Inference Rules

Basic rules:

mp

```
ϕ → ψ
ϕ
______
ψ
```

---

mt

```
ϕ → ψ
¬ψ
______
¬ϕ
```

---

dn

```
ϕ                ¬¬ϕ
______           ______
¬¬ϕ              ϕ
```

r

```
ϕ
______
ϕ
```

---

s

```
ϕ ∧ ψ            ϕ ∧ ψ
______           ______
ϕ                ψ
```

---

adj

```
ϕ                ϕ
ψ                ψ
______           ______
ϕ ∧ ψ            ψ ∧ ϕ
```

---

add

```
ϕ                ϕ
______           ______
ϕ ∨ ψ            ψ ∨ ϕ
```

---

mtp

```
ϕ ∨ ψ            ϕ ∨ ψ
¬ψ               ¬ϕ
______           ______
ϕ                ψ
```

cb

```
ϕ → ψ
ψ → ϕ
______
ϕ ↔ ψ
```

bc

```
ϕ ↔ ψ            ϕ ↔ ψ
______           ______
ϕ → ψ            ψ → ϕ
```

ui

```
∀αϕα
______
ϕβ
```

`Provided that β is a name (or a variable) and ϕβ comes from ϕα by proper substitution of β for α.`

eg

```
ϕβ
______
∃αϕα
```

`Provided that α is a variable and ϕβ comes from ϕα by proper substitution of β for α.`

ei

```
∃αϕα
______
ϕβ
```

`Provided that β is a new variable and ϕβ comes from ϕα by proper substitution of β for α.`

Derived rules:

nc (T40)

```
¬(ϕ → ψ)         ϕ ∧ ¬ψ
_________        _________
ϕ ∧ ¬ψ           ¬(ϕ → ψ)
```

cdj (T45,T46)

```
ϕ → ψ            ¬ϕ ∨ ψ            ¬ϕ → ψ          ϕ ∨ ψ
_________        _________        _________       _________
¬ϕ ∨ ψ           ϕ → ψ             ϕ ∨ ψ          ¬ϕ → ψ
```

sc (T33,T49)

```
ϕ ∨ ψ
ϕ → χ            ϕ → χ
ψ → χ            ¬ϕ → χ
_________        _________
χ                χ
```

dm (T63-T66)

$$\frac{\neg(\phi \wedge \psi)}{(\neg\phi \vee \neg\psi)} \qquad \frac{(\neg\phi \vee \neg\psi)}{\neg(\phi \wedge \psi)} \qquad \frac{(\phi \wedge \psi)}{\neg(\neg\phi \vee \neg\psi)} \qquad \frac{\neg(\neg\phi \vee \neg\psi)}{(\phi \wedge \psi)}$$

$$\frac{(\phi \vee \psi)}{\neg(\neg\phi \wedge \neg\psi)} \qquad \frac{\neg(\phi \vee \psi)}{(\neg\phi \wedge \neg\psi)} \qquad \frac{\neg(\neg\phi \wedge \neg\psi)}{(\phi \vee \psi)} \qquad \frac{(\neg\phi \wedge \neg\psi)}{\neg(\phi \vee \psi)}$$

nb (T90)

$$\frac{\neg(\phi \leftrightarrow \psi)}{(\phi \leftrightarrow \neg\psi)} \qquad \frac{(\phi \leftrightarrow \neg\psi)}{\neg(\phi \leftrightarrow \psi)}$$

qn (T203-T206)

$$\frac{\neg\forall x\phi x}{\exists x\neg\phi x} \qquad \frac{\neg\exists x\phi x}{\forall x\neg\phi x} \qquad \frac{\forall x\phi x}{\neg\exists x\neg\phi x} \qquad \frac{\exists x\phi x}{\neg\forall x\neg\phi x}$$

$$\frac{\neg\forall x\neg\phi x}{\exists x\phi x} \qquad \frac{\neg\exists x\neg\phi x}{\forall x\phi x} \qquad \frac{\forall x\neg\phi x}{\neg\exists x\phi x} \qquad \frac{\exists x\neg\phi x}{\neg\forall x\phi x}$$

av (T231-T232)

$$\frac{\forall x\phi x}{\forall y\phi y} \qquad \frac{\exists x\phi x}{\exists y\phi y}$$

`Given the total proper substitution of y for x, and provided no variable capturing arises in ϕy.`