What is calculus?

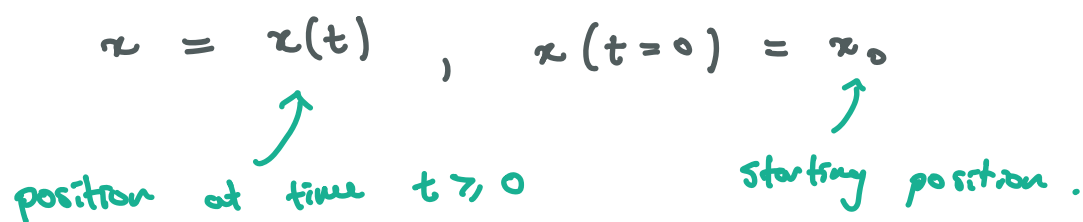One answer: Calculus is the mathematics behind (continuously) changing quantities.

Originally it was called the "calculus of infinitesimals". The idea is that to understand how something changes with respect to time (say), one needs to understand how it changes over an infinitesimally small amount of time.

An example of this is velocity. The slogan is

"Velocity is the rate of change of position".

Suppose a particle is moving on a straight line and its position x away from its starting location is a function of time:

$$x = x(t) \quad , \quad x(t=0) = x_0$$

position at time $t \geqslant 0$

starting position.

Average velocity from time $t = t_1$ to $t = t_2$ $=$ $\dfrac{\text{change in position}}{\text{change in time}}$

$$= \frac{x(t_2) - x(t_1)}{t_2 - t_1} \quad .$$

Ex: Sps $x(t) = t^3 - 2t$, $t \geq 0$.

- What is the starting position?

- What is the particles average velocity from time $t = 1$ to $t = 2$?

Succinctly, we will sometimes write

$$\Delta x = x(t_2) - x(t_1) = \begin{matrix} \text{change in} \\ \text{position} \end{matrix} \quad t_1 \rightarrow t_2$$
$$\Delta t = t_2 - t_1 = \text{change in time}$$

So average velocity is $\dfrac{\Delta x}{\Delta t}$.

Graphically, average velocity can be understood as the slope of a line. This is the so-called "secant" line through the points

$$\left(t_1 , x(t_1) \right) \text{ and } \left(t_2 , x(t_2) \right)$$

$$\text{Slope is } \frac{\text{rise}}{\text{run}} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \begin{matrix} \text{avg} \\ \text{velocity} \end{matrix} !$$