

The Yangian and four dimensional gauge theory

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There is a deep connection between gauge theory and quantum groups. Perhaps the most ubiquitous is the relationship between the category of line operators in Chern-Simons theory on three manifolds and the braided monoidal category of representations of the quantum group. In the seminal work, Costello [[Cosa](#), [Cosb](#)] has shown how certain infinite dimensional quantum groups, Yangians, arise from the algebra of operators of a *four* dimensional (supersymmetric) gauge theory. The key to this formalism is a result of Costello-Gwilliam that the factorization algebras of a quantum field theory form a factorization algebra. This is an enhancement of the well-known description of a conformal field theory in terms of vertex, or chiral, algebras. The primary goal of this seminar is to outline Costello's construction which starts from the factorization algebra description of the observables of a four dimensional gauge theory and recovers the Yangian quantum group of the gauge Lie algebra. Time permitting, we can focus on more concrete and computational sides of the program began in the works [[CWYa](#), [CWYb](#)].

Some keywords: Factorization algebras, Koszul duality, quantum groups, Yangians.

- An introduction to factorization algebras valued in symmetric monoidal ∞ -categories. Lurie's result that locally constant factorization algebras recover algebras over the operad of little disks. Modules for E_n -algebras. [[CG17](#), [Lur](#), [AF15](#)]
- Holomorphic factorization algebras and their relationship to chiral and vertex algebras. [[CG17](#), [BD04](#)]
- Koszul duality for E_n -algebras. Tamarkin's [[Tam03](#)] result that the Koszul dual of an E_2 algebra is a Hopf algebra. The Hochschild homology of E_n algebras.
- An introduction to quantum groups and the Yangian. Drinfeld's universal R -matrix. [[ES02](#), [CP94](#)] Relationship to integrable systems and lattice models.
- The classical moduli space of holomorphic partially flat connections on complex surfaces.¹ The Batalin-Vilkovisky formalism and quantization. Costello-Gwilliam's result that the operators of a gauge theory (or any QFT) form a factorization algebra. [[Cos11](#), [CG](#), [Cosb](#)]
- Line operators of the four dimensional gauge theory and their interpretation via Koszul duality. Possibly introduce Hochschild homology for monoidal categories.

¹Roughly, a holomorphic partially flat connection on a complex manifold of the form $X \times Y$ is a holomorphic connection that is holomorphically flat in the Y -direction.

- Computing the universal R -matrix from the BV quantization.
- Enhancements and variations of the construction. Realizing various spin systems by tweaking the input data. Coupling to surface operators.

References

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