March 4.

· Orientations: An orientation on a vector space V is on equivalence class of basis

(e,,..,e,) where

7 = Ae, det A70.  $\left[e_{1},\ldots,e_{n}\right]=\left[f_{1},\ldots,f_{n}\right]$ 

An orientation on a vector budle I is on orientation of Ex, xeM exists book triv's

Elu = le x R°

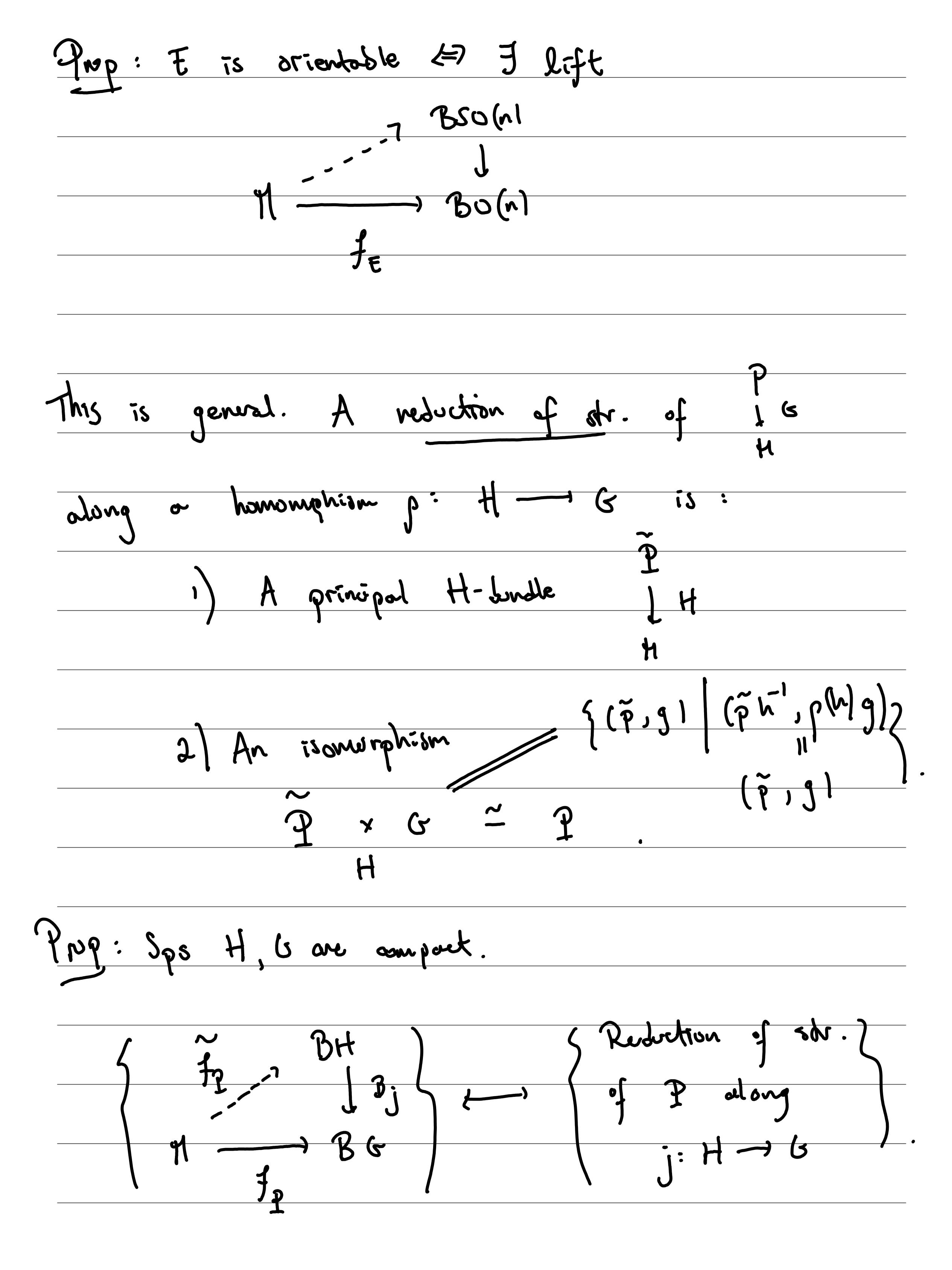
which relate these orientations to the standard orientation of 1R".

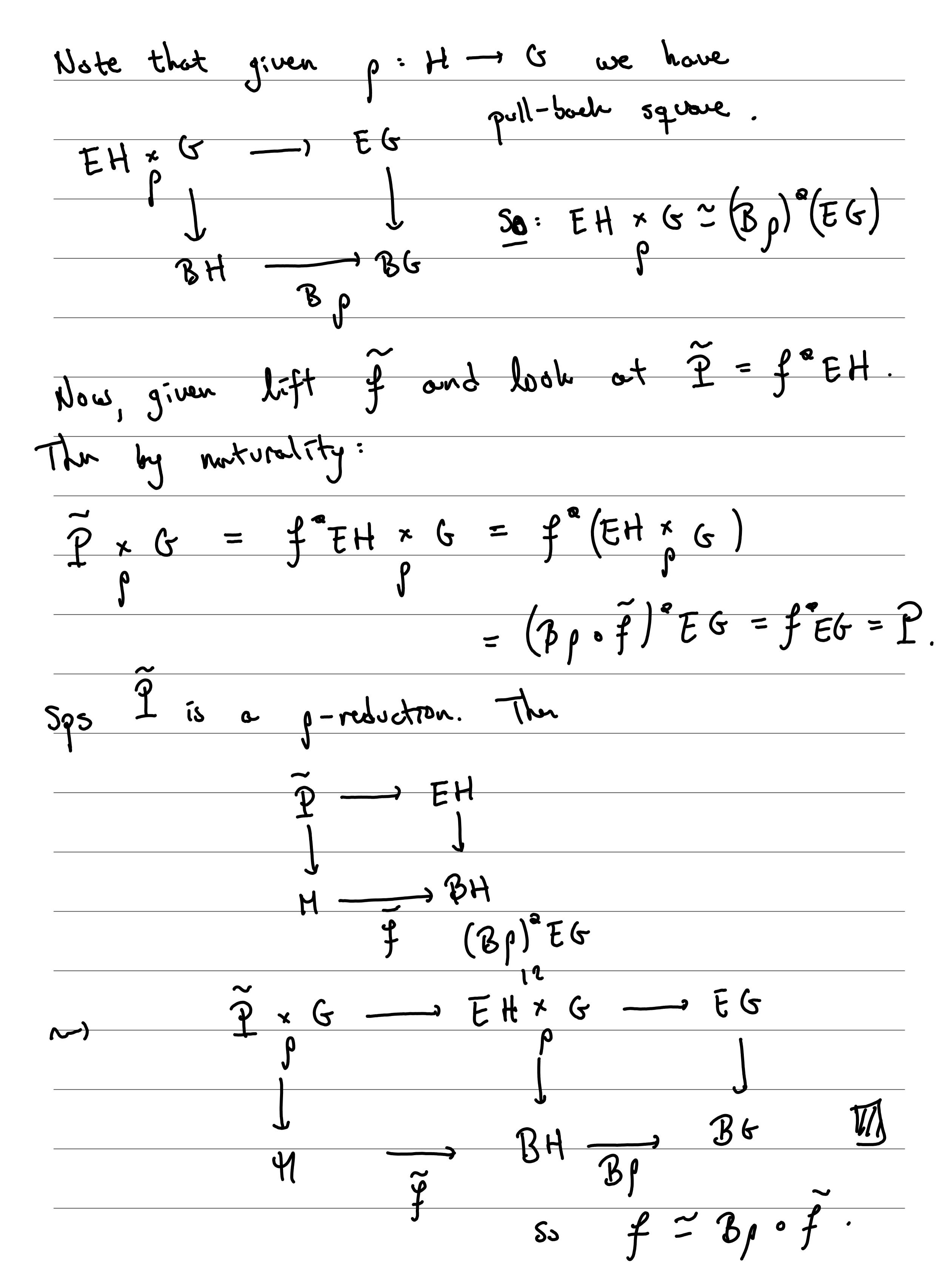
Suppose E is a Riemannian vector bundle.

~>> Fr E is its principal O(n) - bundle of orthonormal frances.

So(n) GFE

We form the quotrent bundle:
$Or_{E} = \frac{2fn}{fr_{E}/So(n)} \qquad 2f/2 = O(n)/So(n) - principal bundle$
a:/ wverting spose.
Let w, (E) = [ Or(E)] E H'(M; 2/2).
$Prop: E = 75$ orientable (=) $w_1(E) = 0$ .
Pt: Sps we have on orientation.
$\chi \in \mathcal{H} \longrightarrow \{e_1 _{\chi_1,\dots,e_n} _{\chi}\}$
deternines a section of Or = since SO(n)
are those orth. matrices of det >0.
(=) $triv. of Ore (=) W_1(E) = 0.$
· Another way to phrase orientability: E Riem
=) classified by f: 11 -> BO(n)





$$\frac{2}{3} \times \left(\frac{O(n)}{So(n)}\right)^{2} = \frac{Or}{E}$$

So:

$$W_1(E) = 0$$
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 $W_1(E) =$ 

$$\frac{f_{E}}{W_{1}(E)} \leftarrow \frac{f_{E}}{W_{1}} = \frac{W_{1}(E)(N)}{SO(N)}$$

$$\frac{f_{E}}{N} = \frac{W_{1}(E)(E)(N)}{SO(N)}$$

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It turns out:

Tw-1=:

· Spin structures:

Riem.

Ifn: A spin str. on on oriented vector budle E

is a reduction of str. of Fr along

 $p: Spin(n) \longrightarrow So(n).$ 

In other words, this is principal Spin(n) bundle  $\widetilde{P}$  on M s.t.:

 $\frac{2\cdot 1}{2\cdot 1} \xrightarrow{\text{Spin}(n) - \text{equivariant}}.$ 

Equivalent if:

