Solutions to selected exercises from §1.4

Question 12

First, suppose that $\mathsf{W} \subset \mathsf{V}$ is a subspace. We will show that $\operatorname{span}(\mathsf{W}) = \mathsf{W}$. For any set S it is always true that $S \subset \operatorname{span}(S)$, so to prove the assertion we must only show that $\operatorname{span}(\mathsf{W}) \subset \mathsf{W}$. For this, start with $v \in \operatorname{span}(\mathsf{W})$. By definition of span, there exists $\lambda_1, \dots, \lambda_m \in \mathbb{F}$ and $u_1, \dots, u_m \in \mathsf{W}$ such that

$$v = \lambda_1 u_1 + \cdots + \lambda_m u_m. \tag{1}$$

On the other hand, since W is a subspace we know that it is closed under arbitrary addition and scalar multiplication. Thus, we see that $v \in \mathsf{W}$ as desired.

In the other direction, assume that $\mathsf{W} \subset \mathsf{V}$ is a subset such that $\operatorname{span}(\mathsf{W}) = \mathsf{W}$. Since we have shown that $\operatorname{span}(S)$ is a subspace for any S, the result follows.

Question 16

In class on 02/04.