## GAUGING VERSUS THE BACKGROUND FIELD METHOD

0.1. **Gauging = local BRST cohomology.** BW: recall ordinary BRST cohomology for symplectic actions

We now explain, in more detail, the process of gauging a symmetry of a BV theory. We will find that the observables of the gauged theory introduce a "local" version of BRST cohomology. In our setup, of course, this will be a factorization algebra. As in the case of the equivariant observables, the process of gauging is defined whenever we have a symmetry of a theory by some local Lie algebra, but there are many key differences as we have explained in Table BW: ref. First, we focus on the classical case.

0.1.1. Indeed, let $\mathcal{E}$ be the fields of a classical BV theory on a manifold $M$ described by some action functional

$$S = S^{free} + I \in \mathcal{O}_{\mathrm{loc}}(\mathcal{E}),$$

satisfying the classical master equation $\{S, S\} = 0$. Let $\mathcal{L}$ be a local Lie algebra that acts on the theory $\mathcal{E}$ with corresponding Noether functional $I^{\mathcal{L}} : \mathcal{L} \to \mathcal{O}_{\mathrm{loc}}(\mathcal{E})$. Recall, this means that the equivariant classical master equation $(\mathrm{d}_{\mathcal{L}} + Q)(I + I^{\mathcal{L}}) + \frac{1}{2}\{I + I^{\mathcal{L}}, I + I^{\mathcal{L}}\} = 0$, where $\mathrm{d}_{\mathcal{L}}$ is the Chevalley–Eilenberg differential on $\mathcal{L}$, is satisfied.

We define a new BV theory built from this action of $\mathcal{L}$ on $\mathcal{E}$. The space of fields is

$$\mathcal{E}^{\mathcal{L}}_{BRST} = \mathcal{E} \oplus \mathcal{L}[1] \oplus \mathcal{L}^![-2].$$

The $(-1)$-shifted symplectic pairing is given by the sum of the pairing defining the theory $\mathcal{E}$ together with the natural pairing

$$\langle -, - \rangle_{\mathcal{L}} : \mathcal{L}[1] \times \mathcal{L}^![-2] \xrightarrow{\mathrm{ev}} \mathrm{Dens}_M \xrightarrow{\int_M} \mathbb{C}.$$

The first arrow is the density valued evaluation pairing between $\mathcal{L}$ and its !-dual $\mathcal{L}^! = \mathcal{L}^\vee \otimes \mathrm{Dens}_M$. Write $X, X^!$ for elements of $\mathcal{L}, \mathcal{L}^!$, respectively. The action functional for this new theory is

$$S^{\mathcal{L}}_{BRST} = S + I^{\mathcal{L}} + \sum_{n \geq 0} \frac{1}{n!} \langle X^!, \ell_n(X, \ldots, X) \rangle_{\mathcal{L}} \in \mathcal{O}_{\mathrm{loc}}(\mathcal{E}^{\mathcal{L}}_{BRST}).$$

If we denote by $\{\ell_n\}$ the $L_\infty$ structure maps for $\mathcal{L}$ we can think about this new theory as splitting up into a free part

$$S^{\mathcal{L}, free}_{BRST} = S^{free} + \langle X^!, \ell_1(X) \rangle_{\mathcal{L}}$$

and an interacting part

$$I^{\mathcal{L}}_{BRST} = I + I^{\mathcal{L}} + \sum_{n \neq 1} \langle X^!, \ell_n(X, \ldots, X) \rangle_{\mathcal{L}}.$$

**Lemma 0.1.** *Let $\{-, -\}_{BRST}$ denote the bracket on $\mathcal{O}_{\mathrm{loc}}(\mathcal{E}^{BRST})$ induced by the $(-1)$-shifted symplectic structure on $\mathcal{E}^{BRST}$ defined above. The functional $S^{\mathcal{L}}_{BRST}$ satisfies the curved classical master equation*

$$\{S^{\mathcal{L}}_{BRST}, S^{\mathcal{L}}_{BRST}\} = \ell_0.$$

**Definition 0.2.** The *local BRST complex* for the action of $\mathcal{L}$ on $\mathcal{E}$ is the factorization algebra $\mathrm{Obs}^{cl}_{BRST}$ of classical observables for the theory $(\mathcal{E}^{\mathcal{L}}_{BRST}, S^{\mathcal{L}}_{BRST})$.

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*Example* 0.3. Suppose that $\Sigma$ is a Riemann surface with fixed metric $g_0$. We have already seen in Example **??** how the local Lie algebra controlling deformations of Riemannian metrics $\mathcal{L}^{Riem}$ acts on the free boson on $\Sigma$. The corresponding BRST theory

$$\mathcal{E}_{BRST} = \mathcal{E}^{boson}_{\Sigma} \oplus \mathcal{L}^{Riem}_{(\Sigma,g_0)}[1] \oplus (\mathcal{L}^{Riem}_{(\Sigma,g_0)})^![-2],$$

is precisely the perturbative expansion of the Polchinski string **??** near the fixed metric $(\Sigma, g_0)$. Explicitly, the action functional is BW: finish, copy from string

*Example* 0.4. Suppose, again, that $\Sigma$ is a Riemann surface. Consider local Lie algebra given by the Dolbeualt complex of holomorphic vector fields $\mathcal{L} = \mathcal{T}_\Sigma = \Omega^{0,*}(\Sigma, T^{1,0}_\Sigma)$.

0.1.2. BW: Quantum story. Formula in the case that the theory is free.

### 0.2. **Examples of BRST cohomology.**

### 0.3. **Chiral gravity.**