MA 442 - Quiz

February 11

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Question 1. Suppose that $\{u, v, w\}$ form a basis for some vector space V. Show that $\{u, u + v, u + v + w\}$ is also a basis for V.

Solution. The easiest way to do this is to show that the new set is linearly independent. Since it still has three elements it must be a basis. To show linear independence, assume that a, b, c are scalars with the property that

$$au + b(u + v) + c(u + v + w) = \mathbf{0}.$$

We can rearrange this to write the left hand side in terms of the original basis:

$$(a + b + c)u + (b + c)v + cw = 0. \tag{1}$$

But, since $\{u, v, w\}$ is linearly independent it follows that

$$a + b + c = b + c = c = 0. \tag{2}$$

This implies $a = b = c = 0$, hence $\{u, u + v, u + v + w\}$ is linearly independent.

Question 2. Let W_1, W_2 be subspaces of a vector space V. Prove that

$$\dim(W_1 + W_2) \leq \dim W_1 + \dim W_2. \tag{3}$$

Solution.

$$\begin{aligned} \dim(W_1 + W_2) &= \dim W_1 + \dim W_2 - \dim W_1 \cap W_2 && (4) \\ &\leq \dim W_1 + \dim W_2. && (5) \end{aligned}$$

The first equality was the result we proved in class. The inequality follows from

$$\dim W_1 \cap W_2 \geq 0.$$

(This is just an observation: *the dimension of any vector space is a non-negative integer.*)