GRAPH THEORY FOR PHILOSOPHY

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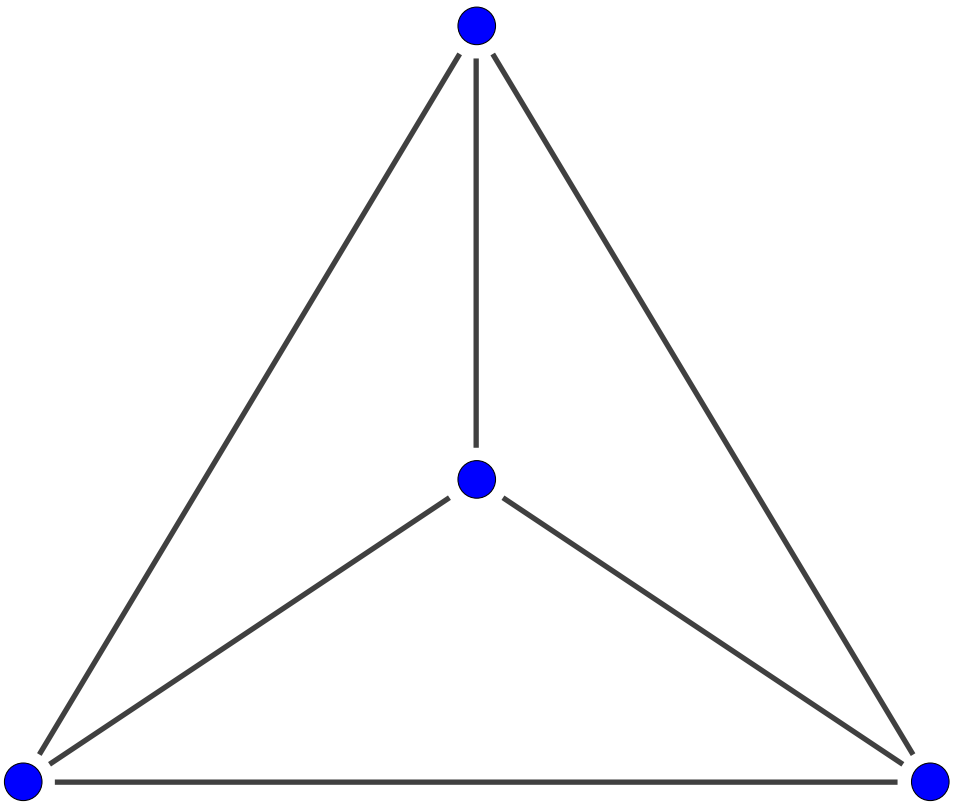

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A brief history of graph theory

A *graph* is a collection of dots we call *vertices* some of which are connected by curves we call *edges*. The relative location of the dots and the shape of the curves are not relevant, we are only concerned with whether or not a given pair of dots is connected by a curve. Initially, we forbid edges from a vertex to itself and multiple edges between two vertices. If G is a graph, then $V(G)$ is its set of vertices and $E(G)$ its set of edges. We write $|G|$ for the number of vertices in $V(G)$ and $\|G\|$ for the number of edges in $E(G)$. Two vertices are *adjacent* if they are connected by an edge. The set of vertices to which v is adjacent is its *neighborhood*, written $N(v)$. For the size of v's neighborhood $|N(v)|$, we write $d(v)$ and call this the *degree* of v. We write $E(v)$ for the set of edges containing v, these are the edges *incident* to v.

We use the shorthand $[k] := \{1, 2, \ldots, k\}$. A *path* in G is a sequence of different vertices $x_1, x_2, \ldots, x_r$ such that x_i is adjacent to x_{i+1} for all $i \in [r-1]$. We say this is a path from x_1 to x_r. If x_r is adjacent to x_1 as well, then we have a *cycle*. A graph G is *connected* if for all $x, y \in V(G)$, there is a path from x to y. Figure **??** shows all the connected graphs with at most five vertices.

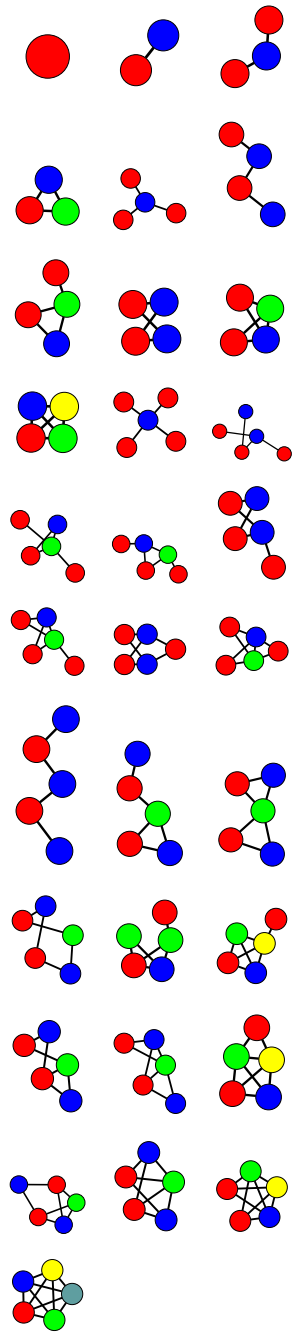

Figure 1: The connected graphs with at most five vertices.

The basics of graph theory