HIGHER DIMENSIONAL HOLOMORPHIC QUANTUM FIELD THEORY

Theorem 0.1. *Let $\mathcal{E}$ be the fields of any holomorphically translation invariant field theory on $\mathbb{C}^d$ with classical interaction $I \in \mathcal{O}_{\mathrm{loc}}(\mathcal{E})$. Then, there exists a one-loop prequantization $\{I[L] \mid L > 0\}$ of I involving no counterterms. Moreover, we can pick the family $\{I[L]\}$ to be holomorphically translation invariant.*

Theorem 0.2. *Let $d \geq 1$, and let X be a Kähler manifold. For every trivialization of*

$$\mathrm{ch}_{d+1}(T^{1,0}X) \in H^{d+1}(X; \Omega^{d+1}_{cl}) \hookrightarrow H^{2d+2}_{dR}(X),$$

there exists a unique (up to homotopy) cotangent quantization of the holomorphic σ-model of maps $\mathbb{C}^d \to X$ that is compatible with the action of translations and the unitary group $U(d)$ on $\mathbb{C}^d$.

Theorem 0.3. *(Chiral algebraic index theorem) Let $q \in D(0,1)^\times$, X be a compact complex manifold, and α a trivialization of $\mathrm{ch}_2(TX) \in H^2(X, \Omega^2_{cl})$, so that the factorization algebra $\mathrm{Obs}^{\mathrm{q}}_{X,\alpha}$ of quantum observables of the two-dimensional holomorphic σ-model from E_q to X is defined (Theorem* **??***). Then:*

(1) there is quasi-isomorphism of sheaves of cochain complexes on X

$$\Phi^q : \mathrm{Hoch}(D^\hbar_{LX^{alg}}; q) \xrightarrow{\simeq} \int_{E_q} \mathrm{Obs}^{\mathrm{q}}_{X,\alpha};$$

(2) there is a q-twisted trace map

$$\mathrm{Tr}^q_X : \mathrm{Hoch}(D^\hbar_{LX^{alg}}; q) \to \mathbb{C}[[\hbar, \hbar^{-1}];$$

(3) if

$$1 \in H^* \int_{E_q} \mathrm{Obs}^q_{X,\alpha} \cong HH^*(D^\hbar_{LX^{alg}}; q)$$

denotes the unit observable, then in cohomology the trace map satisfies

$$\mathrm{Tr}^q_X(1) = \int_X \mathrm{Wit}(X, q).$$

Theorem 0.4. *Suppose $\mathcal{E}$ is a free holomorphic theory on a complex d-fold X that is invariant for holomorphic diffeomorphisms. Then, associated to $\mathcal{E}$ there is a Gelfand-Fuks cohomology class*

$$\alpha_{\mathcal{E}} \in H^{2d+1}_{GF}(\mathrm{W}_d),$$

and a map of factorization algebras on X

$$\Phi : \mathbb{U}_{\alpha_{\mathcal{E}}}(\mathcal{T}_X) \to \mathrm{Obs}^{\mathrm{q}}_{\mathcal{E}}.$$

In other words, an extension of $\mathcal{T}_X$ parametrized by the class $\alpha_{\mathcal{E}}$ is a quantum symmetry of $\mathcal{E}$.

Theorem 0.5. *The A_∞-algebra of operators on $S^3 \subset \mathbb{C}^2 \setminus \{0\}$ supported on the boundary of the twist of the $\mathcal{N} = 1$ supersymmetric gauge theory on the 5-manifold*

$$\left(\mathbb{C}^2 \setminus \{0\}\right) \times \mathbb{R}_{\geq 0}$$

is equivalent to the enveloping algebra $U^{\mathrm{Lie}}(\widehat{A_2 \otimes \mathfrak{g}})$, where the central extension is associated to the element

$$\mathrm{ch}_3(\mathfrak{g}^{ad}) \in \mathrm{Sym}^3(\mathfrak{g}^\vee)^{\mathfrak{g}}.$$