

HOMEWORK 5
DUE OCTOBER 13

There are two problems to turn in.

- (1) This problem utilizes notation and constructions from problem 2 of homework 3. Define $h_w: U_w \rightarrow \mathbf{R}$ by the formula

$$h_w([z : w]) = \frac{|z/w|^2 - 1}{|z/w|^2 + 1}.$$

(You should think for a moment about why h_w is well-defined). Also, define $h_z: U_z \rightarrow \mathbf{R}$ by the formula

$$h_z([z : w]) = -\frac{|w/z|^2 - 1}{|w/z|^2 + 1}.$$

- (a) Show that there is a unique smooth function $h: \mathbf{CP}^1 \rightarrow \mathbf{R}$ such that $h|_{U_w} = h_w$ and $h|_{U_z} = h_z$.
- (b) Classify the critical values of h .
- (c) If $c \in \mathbf{R}$ is a regular value of h , show the the level set $\Phi^{-1}(c)$ is diffeomorphic to S^1 .
- (d) For $c \in \mathbf{R}$ a critical value of h , describe the set $\Phi^{-1}(c)$.

- (2) Let $f: \mathbf{R}^n \setminus \{0\} \rightarrow \mathbf{R}$ be a smooth function such that for all $\lambda \in \mathbf{R}^\times$ one has

$$f(\lambda x) = \lambda^c f(x)$$

for some real number c . Let

$$E = \sum_i x^i \frac{\partial}{\partial x_i}$$

be the Euler vector field. Show that

$$Ef = cf.$$