

Cash Payments and the Penny Policy Debate*

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Abstract

This article constructs a model of optimal consumer-merchant exchange of cash payments. We use consumer payment choice diary data to quantify the burden of exchanging currency notes and coins. The model is then applied to analyze a policy debate whether to eliminate the penny coin from circulation by quantifying the effects of penny elimination on the burden of exchanging cash under a counterfactual U.S. economy.

Keywords: Optimal currency denominations, cash transactions, burden of change, penny elimination.

JEL Classification Numbers: D12, E42, G59.

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1. Introduction

Despite the availability of alternative means of payment, consumers still depend on cash. U.S. consumers in 2019, prior to the COVID pandemic, used cash for 26 percent of all payments and 47 percent of all payments below \$10, see Kim, Kumar, and O'Brien (2020). Coyle, Kim, and O'Brien (2021) found that, amidst the pandemic, this value declined to 19 percent of all payments in 2020. ECB (2020) finds that in seventeen euro area countries 73 percent by volume (48 percent by value) of all point-of-sale (POS) and person-to-person (P2P) payments were made with cash in 2019. Khiaonarong and Humphrey (2019) provide estimates of cash use for 11 countries and forecasts until 2026.

Cash payments are two-way exchanges of currency notes and coins. The payer (consumer, buyer) hands in currency notes and coins, then, if needed, receives change from the payee (merchant, seller) in the form of currency notes and coins. The burden of exchanging cash is proportional to the total number of coins and notes required to transact. While the (fixed) cost of producing a currency denomination is widely known, the exchange (flow) cost is not.¹ Consequently, policymakers interested in adjustments to the currency system have incomplete information on the total costs. Given the high frequency of cash usage, rectifying this incomplete information is of the utmost importance. The goal of this article is to quantify the burden of exchanging cash. We do this by constructing a framework for computing lower and upper bounds on the number of tokens exchanged in each transaction, where tokens refer to the sum of the number of currency notes and coins exchanged in both directions. The article therefore fills an important gap in the optimal denomination literature by providing policymakers with a framework for quantifying the burden of exchanging cash. Such a framework is necessary for policymakers considering adjustments to the currency system.²

¹For the cost of producing currency notes see https://www.federalreserve.gov/faqs/currency_12771.htm. The cost of minting U.S. coins can be found in the annual reports of the U.S. Mint, see <https://www.usmint.gov/about/reports>.

²The burden of exchanging currency notes and coins is shared by both consumers and merchants. Using multicountry panel data, Png and Tan (2021) show that cashier wages increase with retail cash usage. In addition, 8 of 10 cashiers preferred card payments to cash payments.

We quantify the burden of exchanging cash by first constructing a repeated game of consumer-merchant interactions. In the model both consumers and merchants are able to pay or give change with any currency denomination except \$2 notes.³ They jointly choose, given the dollar amount, which coins and notes to transact with. Using transaction data from consumer payment diary surveys spanning 2015 to 2019, we use the model to compute the burden of cash exchanges for each cash transaction observed. From that, we derive a lower bound distribution of the number of tokens exchanged during the cash transactions. The quantification described represents a lower bound on the burden of exchanging cash because the environment grossly underestimates the true burden. This is because the computations are based on the assumption that consumers and merchants possess all currency and coin denominations in circulation. Therefore, the agents in this environment can transact using the minimum possible number of exchanged tokens in each instance. We view access to all coins and notes as a reasonable assumption for merchants. However, it is likely too strong an assumption for consumers.

We mitigate this issue by considering a second environment where the assumption on consumers is relaxed. In this second environment consumers can only transact using \$20 notes. This assumption represents a consumer first withdrawing cash from an ATM then choosing to transact. We choose to have consumers withdraw \$20 notes because ATMs typically only distribute \$20 notes. Shy (2020) finds significant cash use discontinuities at multiples of \$20 in the U.S. We view this as a reasonable assumption for consumers which brings the model closer to reality. We thus re-solve the model and derive a new distribution for the exchanging burden which we interpret as an upper bound.

The quantification exercises performed are useful for policymakers because they provide reliable bounds on the burden of exchanging cash with respect to U.S. currency denominations. By reliable we mean that the agents (consumers and merchants) are optimizing with respect to the number of tokens exchanged and that the computations are

³Greene and Stavins (2020) find that, from 2015 to 2020, consumers rarely had a \$2 note on hand. More precisely, they find that consumers had on average between \$0.00 and \$0.10 dollars in \$2 notes on hand. Moreover, there is no evidence that ATMs dispense \$2 notes.

based on actual transaction diary data in which consumers report the exact dollar amount of each cash payment. Deriving bounds are also useful for the design of optimal currency and coin denominations. Section 2 reviews this literature.

The second goal of this article is to use the model to analyze counterfactual changes to the currency system. Such currency system changes have recently occurred internationally. In 2014, Singapore demonetized its largest denominated bills while the European Central Bank (ECB) dropped the 500-euro bills in 2016. Moreover, in 2013 Canada eliminated the penny from its currency system. To this end, we choose a long-standing policy debate in the U.S.: whether to drop the one-penny coin from circulation. Beginning in 1989, several bills have been introduced in the U.S. Congress regarding the proposal to drop the one-penny coin from circulation and round prices to their nearest 5¢ value.⁴ Indeed, the COVID-19 coin shortages in the United States have renewed calls to eliminate the penny. Moreover, evidence from a nationally representative survey found that, in both 2019 and 2020, approximately two-thirds of U.S. consumers store coins at their homes or vehicles, see Figure 4. The policy is also not without precedent. The one-penny coin was phased from circulation in Canada on February 4, 2013. Merchants were then instructed to round payment amounts to their nearest multiples of 5¢. For example, payment values \$1.31 and \$1.32 are rounded to \$1.30 whereas \$1.33 and \$1.34 are rounded to \$1.35. The Canadian Mint estimated the cost savings from phasing out the penny to be \$11 million per year.⁵

To address the penny policy debate, we use our model to analyze a counterfactual U.S. economy where no pennies are in circulation and merchants have been instructed to round prices to the nearest 5¢. However, it could be the case that, after the policy is implemented, merchants will reset prices and round all prices up. We therefore consider a second counterfactual economy wherein merchants strictly round prices up to the nearest

⁴See, <https://www.congress.gov/bill/101st-congress/house-bill/3761/> (1989), <https://www.congress.gov/bill/107th-congress/house-bill/2528> (2001), <https://www.congress.gov/bill/109th-congress/house-bill/5818?s=1&r=6> (2006), <https://www.congress.gov/bill/115th-congress/senate-bill/759/actions> (2017).

⁵See, <https://www.mint.ca/store/mint/about-the-mint/phasing-out-the-penny-6900002>.

5¢. We simulate the effects of a hypothetical transition to an economy with no pennies by comparing the burden of exchanging cash in an economy with pennies to the counterfactual economy without pennies. We briefly explore the aggregate price effects of merchant rounding on cash transactions. Importantly, our counterfactual analysis is not restricted to the penny policy. It is general enough to be used by policymakers when analyzing other changes to the currency system.

Section 2 reviews the literature on currency denominations. Section 3 describes the data and provides descriptive statistics on cash payments. Section 4 derives a model of optimal exchange of currency notes and coins in cash transactions. Using transaction-level data, the model is then used to quantify the burden of exchanging cash. Section 5 provides a counterfactual analysis by computing the effects of transitioning to an economy with no pennies. In particular, Section 6 considers the aggregate price effects from transitioning to a penniless economy. Section 7 concludes. All data and the code used in the article are available at <https://github.com/brian-prescott/change-burden>.

2. Overview of the literature on currency denomination

The literature on the use of cash is surveyed in Shy (Forthcoming). Table 1 displays currency and coin denominations currently in circulation in selected countries. Denominations of medieval coinage systems are described in Sargent and Velde (1999, 2003). We divide the literature into three subjects: (i) The (mostly theoretical) literature that computes optimal currency denominations for a given distribution of payment amounts. (ii) An empirical literature that quantifies how existing currency denominations affect consumers' choice of whether to pay cash or with debit and credit cards. (iii) How merchants in some lines of business round prices to facilitate cash transactions.

Cramer (1983) computes optimal currency denominations using the *principle of least effort*. According to this principle, denominations of notes and coins are properly designed so that the settlements of cash transactions minimize the number of tokens (notes and coins) exchanged between the trading parties. Using 1984 to 1986 survey data of

Dutch households, Boeschoten and Fase (1989) apply the observed distribution of cash payments and find that, on average, efficient payments involved the use 3.3 notes and coins per payment.

Sumner (1993) and Telser (1995) formulate the problem differently suggesting that the problem of choosing the optimal spacing of currency and coin denominations is related to the problem of spacing units of standard weight measurements (the problem of Bâchet). For a uniform distribution of transaction amounts, this approach yields denominations that are powers of 3, such as 1, 3, 9, 27, and so on when overpayments and the return of change are allowed. Shallit (2003) solves a different problem by focusing on denominations of coins that minimize the average number of coins needed to hand out change.

Using the principle of the least effort, Van Hove and Heyndels (1996) show that, for a uniform distribution of transaction amounts, the average number of notes and coins exchanged in a transaction is minimized by spacing denominations apart by a factor of two (rather than three), even when allowing for overpayments and the return of change. Van Hove (2001) argues that Bâchet's problem of weights cannot be transposed blindly to the problem of finding optimal currency denominations. The reasons are: (i) Bâchet's problem minimizes the number of standard weights rather than the number of weights used in a typical weighing; and (ii) it is based on the assumption that the set of weights contains only one weight of each size. The author concludes that the problem of finding optimal denominations involves a multicriteria optimization problem in which the principle of the least effort should be given the greatest weight. Several papers examine the two theories of optimal denominations by comparing the results to denominations in circulations in various countries. Wynne (1997) finds that only five countries have denominations that are either powers or integer multiples of three. Tschoegl (1997) finds that the average ratio of adjacent denominations is 2.6 for coins and 2.62 for notes.

The second line of research examines how currency denominations affect consumers' decisions whether to pay cash. Using payment diary data from Canada and the U.S., Chen, Kim, and Shy (2019), Shy (2020), and Van Hove (2020) show how currency de-

nominations generate discontinuities in the percentage of cash payments. Lee, Wallace, and Zhu (2005) construct a model of random pairwise meetings in which trading parties choose the portfolio of denominations they carry. Rogoff (2016) argues that large denomination notes should be eliminated since they are mostly used for illegal trading. In contrast, McAndrews (2017) and Hendrickson and Park (2021) argue that eliminating large denominations is sub-optimal for reducing illegal trading and that allowing illegal traders to use large bills with a lower rate of return will produce more seigniorage. Lee (2010) studies how different denominations affect the carrying cost of money in a monetary economy and presents welfare estimates of the gains from different denomination structures.

A third line of research examines some markets where sellers set prices to reduce the cost of paying cash. Levy and Young (2004) provide evidence for *convenient prices* reporting that the price of a 6.5-oz Coca-Cola was 5¢ (a nickel) for 73 years (1886 to 1959). Other examples are local weekday newspapers that during the 1980s were sold for 25¢ and parking meters that still utilize quarters. In all three examples, sellers utilized low-tech vending machines with no need to provide change. Further empirical support for this behavior is provided in Knotek (2008, 2011) and Bouhdaoui, Bounie, and François (2014).

This article is most closely related to Lee (2010) and Chen, Kim, and Shy (2019). It contributes to the optimal denominations literature in two ways. First, we formulate a model based on the principle of least effort and apply it to the empirical distribution of cash transactions from 2015 to 2019. Our quantification of the burden of exchanging cash from the empirical distribution of cash payments is, to the best of our knowledge, the first of its kind in the U.S. Furthermore, we demonstrate how the model can be used as a framework for policymakers when evaluating counterfactual changes to the currency system. We demonstrate this by using the model to document how the burden of cash payments would evolve if the penny were dropped from circulation.

Second, we contribute to the literature by studying the price effects resulting from

penny elimination. We estimate how merchants' price rounding, as a result of the policy, would impact the distribution of consumer prices. Our results provide valuable information to policymakers considering changes to the U.S. currency system. Importantly, the frameworks presented in this article could be used to study further changes to the currency system beyond penny elimination.

3. Data and descriptive statistics

The data used throughout this article are taken from the 2015 to 2019 Survey of Consumer Payment Choice (SCPC) and Diary of Consumer Payment Choice (DCPC). The SCPC and DCPC are composed of a representative sample of U.S. consumers age 18 and older taken from the University of Southern California's Understanding America Study.⁶

The SCPC is a recall-based survey where respondents are asked to estimate the number of payments made with a specific type of method during a normal month. The SCPC also collects a wide range of demographic information, which is the information we leverage from it. The DCPC asks respondents to record all of their transactions during three consecutive days in October. The transactions data collected in the DCPC include purchases, bill payments, p2p payments, ATM withdrawals, deposits, and income receipts. Respondents in the DCPC report how much they paid, the means of payment, and merchant or person they paid on each assigned diary day. Respondents' three-day periods were evenly distributed throughout each day in October from 2015 to 2019.⁷ The data contain weights for all respondents that can be used to produce monthly population estimates of the adult U.S. population for the given year.

The data used for our exchange burden quantification are the 16,906 in-person cash transactions conducted by 3,452 respondents from 2015 to 2019. Figure 1 displays the distribution of transaction amounts for all cash payments not exceeding \$100 (96.6 percent

⁶See Foster, Greene, and Stavins (2020) for more information on the representativeness of the panel in 2019.

⁷The panel is not balanced as there are many respondents who drop out from one year to the next. See Greene and Stavins (2020) for more information on the panel dimension of the data.

of all cash payments). This distribution achieves its global maximum at \$5 with local maxima at \$1 and then every \$5 interval up until \$60 and then at \$100. This observation is consistent with Shy (2020) who found spikes in the use of cash at \$1, \$5, \$20, and \$40 payment amounts.

The data also provide information on which merchants receive the most cash payments and which consumers make the most cash payments. Table 2 provides all summary statistics of the cash transactions sample. We find that, from 2015 to 2019, retail stores received the most cash payments, 72 percent. They were followed by merchant business service providers (11 percent) and person-to-person (9 percent), respectively. We also know which consumers make the most cash payments. Approximately 40 percent of cash transactions are conducted by consumers with only a high school diploma while 43 percent are made by those with household income less than \$50,000. The average age of a consumer making a cash transaction was 54 while the most common decade consumers were born in was the 1950s. Now that we know who makes and receives cash payments in our sample, we proceed to characterizing the distribution of the burden of exchanging cash borne by consumers and merchants.

4. Optimal cash transactions and the burden of exchanging cash

This section quantifies the burden of exchanging cash during cash transactions. We construct an analytical model to explain how cash transactions are settled in an economy where payers and payees minimize their exchange burden of cash payments. Similar to Cramer (1983), we say that *a cash transaction is optimal* if, for given currency denominations and for a given payment amount, the chosen currency notes and coins minimize the sum of tokens (currency notes plus coins) that the parties need to exchange. That includes the initial payment and the return of change. For such payments, we say that *the burden of exchanging cash is minimized*.

For example, given the denominations of U.S. currency displayed in Table 1, there are two potential ways to pay for a 43¢ transaction: (i) to pay with 25¢ + 10¢ + 10¢ coins and

receive $1\text{¢} + 1\text{¢}$ coins as change, or (ii) to pay with a 50¢ coin and receive $5\text{¢} + 1\text{¢} + 1\text{¢}$ coins as change. We consider the second option to be optimal because it minimizes the number of tokens exchanged between the consumer and the merchant: the first option corresponds to an exchange of 5 coins whereas the second option corresponds to an exchange of 4 coins.

The top panel of Figure 2 displays the optimal burden for transaction amounts between \$4.00 and \$5.00. The figure shows that payment amounts that are multiples of 5¢ have the lowest burdens while amounts ending in $2\text{¢}, 3\text{¢}, 7\text{¢}$, and 8¢ have higher burdens. For example, the top panel of Figure 2 shows that the burden of paying \$4.72 is 5: the buyer pays with a \$5 bill and receives $25\text{¢} + 1\text{¢} + 1\text{¢} + 1\text{¢}$ coins as change. Similarly, the burden of paying \$4.13 is 6: the buyers pays with a \$5 bill and receives $50\text{¢} + 25\text{¢} + 10\text{¢} + 1\text{¢} + 1\text{¢}$ coins as change.

4.1 A model of the principle of the least effort

Let δ denote the exogenously-given vector of denominations of currency notes and coins in circulation ordered by denomination value. For example, Table 1 shows that $\delta^{\text{U.S.}} = [0.01, 0.05, 0.10, 0.25, 0.50, 1, 2, 5, 10, 20, 50, 100]$, which implies that there are $D = 12$ denominations in the U.S. We use the term *token* to refer to one unit of one denomination. For example, a token could be a \$0.01 (1¢) coin or a \$5 note.

Consider a consumer (payer) and a merchant (payee) who engage in a transaction that requires a payment of a , where a is quoted in multiples of some of the denominations listed in δ . For example, in the U.S., transaction amounts must be in multiples of \$0.01 (1¢) or higher denominations (but cannot be a fraction of 1¢).

The payment involves a two-way exchange of tokens. First, the consumer pays in the form of a non-negative vector of integers θ^p of multiples of denominations. Second, the merchant returns a non-negative vector of integers θ^r of multiple denominations. Both, θ^p and θ^r , are vectors of dimension D which is the number of denominations in circulation. For example, $\theta^p = [2, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 3]$ in the U.S. means that the payer hands in to

the merchant two 1¢ coins, one 50¢ coin, one \$5 bill, and three \$100 bills, which sum up to an initial payment of \$305.52. The vector of returned change θ^r (which could be a vector of zeros) is similarly defined.

For each transaction amount a , the consumer initially pays $\theta^p \cdot \delta \geq a$. That is, the initial payment must be greater than or equal to the transaction amount. Then, in the case of an over-payment ($\theta^p \cdot \delta > a$), the merchant returns change as a vector θ^r that satisfies $(\theta^p - \theta^r) \cdot \delta = a$. We define the *the burden of exchanging cash* by $b = \sum_{d=1}^D (\theta_d^p + \theta_d^r)$, which is the total number of tokens exchanged between the payer and the payee in both directions.

The consumer and the merchant jointly decide on the exact denominations θ^p and θ^r to be exchanged in both direction to minimize the burden. Formally, for a given transaction amount a and a vector of denominations in circulation δ , they choose non-negative vectors of integers θ^p and θ^r to solve

$$\begin{aligned} b^*(a) = \min_{\theta^p, \theta^r} & \sum_{d=1}^D (\theta_d^p + \theta_d^r) \\ \text{s.t. } & \theta^p \cdot \delta \geq a \\ & (\theta^p - \theta^r) \cdot \delta = a. \end{aligned} \tag{1}$$

Note that a solution for (1), denoted by $b^*(a)$, always exists because the transaction amount a is quoted in multiples of some denominations in the vector of all denominations δ . However, the solution need not be unique as there may be several combinations to exchange tokens that yield the same minimum burden $b^*(a)$.

4.2 Quantifying the burden of exchanging cash

We now approach the first set of results, which are the derivations of the distribution of the burden of exchanging cash. Recall from Footnote 3 that we exclude the \$2 notes from the set of available denominations. For each cash payment amount a in the DCPC data described in section 3, we use a branch-and-bound algorithm that solves the burden min-

imization problem defined in (1).⁸ We denote the resulting minimum burden values by $b^*(a)$ for every payment amount a in the data. From these values we compute the distribution of the burden of exchanging cash, b^* , derived from all the 16,906 cash payment amounts a in the DCPC.

The distribution of optimal burdens derived from the cash payment amounts in the data is displayed in the left panel of Figure 3. We find that the optimal burden for approximately one-fifth of cash transactions is $b^* = 2$ and that nearly 50 percent of cash transactions (22.1 + 22.5 percent) result in $b^* \leq 2$. The expected burden for any given cash transaction is 3.14 coins and notes. Transactions with $a > \$100$, on average, require approximately 5 tokens and have a median value of 8 which is approximately 2.5 times as large transactions of at most \$100. The difference in the maximum burden observed for transactions exceeding \$100 and transactions not exceeding \$100 is $b^*(\$6420) - b^*(\$100) = 65 - 9 = 56$.

It is important to re-emphasize that the above results and the derived distribution of burdens displayed on the left panel in Figure 3 constitute only lower bounds of the actual burden of exchanging cash faced by payers and payees. This is because the optimization problem defined in (1) is based on the assumption that the transacting parties possess the exact denominations required to achieve the minimum exchange burden of cash for each particular transaction they are engaged in. If this assumption does not hold for a transaction, then buyers and sellers may have to exchange a larger number of tokens than what is optimal.

To obtain an estimation of the exact burden of exchanging cash, one needs to conduct surveys in which buyers (or sellers) describe the exact denominations that they actually exchange in both directions for each payment they record. Such surveys, particularly those with a sufficiently-large number of observations, are extremely expensive to conduct. However, the distribution of burdens that we compute in this article are still very useful because they provide reliable lower bounds which can form the basis for estimat-

⁸We implement this algorithm via the `lpSolve` R-package.

ing the true burden of exchanging cash. Nevertheless, we are also interested in computing upper bounds on the these burdens during cash transactions.

4.3 Quantifying the burden of exchanging cash with restricted choice

If the assumption that payers and merchants have access to all denominations does not hold, then they will have to exchange a sub-optimal number of tokens. Thus, in order to provide an upper bound on the exchange burden of cash we impose additional structure on the model. Recent work by Shy (2020) identified a correlation between ATM currency denominations and payment choice. Figure 1 displays significant payment discontinuities at \$20 and other multiples of \$5. With this information in hand, we re-solve the model requiring payers to use \$20 notes while allowing merchants access to all denominations. This brings the model closer to reality by assuming a payer does not have access to all denominations at the point-of-sale. It also provides an upper bound scenario since it reflects an exchange wherein the payer prefers to use cash yet has none on hand and must make a withdraw prior to transacting.

The resulting distribution of exchange burdens from this simulation is presented in the right panel of Figure 3. In general, we find that restricting payers' choice set shifts the expected exchange burden from 3.14 to 4.96 tokens. This 57.8 percent increase in the expected exchange burden also includes a 58 percent reduction in the number of cash transactions that require the use of only two tokens (a reduction from 22.5 percent to 9.5 percent). To put these changes into perspective, prior to restricting payers, we found that the share of transactions resulting in $b^* \leq 2$ was approximately 45 percent. However, after restricting payers, the share for these transactions dropped to approximately 15 percent. Finally, the single largest increase in this distribution is observed for the exchange burden of $b^* = 4$ exchanged tokens. The share of this exchange burden increases by 57 percent (from 13 percent to 20.5 percent).

5. Policy counterfactual: Elimination of the penny

An important policy question in the U.S. is whether to cease production and circulation of the penny. As discussed in the introduction, the proposal to eliminate the penny was first submitted to Congress in 1989 and has been resubmitted several times with the last motion occurring in 2017. The debate has received renewed attention due to the COVID-19 coin shortage.⁹ Additionally, as shown in Figure 4, approximately two-thirds of U.S. consumers store coins at their homes or vehicles. In light of this, we use the model constructed in Section 4 to estimate how the distribution of the burden of exchanging cash would evolve if the penny were eliminated.

We consider a counterfactual economy wherein the penny has been removed from circulation and all cash transaction amounts are to be rounded.¹⁰ We consider the counterfactual economy under two rounding policies. The first policy, which we label "symmetric", assumes that merchants round their prices evenly to the nearest 5¢. For example, a transaction of amount \$1.07 would round to \$1.05 while one valued at \$1.08 would round to \$1.10. This is the same rounding rule used in Canada during their penny elimination, see footnote 5. However, it might be the case that, without government direction, merchants will reset all prices and round up. Thus, the second policy, labeled "asymmetric", is a rounding rule where merchants round all prices up to the nearest \$0.05. For example, a transaction valued at \$10.01 would round up to \$10.05 rather than down to \$10.00.

Under this counterfactual economy we repeat the exercise of computing the optimal burden of cash exchange for each of the 16,906 cash transactions in the data while assuming that all payment amounts a are rounded according to the symmetric or asymmetric rounding rules. The theoretical implications for the optimal burden of cash exchanges are displayed in the bottom panel of Figure 2. This figure shows that the maximum burden for cash payment amounts in the [\$4.01, \$5] interval would decline to 4 in the penniless

⁹<https://www.nytimes.com/2020/07/29/business/coin-shortage-penny.html>

¹⁰This exercise may seem similar to the restriction of payers' tokens on hand. However, the fact that merchants round their prices in response to the penny being eliminated is what differentiates this from restricting payer choice sets.

economy compared with 6 in the current economy. Moreover, Figure 2 shows that there is less variation in the burden over this interval after the penny is removed and prices are rounded.

5.1 Penniless economy with unrestricted choice

We now present the results from eliminating the penny in the economy. Recall that we are quantifying both the lower and upper bound of the burden of cash exchange resulting from penny elimination. We first consider the lower bound of this burden. The lower bound is represented by the economy wherein both payer and merchant can transact with all tokens (except for \$2 notes). The resulting b_c^* distributions of the burden of cash exchange are displayed in left panels of Figure 5. The top panel reflects the changes when merchants are instructed to round prices symmetrically. The bottom panel shows the changes when merchants are able to round all prices up. Recall that 96.6 percent of all cash payments do not exceed \$100. For these payments, we find that the density for all $b_c^* \geq 7$ drops to about 0.3 percent when pennies are removed and prices are rounded, where subscript “ c ” stands for counterfactual. Therefore, the new maximum burden b_c^* with density greater than 1 percent would be 6 tokens.

First, we consider the lower bound case under symmetric price rounding (upper-left panel in Figure 5). The share of all cash transactions requiring 3 and 4 tokens each increases by 30 percent, from 18.5 to 24 percent and from 13 to 17 percent, respectively. Importantly, the cumulative share for bins with $b_c^* \leq 4$ increases by 17.2 percent while the share for bins with $b_c^* \geq 5$ decreases by 55 percent. The expected burden of cash exchange decreases from 3.14 to 2.729 tokens per transaction. In other words, there is a 13 percent decrease in the expected burden per cash transaction.

Next, we analyze the case where merchants engage in asymmetric price rounding (lower-left panel in Figure 5). Under this counterfactual economy, the expected burden would decline to 2.727 tokens per cash transaction. Indeed, we find that the differences in the burden under asymmetric price rounding, relative to symmetric, are small. Consider

the share of transactions requiring at least 3 tokens. In the symmetric pricing counterfactual this share would be 24 percent, however, in the asymmetric case it is 23.7 percent. Furthermore, looking at the share of all $b_c^* \leq 4$, the asymmetric counterfactual has a share of 89.21 percent while the symmetric counterfactual has a share of 89.18 percent. Thus, we find that merchants rounding prices up has little impact on the lower bound of the burden of cash exchange in a penniless economy when consumers have access to all tokens.

5.2 Penniless economy with restricted consumer choice

We now quantify changes to the upper bound of the exchange burden of cash in a penniless economy. This is the economy wherein payers can only transact with \$20 notes while merchants have access to all coins and notes. The results from these counterfactual simulation are displayed in the right panels of Figure 5.

We first look at this counterfactual economy when there is symmetric price rounding (upper-right panel in Figure 5). Under this counterfactual, the expected burden declines by approximately 8 percent (from 4.96 to 4.55). Additionally, the share of cash transactions requiring 4 tokens increases from 20.5 to 25.5 percent. This is in contrast to those requiring $b^* \geq 7$, which declines from 18.1 to 6.5 percent. Interestingly, the share of transactions resulting in $b^* \in \{1, 2\}$ is relatively unchanged when the penny is eliminated.

Finally, looking at the counterfactual under asymmetric price rounding (lower-right panel in Figure 5), we find that the expected exchange burden declines to 4.54 tokens per transaction. This 8.4 percent decrease is qualitatively similar to the 8.3 decline under symmetric price rounding. Indeed, much like the results from Section 5.1, we find that the form of price rounding makes little difference in the counterfactual burden of cash exchange.

6. Aggregate price effects of penny elimination

If the penny were eliminated from the U.S. economy then the form of price rounding engaged in by merchants could potentially have inflationary effects. Thus, the price effects

resulting from rounding policies after penny elimination are of interest to policymakers. Therefore, we now analyze the price effects of penny elimination. We do so by first estimating the share of transactions in which merchants would have to round prices.

6.1 Symmetric price rounding

First, we look at the price effects under symmetric price rounding. We analyze the effect of penny elimination on the prices by subtracting the counterfactual empirical distribution (after prices are rounded without the penny) from the observed empirical distribution (with the penny). We define a new variable $\Delta^a = a_c - a$ for each observed transaction where a_c is the transaction amount after rounding. Note that, because of the rounding policy, the variable Δ^a is restricted to the set $\{-\$0.02, -\$0.01, \$0, \$0.01, \$0.02\}$. That is, since each transaction is rounded to the nearest \$0.05, prices cannot change by more than \$0.02. The effects of price rounding by merchant type are detailed in Figure 6.

We find that, under the symmetric rounding rule, 73 percent of transactions would experience no price change. Moreover, the fraction of price decreases (13 percent) and increases (15 percent) approximately offset each other. Investigating these price changes further, we perform the same calculation except we now condition on the merchant being paid. We find that the only merchant who would have noticeable non-zero price changes are retail stores. Nonetheless, of these stores, the price increases and decreases exhibit the same offsetting behavior observed in the aggregate distribution. That is, 20 percent of transactions would experience price increases while 17 would experience decreases. Furthermore, even for these merchants, the share of transactions requiring no price change is still more than 60 percent. It is important to note, however, that the lack of price changes may reflect pre-existing price rounding conducted by merchants and respondents. Nonetheless, given the symmetry of the rounded prices, our findings suggest that, with symmetric price rounding, penny elimination would not have any significant inflationary consequences. These results are consistent with earlier findings by Chande and Fisher (2003) in Canada and Whaples (2007) in the U.S.

6.2 Asymmetric price rounding

We now consider what the price effects would be if, following the penny elimination, all merchants rounded their prices up. This is important because there could be inflationary consequences associated with the penny removal if merchants only rounded prices up. We thus repeat the exercise conducted in Section 6.1 under asymmetric price rounding. Note that, because of the rounding policy, the variable Δ^a for this exercise is restricted to the set $\{\$0, \$0.01, \$0.02, \$0.03, \$0.04\}$. It is important to note that the price increases are bounded above by 4¢ due to our assumptions on rounding. The effects of price rounding by merchant type are detailed in Figure 7.

We find that, under the asymmetric rounding rule, 73 percent of transactions would experience no price change. However, unlike the symmetric policy, the percentage of price increases is now 27 percent of transactions. Performing the same calculation conditional on the merchant being paid yields similar results to the symmetric case. Indeed, for all merchants except retailers, we find little evidence of price increases. However, when analyzing retailers we find that approximately 37 percent of these transactions would experience a price increase. These price increases are distributed approximately uniform across the $[\$0.01, \$0.04]$ interval.

7. Conclusion

This article quantifies the burden of exchanging cash. We specify a model of optimizing consumers and merchants and then apply it to an empirical distribution of cash payments from transaction survey diary data spanning 2015–2019. We then construct the distribution of exchange burden for cash payments observed in the current economy. Furthermore, we use the model to conduct a counterfactual policy simulation wherein the penny is eliminated from the economy. The contributions of this research are: (i) we provide an estimate of the exchange burden of cash in the U.S. and (ii) provide a framework for quantifying how the exchange burden would evolve under a change to the currency system. This framework is applied to a currently proposed currency system policy, penny

elimination.

The baseline results show that, on average, the exchange burden per cash transaction is 3.14 coins and notes. This estimate increases to 4.96 when we assume that consumers can only pay with \$20 notes. The counterfactual simulations suggest that eliminating the penny and rounding prices would result in a decrease in the exchange burden of cash. Importantly, this finding is robust to the form of price rounding conducted by merchants. For the baseline model, the expected cash exchange burden would decrease from 3.14 to 2.73 coins and notes while it would decline from 4.96 to approximately to 4.55 coins and notes in the model restricting consumer choice sets.

The paper briefly analyzes the aggregate price effects stemming from penny elimination. We find that more than two-thirds of cash transactions will result in no price change. Furthermore, the positive and negative price changes offset each other almost perfectly across all merchant types under symmetric price rounding. If merchants only round prices up to the nearest \$0.05, then we find that the remaining one-third of transactions would experience evenly distributed price changes in the 1¢ to 5¢ interval. These results suggest that, regardless of the rounding policy, penny elimination would not have any significant inflationary consequences.

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Table 1: Currency denominations of coins and notes in circulation in selected countries.

Country	Denominations									
Argentina (ARS, coins)	0.01	0.05	0.10	0.25	0.50	1	2	5	10	
Argentina (ARS, notes)	2	5	10	20	50	100	200	500	1000	
Canada (CAD, coins)	0.05	0.10	0.25	1	2					
Canada (CAD, notes)	5	10	20	50	100					
Euro area (EUR, coins)	0.01	0.02	0.05	0.10	0.20	0.50	1	2		
Euro area (EUR, notes)	5	10	20	50	100	200				
India (INR, coins)	0.10	0.20	0.25	0.50	1	2	5			
India (INR, notes)	5	10	20	50	100					
Japan (JPY, coins)	1	5	10	50	100	500				
Japan (JPY, notes)	1000	2000	5000	10000						
Mexico (MXN, coins)	0.05	0.10	0.20	0.50	1	2	5	10	20	
Mexico (MXN, notes)	20	50	100	200	500	1000				
U.S. (USD, coins)	0.01	0.05	0.10	0.25	0.50	1				
U.S. (USD, notes)	1	2	5	10	20	50	100			

Notes: The table displays notes and coins that are legal tender. Some of these notes and coins are rarely used, for example, the \$2 note and \$1 coin in the U.S., and low-denomination notes and coins in Argentina.

Source: Central banks of Argentina, Canada, European Central Bank, India, Japan, Mexico, and usa.gov.

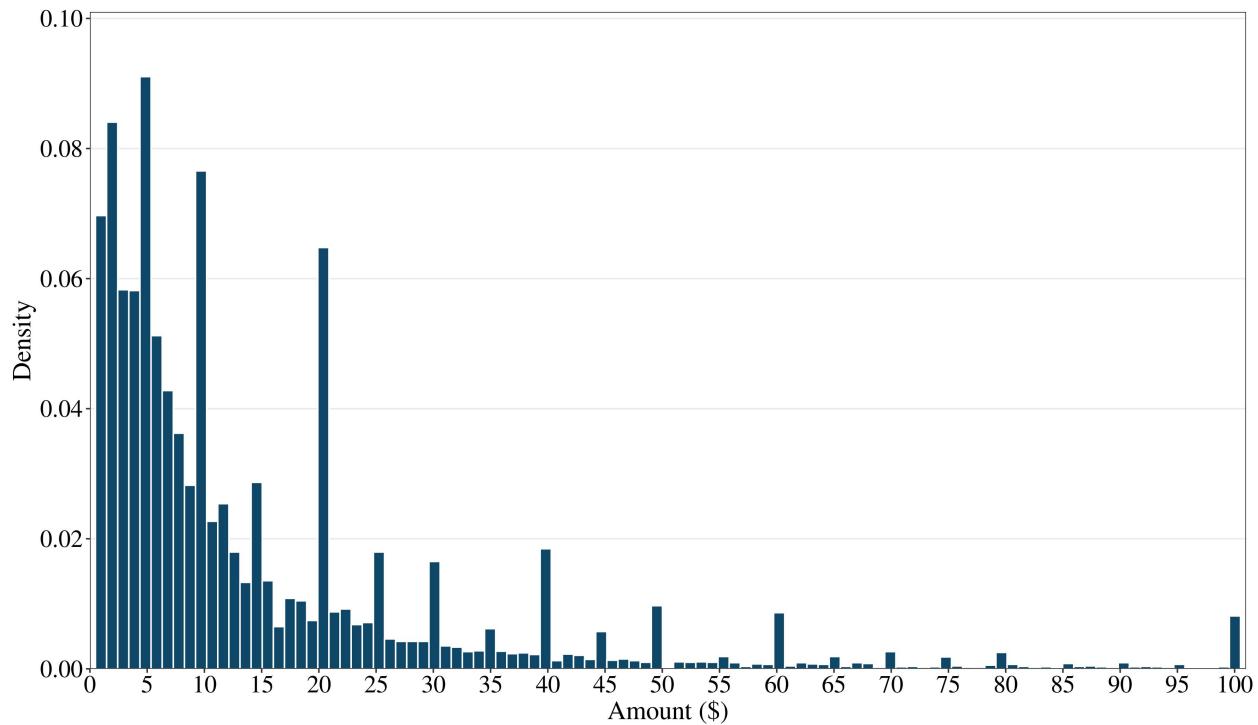
Table 2: Summary statistics of cash transactions sample from 2015 to 2019

	Mean/Fraction	Std. Dev.	Median	Min	Max
<i>Numeric demographic variables</i>					
Age	53.69	14.26	55	18	101
Household size	2.56	1.30	2	1	11
<i>Discrete demographic variables</i>					
Income < \$25k	0.19	0.39	—	—	—
Income ∈ [\$25k, \$50k)	0.24	0.43	—	—	—
Income ∈ [\$50k, \$75k)	0.19	0.40	—	—	—
Income ∈ [\$75k, \$100k)	0.14	0.35	—	—	—
Income ≥ \$100k	0.23	0.42	—	—	—
Education: Less than high school	0.03	0.17	—	—	—
Education: High school diploma	0.41	0.49	—	—	—
Education: Associate's degree	0.15	0.36	—	—	—
Education: Bachelor's degree	0.23	0.42	—	—	—
Education: Graduate degree	0.17	0.38	—	—	—
Birth decade: Before 1950s	0.17	0.37	—	—	—
Birth decade: 1950s	0.27	0.44	—	—	—
Birth decade: 1960s	0.23	0.42	—	—	—
Birth decade: 1970s	0.18	0.38	—	—	—
Birth decade: 1980s	0.11	0.32	—	—	—
Birth decade: After 1980s	0.04	0.19	—	—	—
Ethnicity: Latinx	0.06	0.24	—	—	—
Race: Asian	0.02	0.14	—	—	—
Race: Black	0.09	0.28	—	—	—
Race: Other	0.04	0.18	—	—	—
Race: White	0.86	0.34	—	—	—
Unbanked	0.06	0.24	—	—	—
Employed	0.60	0.49	—	—	—
Male	0.89	0.71	—	—	—
Married	0.59	0.49	—	—	—
<i>Transaction variables</i>					
Amount (a)	\$24.93	\$91.42	\$9.73	\$0.02	\$6420.00
Merchant: Hospital, doctor or dentist	0.01	0.09	—	—	—
Merchant: Nonprofit, charity or religion	0.04	0.19	—	—	—
Merchant: Government	0.02	0.14	—	—	—
Merchant: Retail store	0.72	0.45	—	—	—
Merchant: A person	0.09	0.29	—	—	—
Merchant: Business that primarily sells services	0.11	0.31	—	—	—
Merchant: Education provider	0.01	0.09	—	—	—
Merchant: Financial services provider	0.01	0.07	—	—	—
Number of consumers = 3,452					
Number of transactions = 16,906					

Notes: The discrete demographic variables report the fraction of cash transactions conducted by each demographic group. The discrete merchant variables report the fraction of all cash transactions occurring at that merchant type. The median value is not calculated for discrete variables.

Source: Diary of Consumer Payment Choice, 2015 to 2019.

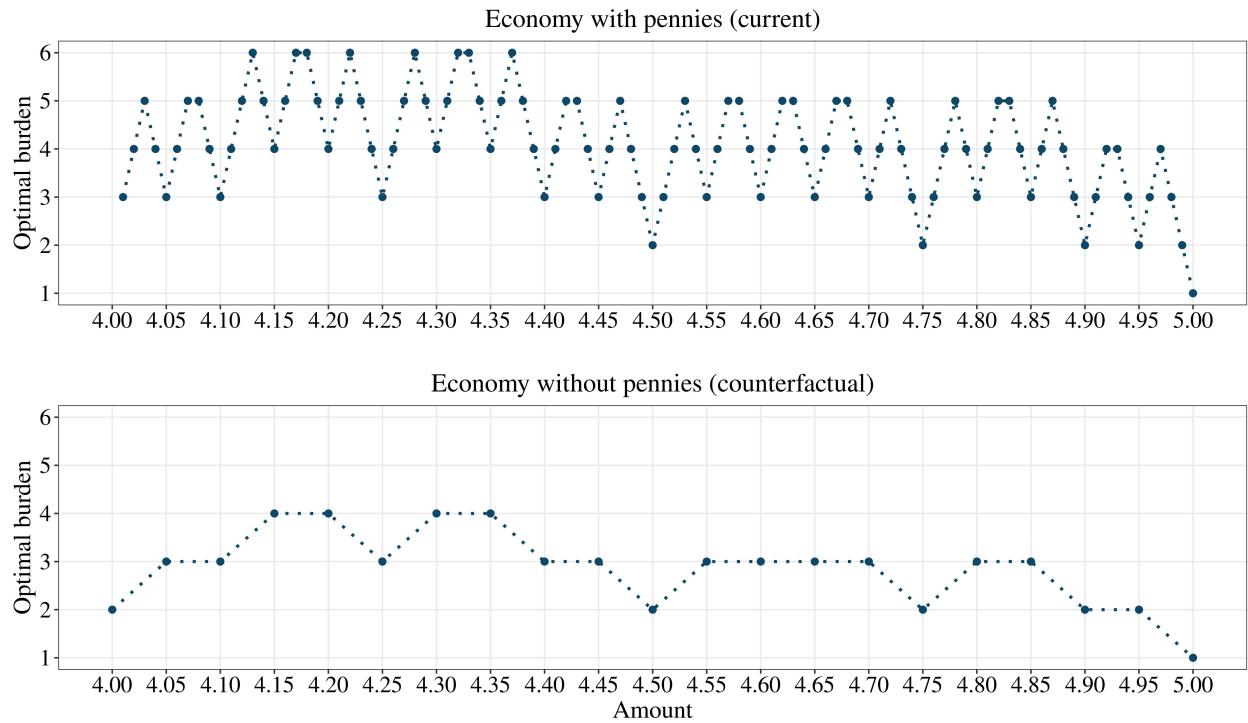
Figure 1: Distribution of all cash payments not exceeding \$100.



Notes: The distribution is constructed using the 16,342 cash payments not exceeding \$100 (96.6 percent of all cash payments). The histogram bins are evenly spaced along \$1 intervals.

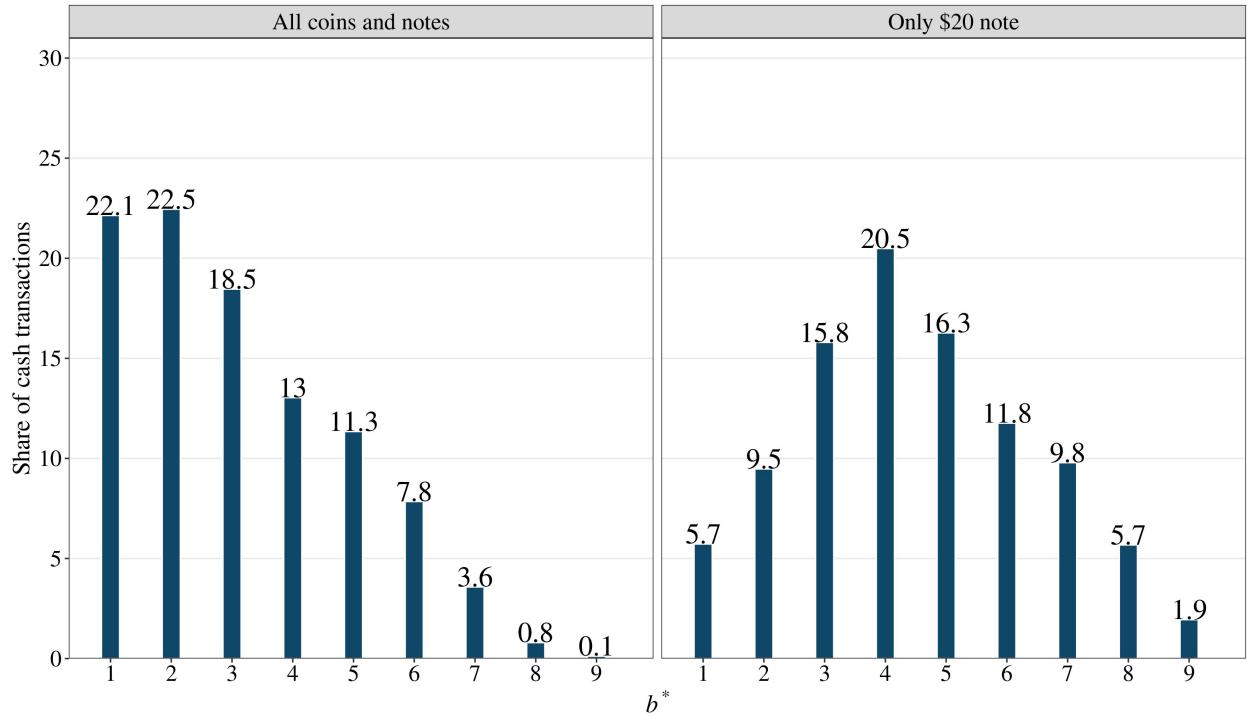
Source: Authors' calculations from the 2015 to 2019 Diary of Consumer Payment Choice.

Figure 2: The optimal burden for a cash payment valued between \$4 and \$5.



Notes: Top: Optimal burden by cash payment amount before the penny is eliminated. Bottom: Optimal burden by cash payment amount after the penny has been eliminated. Recall that all computations are based on the assumption that \$2 bills are not used for payment purposes.
Source: Authors' calculations.

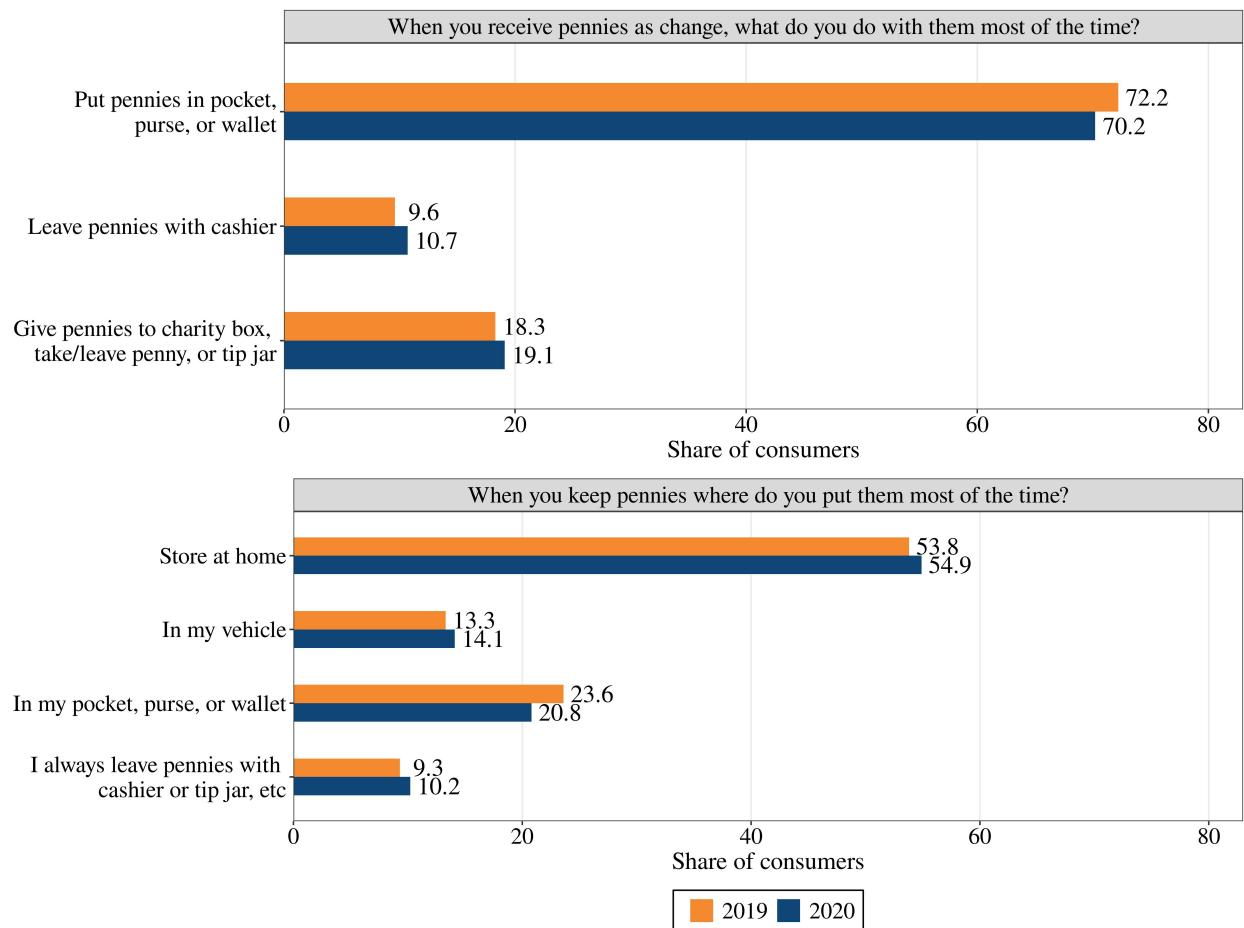
Figure 3: Distribution of optimal burdens of exchanging cash



Note: Both panels of Figure 3 are derived from all the 16,906 cash payments in the data. However, we do not present histogram values beyond burden of 9 because they are too small. *Left:* Distribution of optimal burdens of cash payments for 16,906 cash payments assuming that consumers possess all denominations (except for \$2 bills). *Right:* Distribution of optimal burdens of cash payments assuming that consumers pay with \$20 currency notes.

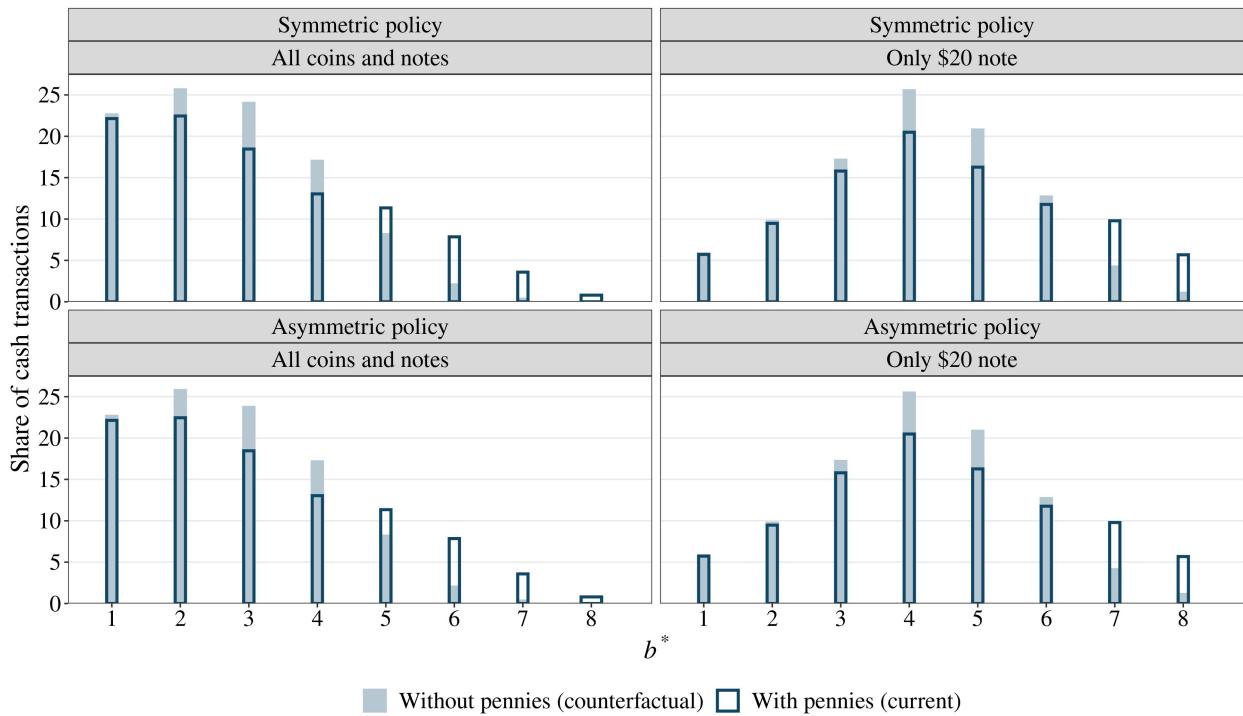
Source: Authors' calculations from the 2015 to 2019 Diary of Consumer Payment Choice.

Figure 4: Where consumers keep pennies and how they use them



Source: Authors' calculations from the 2019 and 2020 Diary of Consumer Payment Choice.

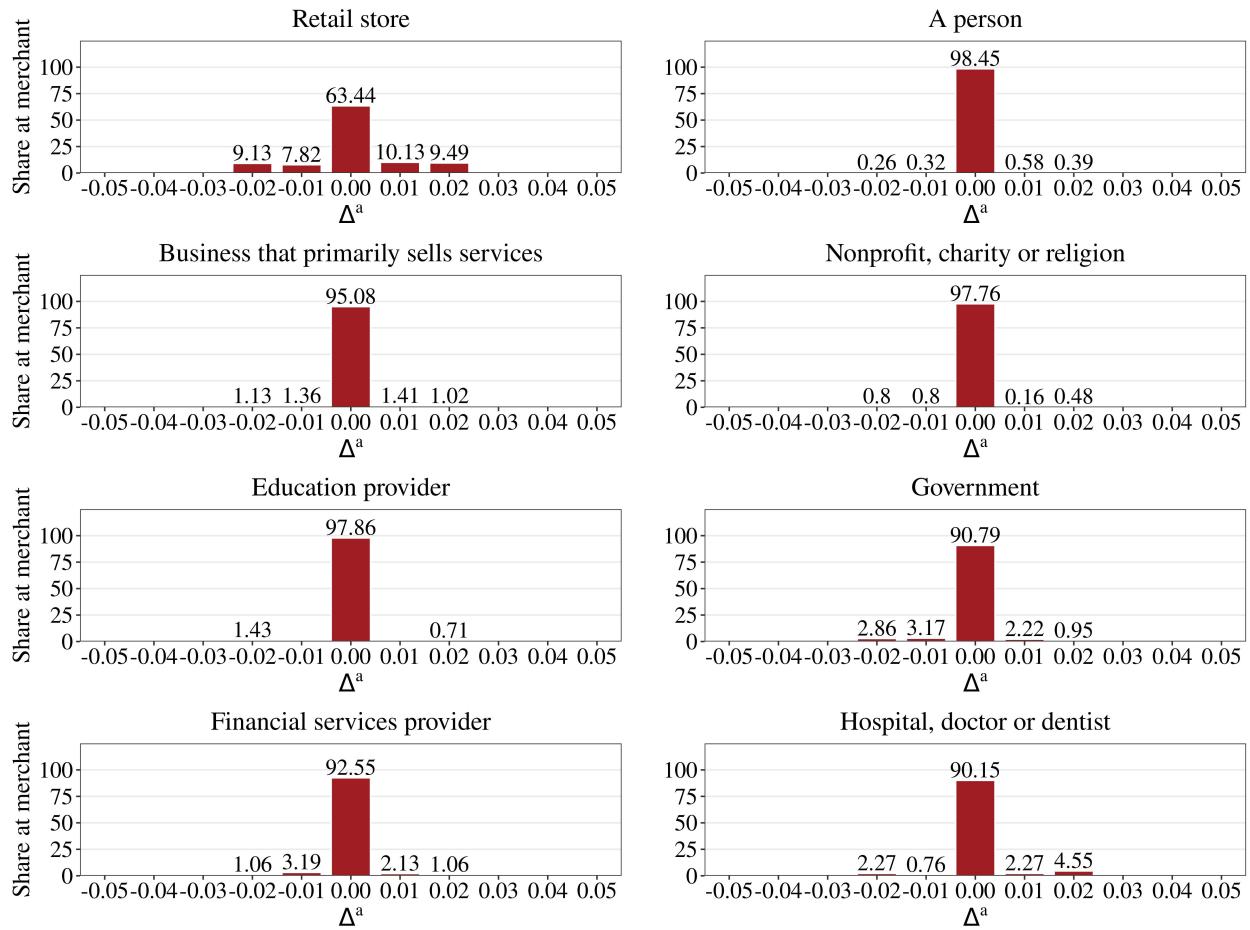
Figure 5: A comparison of distribution of optimal burden of cash exchange in the counterfactual penniless economy to current economy with pennies.



Note: Results are derived from all the 16,906 cash payments in the data. However, we do not present histogram values beyond burden of 9 because they are too small. *Left:* Distribution of optimal burdens of cash exchange using all denominations. *Right:* Distribution of optimal burdens of cash exchange assuming that consumers pay with \$20 currency notes. All panels display burden of the counterfactual economy (without the penny) in solid blue bins and for the current economy (with pennies included) in unfilled bins.

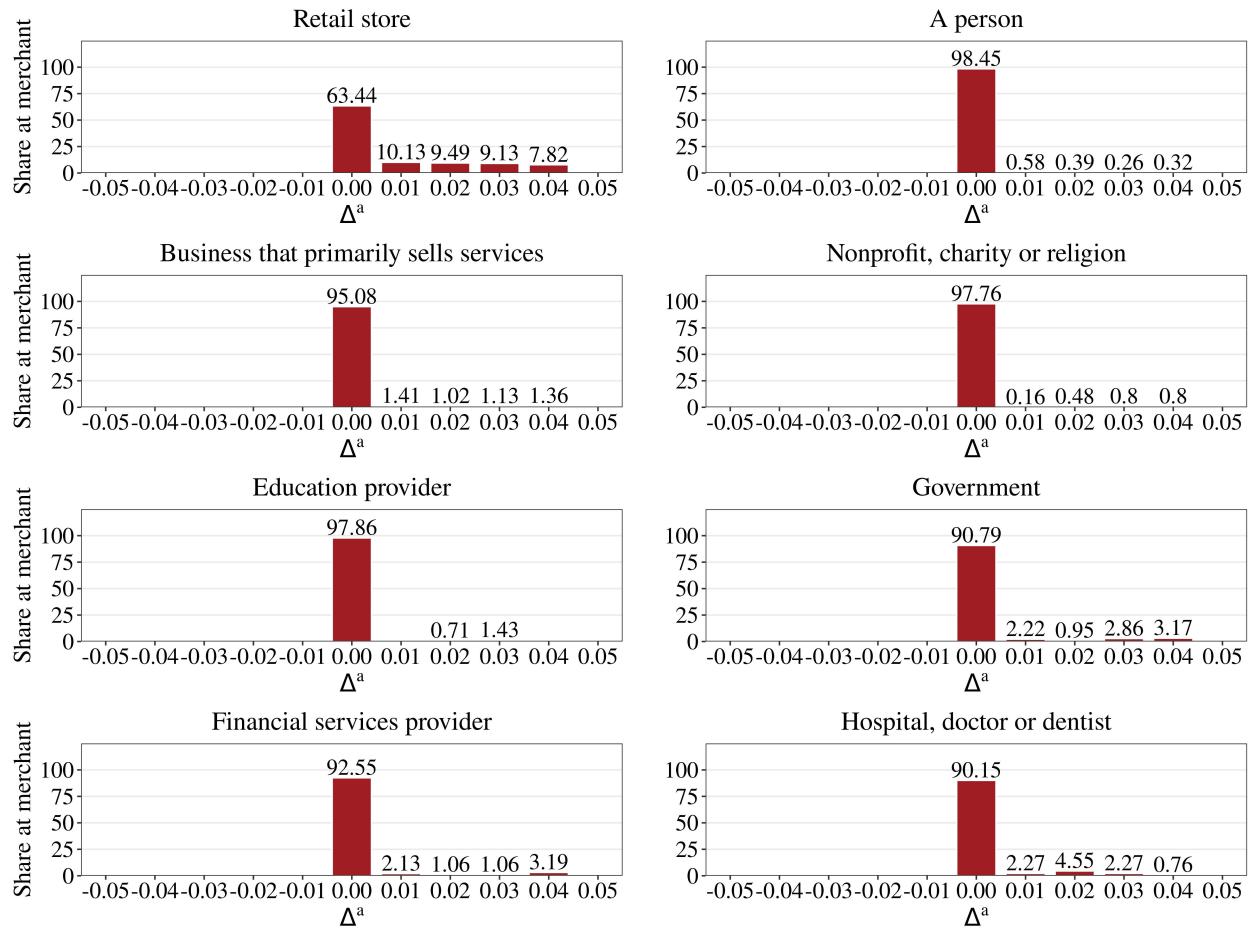
Source: Authors' calculations from the 2015 to 2019 Diary of Consumer Payment Choice.

Figure 6: Distribution of symmetric rounding price effects by merchant type



Source: Authors' calculations from the 2015 to 2019 Diary of Consumer Payment Choice.

Figure 7: Distribution of asymmetric rounding price effects by merchant type



Source: Authors' calculations from the 2015 to 2019 Diary of Consumer Payment Choice.