Wavelet Multi-scale Correlation

The Wavelet Multi-scale Correlation (WMC) uses wavelet coefficients to calculate correlations between time series at different time scales. It has been used in wide range of fields like finance, economics, and climate studies [1, 3].

Fourier transform does not represent abrupt changes efficiently because it is not a local function in time. Wavelets are well localized in time and frequency can be used for simultaneous time and frequency analysis. Using a Discrete Wavelet Transform (DWT), a time series can be decomposed into time and frequency components. The decomposed components are called wavelet coefficients. Here we focus on the methodology of using these wavelet co-efficient to obtain correlations at different time scales. For more detailed introduction of wavelets and its applications, I can recommend reading the books [3, 2].

A type of DWT called Maximal Overlap Discrete Wavelet Transform (MODWT) gives same number of wavelet co-efficient at all the time scales and hence is used in correlation analysis. Consider two discrete time series X and Y of length T. Using MODWT, each of the time series can be decomposed into $J = 1, 2, ... \log_2 T$ time scales. Each of these time scales will have T discrete wavelet coefficients, each represented as $d_X^{j,t}$ and $d_Y^{j,t}$, where j is the time scale and $t = 1, 2, ..., T$ is the discrete time step. Variance and co-variance of these time series can be then calculated as:

$$Var_X^j \equiv \frac{1}{T_j} \sum_{t=M_j-1}^{T-1} \left[d_X^{j,t}\right]^2 \tag{1}$$

$$Var_Y^j \equiv \frac{1}{T_j} \sum_{t=M_j-1}^{T-1} \left[d_Y^{j,t}\right]^2 \tag{2}$$

$$COV_{XY}^j \equiv \frac{1}{T_j} \sum_{t=M_j-1}^{T-1} d_X^{j,t} d_Y^{j,t} \,, \tag{3}$$

where Var_X^j and Var_Y^j are the variances at j^{th} scale of the time series X and Y respectively. COV_{XY}^j is the co-variance between X and Y at j^{th} scale. $T_j = T - M_j - 1$ stands for the number of wavelet coefficients unaffected by the boundary, with $M_j = (2^j - 1)(M - 1)$ and M is the length of the wavelet filter used. The Wavelet Multi-scale Correlation (WMC) at j^{th} scale can then be found using the equation:

$$\rho_{XY}^j \equiv \frac{COV_{XY}^j}{\sqrt{Var_Y^j Var_X^j}}, \tag{4}$$

The values of ρ_{XY}^j can be between -1 and 1, with the extreme values indicating complete correlation or inverse correlation, and 0 indicating no correlation.

Abbreviations

DWT Discrete Wavelet Transform

MODWT Maximal Overlap Discrete Wavelet Transform

WMC Wavelet Multi-scale Correlation

Bibliography

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[3] Ramazan Gençay, Faruk Selçuk, and Brandon J Whitcher. *An introduction to wavelets and other filtering methods in finance and economics*. Elsevier, 2001.