BV-QUANTIZATION

I'm going to start using $\mathscr{E}, \mathscr{L}$ for fields and $\mathcal{E}, \mathcal{L}$ for Lie algebras. Thus $\mathscr{E} = \mathcal{E}[1]$, etc..

1. Renormalization

1.1. **Generalities on renormalization.** existence of counterterms, effective theories, gauge fixing, $\beta\gamma$ examples etc.

1.2. **Effective propagator.** There is a natural choice for the gauge fixing operator for our theory. Namely,

$$Q_{GF} := \overline{\partial}^* : \mathscr{E} \to \mathscr{E}$$

defined using the flat metrix on $\mathbb{C}$ coming from the holomorphic volume element $\mathrm{d}z$.

Lemma 1. *Let Q be the total differential for fields $\mathscr{E}$. Then*

$$D = [\overline{\partial}^*, Q] : \mathscr{E} \to \mathscr{E}$$

is the usual Laplacian acting on the appropriate Dolbeault complex.

The heat kernel is the element $\mathbb{K}_t \in \mathrm{Sym}^2(\mathscr{E})$ defined for $t > 0$ determined by the equation

$$\langle \mathbb{K}_t(z, w), \varphi(w) \rangle = (e^{-tD}\varphi)(z)$$

for all $\varphi \in \mathscr{E}$.

For us, the fields split as $\mathscr{E} = \mathscr{E}_{\beta\gamma} \oplus \mathscr{E}_{\mathrm{grav}}$.

Lemma 2. *The heat kernel is of the form*

$$\mathbb{K}_t = \mathbb{K}_t^{\beta\gamma} \oplus \mathbb{K}_t^{\mathrm{grav}} \in \mathrm{Sym}^2(\mathscr{E}_{\beta\gamma}) \oplus \mathrm{Sym}^2(\mathscr{E}_{\mathrm{grav}})$$

where write down kernels

1.3. **Pre-theory.** naive quantization works, i.e. no counterterms. Reduction to wheels, reduction to trivalent wheels.

2. The quantum master equation

2.1. **Obstruction theory.** introduce deformation complex for bosonic string. By definition, holomorphic string is described by the local curved L_∞-algebra

$$\mathbb{D}_S(\mathcal{L} \ltimes \mathcal{E}) = .$$

Thus, the full obstruction-deformation complex is

$$\mathrm{Def}_X := C^*_{\mathrm{loc}}(\mathbb{D}_S(\mathcal{L} \ltimes \mathcal{E})).$$

2.2. **Symmetries.** symmetries to consider for the deformation complex. Cotangent quantization. $\mathrm{Aff}(\mathbb{C})$-symmetry.