

ORIGINAL ARTICLE

# Propositions and Multiple Indexing

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It is argued that propositions cannot be the compositional semantic values of sentences (in context) simply due to issues stemming from the compositional semantics of modal operators (or modal quantifiers). In particular, the fact that the arguments for double indexing generalize to multiple indexing exposes a fundamental tension in the default philosophical conception of semantic theory. This provides further motivation for making a distinction between two sentential semantic contents—what (Dummett 1973) called “ingredient sense” and “assertoric content”.

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## Semantic values and propositions

The following is a common view in the philosophy of language: a semantics for natural language systematically pairs sentences with propositions in accord with the principle of compositionality. This association of propositions with sentences must be given relative to a context of utterance due to the context-sensitivity of language. The propositions that sentences express relative to a context are: (i) the things we assert and communicate, (ii) the things we believe and know, (iii) the bearers of truth, (iv) the referents of ‘that’-clauses, and (v) the compositional semantic values of sentences (in context). (For recent endorsements of this common view see King 2003 and Cappelen and Hawthorne 2009.)

Consider role (v) (and to some extent role (iv)). It is commonly thought that propositions are the arguments to sentential operators.<sup>1</sup> For example, in the sentence

(1) It is necessary that kangaroos have tails.

the necessity operator takes the proposition expressed by ‘Kangaroos have tails’ as argument and checks whether or not it is true in all accessible worlds. Likewise, it is commonly said that propositional attitude reports relate an individual to a proposition. For example, the sentence

(2) Olivia believes that kangaroos have tails.

says that Olivia stands in the believing relation to a certain proposition, namely the proposition expressed by the embedded sentence ‘Kangaroos have tails’. The key idea is that sentential operators such as ‘It is necessary that’ or ‘Olivia believes that’ operate on the proposition expressed by their embedded sentence (in a context).

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This common conception acknowledges the need to first resolve indexicality. For example, consider the following sentence:

(3) It is necessary that I am here now.

In a context  $\langle w, t, a, l \rangle$  the proposition expressed by ‘I am here now’ is the proposition that  $a$  is at location  $l$  at time  $t$ . In the semantic evaluation of sentence (3) the necessity operator takes this proposition as argument and checks its profile across modal space.

It is also acknowledged that the context must include a world parameter for the resolution of modal indexicals such as ‘actual’. If we ignore the other parameters of context and focus solely on the world parameter, then on the common view sentential truth is relativized to a pair of worlds  $\langle w, w' \rangle$ . In this way, each sentence is associated with a function from world-pairs to truth-values. The proposition expressed by a sentence in a context is what one gets after supplying a contextual world (cf. Kaplan 1989). This is to say that the common view accepts *double indexing* but identifies the proposition expressed by a sentence with the set of worlds in which it is true *relative to a context world*.

My aim in this note is to argue that this common view is untenable. In particular, I will argue that the compositional semantic value of a sentence (in context) cannot be a set of worlds. There is a standard type of argument against this view: The so-called *Operator Arguments* based on arguments in Kaplan (1989) and Lewis (1980). These arguments attempt to establish that sentential values in a context must be relativized to parameters beyond possible worlds, e.g. times, locations, judges, etc. by focusing on the compositional semantics of various non-modal intensional operators (e.g. temporal operators, locative operators, agential operators etc.). The argument presented here is different: I will only rely on issues stemming from the compositional semantics of modal operators (or modal quantifiers).

The argument is simply this: the motivations for modal double indexing generalize to modal *multiple indexing*. Thus, sentential semantic values (in context) must be sets of infinite sequences of worlds. Sets of infinite sequences of worlds are not propositions. So, the compositional semantic value of a sentence (in a context) is not a proposition. Admittedly, the point is absolutely obvious once it is considered—but it needs to be said.

## Multiple indexing

Consider the following sentence:

(4) It is possible for everyone who actually survived the Titanic’s maiden voyage to have died on the maiden voyage.

The correct representation of (4) requires the use of an ‘actuality’-operator (cf. Crossley and Humberstone 1977), since neither of the following logical forms quite capture its meaning (where *survive* stands for ‘survived the Titanic’s maiden voyage’ and *die* stands for ‘died on the Titanic’s maiden voyage’):

4a.  $\Diamond[\forall x(\text{survive}(x) \supset \text{die}(x))]$

4b.  $\forall x[\text{survive}(x) \supset \Diamond(\text{die}(x))]$

Logical form (4a) simply states that there is a world where no one survives the Titanic's maiden voyage. So it says nothing about the fate of actual Titanic survivors in other worlds, whereas (4) clearly does. Logical form (4b) does say something about the fate of actual Titanic survivors in other worlds but it doesn't say anything about their fate as a *collective*. Notice that the truth of (4) requires that there be a world  $w$  where all the actual Titanic survivors die in  $w$  together—logical form (4b) misses this. Whereas if we add the 'actuality'-operator ' $A$ ' to the syntax we have the resources to achieve the right result:

4c.  $\Diamond[\forall x(A(\text{survive}(x)) \supset \text{die}(x))]$

But, of course, the remedy isn't just a matter of syntax. If sentential truth were only relativized to a single world, one still couldn't—given some standard assumptions—provide the correct semantic representation of sentence (4). With a single world in the point of reference  $\llbracket A(\phi) \rrbracket^w = 1$  iff  $\llbracket \phi \rrbracket^w = 1$ . Thus there would be no semantic difference between logical forms (4a) and (4c).<sup>2</sup> For the 'actuality'-operator to make any difference we must relativize sentential truth to two worlds—that is to say that the correct representation of sentence (4) requires a system of double indexing.

$\llbracket (4c) \rrbracket^{w,w} = 1$   
 iff there is a world  $w' R w$  such that  $\llbracket \forall x(A(\text{survive}(x)) \supset \text{die}(x)) \rrbracket^{w,w'} = 1$   
 iff there is a world  $w' R w$  such that for all  $x$ , either  $x$  doesn't survive in  $w$  or  $x$  dies in  $w'$ .<sup>3,4</sup>

The fact that the semantics of indexicals embedded under intensional operators requires double indexing was first pointed out by Kamp (1971) with regard to tense logic. Consider the following sentence:

(5) Everyone now alive will be dead.

The logical form (or at least the metalanguage truth conditions) can be represented as such:

$[\exists t' : t' > t](\forall x(\text{alive}(x, t) \supset \text{dead}(x, t')))$

And this requires two times in the point of reference (i.e. an assignment of times to *two* distinct temporal variables). Vlach (1973) upped the ante by focusing on sentences such as the "past tense" version of (5):

(6) Once everyone then alive would be dead.

We can represent the logical form of (6) as follows:

$[\exists t' : t' < t][\exists t'' : t'' > t'](\forall x(\text{alive}(x, t') \supset \text{dead}(x, t'')))$

Given the three temporal variables in the syntax this requires *three* times in the point of reference. With increasingly more complex sentences involving further temporal embedding there is a need for further temporal parameters.<sup>5</sup> The upshot being that natural language has the full expressive power of object language quantification over times such that points of reference must include an infinite sequence of times (Cresswell 1990).<sup>6</sup>

Let's now set aside temporal constructions and focus only on world-shifting devices. There is a completely analogous situation in the modal realm. Consider the following "counterfactual version" of sentence (4).

(7) If the Titanic had not hit an iceberg on its maiden voyage, it would still have been possible for everyone who would then have survived the maiden voyage to die on the maiden voyage.<sup>7</sup>

Whereas sentence (4) required that sentential truth be relativized to two worlds, sentence (7) requires that sentential truth be relativized to *three* worlds. Sentence (7) says that in the closest worlds where the Titanic doesn't hit an iceberg on its maiden voyage, the survivors of that voyage are not *essentially* survivors. This cannot be captured by the following logical form (where *no-iceberg* stands for the sentence "The Titanic did not hit an iceberg on its maiden voyage"):

$$7a. \text{no-iceberg} \Box \rightarrow \Diamond [\forall x (\text{survive}(x) \supset \text{die}(x))]$$

This simply says that in the closest worlds where the Titanic doesn't hit an iceberg on its maiden voyage, there is a world accessible from that world where there are survivors of the Titanic's maiden voyage. Thus, it says nothing about the other-worldly fate of the survivors in the closest *no-iceberg* worlds. More explicitly (7a) is true at *w* if and only if in all the closest-to-*w* *no-iceberg* worlds *w'*, there is a world *w''* accessible from *w'* such that for all *x*, either *x* doesn't survive in *w''* or *x* dies in *w''*.

Whereas (7) instead looks to a close world where the Titanic doesn't hit an iceberg, and then says of the Titanic survivors in *that* world, that there is another world where they don't survive. So it says something about the other-worldly fate of the survivors in the closest *no-iceberg* worlds. To put the point a different way, the consequent of the counterfactual must back-reference the world introduced by the counterfactual. (This is analogous to the way that 'then' back-references the time introduced by 'Once' in (6).) So sentence (7) is true at *w* if and only if in all the closest-to-*w* *no-iceberg* worlds *w'*, there is a world *w''* accessible from *w'* such that for all *x*, either *x* doesn't survive in *w'* or *x* dies in *w''*. Notice that there are *three* worlds involved in the semantic evaluation of sentence (7).

With a few harmless assumptions we can get a different and more perspicuous view of the situation. Assume that a counterfactual [ $\phi \Box \rightarrow \psi$ ] is true at *w* if and only if for all the closest-to-*w*  $\phi$ -worlds *w'*,  $\psi$  is true at *w'*.<sup>8</sup> Let's abbreviate "the closest-to-*w*  $\phi$ -worlds" as  $C(\phi, w)$  and let's go ahead and use object language modal quantifiers and variables to represent the logical form of counterfactual sentences. We translate a counterfactual sentence [ $\phi \Box \rightarrow \psi$ ] into syntax using world variables and a restricted modal quantifier

as follows:

$$[\forall w' : w' \in C(\phi, w)]\psi(w')$$

Under these assumptions the correct logical form of (7) is as follows:

$$7b. [\forall w' : w' \in C(\text{no-iceberg}, w)][\exists w'' : w'' R w'](\forall x(\text{survive}(x, w') \supset \text{die}(x, w'')))$$

Logical form (7b) contains three distinct world variables and thus the semantic evaluation of (7b) requires three worlds in the point of reference.

We see that the argument for double world indexing generalizes to triple and quadruple, and ultimately to infinite world indexing (see Cresswell 1990, pp. 34–46).<sup>9</sup> Which is to say that natural language has the full expressive power of object language quantification over worlds such that points of reference must include an infinite sequence of worlds—a mere doubly-indexed semantics (i.e. a semantics where sentential values are mere sets of worlds relative to a context) is insufficient. This is to say that the compositional semantic values of sentences (in context) must be *sets of infinite sequences of worlds* (or functions from infinite sequences of worlds to truth-values).

### Propositions under multiple indexing

The preceding section was a reminder.<sup>10</sup> Multiple indexing is a fact that theorists generally accept, e.g. Kratzer (2011) states:

[Cresswell (1990) has shewn] that natural languages have the full expressive power of object language quantification over worlds and times. Quantification over worlds or times is thus no different from quantification over individuals, and should be accounted for in the same way.<sup>11</sup>

But it hasn't been emphasized that multiple indexing conflicts with the common view that propositions are both the objects of assertion and the compositional semantic values of sentences. If we make the plausible assumption that propositions are not sets of infinite sequences of worlds, then it immediately follows that due to multiple indexing propositions are not the compositional semantic values of sentences.<sup>12</sup> One could, of course, apply *modus tollens* but do we have any independent reason to think that the things we say and believe are (or are best modeled by) sets of infinite sequences of worlds?

Theorists tend to focus on the case of *double* indexing. And this lines up well with the Kaplanian distinction between content *generating* versus content *evaluating* parameters (Kaplan 1989). But the motivations for this Kaplanian distinction are really quite different from the motivations for double indexing from compositional semantics. And once we take the latter motivations to their logical consequence we are left with a system of infinite indexing, which doesn't cohere nicely with Kaplan's two-level picture. There is a fundamental tension in our default philosophical conception of semantic theory.<sup>13</sup>

Yet, one might think that this discussion is misguided for the following reason: If modal language is treated by means of object language quantification and variables, then

the semantics of modal constructions are handled by the assignment function (Schaffer 2012). And for this reason it might be thought that there is no threat to the idea that propositions are semantic values. After all, if modal quantifiers present a problem for the common view, then we already have the problem due to individual quantifiers and variables.

Indeed we do! Whether we call the parameters relative to which expressions have extensions “the index” or “the assignment” makes no essential difference. For example, with an “intensional” treatment of tense extensions are relative to an index  $\langle t_1, t_2, t_3 \dots \rangle$ , whereas with an “extensional” treatment of tense extensions are relative to an assignment  $\langle t_1, t_2, t_3 \dots \rangle$ .<sup>14</sup> It is simply a confusion to think that the semantics of object language quantification is somehow of a different kind than the semantics of “intensional operators”. In an important sense it’s the same semantics just in a different syntactic package.<sup>15</sup> This is why I have been confidently jumping back and forth between intensionalist logical forms and extensionalist logical forms—the conclusion holds regardless of whether we adopt intensionalist or extensionalist syntactic treatments.

No doubt there remains the schmentencite way out (Lewis 1980, pp. 32–33). The schmentencite claims that sentences do not embed under modal operators/quantifiers. If not, then the semantic values of sentences needn’t be multiply indexed. Yet non-sentential expressions—schmentences—*do* embed under modal operators/quantifiers and the arguments for multiple indexing will apply to their semantic values. The schmentencite might as well have the semantic values of sentences be truth-values, since if the values of sentences are never actually the input to compositionality it will be trivially true that they are the values apt for compositional semantics. The schmentencite thesis is a syntactic one, and as such I will leave it to future syntax theory.

The real lesson I think is a familiar yet underemphasized one: we must distinguish between the compositional semantic value of a sentence (what Dummett called “ingredient sense”) and the proposition expressed by a sentence in a context (what Dummett called “assertoric content”).<sup>16</sup> The argument from multiple indexing rests on premises that we all generally accept, thus we can no longer turn a blind eye to this fundamental tension in the default philosophical conception of semantic theory.

## Notes

- 1 The claim that propositions are the compositional semantic values of a sentence (in context) is so orthodox that it is hardly ever explicitly defended. For explicit endorsement see King (2003, p. 206): “... sentences *can* be assigned semantic values relative to contexts in such a way that propositions are compositionally assigned to sentences relative to context and are the semantic values relative to those contexts of the sentences in question. And we need not assign sentences any second sort of semantic value.” and see Cappelen & Hawthorne (2009, p. 1): “The semantic values of declarative sentences relative to contexts of utterance are propositions.”
- 2  $\llbracket 4c \rrbracket^w = 1$  iff there is a world  $w' R w$  such that  $\llbracket \forall x[A(\text{survive}(x)) \supset \text{die}(x)] \rrbracket^{w'} = 1$  iff there is a world  $w' R w$  such that for all  $x$ , either  $x$  doesn’t survive in  $w'$  or  $x$  dies in  $w'$ .
- 3 For dialectical purposes, I am intentionally ignoring the semantics of quantification over individuals and the *assignment function* (i.e. the infinite sequence of individuals) required for

the semantic evaluation of free variables and variable binding. One might, however, argue that assignment-shifting devices of this sort already provide an argument against the philosophically entrenched view of propositions (see Rabern 2012). Relatedly, one might already be suspect of the view that propositions are the arguments to sentential operators given the phenomena of “quantifying in”. For example, consider sentences such as “Ralph believes some man is a spy” (focus on the reading where the QNP takes wide scope: “Some man is such that Ralph believes that he is a spy”), “Every prime number is such that necessarily it is greater than one”, or “Every man believes that he is misunderstood”. The compositional semantic values of the embedded sentences—containing bound pronouns—cannot be propositions. In this connection it is worth pointing out that Kaplan (1989) introduces and motivates direct reference theory by considering the following quantified modal formula:  $\exists x(Fx \wedge \neg \Box Fx)$ . Kaplan states that in order to evaluate the truth-value of the component formula ‘ $\Box Fx$ ’ (at an assignment) we must first determine what *proposition* is expressed by its component formula ‘ $Fx$ ’ (at an assignment) and then determine whether or not it is a necessary proposition. It is interesting that Kaplan seems to be endorsing the thesis that propositions are the arguments to sentential operators (e.g.  $\Box$ ) with the type of “quantifying in” example that might be thought to expose its falsity. (Thanks to Daniel Nolan and Jonathan Schaffer for helping me to make these connections.)

- 4 I should also note that I am assuming a constant domain semantics to keep things simple. Moving to a variable-domain semantics makes things more complicated and some subtle issues arise but it does not affect the essential point that double indexing is required.
- 5 The appendix to Vlach (1973) briefly outlines the need to generalize to the infinite case (see pp. 183–85) and Cresswell (1990) has a detailed and careful treatment of such a generalization for both the temporal and modal case.
- 6 Note that due to multiple indexing those who motivate temporalism about propositions via arguments from temporal embedding must instead conclude that propositions are sets of tuples consisting of a world followed by an infinite sequence of times, i.e. sets of pairs of the form  $\langle w, \langle t_1, t_2, t_3, \dots \rangle \rangle$ . Eternalists may view this as a reductio of such arguments.
- 7 I owe examples of this general form to Wolfgang Schwarz. It might sound more natural to consider this dialogue concerning counterfactual histories of the Titanic: “A: Everyone who actually survived the Titanic’s maiden voyage could have died on the maiden voyage. B: I agree. And even if the Titanic had not hit an iceberg on its maiden voyage, it would still have been possible for everyone who would then have survived the maiden voyage to die on the maiden voyage.”
- 8 Note that I’ve made certain simplifying assumptions about the semantics of counterfactual conditionals, e.g. I’ve made the limit assumption, I’ve assumed that the quantification involved is “all” instead of “most”, and I’ve assumed that the relevant accessibility relation is tightly constrained by the antecedent. But all that matters for the argument I provided here is that the context world  $w$  is represented in the metalanguage truth conditions. Any semantics that has the truth of  $\phi \Box \rightarrow \psi$  at  $w$  depend on the truth of  $\psi$  at worlds somehow related to  $w$  will share this essential feature.
- 9 For a sentence requiring three worlds in the index Cresswell provides “If the economic climate had been favorable it would have been desirable that some who are not actually rich but would have been rich be poor” (p. 40). See Cresswell (1990, p. 40) sentence (36) for a sentence with four worlds in play.
- 10 See Cresswell (1990, pp. 34–46); and see also Forbes (1989, chapters 1–2).

- 11 Kratzer (2011, sec. 5). See also Schlenker (2006, p. 509) and Schaffer (2012, sec. 2).
- 12 If propositions are understood to be Russellian or structured (instead of simple sets of points of reference) the conclusion still holds but it is not as straightforward. If Russellian propositions only determine a set of worlds, then they are informationally insufficient. In addition to a world-neutral Russellian content the semantics requires a “content-base” that is neutral with respect to an infinite sequence of worlds (this is analogous to the situation with tense in the *propositional semantics* provided in (Salmon 1986), pp. 143–51). We must also bear in mind that providing a recursive pairing of Russellian propositions with sentences does not actually provide a compositional semantics—its more akin to a translation into another language. To complete the semantics one must provide a recursive definition of the truth-values of propositions relative to points of reference.
- 13 We can, of course, recover a set of worlds from a set of sequences of worlds by taking the diagonal. This, however, does not retain the identification of compositional values with propositions, so it would be wrong to insist that diagonalization saves the common view. Likewise, one could assign sentences sets of worlds and give a non-compositional yet recursive account of modal operators. But I am working under the assumption that the semantics for modal constructions is genuinely compositional.
- 14 We could construe the assignment as a function from temporal variables to times instead of as a sequence of times (to which the temporal variables are indexed). In this way there would be a (superficial) difference between the “index” and the “assignment”. But there is no essential difference between these two devices as is demonstrated by the fact that they can both be a sequence of times. The difference is merely verbal and sociological.
- 15 See (Rabern 2012) for the problem under its other guise of individual variable binding.
- 16 This distinction has been made for various reasons by (Dummett 1973), (Lewis 1980), (Stanley 1997), and (Ninan 2010), among others.

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