

## RESPONSE TO REFEREE REPORT

I thank the referee for their detailed reading of my manuscript and for the interesting and very helpful questions and comments. Below, I have responded to each of these questions and comments.

- I have added a remark after the definition of  $\mathcal{L}$ -background to point to the familiar notion of "background fields". I have also added a paragraph in the introduction which more explicitly relates this work to the untwisted superconformal setting.
- We only consider theories which admit quantizations that are exact (or truncate) at one-loop in perturbation theory. For this reason, all of our anomalies necessarily occur at one-loop. I do not know of any theories which admit a one-loop symmetry of holomorphic vector fields but are obstructed to having a higher-loop symmetry. It is an interesting question. I think to construct such an example one needs to consider more exotic theories that do not admit a strict action of holomorphic vector fields, but rather a homotopical one.
- There is no problem extending the results to non-rational  $\lambda$  on flat space (which is all we consider here). But, we lose the geometric interpretation in this case. I have added a paragraph at the end of section 3.2 detailing this.
- Proposition 5.2 proves that if we choose the bundle where the holomorphic quarks live correctly, then the theory has a quantum background for all holomorphic vector fields. Since holomorphic symplectic vector fields are automatically divergence-free, we see that symplectic vector fields will be a quantum symmetry for *any* choice of  $\lambda$  (hence for any choice of line bundle to twist the holomorphic quarks by). At the present time, I am unaware how to see the superconformal window at the level of the holomorphic twist. What I can prove is that the twist of any superconformal theory has a quantum background for all holomorphic vector fields. On the other hand, this is not a necessary condition; there are many supersymmetric theories (like QCD) which are non-conformal yet whose holomorphic twists admit a symmetry by holomorphic vector fields. I think what needs to be uncovered is how the process of RG flow intertwines with twisting. This would be a big breakthrough towards a holomorphic understanding of Seiberg duality.

- As indicated in the last bullet point, I know of many theories which are not conformal yet twist to a theory which admits a holomorphic vector field symmetry. My conjectural interpretation is that  $a^{hol}, c^{hol}$ , and hence  $a, c$ , for these theories should be the ones associated to the superconformal theory that the original theory flows to (if such a flow exists). I am happy to put this in the draft if the referee thinks it is illuminating.
- For the last typo suggestion, I believe that in equation (4.7) the space that  $\beta$  lives is correct. The factor of the canonical bundle is absorbed into the coefficients of the Dolbeault complex.