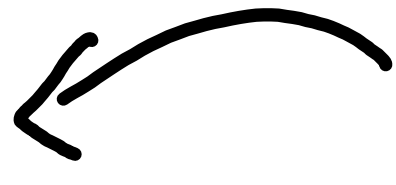
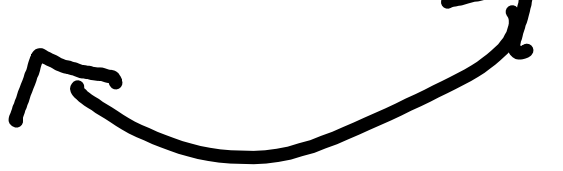


Let V be a vector space.

Then: a) $0 \cdot v = 0$.  The zero vector.

$$b) (-\lambda) \cdot v = -(\lambda \cdot v) = \lambda(-v) \\ \text{for all } \lambda \in \mathbb{F}, v \in V.$$

$$c) \lambda \cdot 0 = 0 \text{ for all } \lambda \in \mathbb{F}$$

 zero vector.

Pf: a) By VS 8, VS 3, VS 1:

$$0 \cdot v + 0 \cdot v = (0 + 0) \cdot v = 0 \cdot v = 0 \cdot v + 0$$

$\Rightarrow 0 \cdot v = 0$ by the cancellation lemma.

b) From last time, we know $-(\lambda \cdot v)$ is the unique vector for which

$$\lambda \cdot v + (-(\lambda \cdot v)) = 0.$$

So, if $\lambda \cdot v + (-\lambda) \cdot v = 0$ we have

that $(-\lambda) \cdot v = -(\lambda \cdot v)$. In particular,

$$(-1) \cdot v = -v$$

c) Exercise.



Exercises for §1.2:

1, 4(e) - 4(h), 7, 8, 10, 12, 13, 17, 20, 21.

- Subspaces.

A subspace of a vector space is a subset which is, itself, a vector space. Here is the precise definition.

↙ V is a vector space

Dfn: A subset $W \subset V$ is a subspace if it is a vector space according to the operations of addition and multiplication in V .

[X] Note: if $W \subset V$ is a subspace then it must contain the zero vector!

Ex: $W = \{0\} =$ the single element set containing the zero vector
 \cap
 V is a subspace.

Ex: $W = V$ is a subspace.
 \cap
 V

• To check whether $W \subset V$ is a subspace:

1) $v + w \in W$ whenever $v, w \in W$.
(closed under addition)

2) $\lambda v \in W$ whenever $v \in W$ and $\lambda \in \mathbb{F}$.
(closed under scalar mult.)

3) $W \ni 0$.

4) Each vector $v \in W$ has $-v \in W$
as well.

In fact, 4) is repetitive.

Theorem: $W \subset V$ is a subspace if and only if the following three conditions are satisfied:

a) $0 \in W$

b) $v + w \in W$ whenever $v, w \in W$

c) $\lambda v \in W$ whenever $v \in W, \lambda \in \mathbb{F}$.

Pf: (\Rightarrow) Assume W is a subspace. Then
 - (b) and (c) automatically hold. Also, we know there is $0' \in W$ st. $v + 0' = v$ for all $v \in W$. But also $v + 0 = v$ for all $v \in V$. So, if $v \in W$ then

$$v + 0' = v + 0 \Rightarrow 0 = 0'.$$

Thus $0 = 0' \in W$.

(\Leftarrow) Conversely, suppose (a)-(c) hold. To see W is subspace we need to show that the additive inverse of each $v \in W$ is in W . But by (c) we have $-v = (-1) \cdot v \in W$. Hence W is a subspace of V . \square

Ex: $V = \mathbb{R}^3 = \{ (x, y, z) \mid x, y, z \in \mathbb{R} \}.$

We know \mathbb{R}^3 is a vector space using the usual rules.

$\leadsto W = \{ z = 0 \} = \{ (x, y, 0) \mid x, y \in \mathbb{R} \}$
 "xy-plane" $\cap \mathbb{R}^3$ is a subspace.

Ex: $\mathbb{R}[x]$ = set of all polynomials with real coefficients.

We know this is a vector space.

$W_d = \left\{ \begin{array}{l} \text{set of all polynomials} \\ \text{of degree } \leq d \end{array} \right\}$

E.g.: $W_2 = \{ a_0 + a_1x + a_2x^2 \mid a_0, a_1, a_2 \in \mathbb{R} \}$

Then, $W_d \subset \mathbb{R}[x]$ is a subspace.

• Intersection: If A is a set and

$B, C \subset A$

are subsets. The intersection is

$B \cap C = \left\{ \begin{array}{l} \text{elements } a \in A \text{ s.t.} \\ a \in B \text{ and } a \in C \end{array} \right\}.$

Theorem: If V is a vector space and

$W, Z \subset V$ are two subspaces. Then

$$W \cap Z \subset V$$

is a subspace.

Pf: Let $v, w \in W \cap Z$. Then

$$v + w \in W \text{ and } v + w \in Z$$

$$\Rightarrow v + w \in W \cap Z.$$

Similarly, $\lambda v \in W \cap Z$ for all $\lambda \in \mathbb{F}$.

