(http://www.cut-the-knot.org/manifesto/index.shtml)

Morley's Miracle
J. Conway's proof

John Conway (the inventor of the Game of Life) of Princeton University floated his proof on the geomtery.puzzles newsgroup in 1995. Following is his message (I only replaced his text-based graphics with something more decent and changed his notations to confirm with those I use in other proofs.)

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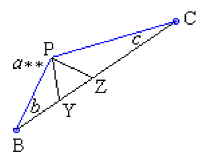
I have the undisputedly simplest proof of Morley's Trisector Theorem. Here it is:

Let your triangle have angles 3a, 3b, 3c and let x mean x + 60°, so that a + b + c = 0*. Then triangles with angles*

	0*,0*,0*	
a,b,c**	*a*,b,c**	*a*,b*,c*
*a**,b,c*	*a,b**,c*	*a,b,c***

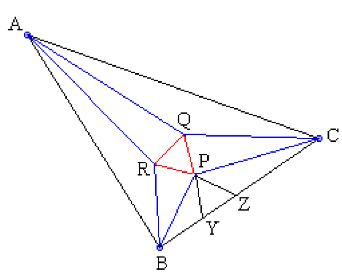
exist abstractly, since in every case the angle-sum is 180°. Build them on a scale defined as follows:

0*,0*,0*	- this is equilateral - make it have edge 1
a,b,c**	- make the edge joining the angles *b** and c* have length 1
	- similarly for *a*,b,c** and *a*,b*,c*
*a**,b,c* (and the other two like it)	- let me draw this one:



*Let the angles at B, P, C be b, a**, c, and draw lines from P cutting BC at angle a* in the two senses, so forming an isosceles triangle PYZ. Choose the scale so that PY and PZ are both 1.*

Now just fit all these 7 triangles together! They'll form a figure like:-



(in which the points X,Y should really be omitted). The points Y,Z are what I meant.

To make it a bit more clear, let me say that the angles of ΔBPR are b (at B), c (at P), a* (at R).*

Why do they all fit together? Well, at each internal vertex, the angles add up to 360°, as you'll easily check. And two coincident edges have either both been declared to have length 1, or are like the common edge BP of triangles BPR and BPC.

But ΔBPR is congruent to the subtriangle BPZ of ΔBPC, since PR = PZ = 1, ∠PBR = ∠PBZ = b, and ∠BRP = ∠BZP = a.*

So the figure formed by these 7 triangles is similar to the one you get by trisecting the angles of your given triangle, and therefore in that triangle the middle subtriangle must also be equilateral.

John Conway

Morley's Miracle (http://www.cut-the-knot.org/triangle/Morley/index.shtml)

On Morley and his theorem

1. ***Doodling and Miracles (http://www.cut-the-knot.org/triangle/Morley/Morley.shtml)***
2. ***Morley's Pursuit of Incidence (http://www.cut-the-knot.org/triangle/Morley/CenterCircle.shtml)***
3. ***Lines, Circles and Beyond (http://www.cut-the-knot.org/triangle/Morley/Beyond.shtml)***
4. ***On Motivation and Understanding (http://www.cut-the-knot.org/triangle/Morley/MorleyFinal.shtml)***
5. ***Of Looking and Seeing (http://www.cut-the-knot.org/ctk/MorleyConc.shtml)***

Backward proofs

1. J.Conway's proof
    - ***Remarks on J. Conway's proof (http://www.cut-the-knot.org/triangle/Morley/remarks_c.shtml)***
2. ***D. J. Newman's proof (http://www.cut-the-knot.org/triangle/Morley/newman.shtml)***
3. ***B. Bollobás' proof (http://www.cut-the-knot.org/triangle/Morley/Bollobas.shtml)***
4. ***G. Zsolt Kiss' proof (http://www.cut-the-knot.org/triangle/Morley/MorleyZsolt.shtml)***
5. ***Backward Proof by B. Stonebridge (http://www.cut-the-knot.org/triangle/Morley/sb3.shtml)***
6. ***Morley's Equilaterals, Spiridon A. Kuruklis' proof (http://www.cut-the-knot.org/m/Geometry/Morley5.shtml)***
7. ***J. Arioni's Proof of Morley's Theorem (http://www.cut-the-knot.org/triangle/Morley/Arioni.shtml)***

Trigonometric proofs

1. ***Bankoff's proof (http://www.cut-the-knot.org/triangle/Morley/BankoffProof.shtml)***
2. ***B. Bollobás' trigonometric proof (http://www.cut-the-knot.org/triangle/Morley/BollobasTrig.shtml)***
3. ***Proof by R. J. Webster (http://www.cut-the-knot.org/triangle/Morley/Webster.shtml)***
4. ***A Vector-based Proof of Morley's Trisector Theorem (http://www.cut-the-knot.org/triangle/Morley/VectorProof.shtml)***
5. ***L. Giugiuc's Proof of Morley's Theorem (http://www.cut-the-knot.org/triangle/Morley/Giugiuc.shtml)***
6. ***Dijkstra's Proof of Morley's Theorem (http://www.cut-the-knot.org/triangle/Morley/Dijkstra.shtml)***

Synthetic proofs

1. ***Another proof (http://www.cut-the-knot.org/triangle/Morley/yours_truly.shtml)***
2. ***Nikos Dergiades' proof (http://www.cut-the-knot.org/triangle/Morley/Dergiades.shtml)***
3. ***M. T. Naraniengar's proof (http://www.cut-the-knot.org/triangle/Morley/Naraniengar.shtml)***
4. ***An Unexpected Variant (http://www.cut-the-knot.org/triangle/Morley/Larry.shtml)***
5. ***Proof by B. Stonebridge and B. Millar (http://www.cut-the-knot.org/triangle/Morley/sb.shtml)***
6. ***Proof by B. Stonebridge (http://www.cut-the-knot.org/triangle/Morley/sb2.shtml)***
7. ***Proof by Roger Smyth (http://www.cut-the-knot.org/triangle/Morley/Smyth.shtml)***
8. ***Proof by H. D. Grossman (http://www.cut-the-knot.org/triangle/Morley/Grossman.shtml)***
9. ***Proof by H. Shutrick (http://www.cut-the-knot.org/wiki-math/index.php?n=Geometry.MorleysTheorem)***
10. ***Original Taylor and Marr's Proof of Morley's Theorem (http://www.cut-the-knot.org/m/Geometry/Morley2.shtml)***
11. ***Taylor and Marr's Proof - R. A. Johnson's Version (http://www.cut-the-knot.org/m/Geometry/Morley.shtml)***
12. ***Morley's Theorem: Second Proof by Roger Smyth (http://www.cut-the-knot.org/triangle/Morley/Smyth2.shtml)***
13. ***Proof by A. Robson (http://www.cut-the-knot.org/triangle/Morley/Robson.shtml)***

Algebraic proofs

1. ***Morley's Redux and More, Alain Connes' proof (http://www.cut-the-knot.org/ctk/MorleysRedux.shtml)***

Invalid proofs

1. ***Bankoff's conundrum (http://www.cut-the-knot.org/triangle/Morley/bankoff.shtml)***
2. ***Proof by Nolan L Aljaddou (http://www.cut-the-knot.org/triangle/Morley/Aljaddou.shtml)***
3. ***Morley's Theorem: A Proof That Needs Fixing (http://www.cut-the-knot.org/triangle/Morley/MorleyFalse.shtml)***

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http://www.cut-the-knot.org/triangle/Morley/conway.shtml (http://www.cut-the-knot.org/triangle/Morley/conway.shtml), Accessed 08 June 2017

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Green Apple
· 31 Mar, 2015

I like how you presented this with an unusual notation — using asterisks — because it highlights how looking at a problem in a different angle, and, more specifically, using another notation, might help you solve the problem easier. (This is really what's behind a lot of math. For example, calculus is what happens when you give specific notations to the notions of limit and derivative.)

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abogom Green Apple
· 31 Mar, 2015

I'd like to claim the notations, but they John Conway's. From what I read and heard he pays a particular heed to inventing convenient notation, in triangle geometry, for one.

Here's an independent example in the same vein:

http://www.cut-the-knot.org/wiki-math/index.php?n=Geometry.ConicInATriangle2UsingCoordinates

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