

# MA 442 - Quiz

February 18

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There are two questions, you must answer both.

**Question 1.** Is there a linear transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  such that  $T(1, 0, 1) = (1, 1)$ ,  $T(1, 0, -1) = (1, 1)$  and  $T(0, 0, 2) = (2, 1)$ ? You must justify your answer.

*Solution.* There is no linear transformation since  $(1, 0, 1) - (1, 0, -1) = (0, 0, 2)$  yet

$$T(1, 0, 1) - T(1, 0, -1) \neq T(0, 0, 2) \quad (1)$$

Thus the property  $T(x - y) = T(x) - T(y)$  would fail for this function.

**Question 2.** Let  $P_k$  be the vector space of polynomials of degree  $\leq k$ . Recall that this is a vector space of dimension  $k + 1$ . Let  $T: P_2 \rightarrow \mathbb{R}[x]$  be the linear map defined by

$$T(f(x)) = f'(x) + xf(x). \quad (2)$$

For example  $T(x^2) = 2x + x^3$ . (You do not need to check that this is linear.)

Compute  $\dim \ker T$  and  $\dim \operatorname{Im} T$ . (Hint: You should only need to compute one of these explicitly.)

*Solution.* It suffices to compute  $\dim \ker T$  since by the dimension theorem  $\dim \mathfrak{I}T = 3 - \dim \ker T$ . Suppose  $f = a + bx + cx^2$  is a polynomial in  $P_2$ . Then we compute

$$T(f) = b + (a + 2c)x + bx^2 + cx^3 \quad (3)$$

Thus, if  $T(f) = 0$  we see immediately that  $c = b = 0$ . This implies that  $a = 0$  as well. In particular  $\ker T = \{0\}$  and hence  $\dim \ker T = 0, \dim \operatorname{Im} T = 3$ .