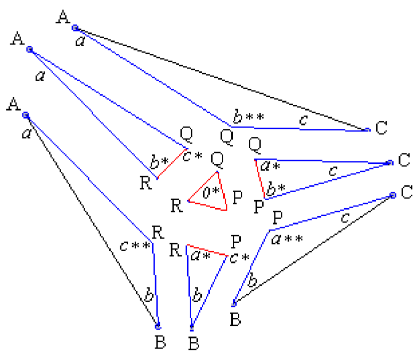
(http://www.cut-the-knot.org/manifesto/index.shtml)

Morley's Miracle
Remarks on J. Conway's proof

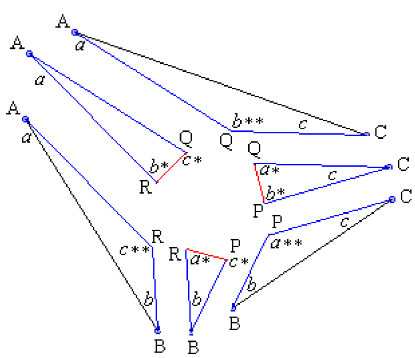
As other "backward" proofs, ***Conway's (http://www.cut-the-knot.org/triangle/Morley/conway.shtml)*** ends up with a triangle that at best is similar to ΔABC which is of course fine.



The seven triangles fit together for two reasons:

1. At the vertices of the equilateral triangle the angles sum up to 360°.
2. The line segments that are to be common sides of two triangles are equal by construction. This is clear for the sides of the equilateral triangle that have been chosen to be 1. The add-on points Y and Z allow to select triangles BPC, AQC, and ARB so that the potential trisectors also match each other.

The whole point of introducing points Y and Z is to eschew trigonometry. Building on the rigidity of the configuration and the fact that all angles in all triangles are known, it is possible to follow the chain of six triangles (the middle one excluded) using the ***Law of Sines (http://www.cut-the-knot.org/proofs/sine_cosine.shtml#law)***:



Start, for example with ΔABR. Fix, say, AR. Then

(1) BR = AR·sin(a)/sin(b).

Construct ΔBPR with BR given above. Then

(2) BP = BR·sin(a*)/sin(c*).

Keep constructing triangles so that their sides match. By the ***Law of Sines (http://www.cut-the-knot.org/proofs/sine_cosine.shtml#law)***, we'll sequentially get

(3) CP = BP·sin(b)/sin(c)
(4) CQ = CP·sin(b*)/sin(a*)
(5) AQ = CQ·sin(c)/sin(a)
(6) AR = AQ·sin(c*)/sin(b*)

The last one should actually read "AR' = AQ·sin(c*)/sin(b*)". For we do not know in advance whether it has the same length as AR. To see that it does, multiply (1)-(6): after reducing like factors we end up with AR' = AR. Therefore, putting together the six triangles fills a triangle with a triangular hole. Counting angles we see that the hole is equiangular. Which is what was needed.

One should appreciate the simplicity of Conway's proof that not only avoids the trigonometry but makes it also unnecessary to handle all 6 triangles separately. ***D.J.Newman's proof (http://www.cut-the-knot.org/triangle/Morley/newman.shtml)*** is a simplified trigonometric variant where only 3 (out of 6) triangles are handled with the ***Law of Sines (http://www.cut-the-knot.org/proofs/sine_cosine.shtml#law)***.

Morley's Miracle (http://www.cut-the-knot.org/triangle/Morley/index.shtml)

On Morley and his theorem

1. ***Doodling and Miracles (http://www.cut-the-knot.org/triangle/Morley/Morley.shtml)***
2. ***Morley's Pursuit of Incidence (http://www.cut-the-knot.org/triangle/Morley/CenterCircle.shtml)***
3. ***Lines, Circles and Beyond (http://www.cut-the-knot.org/triangle/Morley/Beyond.shtml)***
4. ***On Motivation and Understanding (http://www.cut-the-knot.org/triangle/Morley/MorleyFinal.shtml)***
5. ***Of Looking and Seeing (http://www.cut-the-knot.org/ctk/MorleyConc.shtml)***

Backward proofs

1. ***J.Conway's proof (http://www.cut-the-knot.org/triangle/Morley/conway.shtml)***
   - Remarks on J. Conway's proof
2. ***D. J. Newman's proof (http://www.cut-the-knot.org/triangle/Morley/newman.shtml)***
3. ***B. Bollobás' proof (http://www.cut-the-knot.org/triangle/Morley/Bollobas.shtml)***
4. ***G. Zsolt Kiss' proof (http://www.cut-the-knot.org/triangle/Morley/MorleyZsolt.shtml)***
5. ***Backward Proof by B. Stonebridge (http://www.cut-the-knot.org/triangle/Morley/sb3.shtml)***
6. ***Morley's Equilaterals, Spiridon A. Kuruklis' proof (http://www.cut-the-knot.org/m/Geometry/Morley5.shtml)***
7. ***J. Arioni's Proof of Morley's Theorem (http://www.cut-the-knot.org/triangle/Morley/Arioni.shtml)***

Trigonometric proofs

1. ***Bankoff's proof (http://www.cut-the-knot.org/triangle/Morley/BankoffProof.shtml)***
2. ***B. Bollobás' trigonometric proof (http://www.cut-the-knot.org/triangle/Morley/BollobasTrig.shtml)***
3. ***Proof by R. J. Webster (http://www.cut-the-knot.org/triangle/Morley/Webster.shtml)***
4. ***A Vector-based Proof of Morley's Trisector Theorem (http://www.cut-the-knot.org/triangle/Morley/VectorProof.shtml)***
5. ***L. Giugiuc's Proof of Morley's Theorem (http://www.cut-the-knot.org/triangle/Morley/Giugiuc.shtml)***
6. ***Dijkstra's Proof of Morley's Theorem (http://www.cut-the-knot.org/triangle/Morley/Dijkstra.shtml)***

Synthetic proofs

1. ***Another proof (http://www.cut-the-knot.org/triangle/Morley/yours_truly.shtml)***
2. ***Nikos Dergiades' proof (http://www.cut-the-knot.org/triangle/Morley/Dergiades.shtml)***
3. ***M. T. Naraniengar's proof (http://www.cut-the-knot.org/triangle/Morley/Naraniengar.shtml)***
4. ***An Unexpected Variant (http://www.cut-the-knot.org/triangle/Morley/Larry.shtml)***
5. ***Proof by B. Stonebridge and B. Millar (http://www.cut-the-knot.org/triangle/Morley/sb.shtml)***
6. ***Proof by B. Stonebridge (http://www.cut-the-knot.org/triangle/Morley/sb2.shtml)***
7. ***Proof by Roger Smyth (http://www.cut-the-knot.org/triangle/Morley/Smyth.shtml)***
8. ***Proof by H. D. Grossman (http://www.cut-the-knot.org/triangle/Morley/Grossman.shtml)***
9. ***Proof by H. Shutrick (http://www.cut-the-knot.org/wiki-math/index.php?n=Geometry.MorleysTheorem)***
10. ***Original Taylor and Marr's Proof of Morley's Theorem (http://www.cut-the-knot.org/m/Geometry/Morley2.shtml)***
11. ***Taylor and Marr's Proof - R. A. Johnson's Version (http://www.cut-the-knot.org/m/Geometry/Morley.shtml)***
12. ***Morley's Theorem: Second Proof by Roger Smyth (http://www.cut-the-knot.org/triangle/Morley/Smyth2.shtml)***
13. ***Proof by A. Robson (http://www.cut-the-knot.org/triangle/Morley/Robson.shtml)***

Algebraic proofs

1. ***Morley's Redux and More, Alain Connes' proof (http://www.cut-the-knot.org/ctk/MorleysRedux.shtml)***

Invalid proofs

1. ***Bankoff's conundrum (http://www.cut-the-knot.org/triangle/Morley/bankoff.shtml)***
2. ***Proof by Nolan L Aljaddou (http://www.cut-the-knot.org/triangle/Morley/Aljaddou.shtml)***