MA 442 - Quiz

January 28

Name: ______________________________ **BUID:** ____________________

There are two questions, you must answer **both** of them. Write your answers in a clear and well-organized way to receive full credit.

Question 1. Recall that the set of real functions $\mathbb{R}$ to $\mathbb{R}$ has the natural structure of a vector space that we denoted $\mathcal{F}(\mathbb{R}, \mathbb{R})$. Let $\mathsf{V} \subset \mathcal{F}(\mathbb{R}, \mathbb{R})$ be the subset of functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(3) = 0$. Is V a subspace? (If it is, you must prove it. If it is not, you must justify why.)

Solution. V is a subspace of $\mathcal{F}(\mathbb{R}, \mathbb{R})$. First, observe that $\mathbf{0}(3) = 0$ where $\mathbf{0} : \mathbb{R} \to \mathbb{R}$ is the zero function. Thus $\mathbf{0}$ is in V. Now, suppose that $f, g : \mathbb{R} \to \mathbb{R}$ are functions such that $f(3) = g(3) = 0$. Then $(f + g)(3) = f(3) + g(3) = 0 + 0 = 0$, so $f + g \in \mathsf{V}$ (in other words, V is closed under addition). Finally, suppose $\lambda \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ is a function such that $f(3) = 0$. Then $(\lambda f)(3) = 0$ so $\lambda f \in \mathsf{V}$ as well (in other words, V is closed under scalar multiplication). We have shown that V is a subspace.

Question 2. Consider the set $\mathsf{V} = \mathbb{R}^2$ of pairs of real numbers. Define "weird" addition on this set by the rule

$$(a_1, a_2)\tilde{+}(b_1, b_2) = (a_1 + b_2, a_2 + b_1)$$

Is V together with the rule of weird addtion $\tilde{+}$ and ordinary scalar multiplication[1] a vector space? (If yes, you must prove it. If it is not you must explain why not.)

Solution. Weird addition does not satisfy commutativity. For example, $(1, 0)\tilde{+}(0, 1) = (2, 0)$ while on the other hand $(0, 1)\tilde{+}(1, 0) = (0, 2)$. Since $(2, 0) \neq (0, 2)$ we see that weird addition is not commutative. Therefore, it is not part of a vector space structure on $\mathbb{R}^2$.

[1] So, $\lambda \cdot (a_1, a_2) = (\lambda a_1, \lambda a_2)$ for all scalars $\lambda \in \mathbb{R}$.