.519911
n2612	-1.768758	n4892	-2.146877	n7287	-1.706507	n9383	-1.181504
n262	-0.986539	n4895	-2.459466	n7312	-1.941602	n9384	-2.344800
n2635	-0.342199	n491	-0.582952	n7316	-1.904801	n9393	-2.816389
n2656	-3.928844	n4915	-2.905127	n7321	-2.588073	n9400	-2.734621
n2663	-3.519911	n4917	-0.734963	n7323	-2.734621	n9410	-2.816389
n2666	-3.228767	n4921	-3.109254	n7326	-1.346254	n9412	-2.019133
n2680	-2.905127	n4953	-3.002165	n7348	-2.291788	n9428	-1.125154
n2682	-1.676632	n4958	-2.521812	n7349	-2.400587	n9429	-0.414355
n2685	-2.905127	n4959	-2.400587	n7354	-2.241281	n9433	-2.734621
n2690	-3.002165	n4966	-3.002165	n7368	-3.109254	n9437	-2.344800
n2691	-1.834692	n4968	-2.459466	n7372	-2.521812	n9443	-2.060067
n2705	-2.344800	n4990	-0.970262	n7376	-2.816389	n9447	-2.588073
n2718	-0.861908	n5009	-1.220597	n7380	-2.816389	n9450	-3.364029
n2722	-1.125154	n501	-1.706507	n7398	-2.241281	n9457	-3.228767
n2726	-3.109254	n5010	-2.658789	n7402	-3.364029	n9468	-0.524405
n2729	-3.002165	n5018	-2.816389	n7407	-2.291788	n9473	-1.391314
n2730	-2.060067	n5021	-2.146877	n7419	-0.154039	n9478	-1.801236
n2741	-1.941602	n5027	-1.591482	n7424	-1.979681	n9498	-2.102603
n2745	-2.344800	n5051	-2.102603	n7431	-2.588073	n9541	-1.003033
n2755	-2.060067	n5052	-3.364029	n7436	-2.521812	n9548	-2.734621
n2760	-2.734621	n5069	-2.102603	n7440	-2.344800	n9554	-0.748464
n2772	-2.658789	n5073	-0.789836	n7441	-1.414515	n9556	-2.734621
n2780	-3.109254	n5079	-2.019133	n745	-2.658789	n9559	-2.291788
n2802	-0.970262	n5086	-2.193044	n7453	-2.459466	n9566	-1.346254
n2810	-1.368567	n5091	-1.904801	n7458	-2.291788	n9567	-2.521812
n2818	-2.588073	n5101	-2.521812	n7464	-0.803929	n9569	-3.519911
n2820	-3.228767	n5108	-1.941602	n7468	-2.459466	n9573	-3.519911
n2821	-1.512275	n511	-1.979681	n7477	-1.003033	n9579	-3.519911
n2824	-2.658789	n5121	-1.512275	n7481	-3.109254	n9590	-2.905127
n2830	-0.512994	n5147	-1.676632	n7482	-2.344800	n9593	-3.703966
n2835	-2.658789	n5158	-3.519911	n7488	-0.876826	n9605	-0.803929
n2844	-3.228767	n5162	-1.261017	n749	-1.438190	n9610	-2.459466
n2845	-2.734621	n5191	-2.291788	n7498	-1.391314	n9618	-1.647530

node	beta	node	beta	node	beta	node	beta
n2857	-0.938334	n5208	-1.487046	n7503	-1.941602	n9624	-0.876826
n2862	-0.414355	n5213	-2.102603	n7533	-2.816389	n9636	-1.869190
n287	-2.816389	n5223	-2.102603	n7534	-1.941602	n9642	-2.905127
n2871	-2.658789	n5228	-0.594974	n7539	-2.816389	n9643	-3.519911
n2875	-2.459466	n5229	-2.344800	n7558	-3.109254	n9653	-2.816389
n2878	-2.905127	n5253	-3.364029	n7562	-2.521812	n9656	-1.302864
n288	-2.019133	n5259	-1.941602	n7564	-2.344800	n9662	-1.768758
n2880	-2.734621	n5289	-2.193044	n7571	-2.193044	n9666	-4.218261
n2894	-3.364029	n5294	-2.019133	n7574	-2.102603	n9671	-2.146877
n2899	-1.125154	n5297	-1.240635	n7577	-3.364029	n9675	-3.002165
n290	-1.619159	n5303	-0.145235	n7578	-2.588073	n9684	-0.559228
n2901	-2.658789	n5312	-1.869190	n7583	-1.941602	n9690	0.262059
n2902	-2.459466	n5328	-3.519911	n7589	-2.588073	n9693	-2.658789
n2907	-4.625462	n5345	-1.676632	n7590	-2.521812	n9694	-1.801236
n2914	-2.459466	n5348	-2.459466	n7597	-1.647530	n9696	-2.146877
n2919	-2.816389	n535	-1.619159	n7598	-1.706507	n9707	-2.193044
n2927	-2.019133	n5357	-2.905127	n7602	-2.459466	n9717	-1.647530
n2941	-1.324359	n5374	-1.647530	n7605	-1.368567	n9718	-2.019133
n2943	-2.146877	n5392	-3.364029	n7625	-3.109254	n9742	-2.588073
n2960	-2.060067	n5404	-3.109254	n7631	-2.905127	n9745	-2.241281
n2964	-3.519911	n5425	-2.241281	n7632	-2.459466	n9751	-3.002165
n300	-3.228767	n5435	-1.676632	n7634	-2.816389	n9755	-1.941602
n3000	-2.060067	n5447	-2.459466	n7638	-2.344800	n9765	-2.905127
n3011	-0.986539	n5458	-4.218261	n7643	-1.071314	n9768	-4.218261
n3017	-1.941602	n5464	-2.102603	n7644	-0.669480	n9774	-3.228767
n3019	-2.905127	n5475	-2.193044	n7646	-2.193044	n9785	-0.424974
n3026	-2.344800	n5487	-0.986539	n7650	-2.400587	n9805	-1.125154
n304	-3.002165	n5497	-2.521812	n7657	-2.102603	n9811	-3.002165
n3047	-2.291788	n5499	-1.324359	n7662	-2.400587	n9812	-2.734621
n3053	-2.905127	n55	-2.241281	n7666	-1.487046	n9815	-3.002165
n306	-3.703966	n550	-2.521812	n7681	-0.669480	n9828	-3.519911
n3061	-2.146877	n5503	-2.400587	n7685	-1.869190	n9851	-2.344800
n3062	-0.619347	n5522	-1.003033	n7691	-2.816389	n9852	-3.928844
n3068	-2.905127	n5523	-1.324359	n7703	-2.905127	n9858	-2.734621
n3070	-2.400587	n5525	-2.400587	n7704	-1.591482	n9865	-2.588073
n3073	-2.658789	n5528	-1.125154	n7713	-2.905127	n9868	-3.109254
n3080	-2.658789	n5549	-2.588073	n7724	-1.647530	n9872	-3.109254
n3084	-2.734621	n5554	-2.816389	n7725	-4.218261	n9873	-1.438190
n31	-1.240635	n5555	-1.564464	n7727	-3.228767	n9879	-1.591482
n3108	-1.391314	n5558	-3.364029	n7730	-1.564464	n9887	-2.344800
n3116	-2.459466	n5567	-1.801236	n7746	-2.193044	n9891	-2.588073
n3119	-3.364029	n558	-2.734621	n7764	-2.658789	n9893	-1.676632
n3120	-0.457325	n5580	-3.002165	n7778	-2.060067	n9904	-3.703966
n3141	-2.459466	n5590	-2.905127	n7779	-1.487046	n9907	-3.228767
n3149	-1.512275	n5600	-2.521812	n7789	-1.181504	n9914	-3.109254
n3153	-2.734621	n5602	-2.588073	n7795	-2.146877	n9916	-2.521812
n3161	-2.816389	n5604	-2.241281	n7796	-3.519911	n9923	-3.109254
n3165	-2.521812	n5612	-3.228767	n7805	-3.228767	n9939	-3.928844
n3171	-3.109254	n5614	-2.193044	n7806	-1.834692	n9948	-2.102603
n3175	-0.708374	n5621	-1.647530	n7821	-2.102603	n9953	-3.228767
n3181	-2.344800	n5629	-4.625462	n7823	-3.703966	n9959	-4.218261
n3184	-3.364029	n5632	-3.928844	n7825	-2.019133	n9960	-2.734621

node	beta	node	beta	node	beta	node	beta
n319	-2.816389	n5634	-1.904801	n7827	-2.521812	n9961	-2.060067
n3192	-2.905127	n5638	0.220741	n7829	-3.109254	n9963	-2.291788
n3199	-4.625462	n5648	-2.146877	n783	-2.459466	n9980	-1.979681
n3213	-2.816389	n565	-3.703966	n7854	-2.734621	n9985	-1.904801
n3214	-2.734621	n5653	-2.905127	n7866	-3.109254	n9992	-2.344800
n3224	-2.588073	n5664	-0.535914	n7884	-0.775897	n9993	-2.344800
n3227	-4.218261	n5675	-2.459466	n7889	-2.193044	n9994	-3.109254
n3237	-2.459466	n5678	-2.658789	n79	-1.941602	-	0.000000
n3270	-0.734963	n5693	-2.241281	n7906	-2.459466	-	0.000000
n3272	-2.658789	n5694	-1.869190	n7912	0.206743	-	0.000000
n3297	-2.658789	n57	-1.676632	n7914	-3.109254	-	0.000000
n3298	-2.521812	n5705	-1.647530	n7923	-1.462359	-	0.000000
n330	-0.938334	n5706	-0.970262	n7926	-2.521812	-	0.000000