

1 Descartes' rule of signs

Lemma 1.1 (base case) If the polynomial

$$f(x) = a_n x^n + \cdots + a_0$$

has a sign change from a_n to a_0 , then it has a positive real root.

Proof. We can rewrite $f(x) = xp(x) - q(x)$ where $p(x)$ and $q(x)$ are polynomials with nonnegative coefficients. Then

$$f(0) = -q(0) < 0$$

and

$$\lim_{x \rightarrow \infty} f(x) = xp(x) - q(x) = \infty > 0$$

hence $f(x)$ must have a real root. □

Theorem 1.2 (Descartes's rule of signs) Consider

$$f(x) = a_n x^n + \cdots + a_0$$

Let C be the number of sign changes from a_n to a_0 . There are at most C positive roots of f , and if there are fewer, there must be $C - 2n$ positive roots.

Proof. By [base case](#), we know a polynomial with no sign changes has no positive roots.