

MA 442 - Quiz

February 18

Name: _____ BUID: _____

There are two questions, you must answer both.

Question 1. Is there a linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ such that $T(1, 0, 1) = (1, 1)$, $T(1, 0, -1) = (1, 1)$ and $T(0, 0, 2) = (2, 1)$? You must justify your answer.

Solution. There is no linear transformation since $(1, 0, 1) - (1, 0, -1) = (0, 0, 2)$ yet

$$T(1, 0, 1) - T(1, 0, -1) \neq T(0, 0, 2) \quad (1)$$

Thus the property $T(x - y) = T(x) - T(y)$ would fail for this function.

Question 2. Let P_k be the vector space of polynomials of degree $\leq k$. Recall that this is a vector space of dimension $k + 1$. Let $T: P_2 \rightarrow \mathbb{R}[x]$ be the linear map defined by

$$T(f(x)) = f'(x) + xf(x). \quad (2)$$

For example $T(x^2) = 2x + x^3$. (You do not need to check that this is linear.)

Compute $\dim \ker T$ and $\dim \text{Im } T$. (Hint: You should only need to compute one of these explicitly.)

Solution. It suffices to compute $\dim \ker T$ since by the dimension theorem $\dim \text{Im } T = 3 - \dim \ker T$. Suppose $f = a + bx + cx^2$ is a polynomial in P_2 . Then we compute

$$T(f) = b + (a + 2c)x + bx^2 + cx^3 \quad (3)$$

Thus, if $T(f) = 0$ we see immediately that $c = b = 0$. This implies that $a = 0$ as well. In particular $\ker T = \{0\}$ and hence $\dim \ker T = 0$, $\dim \text{Im } T = 3$.