# *MA 442 - practice problems for exam 1*

February 13

**Name:** ______________________ **BUID:** ______________________

A collection of problems designed to help you prepare for midterm exam 1. These problems are by no means exhaustive, but they are in the same style that will appear on the exam.

**Question 1.** Let $\mathsf{V}$ be a finite-dimensional vector space over $\mathbb{R}$. In class we (will) show that the set of *all* linear maps $\mathsf{V} \to \mathbb{R}$ is naturally a vector space; we denote this vector space by $\operatorname{Hom}(\mathsf{V}, \mathbb{R})$. Express

$$\dim \operatorname{Hom}(\mathsf{V}, \mathsf{W})$$

in terms of $\dim V$. (Hint: Given a basis for $\mathsf{V}$ can you construct a basis for $\operatorname{Hom}(\mathsf{V}, \mathbb{R})$?)

**Question 2.** Let $\mathsf{V}$ be any vector space and suppose that $B = \{v_1, v_2, v_3, v_4\}$ is an ordered basis. Define the linear transformation $\mathsf{T}\colon \mathsf{V} \to \mathsf{V}$ by the formulas

$$\mathsf{T}(v_1) = v_1 + v_2,\ \mathsf{T}(v_2) = v_2 + v_3,\ \mathsf{T}(v_3) = v_3 + v_4,\ \mathsf{T}(v_4) = v_4.$$

Find the matrix representation of $T$ with respect to this ordered basis. (So, compute $[\mathsf{T}]_B$.)

**Question 3.** Suppose that $\mathsf{T}\colon \mathbb{R}^2 \to \mathbb{R}^2$ linear transformation with the property that $\dim \ker \mathsf{T} = 1$. Prove that there exists bases $B$ and $B'$ of $\mathbb{R}^2$ such that the matrix representation of $\mathsf{T}$ is

$$[\mathsf{T}]_B^{B'} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \tag{1}$$

**Question 4.** Recall that $\mathbb{R}[x]$ is the vector space of all real polynomials. Let $\mathsf{E} \subset \mathbb{R}[x]$ be the subset of *even* polynomials (so, only even powers of $x$ can appear, like $3 + 4x^6 + 9x^{38}$). Let $\mathsf{U} \subset \mathbb{R}[x]$ be the subset of *odd* polynomials (so, only odd powers of $x$ can appear like $2x + x^7 + 6x^{29}$.)

(a) Prove that both $\mathsf{E}$ and $\mathsf{U}$ are subspaces of $\mathbb{R}[x]$.

(b) Prove that $\mathbb{R}[x] = \mathsf{E} \oplus \mathsf{U}$.

**Question 5.** Prove that there exists a linear transformation $\mathsf{T}\colon \mathbb{R}^2 \to \mathbb{R}^3$ such that $\mathsf{T}(1, 1) = (1, 0, 2)$, $\mathsf{T}(2, 3) = (1, 1, 3)$. Also, compute $\mathsf{T}(7, 10)$.

**Question 6.** This question has two parts.

(a) Suppose that $\mathsf{W}_1$ and $\mathsf{W}_2$ are two two-dimensional subspaces in $\mathbb{R}^3$. Prove that their intersection must contain a nonzero vector. In other words, prove that $\mathsf{W}_1 \cap \mathsf{W}_2 \neq \{0\}$.

(b) Provide an example of two two-dimensional subspaces in $\mathbb{R}^4$ which intersect at single point.

In other words, two planes cannot intersect at a single point in $\mathbb{R}^3$ but they can in $\mathbb{R}^4$!