Two-dimensional quantum (0, 2) supergravity

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We derive the current algebra of two-dimensional (0, 2) supergravity, using the light-cone gauge approach à la Polyakov. It is found that the central charge of the current algebra is real for any d, in agreement with the observation by Polyakov and Zamolodchikov.

Since the pioneering work of Polyakov [1], a considerable amount of progress has been made in understanding two-dimensional quantum gravity. The most remarkable one is the large N random matrix formulation of two-dimensional quantum gravity [2]. One can obtain non-perturbative information about the fluctuation of the topology of the two-dimensional surface using this formulation. However, for conformal matter coupled to gravity, there is a generic singularity at $d=1$ preventing us from solving $d\geqslant 1$ theories. As observed by Polyakov and Zamolodchikov [3], supersymmetry may cure this pathological behavior, especially $N=2$ supersymmetry. The applicability of the powerful large N random matrix model, however, is limited to bosonic theory because of the lack of a discretized version of supersymmetric theories. Thus we are forced to use continuum theory to study two-dimensional supergravity. There are two continuum formulations of two-dimensional quantum gravity. One is the (super-)light-cone gauge approach [1,3–5], the other is the (super-)conformal gauge method [6]. The light-cone gauge approach is simple, and it has a nice analogy with the Wess–Zumino–Witten model. But the applicability of this approach is limited to surfaces with the simplest topology. In the conformal gauge, the theory is the Liouville model. It is applicable to any two-dimensional Riemann surfaces, but the theory is more complicated because of the scale factor dependence of the functional measure.

In this paper, we study (0, 2) quantum supergravity using the light-cone gauge approach. The current algebra of the residual super-coordinate transformation will be derived. It corresponds to an OSP(2|2) Kac–Moody algebra. We will also show that the central charge of the Kac–Moody algebra is real for any d, unlike the cases in bosonic and (0, 1), (1, 1) supersymmetric theories. The result agrees with the observation by Polyakov and Zamolodchikov [3], and it also agrees with the result obtained from the super-Liouville model [6].

(0, 2) superspace is parametrized by two bosonic coordinates $(x^{\neq}, x^{=})$, and two fermionic coordinates $(\theta^+, \bar{\theta}^+)$. Translation in superspace is generated by superderivatives

$$D_A=(\partial_{\neq}, \partial_{=}, D_+ = \partial/\partial\theta^+ + \bar{\theta}^+\partial_{\neq}, \bar{D}_+ = \partial/\partial\bar{\theta}^+ + \theta^+\partial_{\neq}) . \tag{1}$$

D_+ and $\bar{D}_+$ satisfy

$$\{D_+, \bar{D}_+\} = 2\partial_{\neq} . \tag{2}$$

In curved superspace, the super-derivatives are replaced by super-covariant derivatives

$$\nabla_A = E_A^M D_M + \omega_A \mathscr{M} , \tag{3}$$

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where the E_A^M are vielbeins, ω is the Lorentz connection and $\mathscr{M}$ is the Lorentz generator. Note that not all the vielbeins are independent, constraints have to be imposed in order for them to form an irreducible representation of (0, 2) supergravity multiplets. We use the constraints [7]

$$\{\nabla_+, \nabla_+\} = 0\,, \quad \{\bar{\nabla}_+, \bar{\nabla}_+\} = 0\,, \quad \{\nabla_+, \bar{\nabla}_+\} = 2\nabla_{\neq}\,, \tag{4}$$

$$[\nabla_+, \nabla_=] = G_= \nabla_+ + 2\bar{\Sigma}^+ \mathscr{M}\,, \quad [\bar{\nabla}_+, \nabla_=] = -G_= \bar{\nabla}_+ + 2\Sigma^+ \mathscr{M}\,. \tag{5,6}$$

Solving the Bianchi identity, we obtain

$$[\nabla_{\neq}, \nabla_=] = \Sigma^+ \nabla_+ + \bar{\Sigma}^+ \bar{\nabla}_+ + R\mathscr{M}\,, \tag{7}$$

where

$$R = \nabla_+ \Sigma^+ + \bar{\nabla}_+ \bar{\Sigma}^+\,. \tag{8}$$

The infinitesimal form of the general coordinate transformation of the gravitational superfields can be read off from the infinitesimal transformation of the super-covariant derivatives

$$\delta\nabla_A = [\nabla_A, K]\,, \tag{9}$$

where

$$K = K^N \mathrm{D}_N + \Lambda \mathscr{M} \tag{10}$$

is the transformation parameter. Expanding the vielbein around a flat background

$$E_A^M = \delta_A^M + H_A^M\,, \tag{11}$$

we find

$$\delta H_+^{\,+} = \mathrm{D}_+ K^+ - \tfrac{1}{2}\Lambda + H_+^M \mathrm{D}_M K^+ - K^N \mathrm{D}_N H_+^{\,+} - \tfrac{1}{2}\Lambda H_+^{\,+}\,, \tag{12}$$

$$\delta H_+^{\dot{+}} = \mathrm{D}_+ K^{\dot{+}} + H_+^M \mathrm{D}_M K^{\dot{+}} - K^N \mathrm{D}_N H_+^{\dot{+}} - \tfrac{1}{2}\Lambda H_+^{\dot{+}}\,, \tag{13}$$

$$\delta H_+^{=} = \mathrm{D}_+ K^{=} + H_+^M \mathrm{D}_M K^{=} - K^N \mathrm{D}_N H_+^{=} - \tfrac{1}{2}\Lambda H_+^{=}\,, \tag{14}$$

$$\delta H_+^{\neq} = \mathrm{D}_+ K^{\neq} - K^{\dot{+}} + H_+^M \mathrm{D}_M K^{\neq} - (H_+^{\dot{+}} K^+ + H_+^{\,+} K^{\dot{+}}) - K^N \mathrm{D}_N H_+^{\neq} - \tfrac{1}{2}\Lambda H_+^{\neq}\,, \tag{15}$$

$$\delta\omega_+ = \mathrm{D}_+ \Lambda + H_+^M \mathrm{D}_M \Lambda - K^M \mathrm{D}_M \omega_+ - \tfrac{1}{2}\Lambda\omega_+\,. \tag{16}$$

We can use

$$\Lambda \text{ to set } \phi_+ = 0\,, \quad K^+ \text{ to set } H_+^{\,+} = 0\,, \quad K^{\dot{+}} \text{ to set } H_+^{\dot{+}} = 0\,, \quad K^{=} \text{ to set } H_+^{=} = 0\,, \quad K^{\neq} \text{ to set } H_+^{\neq} = 0\,.$$

In this superspace light-cone gauge, the solution to the constraints is

$$\nabla_+ = \mathrm{D}_+\,, \quad \bar{\nabla}_+ = \bar{\mathrm{D}}_+\,, \quad \nabla_{\neq} = \partial_{\neq}\,, \tag{17}$$

$$\nabla_= = \partial_= + H_=^{\neq}\partial_{\neq} + \tfrac{1}{2}(\bar{\mathrm{D}}_+ H_=^{\neq})\mathrm{D}_+ + \tfrac{1}{2}(\mathrm{D}_+ H_=^{\neq})\bar{\mathrm{D}}_+ + (\partial_{\neq} H_=^{\neq})\mathscr{M}\,, \tag{18}$$

$$G_= = \tfrac{1}{4}[\mathrm{D}_+, \bar{\mathrm{D}}_+] H_=^{\neq}\,, \quad \omega_= = \partial_{\neq} H_=^{\neq}\,, \tag{19}$$

$$\Sigma^+ = \tfrac{1}{2}\bar{\mathrm{D}}_+ \partial_{\neq} H_=^{\neq}\,, \quad \bar{\Sigma}^+ = \tfrac{1}{2}\mathrm{D}_+ \partial_{\neq} H_=^{\neq}\,, \quad R = \partial_{\neq}^2 H_=^{\neq}\,. \tag{20}$$

The only independent superfield left after gauge fixing is

$$H_=^{\neq} = h_=^{\neq} + \theta^+ \psi_=^{\dot{+}} + \bar{\theta}^+ \psi_=^{+} + \bar{\theta}^+ \theta^+ a_=\,, \tag{21}$$

where $h_=^{\neq}$ is the graviton, $\psi_=^{+}$ and $\psi_=^{\dot{+}}$ are the two gravitini corresponding to the two supersymmetries, $a_=$ is the auxiliary vector field. After gauge fixing, there is still a residual gauge symmetry, under which $H_=^{\neq}$ transforms as

$$\delta H^{\neq}_{=} = -[\partial_{=} + H^{\neq}_{=}\partial_{\neq} + \tfrac{1}{2}(\bar{\mathrm{D}}_{+}H^{\neq}_{=})\mathrm{D}_{+} + \tfrac{1}{2}(\mathrm{D}_{+}H^{\neq}_{=})\bar{\mathrm{D}}_{+} - (\partial_{\neq}H^{\neq}_{=})]K^{\neq} . \tag{22}$$

In fact $H^{\neq}_{=}$ can be replaced by a scalar superfield Φ, satisfying

$$\nabla_{=}\Phi = 0 . \tag{23}$$

In accordance with the transformation property of $H^{\neq}_{=}$, Φ transforms as

$$\delta\Phi = K^{\neq}\partial_{\neq} + \tfrac{1}{2}(\mathrm{D}_{+}K^{\neq})\bar{\mathrm{D}}_{+}\Phi + \tfrac{1}{2}(\bar{\mathrm{D}}_{+}K^{\neq})\mathrm{D}_{+}\Phi . \tag{24}$$

A super-gravitational analogue of the Wess–Zumino–Witten model can be constructed from Φ.

The effective action of two-dimensional supergravity is defined as

$$Z = \int [\mathscr{D}H^{\neq}_{=}]\exp[\mathrm{i}\Gamma_{\text{eff.}}(H^{\neq}_{=})] , \tag{25}$$

where Z is the partition function of $(0, 2)$ superstring theory. The supercurrent $T_{\neq}$ is defined from

$$\delta\Gamma_{\text{eff.}}(H^{\neq}_{=}) = \int \mathrm{d}^2x\,\mathrm{d}\theta_{+}\,\mathrm{d}\bar{\theta}_{+}\,T_{\neq}(-\nabla_{=}K^{\neq}) . \tag{26}$$

The supercurrent $T_{\neq}$ satisfies

$$\nabla_{=}T_{\neq} = \frac{d-2}{8\pi}\partial_{\neq}[\mathrm{D}_{+}, \bar{\mathrm{D}}_{+}]H^{\neq}_{=} . \tag{27}$$

Here we have used the Weyl anomaly relation.

Now we derive the Ward identity for the residual gauge symmetry, which will enable us to find correlation functions, or equivalently operator product expansions, of field operators. For a scalar field with the Lorentz weight λ, it transforms as

$$\delta\phi = K^{\neq}\partial_{\neq}\phi + \tfrac{1}{2}(\mathrm{D}_{+}K^{\neq})\bar{\mathrm{D}}_{+}\phi + \tfrac{1}{2}(\bar{\mathrm{D}}_{+}K^{\neq})\mathrm{D}_{+}\phi - \lambda(\partial_{\neq}K^{\neq})\phi \tag{28}$$

under the residual general coordinate transformation in $(0, 2)$ superspace. The correlation function of n scalar fields satisfies

$$\sum_{j=1}^{n}\langle\phi(x_1)\ldots\delta\phi(x_j)\ldots\phi(x_n)\rangle = \mathrm{i}\int \mathrm{d}^2x\,\mathrm{d}\theta_{+}\,\mathrm{d}\bar{\theta}_{+}\,\nabla_{=}K^{\neq}\langle T_{\neq}\phi(x_1)\ldots\phi(x_n)\rangle . \tag{29}$$

More explicitly,

$$\sum_{j=1}^{n}\langle\phi(x_1)\ldots[K^{\neq}\partial_{\neq}\phi + \tfrac{1}{2}(\mathrm{D}_{+}K^{\neq})\bar{\mathrm{D}}_{+}\phi + \tfrac{1}{2}(\bar{\mathrm{D}}_{+}K^{\neq})\mathrm{D}_{+}\phi - \lambda(\partial_{\neq}K^{\neq})\phi]\ldots\phi(x_n)\rangle$$

$$= -\mathrm{i}\frac{d-2}{8\pi}\int \mathrm{d}^2x\,\mathrm{d}\theta_{+}\,\mathrm{d}\bar{\theta}_{+}\,K^{\neq}\langle(\partial_{\neq}[\mathrm{D}_{+}, \bar{\mathrm{D}}_{+}]H^{\neq}_{=})\phi(x_1)\ldots\phi(x_n)\rangle . \tag{30}$$

Now we need the following super-contour integrals:

$$K^{\neq}(z_k) = \frac{1}{2\pi\mathrm{i}}\int \mathrm{d}z_0\,K^{\neq}(z_0)\,\partial^2_{\neq}\frac{x^{\neq}_{0k}\bar{\theta}^{+}_{0k}\theta^{+}_{0k}}{x^{=}_{0k}} ,$$

$$D_{+}K^{\neq}(z_k) = \frac{1}{2\pi\mathrm{i}}\int \mathrm{d}z_0\,K^{\neq}(z_0)\,\partial^2_{\neq}\frac{x^{\neq}_{0k}\bar{\theta}^{+}_{0k}}{x^{=}_{0k}} , \quad \bar{\mathrm{D}}_{+}K^{\neq}(z_k) = \frac{-1}{2\pi\mathrm{i}}\int \mathrm{d}z_0\,K^{\neq}(z_0)\,\partial^2_{\neq}\frac{x^{\neq}_{0k}\theta^{+}_{0k}}{x^{=}_{0k}} ,$$

$$\partial_{\neq}K^{\neq}(z_k) = \frac{-1}{2\pi\mathrm{i}}\int \mathrm{d}z_0\,K^{\neq}(z_0)\,\partial^3_{\neq}\frac{x^{\neq}_{0k}\bar{\theta}^{+}_{0k}\theta^{+}_{0k}}{x^{=}_{0k}} , \quad \partial_{=}K^{\neq}(z_k) = \frac{1}{2\pi\mathrm{i}}\int \mathrm{d}z_0\,K^{\neq}(z_0)\,\partial^2_{\neq}\frac{(x^{\neq}_{0k})^2\bar{\theta}^{+}_{0k}\theta^{+}_{0k}}{(x^{=}_{0k})^2} . \tag{31}$$

where $z = (x^{\neq}, x^{=}, \theta^{+}, \bar{\theta}^{+})$, $\mathrm{d}z = \mathrm{d}^2x\,\mathrm{d}\theta_{+}\,\mathrm{d}\bar{\theta}_{+}$ and

$$x_{0k}^{\neq}=x_0^{\neq}-x_k^{\neq}-(\bar\theta_0\theta_k^+ +\theta_0^+\bar\theta_k^+)\,,\quad x_{0k}^{=}=x_0^{=}-x_k^{=}\,,\tag{32}$$

$$\theta_{0k}^+=\theta_0^+-\theta_k^+\,,\quad \bar\theta_{0k}^+=\bar\theta_0^+-\bar\theta_k^+\,.\tag{33}$$

With the help of the super-contour integrals, eq. (30) can be written as

$$\tfrac{1}{2}c\langle H_{=}^{\neq}(x)\phi(x_1)...\phi(x_n)\rangle=\sum_{k=1}^{n}\left(\frac{(x_{0k}^{\neq})^2}{x_{0k}^{=}}\partial_{\neq}+\frac{x_{0k}^{\neq}\bar\theta_{0k}^+}{x_{0k}^{=}}\bar{\mathrm{D}}_+ +\frac{x_{0k}^{\neq}\theta_{0k}^+}{x_{0k}^{=}}\mathrm{D}_+ +2\lambda\frac{x_{0k}^{\neq}}{x_{0k}^{=}}\right)\langle\phi(x_1)...\phi(x_n)\rangle\,.\tag{34}$$

If we write the solution of the equation of motion of $H_{=}^{\neq}$ as

$$H_{=}^{\neq}=x^{\neq 2}j^{-1}-x^{\neq}\bar\theta^+j^{-1/2}-x^{\neq}\theta^+\tilde{j}^{-1/2}+2\bar\theta^+\theta^+\tilde{j}^0-2x^{\neq}j^0+\bar\theta^+j^{1/2}+\theta^+\tilde{j}^{1/2}+j^1\,,\tag{35}$$

then eq. (34) can be further written as

$$\langle j^a\phi(x_1)...\phi(x_n)\rangle=\sum_{k=1}^{n}\frac{L_k^a}{x_{0k}^{=}}\langle\phi(x_1)...\phi(x_n)\rangle\,,\tag{36}$$

where the L are

$$L^{-1}=\partial_{\neq}\,,\quad L^{-1/2}=2\theta^+\partial_{\neq}-\bar{\mathrm{D}}_+\,,\quad \tilde{L}^{-1/2}=2\bar\theta^+\partial_{\neq}-\mathrm{D}_+\,,\quad L^0=x^{\neq}\partial_{\neq}+\tfrac{1}{2}\bar\theta^+\bar{\mathrm{D}}_+\tfrac{1}{2}\theta^+\mathrm{D}_+-\lambda\,,$$

$$\tilde{L}^0=\bar\theta^+\theta^+\partial_{\neq}+\tfrac{1}{2}\theta^+\mathrm{D}_+-\tfrac{1}{2}\bar\theta^+\bar{\mathrm{D}}_+\,,\quad L^{1/2}=2x^{\neq}\theta^+\partial_{\neq}-(x^{\neq}+\bar\theta^+\theta^+)\bar{\mathrm{D}}_+-2\lambda\theta^+\,,$$

$$\tilde{L}^{1/2}=2x^{\neq}\bar\theta^+\partial_{\neq}-(x^{\neq}-\bar\theta^+\theta^+)\mathrm{D}_+-2\lambda\bar\theta^+\,,\quad L^1=(x^{\neq})^2\partial_{\neq}+x^{\neq}\bar\theta^+\bar{\mathrm{D}}_+ +x^{\neq}\theta^+\mathrm{D}_+-2\lambda x^{\neq}\,.\tag{37}$$

They are generators of the OSP (2|2) algebra:

$$[L^m,L^n]=(n-m)L^{m+n}\,,\quad \{G^m,G^n\}=0=\{\tilde G^m,\tilde G^n\}\,,\quad \{G^m,\tilde G^n\}=-2L^{n+m}-2(n-m)\tilde L^{n+m}\,,$$

$$[L^m,G^n]=-(\tfrac{1}{2}m-n)G^{m+n}\,,\quad [L^m,\tilde G^n]=-(\tfrac{1}{2}m-n)\tilde G^{m+n}\,,\quad [\tilde L^m,G^n]=\tfrac{1}{2}G^{m+n}\,,$$

$$[\tilde L^m,\tilde G^n]=-\tfrac{1}{2}\tilde G^{m+n}\,,\quad [\tilde L^m,\tilde L^n]=0\,,\quad [L^m,\tilde L^n]=n\tilde L^{n+m}\,.\tag{38}$$

The j^a are the corresponding currents. Now we derive the current correlation functions.
From the Ward identity, we have

$$-\tfrac{1}{8}c\partial_{\neq}[\mathrm{D}_+,\bar{\mathrm{D}}_+]\langle H_{=}^{\neq}(x)H_{=}^{\neq}(x_1)...H_{=}^{\neq}(x_n)\rangle=\sum_{k=1}^{n}\langle H_{=}^{\neq}(x_1)...\left[\partial_{\neq}^2\left(\frac{x_{0k}^{\neq}\bar\theta_{0k}^+\theta_{0k}^+}{(x_{0k}^{=})^2}\right)-\partial_{\neq}^3\left(\frac{x_{0k}^{\neq}\bar\theta_{0k}^+\theta_{0k}^+}{x_{0k}^{=}}\right)H_{=}^{\neq}\right.$$

$$\left.+\tfrac{1}{2}\partial_{\neq}^2\left(\frac{x_{0k}^{\neq}\theta_{0k}^+}{x_{0k}^{=}}\right)\mathrm{D}_+H_{=}^{\neq}-\tfrac{1}{2}\partial_{\neq}^2\left(\frac{x_{0k}^{\neq}\bar\theta_{0k}^+}{x_{0k}^{=}}\right)\bar{\mathrm{D}}_+H_{=}^{\neq}-\partial_{\neq}^2\left(\frac{x_{0k}^{\neq}\bar\theta_{0k}^+\theta_{0k}^+}{x_{0k}^{=}}\right)\partial_{\neq}H_{=}^{\neq}\right]...H_{=}^{\neq}(x_n)\rangle\,.\tag{39}$$

Rescale all the $H_{=}^{\neq}$ by $\frac{1}{2}c$ and integrate the equation, then we arrive at

$$\langle H_{=}^{\neq}(x)H_{=}^{\neq}(x_1)...H_{=}^{\neq}(x_n)\rangle=-\tfrac{1}{2}c\sum_{k=1}^{n}\frac{(x_{0k}^{\neq})^2}{(x_{0k}^{=})^2}\langle H_{=}^{\neq}(x_1)...\cancel{H_{=}^{\neq}(x_k)}...H_{=}^{\neq}(x_n)\rangle$$

$$+\sum_{k=1}^{n}\left(\frac{(x_{0k}^{\neq})^2}{x_{0k}^{=}}\partial_{\neq}+\frac{x_{0k}^{\neq}\bar\theta_{0k}^+}{x_{0k}^{=}}\bar{\mathrm{D}}_+ +\frac{x_{0k}^{\neq}\theta_{0k}^+}{x_{0k}^{=}}\mathrm{D}_+ +2\frac{x_{0k}^{\neq}}{x_{0k}^{=}}\right)\langle H_{=}^{\neq}(x_1)...H_{=}^{\neq}(x_n)\rangle\,,\tag{40}$$

where a $\nearrow$ means the omission of the term. Using eq. (35), the above equation can be written in terms of currents,

$$\langle j^a(x)j^{b_1}(x_1)...j^{b_n}(x_n)\rangle$$

$$=-\tfrac{1}{2}\kappa\sum_{i=1}^{n}\frac{\eta^{ab_i}}{(x_{0i}^{=})^2}\langle j^{b_1}(x_1)...\cancel{j^{b_i}(x_i)}...j^{b_n}(x_n)\rangle+\sum_{i=1}^{n}\frac{f^{a,\,b_i}_{c_i}}{x_{0i}^{=}}\langle j^{b_1}(x_1)...j^{c_i}(x_i)...j^{b_n}(x_n)\rangle\,.\tag{41}$$

where

$$\eta^{-1,+1}=\eta^{+1,-1}=2\,,\quad \eta^{-1/2,\overline{1/2}}=-\eta^{\overline{1/2},-1/2}=-4\quad \eta^{-\overline{1/2},1/2}=-\eta^{1/2,-\overline{1/2}}=-4\,,\quad \eta^{0,0}=-1\,,\quad \eta^{\bar{0},\bar{0}}=1$$

and $\kappa\equiv\frac{1}{2}c$. The structure function $f_{ci}^{\ ab_i}$ defined here differs from the one in eq. (38) by a minus sign.

Now we construct the super-gravitational energy–momentum tensor. $\Gamma_{\rm eff}(H_{=}^{\neq})$ in eq. (25) is a gauge fixed, non-local expression. To recover the covariance, local counter terms have to be added. We are especially interested in the terms linear in $H_{=}^{=}$, $H_{+}^{=}$ and $H_{\bar{+}}^{=}$,

$$\Gamma_c=H_{=}^{=}(a\partial_{\neq}H_{=}^{\neq}+b[\mathrm{D}_+,\bar{\mathrm{D}}_+]H_{=}^{\neq})+H_{+}^{=}[c\partial_{=}\bar{\mathrm{D}}_+H_{=}^{\neq}+dH_{=}^{\neq}\partial_{\neq}\bar{\mathrm{D}}_+H_{=}^{\neq}+e(\bar{\mathrm{D}}_+H_{=}^{\neq})(\mathrm{D}_+\bar{\mathrm{D}}_+H_{=}^{\neq})]$$
$$+H_{\bar{+}}^{=}[f\partial_{=}\mathrm{D}_+H_{=}^{\neq}+gH_{=}^{\neq}\partial_{\neq}\mathrm{D}_+H_{=}^{\neq}+h(\mathrm{D}_+H_{=}^{\neq})(\bar{\mathrm{D}}_+\mathrm{D}_+H_{=}^{\neq})]+\dots\,, \tag{42}$$

where a, b, c, d, e, f, g, h are coefficients to be determined. The relative coefficients can be determined by requiring $\Gamma_{\rm c}$ to be invariant under

$$\delta H_{=}^{=}=\nabla_{=}K^{=}\,,\quad \delta H_{+}^{=}=\nabla_{+}K^{=}\,,\quad \delta H_{\bar{+}}^{=}=\bar{\nabla}_{+}K^{=}\,.$$

The overall coefficient can be fixed by including the non-local term, and requires Lorentz invariance. Finally, we have

$$\Gamma_c=-\tfrac{1}{4}\kappa\{H_{=}^{=}[\mathrm{D}_+,\bar{\mathrm{D}}_+]H_{=}^{\neq}-H_{+}^{=}[\partial_{=}\bar{\mathrm{D}}_+H_{=}^{\neq}+H_{=}^{\neq}\partial_{\neq}\bar{\mathrm{D}}_+H_{=}^{\neq}-\tfrac{1}{2}(\bar{\mathrm{D}}_+H_{=}^{\neq})(\mathrm{D}_+\bar{\mathrm{D}}_+H_{=}^{\neq})]$$
$$+H_{\bar{+}}^{=}[\partial_{=}\mathrm{D}_+H_{=}^{\neq}+H_{=}^{\neq}\partial_{\neq}\mathrm{D}_+H_{=}^{\neq}-\tfrac{1}{2}(\mathrm{D}_+H_{=}^{\neq})(\bar{\mathrm{D}}_+\mathrm{D}_+H_{=}^{\neq})]\}+\dots\,. \tag{43}$$

The energy–momentum tensor is

$$T_{==}=\mathrm{D}_+\left(\frac{\delta\Gamma_{\rm c}}{\delta H_{+}^{=}}\right)\bigg|_{H_{\bar{+}}^{=}=0}-\bar{\mathrm{D}}_+\left(\frac{\delta\Gamma_{\rm c}}{\delta H_{\bar{+}}^{=}}\right)\bigg|_{H_{+}^{=}=0}=-\frac{1}{\kappa}\eta_{ba}j^aj^b+\partial_{=}j^0\,. \tag{44}$$

This is a classical result. In general, κ will be renormalized by quantum effects. Let us look at two terms in the energy–momentum separately. The first term is quadratic in currents, we will call it $T_{==}^{\rm sug.}$. The renormalized coupling constant $\tilde{\kappa}$ is defined as

$$-\tilde{\kappa}T_{==}^{\rm sug.}=:\eta_{ba}j^aj^b:\equiv\eta_{ab}j^a(x_0)j^b(x_k)+\frac{\frac{1}{2}\kappa\delta_a^a}{(x_0^{=}-x_k^{=})^2}\,. \tag{45}$$

Consistency condition requires [8]

$$C_{\rm v}-\tfrac{1}{2}\kappa-\tilde{\kappa}=0\,,\quad C=\frac{-\frac{1}{2}\kappa\delta_a^a}{C_{\rm v}-\frac{1}{2}\kappa}\,, \tag{46}$$

where C is the central charge of $T_{==}^{\rm sug.}$, and $C_{\rm v}$ is defined as

$$C_{\rm v}\delta^{ab}\equiv f^{\,acd}f^{\,b}_{\ cd}\,. \tag{47}$$

The term linear in the current will not be modified by quantum effects. Thus the full energy–momentum tensor can be written as

$$T_{==}=\frac{2}{\kappa-C_{\rm v}}:\eta_{ba}j^aj^b:+\partial_{=}j^0\,. \tag{48}$$

In both (0, 2) and (2, 2) supergravities, δ_a^a vanishes. Thus the only non-vanishing contribution from two-dimensional (0, 2) supergravity to the central charge of the Virasoro algebra is from $\partial_{=}j^0$. From eq. (44), we see that

$$j^0(x_0)j^0(x_1)=\frac{-\frac{1}{2}\kappa}{(x_0^{=}-x_1^{=})^2}+(\text{regular terms})\,. \tag{49}$$

This leads to

$$T_{==}(x_0)T_{==}(x_k)=\frac{-3\kappa}{(x_0^{=}-x_k^{=})^4}+\ldots . \tag{50}$$

Therefore

$$C_{\text{grav.}}=-6\kappa . \tag{51}$$

If the residual gauge symmetry is not spoiled by any anomaly, then $T^{\text{tot.}}_{==}$ is a generator of a good gauge symmetry. It is constrained to be weakly zero. This requires

$$C_{\text{matter}}+C_{\text{ghost}}+C_{\text{grav.}}=0 . \tag{52}$$

The ghost contribution to the central charge of the Virasoro algebra can be read off from ref. [9] where the ghost contribution to the central charge for a general ghost lagrangian was calculated using the (0, 1) superspace background field method. (0, 2) supergravity multiplets can be written in terms of (0, 1) multiplets. In (0, 1) supergravity language, setting $H^{=}_{+}=0$ contributes -26 to the central charge; setting $H^{\neq}_{+}$ contributes -2; and setting $S=0$ contributes -1 (S is now a complex scalar superfield). Therefore,

$$C_{\text{matter}}-26-2-1-6\kappa=0 . \tag{53}$$

Using the relation

$$C_{\text{matter}}-26=3(d-2) , \tag{54}$$

we obtain

$$d-1=2(\kappa+1) .$$

This is our final result. It is in agreement with the observation by Polyakov and Zamolodchikov [3]. It also agrees with the result found in the $N=2$ super-Liouville model [6].

Note added. When this paper was being typed, we received a paper by Sabra [10], in which similar results have been obtained.

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